Tuning flexural elastic wave propagation in electroactive phononic crystals

Zhou, Xiaoling; Xu, Yanlong; Wang, Longqi

2018

Zhou, X., Xu, Y., & Wang, L. (2018). Tuning flexural elastic wave propagation in electroactive phononic crystals. *Journal of Applied Physics*, 123(22), 225103-. doi:10.1063/1.5024880

<https://hdl.handle.net/10356/89089>

<https://doi.org/10.1063/1.5024880>

© 2018 The Author(s). All rights reserved. This paper was published by AIP Publishing in *Journal of Applied Physics* and is made available with permission of The Author(s).

Downloaded on 26 Apr 2023 21:46:21 SGT

Tuning flexural elastic wave propagation in electroactive phononic crystals

Xiaoling Zhou,^{1,a)} Yanlong Xu,² and Longqi Wang³

¹Shanghai Institute of Aerospace System Engineering, Shanghai 201109, China

²School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, Shaanxi, China

³School of Civil and Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore, Singapore 639798

(Received 5 February 2018; accepted 24 May 2018; published online 13 June 2018)

In this paper, we investigate the flexural elastic wave propagation properties in phononic crystals (PCs) consisting of co-axial steel and electrorheological (ER) elastomer pillars deposited on a thin plate. The finite element method and the super cell technique are used to calculate the band structures and transmission spectra. It is found that the band gap boundaries of the PCs can be tuned by applying an electric field to the ER elastomer component. The relationship between the band gap boundaries and the electric field is given. It is also found that point and line defects can be created by properly applying an electric field to some of the cells. These defects bring defect bands inside the original band gaps and lead to elastic wave confinement in the point defect or along the line defect. The dependence of the defect band frequencies on the electric field is expounded. Band gap tunability, wave confinement, and wave guiding achieved by applying electric field in this paper are useful in applications of flexural vibration reduction and energy harvesting in engineering.

Published by AIP Publishing. <https://doi.org/10.1063/1.5024880>

I. INTRODUCTION

Elastic wave propagations in phononic crystals (PCs) show some unique properties like band gap,^{1,2} negative refraction,^{3,4} sound focusing, etc.^{5,6} These novel characters have great potentials in engineering applications. For example, the low frequency band gaps in locally resonant phononic crystals bring new insights in noise and vibration control.^{2,7} On the other hand, elastic wave propagations in PCs with defects show acoustical phenomena like energy focusing,^{8,9} wave guiding,^{10,11} wave modulation,¹² and so on. However, the elastic wave propagation properties can hardly be changed once the periodic composites of PCs formed and this restricts the application of PCs.

Previous work showed that both the geometrical parameters and the material constants of the components in PCs have great influences on the band structure properties. For instance, researchers pointed out that band gap boundaries of PCs can be tuned by changing the size of the cells.^{13,14} Jin *et al.*¹⁵ proposed tunable waveguide and cavity in a phononic crystal plate by altering the radius of the hollows. Addouche *et al.*¹⁶ investigated the surface phonon guiding through locally resonant defects in pillar-based PCs and found that the frequency position and mechanisms of the defect modes could be tuned by carefully changing the geometrical parameters. Li *et al.*¹⁷ used different materials to create defects and investigated the effects of the elastic constants on the defect mode frequencies by the super cell method.

In practical applications, it is difficult to change the geometry or replace the components once the structure is manufactured. But it is of great importance that we can control the elastic wave propagation in the manufactured

structure to suit different circumstances. Fortunately, it is possible to tune the wave propagation behaviors in PCs conveniently by using smart materials. For example, piezoelectric materials,^{18,19} magnetostrictive materials,^{20,21} and dielectric materials^{22,23} are utilized to change the band gap frequencies. Zhou and Chen²⁴ studied the effects of electric field on the band gap boundaries of the two-dimensional locally resonant PCs. Chen and Huang²⁵ reported active elastic metamaterials for subwavelength wave propagation control. They tuned the band gap of the metamaterial by shunted piezoelectric patches and electrorheological (ER) elastomer. Li *et al.*²⁶ actively tuned the vibration and wave propagation in elastic beams by periodically placing the piezoelectric actuator/sensor pairs along the beam axis. Wang *et al.*²⁷ reported tunable pass band in one-dimensional PC with a piezoelectric defect layer. They pointed out that the external electric field which changes the strain coefficient of the defect layer could be used to tune the pass band.

As we know, flexural elastic wave propagations in plates are very common in engineering.^{28,29} We propose to utilize electroactive material (ER elastomer³⁰) to tune the flexural elastic wave propagation behaviors in PCs consisting of cylindrical pillars deposited on a plate. By properly applying electric field to the ER components, band gap tunability, wave confinement, and wave guiding can be achieved. The relationships between the electric field and the band gap boundaries, point defect state frequencies, and line defect band boundaries are studied.

II. MODEL AND METHOD

In this paper, PCs consisting of steel and ER elastomer pillars deposited on an Aluminum plate are investigated. Figure 1(a) shows the schematic of the periodic structures with 5×5 cells and Fig. 1(d) displays the Brillouin zone. As

^{a)}Author to whom correspondence should be addressed: zhouxiaoling87@163.com

FIG. 1. Schematic illustrations of the 5×5 structures. (a) Perfect PC. (b) Point defect structure. (c) Line defect structure. (d) The corresponding Brillouin zone.

shown in the figure, the elastomer and steel pillars have the same radius of $r = 0.004$ m. The height of the elastomer and steel pillars is $h_1 = 0.001$ m and $h_2 = 0.002$ m, respectively. The pillars are arranged periodically on the plate with a square lattice. The lattice constant is $a = 0.01$ m. The thickness of the plate is $e = 0.002$ m.

We can apply electric field to some of the cells in the perfect PCs to create defects. In Fig. 1(b), a point defect is created by applying electric field to the ER pillar locating in the middle place of the periodic structure. Figure 1(c) shows the line defect structure with a row of ER pillars in the 5×5 structure being applied electric field. In this paper, the shear modulus G of the ER elastomer changes with external applied electric field E in the form of^{24,30}

$$G = G_0 \left[1 + (E/E_0)^2 \right], \quad (1)$$

where G_0 is the shear modulus without electric field and is 11 kPa, and E_0 is the material constant. For the ER elastomer, $E_0 = 3.3$ kV/mm. The density and Poisson's ratio of the ER elastomer are 1000 kg/m^3 and 0.48, respectively. The viscoelasticity of the elastomer layers is ignored in this paper. For the steel, the density is 7780 kg/m^3 ; the Young's modulus is 210.6 GPa. For the Aluminum, the density is 2600 kg/m^3 and the Young's modulus is 70 GPa. Both the steel and the Aluminum have the Poisson's ratio of 0.3.

To investigate the band structures of the PCs with defects, the finite element method (FEM) in combination with the supercell technique is used. In this paper, the 5×5 unit cells

in Figs. 1(a)–1(c) are considered as the supercells. The elastic wave propagation equation in the solid structure is as follows:

$$\nabla \cdot [\lambda(\mathbf{r}) + 2G(\mathbf{r})](\nabla \cdot \mathbf{u}) - \nabla \times [G(\mathbf{r})\nabla \times \mathbf{u}] = \rho \ddot{\mathbf{u}}. \quad (2)$$

According to the Bloch-Floquet theorem, periodic boundary conditions are imposed on the supercells. The boundary condition is taken as

$$\mathbf{u}(\mathbf{r} + n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2) = \mathbf{u}(\mathbf{r})e^{i\mathbf{k}\mathbf{a}}. \quad (3)$$

Then, the band structures of the PCs with and without defects are investigated by FEM. Since elastic wave propagation in both the in-plane and out-of-plane directions are considered in the simulations, the flexural wave band gaps are identified by the displacement fields of the supercells.

Moreover, the transmission spectra of the perfect PCs are studied via the frequency response simulation. Finite periodic structures with 5×5 , 10×10 , and 15×15 cells are investigated. To test the flexural wave attenuation, we impose a prescribed displacement in Z direction (denoted by S_0) on the left side of the finite periodic plates. We take the boundary integration of the displacement amplitude on the right side of the plate and then divide the integration by the area to get the response S_a . Then, we can calculate the transmission of the periodic structures by the following equation:

$$T = 20 \times \log(S_a/S_0). \quad (4)$$

When doing this, we take the frequency interval as 10 Hz. In all the FEM simulations, we refine the meshes to get stabilized values of frequencies.

FIG. 2. Band structure and transmission spectra of the perfect PC without electric field.

III. RESULTS AND DISCUSSIONS

A. Band gap tunability

Figure 2 displays the band structure and transmission of the perfect PC without electric field. In this paper, only the flexural elastic wave propagation is taken into consideration. So, we emphasize the flexural band gap boundaries by plotting them as dotted dashed lines in the band structures. The solid lines in the band structures represent the flat modes of the flexural wave and the vibration modes in the in-plane directions. As shown in Fig. 2(a), the shadow region between the dotted dashed lines is the flexural band gap which is from 472 Hz to 732 Hz. Figure 2(b) presents the frequency response of the finite periodic plate. Since we impose prescribed displacement in Z direction, the intersections in the transmission spectra of the periodic structures with different number of cells display the boundaries of the flexural band gap. It is found that there is a flexural band gap from 480 Hz to 730 Hz [see the shadow region in Fig. 2(b)]. In this frequency range, the vibration is obviously reduced. Considering that the frequency interval in the transmission simulations is 10 Hz, this result agrees well with the band structure shown in Fig. 2(a).

Figure 3 gives the displacement fields of the eigenmodes of the supercell at the flexural band gap boundaries. At the lower boundary of 472 Hz [see Fig. 3(a)], the plate keeps stationary and the steel pillars move as a whole which works as the masses. The steel pillars vibrate oppositely in the Z direction and the periodic PC model reaches a dynamic

equilibrium state. Figure 3(b) shows the displacement field of the supercell at the upper boundary of the flexural band gap. The steel pillars and the plate move oppositely. It can be seen from the displacement modes that the flexural band gap in the PC is a locally resonant stop band. The ER elastomers act as springs.

To tune the band gap boundaries, we apply electric field to the whole PC. Figure 4 shows the band structure and transmission spectra of the perfect PC under electric field of $E = 3$ kV/mm. It can be seen from Fig. 4(a) that the flexural band gap frequencies increase to 638–990 Hz. The transmission spectra in Fig. 4(b) show that the band gap is from 640 Hz to 990 Hz which again agrees well with the band gap shown in the band structure.

As we know, the modulus of the ER elastomer layers increases when the electric field is applied. So it is reasonable that the band gap frequencies increase to high values. For the locally resonance modes, the band gap boundaries are proportional to the root of the equivalent stiffness. In the PC models, the stiffness of the elastomer is much smaller than the metal. For the flexural band gaps, the vibration of the steel pillars and the Aluminum plate can be seen as rigid motions. The elastomer layers act as springs in the vibration modes. So, the equivalent stiffness of the PCs is dependent on the stiffness of the elastomers. Hence, the band gap boundaries of the PCs are proportional to the root of the elastomer pillars' stiffness. When the geometrical parameters of the PCs keep constant, the change of the band gap boundaries depends on the change of the modulus of the elastomer.

(a) displacement in Z direction, $f=472$ Hz

(b) displacement in Z direction, $f=732$ Hz

FIG. 3. Displacement fields of the eigenmodes of the supercell at the flexural band gap boundaries.

FIG. 4. Band structure and transmission spectra of the perfect PC with electric field of $E = 3$ kV/mm.

FIG. 5. The change of the band gap boundaries as a function of the electric field.

Combining with Eq. (1), we can conclude that the band gap boundaries increase with the electric field in the form of

$$f/f_0 = \sqrt{G/G_0} = \sqrt{1 + (E/E_0)^2}, \quad (5)$$

where f and f_0 are the band gap boundaries with and without electric field, respectively. Figure 5 indicates the changes of the flexural band gap boundaries versus the electric field. It is found from Fig. 5 that the simulated results agree well with the predicted results by Eq. (5).

B. Wave confinement

Besides band gap tunability, it is important to achieve acoustic functionalities conveniently. Point defect is created as shown in Fig. 1(b) by applying electric field to the cell located in the middle place and keeping the others unchanged. Figure 6(a) shows the band structure of the PC with point defect under $E = 3$ kV/mm. It can be seen from Fig. 6(a) that there are three defect states appearing inside the band gap. The band gap (denoted by the shadow region) is from 472 Hz to 757 Hz. The upper boundary is higher than that of the perfect PC without electric field, and the lower boundary is the same with that of the perfect PC in Fig. 2(a). The frequencies of the three defect states (i.e., I, II, and III) are 519 Hz, 522 Hz, and 638 Hz, respectively. Figure 7 gives the displacement fields of the eigenmodes of the supercell corresponding to the point defect states. It is found that the waves are confined in the point defect at the defect state

FIG. 6. (a) Band structure of the 5×5 structure with point defect under $E = 3$ kV/mm. (b) The dependence of the point defect state frequencies on the electric field.

(a) total displacement, defect state I, $f=519$ Hz **(b) total displacement, defect state II, $f=522$ Hz** **(c) total displacement, defect state III, $f=638$ Hz**

FIG. 7. Displacement fields of eigenmodes of the supercell with point defect at the defect frequencies.

frequencies. That means, wave confinement can be achieved by properly applying electric field to the PC.

To show the dependence of the defect frequencies on the electric field, band structures of the PCs with point defect structures under various electric field are investigated. Figure 6(b) shows the flexural band gap boundaries and the point defect frequencies versus the electric field. It is found from Fig. 6(b) that the lower boundary almost keeps unchanged, while the upper boundary increases as the electric field increases. The defect states appear in the band gaps and the corresponding defect frequencies increase with the electric field rises.

C. Wave guiding

By applying electric field to a row of cells in the periodic plate as shown in Fig. 1(c), a line defect is created. Figure 8(a) gives the band structure of the PC with the line defect under $E = 3$ kV/mm. It can be seen from Fig. 8(a) that a pass band appears in the flexural band gap. The band gap (denoted by the shadow region) is from 472 Hz to 818 Hz. The defect states I and III marked in Fig. 8(a) are straight lines, and the frequencies are 515 Hz and 639 Hz, respectively. The dashed line in Fig. 8(a) leads to a pass band inside the band gap. The defect state II locates at the lowest

FIG. 8. (a) Band structure of the 5×5 structure with line defect under $E = 3$ kV/mm. (b) The dependence of the line defect band boundaries on the electric field.

(a) total displacement, defect state I, $f=515$ Hz

(b) total displacement, defect state II, $f=569$ Hz

(c) total displacement, defect state III, $f=639$ Hz

FIG. 9. Displacement fields of the eigenmodes of the supercell with the line defect at the defect band frequencies.

frequency (569 Hz) of the dashed line. Figure 9 shows the displacement fields of the eigenmodes of the defect states I, II, and III. As shown in Fig. 9, the elastic wave propagates along the line defect which can be used as wave guiding.

Figure 8(b) shows the band gap boundaries and the boundaries of the defect-induced pass band as a function of the electric field. As the electric field increases, the upper band gap boundary increases, while the lower boundary keeps unchanged. The boundaries and width of the defect band increase as the electric field rises.

All the above results demonstrate that band gap tunability, wave confinement, and wave guiding in specific frequencies can be achieved by properly applying electric field to the perfect PCs.

IV. CONCLUSIONS

Flexural elastic wave propagations in PCs with ER elastomers are investigated by FEM. The applied electric field is found to tune the flexural band gap frequencies in the form of $f/f_0 = \sqrt{1 + (E/E_0)^2}$. Moreover, by properly applying electric field to some of the cells, we create point and line defect structures in the perfect PCs. The point defect states and line defect pass band appear inside the band gaps of the perfect PCs and lead to wave confinement and wave guiding in those frequencies correspondingly. We also give the relationships between the electric field and the defect frequencies. These results provide good guidance for control of flexural elastic wave in structures.

ACKNOWLEDGMENTS

This work was supported by the Shanghai Sailing Program (17YF1419400).

- ¹M. S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari-Rouhani, *Phys. Rev. Lett.* **71**, 2022 (1993).
- ²Z. Liu, X. Zhang, Y. Mao, Y. Zhu, Z. Yang, C. Chan, and P. Sheng, *Science* **289**, 1734 (2000).
- ³H. Zhu and F. Semperlotti, *J. Appl. Phys.* **122**, 065104 (2017).
- ⁴F. D. Philippe, T. W. Murray, and C. Prada, *Sci. Rep.* **5**, 11112 (2015).
- ⁵J. Zhao, B. Bonello, R. Marchal *et al.*, *New J. Phys.* **16**, 063031 (2014).
- ⁶J. Qian, J. Xia, H. Sun *et al.*, *J. Appl. Phys.* **122**, 244501 (2017).
- ⁷X. Fang, J. Wen, B. Bonello *et al.*, *Nat. Commun.* **8**, 1288 (2017).
- ⁸H. Lv, X. Tian, M. Y. Wang *et al.*, *Appl. Phys. Lett.* **102**, 034103 (2013).
- ⁹W. Jiang, D. Feng, D. Xu, B. Xiong, and Y. Wang, *Appl. Phys. Lett.* **109**, 161102 (2016).
- ¹⁰S. Benchabane, O. Gaiffe, R. Salut *et al.*, *Appl. Phys. Lett.* **106**, 081903 (2015).
- ¹¹F. Ciampa, A. Mankar, and A. Marini, *Sci. Rep.* **7**, 14712 (2017).
- ¹²A. Shelke, S. Banerjee, A. Habib *et al.*, *J. Intell. Mater. Syst. Struct.* **25**, 1541–1552 (2014).
- ¹³Y. F. Wang and Y. S. Wang, *J. Appl. Phys.* **114**, 043509 (2013).
- ¹⁴K. Yu, T. Chen, and X. Wang, *J. Appl. Phys.* **113**, 134901 (2013).
- ¹⁵Y. Jin, N. Fernez, Y. Pennec *et al.*, *Phys. Rev. B* **93**, 054109 (2016).
- ¹⁶M. Addouche, M. A. Al-Lethawe, A. Elayouch *et al.*, *AIP Adv.* **4**, 124303 (2014).
- ¹⁷Y. Li, T. Chen, X. Wang *et al.*, *J. Appl. Phys.* **116**, 024904 (2014).
- ¹⁸S. Degraeve, C. Granger, B. Dubus *et al.*, *Smart Mater. Struct.* **24**, 085013 (2015).
- ¹⁹Y. Wang, F. Li, Y. Wang *et al.*, *Acta Mech. Sin.* **25**, 65–71 (2009).
- ²⁰R. Ding, X. Su, J. Zhang *et al.*, *J. Appl. Phys.* **115**, 074104 (2014).
- ²¹O. Bou Matar, J. F. Robillard, J. O. Vasseur *et al.*, *J. Appl. Phys.* **111**, 054901 (2012).
- ²²M. A. Jandron and D. Henann, *J. Acoust. Soc. Am.* **141**, 3743 (2017).
- ²³E. Bortot and G. Shmuel, *Smart Mater. Struct.* **26**, 045028 (2017).
- ²⁴X. Zhou and C. Chen, *Phys. B* **431**, 23–31 (2013).
- ²⁵Y. Y. Chen and G. L. Huang, *Acta Mech. Sin.* **31**, 349–363 (2015).
- ²⁶F. Li, C. Zhang, and C. Liu, *J. Sound Vib.* **393**, 14–29 (2017).
- ²⁷Y. Wang, W. Song, E. Sun *et al.*, *Phys. E* **60**, 37–41 (2014).
- ²⁸M. Badreddine Assouar and M. Oudich, *Appl. Phys. Lett.* **100**, 123506 (2012).
- ²⁹H.-J. Zhao, H.-W. Guo, B.-Y. Li, Z.-Q. Deng, and R.-Q. Liu, *J. Appl. Phys.* **118**, 044906 (2015).
- ³⁰B. Liu and M. Shaw, *J. Rheol.* **45**, 641 (2001).