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Predicting response of reinforced concrete slabs under low-velocity impact

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A two-degrees-of-freedom (2-DoF) dynamic model is proposed to predict the response of reinforced concrete (RC) slabs subjected to low-velocity impact. In this model, the slab stiffness is calculated based on its dynamic load–displacement curve, which is built using moment–curvature analysis and critical shear crack theory. The contact between the impactor and the impacted RC slab is modelled with an elastic spring and a viscous damper. The effectiveness of the 2-DoF model was verified by comparisons with experimental data. The results obtained from the 2-DoF model are further processed to show the distribution of impact energy during the impact process, and a simple method is recommended for a quick estimation of the energy capacity of RC slabs.

Notation

A_s	area of shear reinforcement intersected by critical shear crack	l	clear span of slab
b_0	punching shear perimeter	m_{cr}	cracking moment per unit width
c	depth of compression zone	m_i	mass of impactor
c_c	contact damping coefficient	m_R	moment capacity per unit width
c_s	damping coefficient of slab	m_r	radial moment per unit width
d	effective depth of slab	m_s	equivalent mass of slab
d_g	maximum aggregate size of concrete	m_t	tangential moment per unit width
d_{g0}	reference aggregate size of concrete (16 mm)	r_c	radius of loaded area
E_c	elastic modulus of concrete	r_{cr}	radius of cracked zone
E_i	impact energy	r_q	radius of support reaction force at the perimeter
E_{ip}	imparted energy	r_s	radius of slab
E_s	elastic modulus of reinforcement (assumed to be $10^4 f_c^{1/3}$)	r_y	radius of yielded zone
EI_0	flexural stiffness before cracking	r_0	radius of critical shear crack
EI_1	tangential flexural stiffness after cracking	r_1	radius of zone in which cracking is stabilised
e	coefficient of restitution	T_c	duration of impact
F_c	impact force	u_i	displacement of impactor
$F_{c,max}$	maximum impact force	$\dot{u}_i$	velocity of impactor ($\dot{u}_i(0)$ = initial velocity; $\dot{u}_i(t_r)$ = rebound velocity)
f_c	compressive strength of concrete	$\ddot{u}_i$	acceleration of impactor
f_{cd}	dynamic compressive strength of concrete	u_{max}	midpoint displacement of slab corresponding to V_{max}
f_{ct}	tensile strength of concrete (assumed $f_{ct} = 0.3 f_c^{2/3}$)	u_s	midpoint displacement of slab
f_{ctd}	dynamic tensile strength of concrete	$\dot{u}_s$	midpoint velocity of slab
f_u	ultimate strength of reinforcement	$\ddot{u}_s$	midpoint acceleration of slab
f_y	yield strength of reinforcement	$u_{s,max}$	maximum slab displacement under impact load
f_{yd}	dynamic yield strength of reinforcement	$u_{s,res}$	residual slab displacement under impact load
g	gravitational acceleration	V	static strength of RC slab
H	slab depth	V^d	dynamic strength of slab
I	impulse	V_{max}	maximum static strength of slab
k_c	contact stiffness	V_R	punching shear strength of slab
k_e	elastic stiffness of slab	V_s	shear force carried by shear reinforcement
k_s	slab stiffness	w	width of critical crack
		β	efficiency factor
		$\Delta\phi$	angle of slab sector

$\dot{\epsilon}$	strain rate
ζ	damping ratio of slab
ζ_c	damping ratio of contact model
ρ	reinforcement ratio
ρ_s	slab density
$\phi(x)$	shape function of slab
χ_{cr}	curvature at cracking
χ_r	curvature in radial direction
χ_t	curvature in tangential direction
χ_{TS}	decrease in curvature due to tension stiffening
χ_y	yielding curvature
χ_1	curvature at stabilised cracking
ψ	slab rotation

Introduction

Some reinforced concrete (RC) structures need to be designed to resist impact loads. Typical examples include military buildings, rock sheds in mountainous areas and the protective walls of nuclear power plants, which may experience impacts from missiles, falling rocks, vehicles, aircraft and so on. It is therefore important to be able to predict the response of and damage to RC structures under impact loads. Conducting impact tests is a reliable way of studying the impact responses of RC members (Adhikary *et al.*, 2015, 2016; Banthia *et al.*, 1989; Kishi *et al.*, 2002; Kojima, 1991; Murtiadi and Marzouk, 2001; Remennikov and Kaewunruen, 2006; Saatci and Vecchio, 2009; Xiao, 2015). However, such tests are complex, costly and time consuming. Additionally, finite-element simulations of impact problems require great modelling and calculation effort. A simplified analytical model is thus needed for predicting the impact responses of structural members. A rational simplification is to use a single-degree-of-freedom (SDoF) model. This method models the structural member as a lumped mass whose stiffness and equivalent mass are determined by assessing the load–displacement relationship and the deflected shape of the member (Biggs and Testa, 1964). Abde-Rohman and Sawan (1985) used a SDoF model to simulate the response of RC slabs under the impact of a steel dropping ball. Rong and Li (2007, 2008) and Stochino and Carta (2014) also used SDoF models to study the response of RC beams under impact and blast loads. However, the main deficiency of SDoF models is that they cannot simulate the interaction between the impactor and the impacted member, and knowing the impact load is a prerequisite for analysis.

To overcome this deficiency, an extra lumped mass that represents the impactor can be added to the SDoF model, thus forming a mode with two degrees-of-freedom (2-DoF). In addition, a contact model that describes the interaction between the impactor and the impacted RC member is needed to determine the impact load. Fujikake *et al.* (2009) employed a 2-DoF model to generate the midspan displacement and impact force of 12 RC beams subjected to low-velocity

impact. Bertrand *et al.* (2015) also used a 2-DoF model to conduct impact analysis of RC members with flexural failure modes. Delhomme *et al.* (2007) used a two-stage 2-DoF model to simulate the impact response of an RC slab, but determination of many of the parameters in this model requires experimental data or finite-element analysis. Micallef *et al.* (2014) also used a two-stage 2-DoF model for impact analysis of an RC slab, but the verification of this model was insufficient and the induced impact forces during analysis were not reported.

The load–displacement curve is very important for a SDoF/2-DoF model to produce reliable results (Crawford *et al.*, 2000) and a good load–displacement curve should rationally reflect the dynamic load–displacement relationship of the impacted member. Punching shear failure has been found to be the dominant failure mode of RC slabs in many impact tests (Chen and May, 2009; Delhomme *et al.*, 2005; Hrynyk and Vecchio, 2014; Zineddin and Krauthammer, 2007). Nevertheless, the load–displacement curves used in most current SDoF and 2-DoF models do not consider the strength softening of RC slabs after punching shear failure. Additionally, an impact load will lead to a dynamic load–displacement relationship of an impacted member whose strength is higher than a static one. In current SDoF or 2-DoF models, this strength enhancement is usually considered by taking account of the strain rate effects on concrete and steel. However, the strain rate of RC members under low-velocity impact (impact velocity less than 10 m/s) has been shown to be very small when compared with the values considered to have significant effects on the mechanical characteristics of concrete and steel (Cotsovos, 2010). Therefore, attributing the strength enhancement solely to strain rate effects may underestimate the strength of the RC slab.

The objective of the work reported in this paper was to develop a 2-DoF model for predicting the response of RC slabs under low-velocity impact. In contrast to previous research, the load–displacement curve used in this model can reasonably consider strength softening of the slab due to punching shear failure and strength enhancement due to loading rate effects. The effectiveness of this model in producing slab deflection and impact force was sufficiently verified by experimental data.

Load–displacement relationship of RC slabs

Load–rotation relationship of RC slabs before punching shear failure

Flexural behaviour of RC slabs has been observed in many low-velocity impact tests before punching shear failure (Hrynyk and Vecchio, 2014; Xiao *et al.*, 2016; Zineddin and Krauthammer, 2007). Therefore, the load–rotation relationship of RC slabs in this stage can be obtained through a moment–curvature analysis. The analytical study carried out by

Muttoni (2008) is now introduced to describe the load–rotation relationship of an RC slab.

According to Muttoni (2008), tangential cracks and the radial curvature of an RC slab are concentrated in the vicinity of the loaded region. Outside r_0 (assumed to be at a distance d (the effective depth of the slab) from the face of the loaded region), the radial curvature decreases rapidly. Therefore, the deformation of the corresponding slab portion is assumed to have a conical shape and a constant slab rotation ψ (Figure 1(a)). Inside r_0 , the slab deformation is assumed to have a spherical shape. With the assumed slab deformation, the radial curvature χ_r (Figure 1(c)) and tangential curvature χ_t (Figure 1(d)) of the slab can be obtained from

$$1a. \quad \chi_t = \frac{\psi}{r}, \quad \chi_r = 0 \quad (r > r_0)$$

$$1b. \quad \chi_t = \chi_r = \frac{\psi}{r_0} \quad (r < r_0)$$

The radial and tangent moments can then be calculated with the quadrilinear moment–curvature relationship shown in Figure 1(e). The following load–rotation relationship of RC slabs can then be obtained through the moment equilibrium of the isolated slab section shown in Figure 1(b) (a detailed derivation of Equation 2 is given in the Appendix)

$$2. \quad V = \frac{2\pi}{r_q - r_c} \left(\frac{-m_r r_0 + m_R (r_y - r_0) + EI_1 \psi (\ln(r_1) - \ln(r_y)) + EI_1 \chi_{TS} (r_1 - r_y) + m_{cr} (r_{cr} - r_1) + EI_0 \psi (\ln(r_s) - \ln(r_{cr}))}{r_q - r_c} \right)$$

where the operator $\langle x \rangle$ is x for $x \geq 0$ and 0 for $x < 0$.

Load–rotation relationship of RC slabs after punching shear strength

The critical shear crack theory (CSCT) presented by Muttoni (2008) is used to obtain the punching shear strength of RC slabs. As shown in Figure 1(a), the CSCT is based on the assumption that the punching shear strength of an RC slab is governed by the roughness and the width of the critical crack (w) developed in a compression strut. When the slab rotation ψ increases, w increases proportionally and the capacity of the compression strut in transmitting shear force decreases. Therefore, the punching shear strength of an RC slab can be expressed as a function of its rotation, given as

$$3. \quad \frac{V_R}{b_0 d \sqrt{f_c}} = \frac{3/4}{1 + 15[\psi d / (d_{g0} + d_g)]}$$

where V_R is the punching shear strength, b_0 is the punching shear perimeter, d is the effective depth, f_c is the compressive strength of concrete, d_g is the maximum aggregate size and d_{g0} is the reference aggregate size ($= 16$ mm).

As shown in Figure 2, the punching shear strength of the slab is reached when the load–rotation curve obtained from Equation 2 intersects (solid curve) with the failure criterion given by Equation 3 (dashed curve). After reaching the punching shear strength, Equation 2 no longer describes the load–rotation relationship of the RC slab because of its inability to consider the strength softening. On the contrary, Equation 3 explains the relationship between reduced punching shear strength and increased slab rotation. Therefore, Equation 3 is

Figure 1. Assumed behaviour of axisymmetric slab. (a) Geometrical parameters and rotation of slab. (b) Internal forces acting on slab sector. (c) Distribution of radial curvature. (d) Distribution of tangential curvature. (e) Quadrilinear moment–curvature relationship of RC slab (Muttoni, 2008)

used to describe the descending branch of the load–rotation curve of RC slabs. The failure mode and corresponding slab damage can be directly known from the constructed load–rotation curve: a slab is expected to have a punching shear failure mode if the slab reaches its punching shear strength before its flexural strength (Figure 2(a)) or a flexural failure mode if the slab reaches its flexural strength before its punching shear strength (Figure 2(b)).

Contribution of shear reinforcement to the shear strength of RC slabs

Application of the CSCT can be further extended to incorporate the contribution of shear reinforcement to the shear strength of RC slabs using the equation (Ruiz and Muttoni, 2009)

$$4. \quad V_s = \frac{E_s \psi}{6} A_s \leq f_y A_s$$

where V_s is the shear force carried by shear reinforcement, E_s is the elastic modulus of the shear reinforcement, f_y is the yield strength of the shear reinforcement and A_s is the area of shear reinforcement intersected by the critical shear crack. For slabs with a punching shear failure mode, the shear force carried by the shear reinforcement is added to the slab's load–rotation curves as shown in Figure 3. However, for slabs with a flexural failure mode, the contribution of shear reinforcement is only added to the failure criterion given by Equation 3 to prevent it from affecting the flexural strength of the slab.

The load–displacement curve of an RC slab can be obtained by assuming the following relationship between slab rotation and displacement of the slab centre u_s

$$5. \quad u_s = r_s \psi$$

where r_s is the radius of the slab.

Residual strength of RC slabs with punching shear failure mode

After punching shear failure, the ability of concrete to carry applied load decreases when the slab displacement increases. With a further increase in slab displacement, the residual strength is mainly carried by flexural reinforcements. During this stage, the residual strength of the RC slab becomes stable and the strength softening behaviour disappears. In this study, strength softening behaviour is assumed to be finished when the slab displacement reaches 20% of the slab depth. After this point, the load–displacement curve becomes a horizontal line. The value of 20% was empirically determined by summarising the experimental load–displacement curves of RC slabs with punching shear failure modes.

Figure 2. Typical load–rotation curves of RC slabs: (a) slab with punching shear failure mode; (b) slab with flexural failure mode

Effect of loading rate

In this study, two methods are used to obtain the load–displacement curve of RC slabs under high loading rates. One method attributes the loading rate effect solely to the strain rate effect of structural materials and uses the dynamic material properties to calculate the load–displacement curve (strain rate method). The influences of strain rate on the compressive and tensile strengths of concrete can be expressed using Equations 6 and 7, provided by the 2010 Model Code (Fib, 2010)

$$6a. \quad \frac{f_{cd}}{f_c} = \left(\frac{\dot{\epsilon}}{30 \times 10^{-6}} \right)^{0.014} \quad \text{for } 30 \times 10^{-6} \text{ s}^{-1} \leq \dot{\epsilon} \leq 30 \text{ s}^{-1}$$

$$6b. \quad \frac{f_{cd}}{f_c} = 0.012 \left(\frac{\dot{\epsilon}}{30 \times 10^{-6}} \right)^{1/3} \quad \text{for } 30 \text{ s}^{-1} < \dot{\epsilon} \leq 300 \text{ s}^{-1}$$

Figure 3. Typical load-rotation curve of RC slab with shear reinforcement

$$7a. \quad \frac{f_{ctd}}{f_{ct}} = \left(\frac{\dot{\epsilon}}{1 \times 10^{-6}} \right)^{0.018} \quad \text{for } 1 \times 10^{-6} \text{ s}^{-1} \leq \dot{\epsilon} \leq 10 \text{ s}^{-1}$$

$$7b. \quad \frac{f_{ctd}}{f_{ct}} = 0.0062 \left(\frac{\dot{\epsilon}}{1 \times 10^{-6}} \right)^{1/3} \quad \text{for } 10 \text{ s}^{-1} < \dot{\epsilon} \leq 300 \text{ s}^{-1}$$

where f_{cd} , f_{ctd} and $\dot{\epsilon}$ are the dynamic compressive strength, dynamic tensile strength and corresponding strain rate. The influence of strain rate on the dynamic strength of steel (both in compression and in tension) is given by Equation 8 (Ceb, 1988)

$$8a. \quad \frac{f_{yd}}{f_y} = \left[1 + \frac{6}{f_y} \ln \left(\frac{\dot{\epsilon}}{5 \times 10^{-5}} \right) \right] \quad \text{for } \dot{\epsilon} \leq 10 \text{ s}^{-1}$$

$$8b. \quad \frac{f_{yd}}{f_y} = \left[1 + \frac{6}{f_y} \ln(2 \times 10^5) \right] \quad \text{for } \dot{\epsilon} > 10 \text{ s}^{-1}$$

where f_{yd} and f_y are the yield strengths of steel under dynamic and static loadings, respectively.

Another method is to use the dynamic increase factor (DIF, defined as the ratio of the RC member's maximum dynamic strength at any loading rate to its maximum static strength) obtained from high loading rate tests to empirically take account of the loading rate effect and to form the dynamic load-displacement curve of RC members (the DIF method). In this study, it is assumed that the slab's maximum dynamic strength equals the product of the DIF and its maximum static strength, and the following expressions are used to account for

the strength enhancement due to the loading rate effect.

$$9a. \quad V^d(u_s) = V(u_s) + u_s \frac{V_{\max}(\text{DIF} - 1)}{u_{\max}} \quad \text{for } u_s \leq u_{\max}$$

$$9b. \quad V^d(u_s) = V^d(u_{\max}) - (u_s - u_{\max}) \frac{V_{\max}(\text{DIF} - 1)}{0.1H - u_{\max}} \quad \text{for } A_s = 0, u_{\max} < u_s \leq 0.1H$$

$$9c. \quad V^d(u_s) = V^d(u_{\max}) - (u_s - u_{\max}) \frac{V_{\max}(\text{DIF} - 1)}{0.2H - u_{\max}} \quad \text{for } A_s \neq 0, u_{\max} < u_s \leq 0.2H$$

Here, V^d is the slab's dynamic strength, $V_{\max}$ and $u_{\max}$ are the maximum static strength and the corresponding displacement of the slab, and H is the slab depth.

Comparison with experimental data

The load-displacement curves formed with the described method were compared with experimental data reported by Xiao *et al.* (2016). In the experiment, six 1.2 m × 1.2 m RC slabs with different slab depths and longitudinal and shear reinforcement ratios were tested under different loading rates (static, 0.0004 m/s; medium, 0.4 m/s; high, 2 m/s). The design of the specimens is shown in Figure 4. The four sides of the slab specimen were clamped and the clear span was 1 m. Each slab was loaded by a 0.2 m diameter circular loading plate placed at its centre. Further details of the specimens are given in Table 1.

The load-displacement curves of all six RC slabs subjected to loadings with a static loading rate (0.0004 m/s) and a high loading rate (2 m/s) were plotted with the described method and they are compared with the experimental data in Figure 5. The analytical results show that all six slabs met their punching shear strengths before their flexural strengths, and therefore failed in punching shear. This estimated failure mode matched the failure mode observed during the experiments. The analytical maximum strengths and corresponding displacements of the RC slabs were also compared with the experimental values (Table 2). As shown in Figure 5, for the static cases, the estimated load-displacement curves generally matched the experimental curves well. However, for the slabs without shear reinforcement, the analytical curves did not capture the sudden decrease in slab strength after the punching shear failure point. This experimental phenomenon was mainly caused by the full formation of a punching shear cone in the loading area and its separation from the surrounding concrete after punching shear failure occurred. In the meantime, the shear cracks suddenly lost their ability to carry shear force. By contrast, for slabs with shear reinforcement, the sudden separation between the punching shear cone and its surroundings after punching shear

Figure 4. Reinforcement layout of slab specimens (Xiao *et al.*, 2016) (dimensions in mm)

Table 1. Summary of slab specimens tested by Xiao *et al.* (2016)

Specimen	H: mm	f_c : MPa	d_g : mm	Flexure					Shear		
				ρ : %	Diameter of rebar: mm	f_y : MPa	f_u : MPa	ρ : %	Diameter of rebar: mm	f_y : MPa	f_u : MPa
10-0-59	100	42.9	10	0.59	10	576	655	—	—	—	—
15-0-25	150	42.9	10	0.25	8	443	504	—	—	—	—
15-0-39	150	45.0	10	0.39	10	576	655	—	—	—	—
15-0-66	150	42.9	10	0.66	13	605	690	—	—	—	—
15-0-39-0-14	150	42.3	10	0.39	10	576	655	0.14	6	282	392
15-0-39-0-28	150	42.3	10	0.39	10	576	655	0.28	6	282	392

failure was prevented by the confining effect of the shear reinforcements and the concrete could still transfer shear force through the shear cracks. This explains why the strength softening of slabs with shear reinforcement is well estimated by the CSCT.

For dynamic cases, the loading rate effect was assessed using both strain rate and DIF methods. In the strain rate method, the dynamic load–displacement curves were formed by assuming a 10 s^{-1} strain rate. As reported by Xiao *et al.* (2016), the maximum strain rates were of the order of 10^0 s^{-1} for the high loading rate tests. Therefore, assuming a 10 s^{-1} strain rate should yield the upper bound of the load–displacement curves, the calculated dynamic strengths are on average 10.6% higher than the calculated static strengths. Similar strength enhancement ratios (4% for 0.1 s^{-1} strain rate, 10% for 10 s^{-1} strain rate and 15% for 300 s^{-1} strain rate) were found by Micallef *et al.* (2014) using the same approach. This enhancement ratio is far less than the experimental value of 52%. As a result, the strain rate effect on a material level cannot sufficiently account for the strength enhancement on a structural level when the loading rate is of the order of 10^0 m/s . In the

DIF method, the average DIF of 1.5 found from the experimental data was used to calculate the loading rate effect. Compared with the strain rate method, the DIF method gives a much better estimation of the dynamic load–rotation curves. Therefore, this method was used to form the resistance function in the 2-DoF model. In addition to the high loading rate test, Xiao *et al.* (2016) also conducted low-velocity impact tests (4.85 m/s) on RC slabs of the same design as those used in the high loading rate tests. The results of the impact tests showed that the deflection rate (loading rate) of the slab centre and the strain rate of reinforcements were very close to those found in the high loading rate test (2 m/s). Therefore, it is possible to use the DIF obtained from the high loading rate test to form the dynamic load–displacement curves of slabs subjected to low-velocity impact loads. Published reports on high loading rate tests on RC slabs are very limited. However, in addition to the work of Xiao *et al.* (2016), Saito *et al.* (1995) tested three RC flat slabs with a loading rate of 3 m/s and the average DIF was also determined to be 1.5. Therefore, the DIF was assumed to be 1.5 in the low-velocity impact analysis conducted in the current study.

Figure 5. Comparison of load–displacement curves of RC slabs obtained from analytical study and experimental data reported by Xiao *et al.* (2016)

Table 2. Summary of maximum strengths and corresponding displacements of RC slabs

Specimen	Static loading rate				High loading rate					
	Test		Analytical method		Test		$\dot{\epsilon} = 10 \text{ s}^{-1}$		DIF = 1.5	
	$V_{\max}$: kN	$u_{\max}$: mm	$V_{\max}$: kN	$u_{\max}$: mm	$V_{\max}$: kN	$u_{\max}$: mm	$V_{\max}$: kN	$u_{\max}$: mm	$V_{\max}$: kN	$u_{\max}$: mm
10-0-59	240	7.4	187	9.9	360	9.5	204	9.9	281	9.9
15-0-25	303	8.9	234	9.8	435	7.7	259	9.6	350	9.8
15-0-39	304	10.4	279	8.2	474	8.1	321	7.7	419	8.2
15-0-66	379	6.3	367	4.5	663	8.3	420	4.3	553	4.5
15-0-39-0-14	387	8.0	343	8.2	569	7.72	371	8.1	514	8.2
15-0-39-0-28	440	11.8	438	8.2	611	10.1	466	8.1	658	8.2

2-DoF model for impact analysis

Governing equation

The 2-DoF mass–spring–damper model shown in Figure 6 was used to analyse the impact response of RC slabs. The dynamic governing equation of this model is

$$10. \quad \begin{bmatrix} m_i \\ m_s \end{bmatrix} \begin{bmatrix} \ddot{u}_i(t) \\ \ddot{u}_s(t) \end{bmatrix} + \begin{bmatrix} c_c & -c_c \\ -c_c & c_c + c_s \end{bmatrix} \begin{bmatrix} \dot{u}_i(t) \\ \dot{u}_s(t) \end{bmatrix} + \begin{bmatrix} k_c & -k_c \\ -k_c & k_c + k_s \end{bmatrix} \begin{bmatrix} u_i(t) \\ u_s(t) \end{bmatrix} = \begin{bmatrix} m_i g \\ 0 \end{bmatrix}$$

where m_i is the mass of the impactor, c_c and k_c are the damping coefficient and stiffness of the contact model that

describes the interaction between the impactor and the RC slab, c_s , k_s and m_s are the damper, stiffness and equivalent mass of the RC slab, and g is the gravitational acceleration (equal to 9.81 m/s^2 when the slab is vertically impacted and zero when the slab is horizontally impacted).

These model parameters depend on the properties of the RC slab (size, depth, material strengths, reinforcement ratio etc.) and the impactor (diameter of contact area and mass). Once these model parameters and the impact velocity $\dot{u}_i(0)$ are determined, the governing equation can be solved to generate the accelerations, velocities and displacements of the impactor and the impacted slab in the time domain ($\ddot{u}_i(t)$, $\ddot{u}_s(t)$, $\dot{u}_i(t)$, $\dot{u}_s(t)$, $u_i(t)$, $u_s(t)$). Equation 10 can be solved using different kinds of numerical methods. In this paper, a solver based on

Figure 6. 2-DoF model

the Newmark-beta method was compiled using the Fortran language to conduct the analysis. This solver also includes a sub-program that forms the load–displacement curves using the method introduced earlier. If the time step is set as 10^{-5} s and the calculation time is set as 10^{-1} s, the analysis of a single impact case can be finished in 1 s.

After introducing the 2-DoF model, the methods for determining the model parameters are illustrated in the following subsections.

Stiffness of the RC slab (k_s)

Before the slab reaches its maximum displacement ($\dot{u}_s(t) \geq 0$), k_s equals the gradient of the load–displacement curve in its ascending branch and the secant slope of the load–displacement curve in its descending branch. After the slab reaches its maximum displacement ($\dot{u}_s(t) < 0$), it begins to rebound and k_s should be the unloading stiffness. Damage to the slab is insignificant during the unloading stage and therefore k_s can be assumed to be constant. In this study, the unloading stiffness is assumed to be the secant slope of the load–displacement curve when the abscissa equals 5% of the slab depth. The value of 5% was chosen because it was found to provide optimal agreement with experimental data.

Equivalent mass (m_s)

The equivalent mass of the RC slab in the 2-DoF model is calculated using the expression

$$11. \quad m_s = \rho_s H \int_0^{2\pi} \int_0^{l/2} x \phi^2(x) dx d\theta$$

where ρ_s is the slab density, l is the clear span of the slab and $\phi(x)$ is a function that describes the deflected shape of the slab. In the earlier discussion of the load–rotation relationship of RC slabs, the slab is assumed to have constant curvature inside r_0 and zero curvature outside r_0 . With this assumption, both the curvature inside r_0 and the rotation ψ of the slab are

Figure 7. Comparison of assumed slab deformation (Equation 12) and experimental data reported by Xiao *et al.* (2016)

required to determine $\phi(x)$, which makes it difficult to calculate m_s . In this study, $\phi(x)$ is assumed to have the simpler format

$$12. \quad \phi(x) = 1 - \frac{12x^2}{l^2} + \frac{16x^3}{l^3}$$

The calculated slab deformations are compared with the recorded slab displacements at different times during impact test I-15-0-39 conducted by Xiao *et al.* (2016) in Figure 7. The figure clearly shows that Equation 12 matches the experimental data well.

Slab damping coefficient (c_s)

Slab damping is considered to account for the energy dissipated by the slab during impact. The damping coefficient is calculated using the following equation, based on structural dynamics

$$13. \quad c_s = 2\zeta \sqrt{k_e m_s}$$

where ζ is the damping ratio and k_e is the elastic stiffness of the RC slab, which is defined as the secant slope of its load–displacement curve when the abscissa equals 1% of slab depth. Several researchers have experimentally measured the damping ratio of RC slabs. For example, Hrynyk (2013) reported values of ζ in the range 1.6–2.3%, while Delhomme *et al.* (2007), measured ζ to be 11%. With experimental studies suggesting a wide range of ζ , the selection of ζ based on previous experimental data is difficult. In this study, the value of ζ was set as 5%; this was found to provide optimal agreement between the experimental and analytical results.

Contact stiffness (k_c) and damping coefficient (c_c)

The contact between the impactor and the impacted RC slab is modelled with the Kelvin–Voigt model, which comprises an elastic spring and a viscous damper. The impact force $F_c(t)$ can therefore be calculated using the expression

$$14a. \quad F_c(t) = [u_i(t) - u_s(t)]k_c + [\dot{u}_i(t) - \dot{u}_s(t)]c_c \\ \text{for } u_i(t) - u_s(t) \geq 0$$

$$14b. \quad F_c(t) = 0 \quad \text{for } u_i(t) - u_s(t) < 0$$

The use of Hertz contact equations to analytically calculate the contact stiffness k_c is commonly reported in the literature (Bertrand *et al.*, 2015; Fujikake *et al.*, 2009). However, the Hertz contact equations are based on the assumption that the impactor has a spherical nose shape and are therefore not suitable for estimating the contact stiffness when the impactor has a different nose shape. In addition, experimental studies of contact stiffness rarely consider concrete. In the absence of sufficient experimental data concerning the contact stiffness of concrete, the rule of thumb is to set the contact stiffness proportional to the stiffness of the impacted RC slab (Muthukumar and DesRoches, 2006). Using a trial and error method, it was found that the estimated impact forces matched the experimental data well when $k_c/k_e = 5$. Therefore, k_c was calculated by multiplying k_e by five in this research.

As a rule, energy would be lost in the form of heat and deformation during impact, and the lost energy can be related to the coefficient of restitution. By equating the lost energy to the energy dissipated by k_c , the following expression of contact damping coefficient can be derived (Jakubowski, 1964)

$$15. \quad c_c = 2\zeta_c \sqrt{\frac{k_c m_i m_s}{(m_i + m_s)}} \\ \zeta_c = -\frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}}$$

where ζ_c is the damping ratio of the contact damper and e is the coefficient of restitution. By assuming the RC slab is stationary during impact, e can be calculated using

$$16. \quad e = \frac{|\dot{u}_i(t_r)|}{|\dot{u}_i(0)|}$$

where $\dot{u}_i(0)$ and $\dot{u}_i(t_r)$ are the impact and rebound velocities of the impactor, respectively. Since the rebound velocity is unknown before the analysis, an iterative process is required for determining e .

Validation of 2-DoF model

The validity of the proposed 2-DoF model was examined by analysing the impact experiment conducted by Xiao *et al.* (2016) and comparing the analytical results with the experimental data. The specimens used in the impact experiment had the same design, material properties and boundary conditions as those used in the aforementioned static and high loading rate tests (Table 1 and Figure 4). The test setup is shown in Figure 8. The four sides of the slab specimen were clamped and its clear span was 1 m. The slab specimens were impacted at their centres by a steel drop weight with a 0.2 m diameter circular flat nose, and the mass of the drop weight was adjustable. The maximum drop height of the drop weight was 3 m and the corresponding maximum impact velocity was 7.7 m/s.

The measured displacement histories of the midpoint (bottom surface) of the tested slabs were not reported in the previous work (Xiao *et al.*, 2016), so these data are reported in this paper. In addition to the six impact tests reported by Xiao *et al.* (2016), data from five extra impact tests (I-10-0-59(2), I-10-0-59(3), I-15-0-39(2), I-15-0-39(3), I-15-0-39(4)) using the same specimens and test setup are also presented here. The impactor masses and impact velocities of these 11 tests are listed in Table 3. The measured midpoint displacements and impact forces are compared with the analytical results in Figure 9. The maximum and residual slab deformations are listed in Table 3, and the characteristics of the impact force histories are given in Table 4.

Midpoint displacement histories ($u_s(t)$)

As shown in Figure 9, the analytical midpoint displacement histories generally matched the experimental data reasonably well in terms of shape, maximum and residual values. On average, the maximum midpoint displacement of the RC slabs was slightly underestimated (by 9%). The largest difference

Figure 8. Drop weight system used in the impact experiment of Xiao *et al.* (2016)

Table 3. Comparison of experimental and analytical displacement histories

Test	m_i : kg	$\dot{u}_i(0)$: m/s	Maximum displacement, $u_{s,max}$: mm			Residual deformation, $u_{s,res}$: mm			Loading rate: m/s	
			Test	2-DoF	Ratio	Test	2-DoF	Ratio	Test	2-DoF
I-10-0-59	500	5.43	—	54.1	—	—	51.7	—	—	2.3
I-10-0-59(2)	300	6.26	43.9	38.1	0.87	35.0	35.8	1.02	2.6	2.4
I-10-0-59(3)	200	7.67	36.0	35.0	0.97	28.6	33.0	1.15	3.0	2.8
I-15-0-25	500	5.43	44.7	46.5	1.04	—	41.7	—	2.1	2.1
I-15-0-39	500	5.43	38.4	28.9	0.75	30.2	26.8	0.89	2.4	1.9
I-15-0-39(2)	500	4.85	20.3	19.7	0.97	14.9	16.8	1.13	1.4	1.9
I-15-0-39(3)	300	5.11	10.7	10.5	0.98	5.9	4.6	0.78	2.3	2.3
I-15-0-39(4)	200	7.67	17.9	15.6	0.87	11.7	12.3	1.05	2.2	2.7
I-15-0-66	500	5.43	26.8	20.7	0.77	20.0	20.2	1.01	2.1	1.9
I-15-0-39-0-28	500	5.43	16.6	16.8	1.01	10.0	11.6	1.16	2.4	2.4
I-15-0-39-0-56	500	5.43	15.0	13.0	0.87	7.9	6.5	0.82	2.6	2.5

between the analytical and experimental maximum midpoint displacements was 9.5 mm, in test I-15-0-39 (underestimated by 25%). The residual midpoint displacement of RC slabs was, on average, overestimated by 0.1%, and the largest difference between the analytical and experimental values was 1.3 mm, in test I-15-0-39(3) (underestimated by 22%). If slab failure is assumed to have happened after the slab reached its maximum strength, damage to the slab can be determined by comparing its maximum impact displacement ($u_{s,max}$ in Table 3) with the displacement when the slab reached its maximum strength (u_{max} in Table 2). Using this method, it was found that all 11 slabs failed during the impact tests.

Loading rate

The loading rates of the impact tests were evaluated by dividing the maximum slab displacements with the corresponding times. As shown in Table 3, the loading rates were found to range from 1.4 m/s to 3.0 m/s and to increase proportionally with impact velocity. These results suggest that it is reasonable to use the DIF taken from the rapid loading test (loading rate of the order of 10^0 m/s) to evaluate the dynamic load–displacement relationship of RC slabs subjected to low-velocity impact ($1 < \dot{u}_i(0) < 10$ m/s).

Impact force histories ($F_c(t)$)

The estimated impact forces also agreed well with the experimental data in terms of shape, maximum value and duration. The maximum impact force was, on average, overestimated by 7%; the maximum difference was found in test I-15-0-39-0-28, with an analytical value 22% higher than the experimental one. The duration of impact was, on average, underestimated by 12% and the maximum difference was in test I-15-0-39(2), where the analytical value was 28% less than the experimental one. The impulse (I) given by the impactor can be obtained by integrating the impact force in the time domain. The calculated impulse can also be used to assess the rebound velocity of the impactor and the amount of energy imparted to the slab (E_{ip}). The calculated impulse energies (E_i) and imparted energies are

listed in Table 4. Both values were accurately estimated: the error between the analytical and experimental results was within 10%.

Generality

In order to test the generality of the 2-DoF model, two low-velocity impact tests were analysed. These were test N-IS4 conducted by Bhatti *et al.* (2011) and test TH2-1 carried out by Hrynyk and Vecchio (2014). For N-IS4, the slab had dimensions of $1.65 \times 1.65 \times 0.15$ m and a longitudinal reinforcement ratio of 0.6%. The compressive strength of the concrete and the yield strength of the reinforcement were 16.7 MPa and 355 MPa, respectively. The initial velocity, mass and diameter of the impactor were 4 m/s, 300 kg and 60 mm, respectively. For TH2-1, the slab had dimensions of $1.8 \times 1.8 \times 0.13$ m and a longitudinal reinforcement ratio of 0.42%. The compressive strength of the concrete and the yield strength of the reinforcement were 69.4 MPa and 489 MPa, respectively. The initial velocity, mass and diameter of the impactor were 180 kg, 8 m/s and 300 mm, respectively. In general, the slab responses were predicted well by the 2-DoF model, as shown in Figure 10.

Distribution of impact energy and fast estimation of slab damage

The results obtained from the 2-DoF model can be further used to assess the distribution of impact energy during the impact process. As an example, the distribution of energy during impact test I-15-0-39(2) is shown in Figure 11. At the end of this impact test, 84% of the impact energy had become the strain energy of the RC slab (actually it had been dissipated due to slab deformation), 7.3% of the impact energy was dissipated by the contact damper, 5.3% of the impact energy was dissipated by the slab damper and 3.4% of the impact energy became the residual kinetic energy of the impactor.

Noting that the majority of the impact energy was absorbed by slab deformation, it is reasonable to simply consider the

Figure 9. Comparison of analytical results and experimental data reported by Xiao *et al.* (2016)

area under the slab's load–displacement curve before the failure point as the energy capacity of the slab (Figure 12). Whether the slab can sustain a given impact can then be quickly estimated by comparing the slab's energy absorption

capacity with the assigned impact energy. Since not all the impact energy is dissipated by way of slab deformation, this method can yield a conservative estimate of the energy capacity of an RC slab. As an example, the energy capacity of

Table 4. Comparison of experimental and analytical impact force histories

Test	m_i : kg	$\dot{u}_i(0)$: m/s	E_i : J	$F_{c,max}$: kN			T_c : ms			I : N.s			$\dot{u}_i(t_r)$: m/s			E_{ip} : J		
				Test	2-DoF	Ratio	Test	2-DoF	Ratio	Test	2-DoF	Ratio	Test	2-DoF	Ratio	Test	2-DoF	Ratio
I-10-0.59	500	5.43	7371	515	559	1.09	33.8	30.0	0.89	3300	3168	0.96	1.17	0.91	0.78	7029	7164	1.02
I-10-0.59(2)	300	6.26	5886	663	642	0.97	22.4	20.3	0.91	2276	2178	0.96	1.33	1.00	0.75	5613	5728	1.02
I-10-0.59(3)	200	7.67	5886	—	767	—	—	16.2	—	—	1756	—	—	1.11	—	—	5760	—
I-15-0.25	500	5.43	7371	785	895	1.14	28.5	29.4	1.03	3134	3278	1.05	0.84	1.13	1.35	7195	7052	0.98
I-15-0.39	500	5.43	7371	984	927	0.94	24.2	22.2	0.92	3391	3140	0.93	1.35	0.85	0.63	6915	7191	1.04
I-15-0.39(2)	500	4.85	5886	794	829	1.04	22.2	15.9	0.72	2895	2951	1.02	0.94	1.05	1.12	5660	5605	0.99
I-15-0.39(3)	300	5.11	3917	679	722	1.06	10.8	8.8	0.81	2279	2272	1.00	2.49	2.46	0.99	2987	3009	1.01
I-15-0.39(4)	200	7.67	5886	—	1227	—	—	9.2	—	—	1865	—	—	1.66	—	—	5607	—
I-15-0.66	500	5.43	7371	1077	1127	1.05	19.0	17.1	0.90	3490	3191	0.91	1.55	0.95	0.61	6771	7146	1.06
I-15-0.39-0.28	500	5.43	7371	687	835	1.22	12.8	11.7	0.91	3353	3668	1.09	1.28	1.91	1.49	6962	6459	0.93
I-15-0.39-0.56	500	5.43	7371	794	917	1.15	11.1	9.3	0.84	3702	3672	0.99	1.97	1.91	0.97	6401	6459	1.01

specimen 15-0.39 estimated by integrating its dynamic load–displacement curve (Figure 12) was 2328 J. The actual energy capacity of the specimen was also evaluated using the 2-DoF model. As shown in Table 5, the slab failed when the impact velocity reached 3.35 m/s and the corresponding impact energy was 2806 J. In this case, the quickly estimated energy capacity underestimated the real energy capacity by 17%, which is conservative and acceptable. Therefore, this quick estimation approach can be used instead of the 2-DoF model for evaluating slab damage in situations where a detailed and accurate slab response is not required.

Steps for predicting the impact response of reinforced concrete slabs

The theoretical background of the proposed method has been thoroughly illustrated. The steps of the method for practical design or analysis works can be summarised as follows.

- Evaluate the dynamic load–displacement curve and the possible failure mode of the RC slab.
- In cases where a detailed slab response is not required, integrate the dynamic load–displacement curve of the RC slab to obtain its energy capacity. This energy capacity is then compared with the assigned impact energy to determine whether or not the RC slab will fail.
- In cases where a detailed slab response is required, the dynamic load–displacement curve of the RC slab and the properties of the impactor are input into the governing equation of the 2-DoF model. By solving the governing equation, the time-domain impact response and the failure condition of the RC slab can be obtained.

Conclusions

The following conclusions are drawn based on the results presented in this paper.

- An approach for constructing the dynamic load–displacement curves of RC slabs has been proposed. In this approach, the load–displacement curve of the RC slab before punching shear failure is developed through moment–curvature analysis. A failure criteria based on critical shear crack theory is then used to determine the punching shear strength and the descending branch of the load–displacement curve. In this way, the strength softening of RC slabs after shear failure can be considered. The effectiveness of this approach was verified by comparing the output with experimental data. For the loading rate effect, it was found that the strain rate method cannot sufficiently consider strength enhancement. By comparison, the strength enhancement can be properly incorporated by using the dynamic increase factor (DIF) obtained from a rapid loading test. One main limitation of this study was that the DIF was set to 1.5 for RC slabs for loading rates of the order of 10^0 m/s, based on available

Figure 10. Comparison of analytical results and experimental data from test N-IS4 (Bhatti *et al.*, 2011) and test TH2-1 (Hrynyk and Vecchio, 2014)

Figure 11. Typical energy distribution of impact problem I-15-0-39(2)

Figure 12. Energy capacity of RC slab specimen 15-0-39

Table 5. Damage to specimen 15-0-39 when subjected to different impact energies

m_i : kg	$\dot{u}_i(0)$: m/s	E_i : J	$u_{s,max}$: (2-DoF): mm	u_{max} : mm	Fail
500	3.3	2723	8.1	8.2	No
500	3.35	2806	8.3	8.2	Yes
500	3.4	2890	8.5	8.2	Yes

experimental data. It is worth mentioning that, in addition to the loading rate, the DIF can also be influenced by other parameters such as the reinforcement ratio, the specimen and so on (Adhikary *et al.*, 2012). However, the current database of rapid loading tests is still limited in terms of obtaining an analytical expression for the DIF. Future work is therefore required in this area to facilitate a better estimation of the DIF.

- A two-degrees-of-freedom (2-DoF) model was presented to analyse the response and damage of RC slabs under low-velocity impact. The model parameters can be easily determined once the properties of the impact load and the slab specimen are known. Using a numerical method, this model can be solved efficiently. In the subsequent validation work, the slab responses in terms of midpoint displacement, impact force, contact duration, impulse and so on were estimated with reasonable accuracy.
- The results from the 2-DoF model were further processed to show RC slab behaviour from an energetic standpoint. It was found that the impact energy can be dissipated by the slab damper, the contact damper and permanent slab deformation. Among these, the energy dissipated by permanent slab deformation was a very high percentage of the impact energy. This result proves why a rational load–displacement curve is necessary for a 2-DoF model to produce good results.
- In situations where a detailed and accurate slab response is not required, a slab’s energy capacity can be estimated by integrating its load–displacement curve until the failure point. Whether the slab will fail or not during impact can then be predicted by comparing this energy capacity with the exerted impact energy.

Appendix

The detailed derivation of Equation 2, proposed by Muttoni (2008) based on the quadrilinear moment–curvature relationship shown in Figure 1(e), is now presented.

This quadrilinear moment–curvature relationship is characterised by the stiffnesses before (EI_0) and after (EI_1) cracking, the cracking moment m_{cr} , the moment capacity m_R and the tension stiffening effect χ_{TS} . Ignoring the effect of reinforcement before cracking, the expressions for m_{cr} , EI_0 and the corresponding curvature χ_{cr} are

$$17. \quad m_{cr} = \frac{f_{ct} H^2}{6}$$

$$18. \quad EI_0 = \frac{E_c H^3}{12}$$

$$19. \quad \chi_{cr} = \frac{m_{cr}}{EI_0} = \frac{2f_{ct}}{HE_c}$$

Assuming linear-elastic behaviour of steel and concrete after cracking, expressions for EI_1 and the depth of the compression zone c can be derived as

$$20. \quad EI_1 = \rho \beta E_s d^3 \left(1 - \frac{c}{d}\right) \left(1 - \frac{c}{3d}\right)$$

$$21. \quad c = \rho \beta \frac{E_s}{E_c} d \left(\sqrt{1 + \frac{2E_c}{\rho \beta E_s}} - 1 \right)$$

where β is an efficiency factor that accounts for the stiffness reduction of the slab after cracking by its orthogonal reinforcement layout. An empirical value of 0.6 is recommended for β .

Assuming perfect plastic behaviour of the reinforcement after yielding, a rectangular stress block for concrete in the compression zone and neglecting compression reinforcement, the moment capacity m_R is given by

$$22. \quad m_R = \rho f_y d^2 \left(1 - \frac{\rho f_y}{2f_c}\right)$$

The decrease in curvature due to tension stiffening can be calculated by

$$23. \quad \chi_{TS} = \frac{f_{ct}}{6H\rho\beta E_s}$$

The curvatures at the beginning of the stabilised cracked regime (χ_1) and at yielding (χ_y) can be calculated by

$$24. \quad \chi_1 = \frac{m_{cr}}{EI_1} - \chi_{TS}$$

$$25. \quad \chi_y = \frac{m_R}{EI_1} - \chi_{TS}$$

The four segments of the quadrilinear moment–curvature relationship correspond to the four regions of the slab shown in Figure 1(d). The radii r_{cb} , r_1 and r_y that divide these four regions can be obtained by substituting Equations 21, 26 and 27 into Equation 1a, and the distribution of the tangent moment (m_t) along the slab section can thus be evaluated. On the other hand, the radial moment is uniformly distributed within the radius r_0 with the value of m_r , which can be obtained with the curvature at r_0 .

Finally, the following expression can be obtained by equating the moments induced by external and internal forces acting on the slab sector in Figure 1(b)

$$26. \quad V \frac{\Delta\phi}{2\pi} (r_q - r_c) = -m_r \Delta\phi r_0 - \Delta\phi \int_{r_0}^{r_s} m_t dr$$

which can be further manipulated to take the form of Equation 2.

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