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MULTI-ROBOT FORMATION CONTROL IN OBSTACLE ENVIRONMENTS

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 3 papers published in the following peer-reviewed journals where I was the first author.

Chapter 3 is published as Y. Wang, D. Wang, and S. Zhu, Formation tracking of multi-vehicle systems in unknown environments using a multi-region control scheme, *International Journal of Control*, 90(12), 2760-2771 (2017). DOI: 10.1080/00207179.2016.1265151.

The contributions of the co-authors are as follows:

- I prepared the manuscript drafts, designed the algorithm and performed the simulations.
- Prof. Wang provided the initial project direction and revised the manuscript drafts.
- Dr. Zhu assisted in the algorithm design and the revision of the manuscript.

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The contributions of the co-authors are as follows:

- I wrote the manuscript drafts, designed the algorithm and performed the experiments.
- Prof. Wang provided the initial project direction and revised the manuscript drafts.
- Dr. Yang assisted in the experiment design and the data collection.
- Dr. Shan assisted in the experiment design and revised the manuscript.

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The contributions of the co-authors are as follows:

- I wrote the manuscript drafts, designed the algorithm and performed the simulations and the experiments.
- Dr. Shan assisted in the experiment design and revised the manuscript.
- Mr. Yue assisted in the experiment design.
- Prof. Wang provided the initial project direction and revised the manuscript drafts.

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Abstract

Multi-robot system has wide applications in both civil and military fields, due to its high efficiency for task completion and robustness with respect to robot failures. Formation keeping, which is one of the fundamental tasks for multi-robot control, has been extensively investigated in the past decade, resulting in a large variety of methodologies. However, most of the existing approaches are subject to certain limitations when implemented for addressing real formation control problems, mainly due to practical constraints introduced, such as sensor limitations, obstacle and inter-robot collision avoidance in unknown environments, connectivity maintenance, unavailability of global pose measurements and so forth. This thesis deals with formation control problem for real robot applications by taking practical constraints into account.

First of all, the virtual-structure based formation tracking problem is studied, where obstacle and inter-robot collision avoidance as well as connectivity maintenance are explicitly considered. To address this problem, a multi-region control scheme is proposed, where three regions, including safe region, dangerous region and transition region, are defined. In different regions, priority is given to different control objectives. In safe region where trajectory tracking holds the priority, the proposed control scheme guarantees the bounded reference tracking for each robot. In dangerous region where avoidance control is the main objective, a new bounded potential function is designed to formulate multiple constraints. By introducing a series of transition functions, smooth switching between trajectory tracking and avoidance

control is achieved in transition region. It has been proven that each robot can track its reference trajectory while satisfying the constraints simultaneously with a bounded controller, implying that the proposed control scheme satisfies input constraints by properly tuning parameters.

Secondly, the vision-based leader-follower formation tracking problem is investigated, while sensor limitations, obstacle and inter-robot collision avoidance and leader-loss situations are considered and tackled in the proposed algorithm. For leader tracking, a class of bounded barrier functions are employed to formulate distance and bearing angle constraints introduced by sensor limitations and leader-follower collision avoidance requirement. To ensure robot safety in unknown environments, a multi-region obstacle avoidance algorithm is proposed which prioritizes different control objectives in different regions. The leader-loss situation has also been studied, which may be caused by illumination variation, motion blurring or visual occlusion by obstacles. To deal with this case, a fault-tolerant strategy is designed to immediately drive the follower to the place where the leader was lost. The proposed control scheme is primarily designed for a communication-free environment where only local pose measurements are available. Furthermore, it has taken control input constraints explicitly into account.

Finally, motion capabilities of rigid and flexible formations are analyzed. In addition, the flexible leader-follower formation tracking problem is addressed. To illustrate the superiority of flexible formation against traditional rigid formation in terms of motion capability, a mathematically rigorous analysis is presented from the motion planning perspective on the basis of the definition of formation flexibility. Unlike the previous studies on flexible formation control, this work is under a more challenging assumption that global position and orientation measurements are not available. To obtain the relative pose relationships amongst robots, a stereo camera is mounted on each follower. In consideration of the fact that visual observations are noise-corrupted and intermittently available, a particle filter based relative pose estimation approach is employed to estimate the position and orientation of the leader

in the local reference frame of the follower using the polluted and discontinuous information. Also, to form a flexible formation, the leader historical trajectory is reconstructed with respect to the current local frame attached on the follower, based on which a reference point is generated. In addition, the situation where robots work in unknown obstacle environments is considered. To ensure robot safety in such environments, a multi-objective control law is proposed to balance reference tracking and collision avoidance in different situations.

In summary, this thesis provides three feasible formation control strategies for multi-robot systems from the viewpoint of practical applications. All of the proposed approaches are well supported by both numerical simulations and real robot experiments.

Symbols and Acronyms

Symbols

A^T	Transpose of matrix A
$\mathbf{0}_n$	$n \times 1$ vector with all zero elements
$\mathbb{R}$	Set of real numbers
$\mathbb{N}$	Set of nonnegative integers
$\ \cdot\ $	2-norm
∇	Gradient operator
Subscript i	The i th robot
Pre-superscript i	Reference coordinate system $\{i\}$

Acronyms

AHRS	Attitude and Heading Reference System
AMCL	Adaptive Monte Carlo localization
APF	Artificial Potential Field
AUV	Autonomous Underwater Vehicle
CLF	Control Lyapunov Function
DVZ	Deformable Virtual Zone
DWA	Dynamic Window Approach
FOV	Field of View
GPS	Global Positioning System
IMU	Inertial Measurement Unit

KKT	KarushKuhnTucker
LOS	Line of Sight
MPC	Model Predictive Control
NTU	Nanyang Technological University
PD	Proportional and Derivative
PDF	Probability Distribution Function
PRM	Probabilistic Roadmap
ROS	Robotic Operating System
RRT	Rapidly-exploring Random Tree
SLAM	Simultaneous Localization and Mapping
UAV	Unmanned Aerial Vehicle
UGV	Unmanned Ground Vehicle
USV	Unmanned Surface Vehicle
WSN	Wireless Sensor Networks

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Chapter 1

Introduction

This chapter presents the introduction of this thesis. First, the background and motivation for conducting the research is stated. Then, major contributions are highlighted. Finally, the outline of the thesis is introduced.

1.1 Background and Motivation

Multi-robot system has wide applications in both civilian and military fields, such as coordinated target tracking, sensitive area surveillance and unknown environment exploration, which has attracted numerous researchers from robotics and control communities devoting to coordination algorithm design for such systems in order to execute various tasks. Up to now, there are already some survey papers that have summarized the existing research results in this area [1–7]. Some major research problems are consensus [3, 4, 8–13], formation control [14–21], flocking [22–24], region/shape control [25–29], task assignment [30–33] and cooperative localization [34–36], to name a few. Among the above-mentioned research hotspots in this area, formation control, which is to form a certain geometric pattern with or without a group reference [1], is one of the brightest topics which has been investi-

gated extensively in the past decade (see [2] for an in-depth survey). As a result, a large variety of formation control approaches have emerged which can be categorized into the following mainstream classes: leader-follower approaches [16, 37–39], virtual-structure approaches [14, 20, 28], behavior-based approaches [15, 19] and so forth. However, most of the existing approaches may suffer from certain limitations when being put into practical applications mainly due to additional constraints introduced. These practical constraints include but not limited to the following items.

1. *Collision avoidance in unknown environments.* Safety is considered the foremost issue for any control problems. For multi-robot formation applications, the obstacle and inter-robot collision avoidance which is closely related to system safety should hold the highest priority among all the control objectives, meaning that the formation objective could be ignored when conflicting with collision avoidance. Important as this issue is, most of the existing formation control approaches have not fully taken into account the obstacle avoidance problem. Although there have been some attempts in the literature, they are under the assumption that the obstacle information is known prior to controller design, which is unrealistic in applications where robots work in an unknown environment.
2. *No global pose measurements.* Up to now, most of the existing literatures on formation control are based on global position and orientation feedback. Generally, in real robot operations, global pose can be obtained by GPS devices, SLAM algorithms, IMU or WSN. To get accurate enough global pose measurements, expensive devices and complicated algorithms should be utilized. Besides, in a communication-free environment, global pose feedback would be useless since robot cannot obtain the global poses of its neighbors.
3. *Sensor limitations.* For leader-follower formation implementation, stereo camera is generally employed for the follower to detect the leader. However, stereo cameras are subject to limited depth range and FOV, which result in the maxi-

mum and minimum distance requirements between the follower and the leader as well as the bearing angle constraints of the leader with respect to the follower. Even though it is considered critical, few researchers have paid attention to this issue as stated in [38].

4. *Control input constraints.* Currently, most of off-the-shelf mobile robot platforms only provide the velocity input interface with specific input constraints, which also add complexity to the control problem.
5. *Communication-free environment.* In many applications, due to military security, environmental constraints or energy saving, there are no communication links among robots. Thus, only relative pose measurements among robots obtained by the on-board sensors are available. In addition, robot does not have the motion information of its neighbors, which is actually an external perturbation in the coordination dynamics. In such situation, controller design becomes significantly more challenging.
6. *Leader-loss situation.* To date, there are already many research outcomes trying to solve the visibility maintenance problem for multi-robot systems. However, maintaining visibility between the leader and the follower in unknown obstacle environments is of great difficulty since occlusion of LOS visibility by obstacles is inevitable. Besides, cameras may suffer from temporary faults in operation raised from change of lighting condition, motion blurring, etc. Thus, from the practical point of view, the leader-loss situation should be taken into consideration and fault-tolerant strategies able to recover visibility again should be designed.
7. *Flexible formation structure.* Classical leader-follower formation control problem is to drive the follower to maintain a fixed relative position relationship with its leader. However, in an obstacle environment, a fixed formation structure seems rather conservative and clumsy. Actually, except in the ornamental applications, it is unnecessary to require the follower to keep a fixed distance

and bearing angle with the leader. The formation can also work well as long as the distance and bearing angle are within specific ranges.

1.2 Contributions

This thesis investigates the formation control problem for multiple nonholonomic mobile robots in unknown obstacle environments. First of all, rigid formation tracking control in unknown obstacle environments is studied under the virtual-structure framework. Secondly, vision-based leader-follower formation tracking control problem is solved, where rigid formation configuration and unknown obstacle environments are also considered. Thirdly, motion capabilities of rigid and flexible formations are analyzed, then a vision-based flexible leader-follower formation tracking control strategy is proposed in unknown obstacle environments without global pose measurements. All of the proposed approaches are well supported by both numerical simulations and real robot experiments.

The main contributions of this thesis are highlighted as follows.

1. *A virtual-structure formation tracking control approach for multiple nonholonomic mobile robots in unknown obstacle environments* (chapter 3)

First, a multi-region control scheme is proposed to address the virtual-structure formation tracking control problem while obstacle avoidance, inter-robot collision avoidance and connectivity maintenance are guaranteed as well. Secondly, a new bounded potential function is introduced to formulate multiple constraints including obstacle avoidance, inter-robot collision avoidance and connectivity maintenance. Boundedness of the potential function can guarantee boundedness of the controller. Thirdly, it has been proven that the proposed controller is bounded and the bounds in different regions are derived. Thus, the proposed control strategy satisfies input constraints by properly setting

parameters.

2. *A vision-based leader-follower formation tracking control approach for multiple nonholonomic mobile robots in unknown obstacle environments* (chapter 4)

First, a leader-tracking control law is proposed, where sensor limitations and leader-follower collision avoidance are modeled as distance and bearing angle constraints, then a bounded barrier function is introduced to formulated these constraints. Secondly, a multi-region obstacle avoidance algorithm is designed which priorities different objectives in different regions, such that robot safety can be guaranteed. Thirdly, the leader-loss situation is pointed out for the first time, while a fault-tolerant solution is provided when the visibility is broken.

3. *Comparative analysis on motion capabilities of rigid and flexible formations* (chapter 5)

First, a new definition called formation flexibility is introduced for the first time as a quantifying metric for formation motion capability analysis, while the criteria for comparison on motion capabilities of different formations is also provided. Then the motion capabilities of perfect rigid, semi-perfect rigid and flexible formations are analyzed, while the ranges of the corresponding formation flexibilities are calculated. Finally, the motion capabilities of perfect rigid, semi-perfect rigid and flexible formations are compared using the introduced criteria.

4. *A vision-based flexible leader-follower formation tracking control approach for multiple nonholonomic mobile robots in unknown obstacle environments* (chapter 5)

First, to account for the nonlinearities and non-Gaussian noise in the process and observation models, a particle filter based relative pose estimation approach is proposed to estimate the local position and orientation of the leader with respect to the follower. Then a leader trajectory reconstruction algorithm is designed to transform the leader historical trajectory to the current local

reference frame attached on the follower, based on which a reference generator is employed to select the reference point for the follower to track. Finally, a multi-objective control scheme is proposed to achieve and coordinate reference tracking, obstacle avoidance and inter-robot collision avoidance.

1.3 Organization of the Thesis

The remainder of this thesis is organized as follows.

Chapter 2 reviews the existing approaches in the literature on formation control, collision avoidance and connectivity/visibility maintenance.

Chapter 3 addresses the virtual-structure formation tracking control problem for multiple nonholonomic mobile robots in unknown obstacle environments. A new bounded potential function is introduced to formulate multiple constraints including obstacle avoidance, inter-robot collision avoidance and connectivity maintenance. Then, a multi-region control scheme is proposed to coordinate different control objectives in different regions. Numerical simulation and real robot experiment have been conducted to validate the efficacy of the proposed method.

Chapter 4 investigates the leader-follower formation tracking control problem for multiple nonholonomic mobile robots in unknown obstacle environments. A class of bounded barrier functions are proposed to formulate distance and bearing angle constraints introduced by sensor limitations and leader-follower collision avoidance requirement. A multi-region obstacle avoidance algorithm is designed to ensure robot safety in unknown environments. The leader-loss situation is pointed out while the corresponding analysis and a fault-tolerant solution are provided. Simulation and real robot experimental results are provided to illustrate the efficacy of the proposed strategy.

Chapter 5 studies the flexible leader-follower formation tracking control problem for

a group of nonholonomic mobile robots without global pose measurements. The motion capabilities of rigid and flexible formations are analyzed mathematically. A vision-based flexible leader-follower formation tracking control approach is proposed under the assumption that global pose measurements are not available. Simulation and real robot experiments have been performed to demonstrate the effectiveness of the proposed algorithm.

Chapter 6 summarizes this thesis and provides recommendations for the future work.

Chapter 2

Literature Review

Multi-robot formation control has been studied extensively in the past decade. However, when considered in obstacle environments, the problem becomes much more challenging as extra requirements, like collision avoidance, connectivity maintenance, visibility preservation, etc., are introduced. In this chapter, an in-depth survey of the existing techniques on related topics, including formation control, collision avoidance and connectivity/visibility maintenance, is presented.

2.1 Formation Control

As many practical applications, such as satellite formation flying, cooperative planetary exploration, agricultural automation, etc., require multi-robot system to maintain a specific geometric shape while executing the assigned task, formation control design has become a major research field in the past decade. Formation control, which is defined as coordination of a group of robots to form a certain geometric pattern, can be classified into two categories according to whether a group reference exists, formation stabilization and formation tracking [1]. Formation stabilization aims at forming a desired geometric shape in the absence of a group reference, while

formation tracking refers to controller design for a group of robots to maintain a desired geometric pattern while following a predesignated group reference simultaneously. Specifically, to drive a group of robots to their corresponding destinations which forms a desired geometric shape, called formation navigation in some literature [40], is a special case of formation tracking with the group reference to be a static point. Compared with formation stabilization, formation tracking is preferable due to a huger demand in practical applications. Additionally, because of the introduction of a group reference, formation tracking is much more challenging than formation stabilization especially when a cluttered environment is considered.

2.1.1 Formation Configuration

In terms of the formation configuration pattern to be formed by the robots, a formation can be divided into two categories, *rigid formation* and *flexible formation*. The configuration of rigid formation is fixed during operation, while that of flexible formation will change with the formation reference path. Until now, almost all of the existing literature dealing with formation control falls into the class of rigid formation due to its simplicity and convenience of formulation. Besides, rigid formation is indeed advantageous in some applications such as cooperative transportation of objects, robot formation parade and so forth. However, in many other applications where geometric shape of the formation is not strictly required, rigid formation may suffer from certain limitations especially when motion constraints of the robots are taken into consideration. In addition, considering motion planning of a rigid formation in obstacle environments, the whole robot group should be treated as a single large robot, which may suffer from collisions or a reduced mobility, especially when passing through a narrow door with a smaller size than the formation.

Reference [41] points out that to maintain a perfect square using four differentially driven mobile robots, the only possible motion is in a straight line. Then, a compromised approach is proposed that is to maintain the formation in curvilinear coordi-

nates rather than in rectilinear coordinate system such that the formation shape can be altered when the formation is turning, which is the embryo of flexible formation. After that, some interesting results along this idea have been published [42–44]. Reference [42] proposes an adaptive and predictive formation control scheme to achieve highly accurate relative positioning of the formation whose desired shape is defined with respect to the formation reference path, in terms of curvilinear distances between robots along the reference path and lateral deviations with respect to this trajectory. Reference [43] investigates the flexible leader-follower formation tracking control problem for nonholonomic mobile robots. The method proposed in [43] consists of two parts, curvilinear-based real-time formation reference generator and single-robot trajectory tracking controller. Reference [44] analyzes the maneuverability of the flexible formation by taking the motion constraints of the robots into account.

Although there are already a few research results on flexible formation control, several problems still remain unsolved. First of all, despite the fact that reference [41] points out the limitation of rigid formation that a perfect square cannot be formed by four moving unicycle-type mobile robots unless the required motion is along a straight line, there are no quantitative analysis on the motion capability of rigid and flexible formation. Consequently, the advantage on the maneuverability of the flexible formation against that of the rigid formation is still not very clear. Secondly, all of the existing approaches which address the flexible formation control problem like [42] and [43] assume that a GPS receiver is installed on each robot and the reference trajectory of the leader is previously learned or received online by the followers. However, in many practical applications, especially those in indoor environments, it is not easy to obtain global position and orientation measurements of multiple robots with respect to a common coordinate system. Therefore, the study of flexible formation control under such situations is of great significance.

2.1.2 Formation Control Approach

Up to now, formation control has been applied to various real applications, such as UGVs [42, 45], UAVs [46, 47], USVs [38, 48, 49], AUVs [50], spacecrafts [20, 51] and so on. According to the strategy utilized for implementation, existing formation control approaches consist of three main classes, leader-follower approaches, virtual-structure approaches and behavior-based approaches, which are detailed as follows.

1. *Leader-follower approaches.* This kind of approaches is inspired by animal behaviors like fish schooling and birds flocking in nature. In these approaches, some robots are assigned as leaders while the others are designated as followers which maintain desired geometric configuration with respect to the leaders. Reference [16] uses the terms $l - \psi$ and $l - l$ control to reflect whether the control laws are based on tracking the position and orientation of the robot relative to a leader robot, or the position relative to two leader robots, respectively. The proposed $l - \psi$ and $l - l$ controllers guarantee the correct formation, while the modified $l - l$ controller can be used to change the shape of the formation in the presence of obstacles. Based on [16], reference [17] defines the concept of a transition matrix, which governs the addition and deletion of edges in the control graph and hence the change in the communication protocol. Upon this, an exhaustive list of all possible transitions that can occur within the robots in the formation is presented. Reference [52] presents a receding horizon leader-follower control framework to solve the formation problem of multiple nonholonomic mobile robots. In particular, a receding horizon scheme is incorporated into the leader-follower controller to yield a fast convergence rate of the formation tracking errors. Reference [38] considers finite-time leader-follower formation control problem for a class of underactuated autonomous surface vessels with LOS range and angle constraints. A novel control structure is proposed that can guarantee the convergence into arbitrarily small neighborhoods around zero in finite time for

the tracking errors while the constraint requirements on the LOS range and angle will not be violated. In [53], formation tracking problem of nonholonomic mobile robots without direct position measurements is addressed. An observer is designed to estimate the local position of the follower robots using the feedback information from a perspective camera, the odometry and AHRS sensors. The closed loop stability of the combined observer-controller system is analyzed by Lyapunov theory. Reference [39] proposes a control method for leader-follower navigation in obstacle environments without data transmission between robots, which takes connectivity maintenance, collision avoidance and input constraints into consideration. A merit of this paper is that the communication topology is changing depending on the environments such that some links are deactivated in order to pass through narrow spaces, while active links are increased in free spaces to keep the group as cohesive as possible.

2. *Virtual-structure approaches.* This kind of approaches is first proposed in [14] where a virtual structure is defined as a collection of elements which maintain a rigid geometric relationship to each other, based on which a general control strategy is developed to force an ensemble of robots to behave as if they are particles embedded in a rigid structure. In [18], the concept of virtual leader is introduced for the virtual-structure formation, whose responsibility is to direct, herd and/or manipulate the group behavior. In [20], a decentralized formation scheme for spacecraft formation flying is presented based on the virtual structure approach, where decentralized formation control strategies are presented for each spacecraft to synchronize the coordination vector instantiation and track its desired states. Reference [45] presents a constructive method to design cooperative controllers that force a group of unicycle-type mobile robots with limited sensing ranges to perform desired formation tracking and guarantee no collisions between the robots. The proposed method is based on the virtual structure approach and artificial potential function. In [54],

formation control strategies are proposed for networked multi-vehicle systems in order to achieve virtual-structure consensus, virtual-structure flocking and virtual-structure flocking with collision avoidance.

3. *Behavior-based approaches.* This kind of approaches is implemented by defining several motor schemas according to the task executed by the robots, each of which represents a behavior response to the current sensory feedback. The overall control action for each robot is a weighted combination of the control for each behavior. Reference [15] defines several new reactive behaviors that implement formations in multi-robot teams. Then the defined formation behaviors are integrated with other navigational behaviors to enable a robotic team to reach navigational goals, avoid hazards and simultaneously remain in formation. Reference [19] proposes a behavior-based approach to address the problem of transitioning a group of robots through a sequence of formation patterns. This paper also provides a rigorous analysis of formation keeping and convergence, which is a contribution to the behavior-based literature. In [55], a null-space-based behavioral control scheme is presented to realized formation control of multi-robot systems, besides, experimental results are provided to validate the proposed method. Reference [29] presents a multilevel-based topology design and a new shape-control approach for the convergence of robot swarms to a desired region. A potential function based controller with a shape regulation control force is developed to drive all the robots to form the desired shape without collisions. The proposed controller is actually a combination of different forces aiming at driving the robot to a desired region and level, to avoid inter-robot collisions and to avoid obstacles, respectively.

The three major formation control approaches have been introduced thoroughly above. Nevertheless, different approaches have different strengths and shortcomings. Leader-follower approaches are simple and easy to be scaled, making it preferable compared with other approaches. However, from the principle of leader-follower design, there is no feedback of the follower to the leader, which weakens the co-

ordination of the whole formation. Furthermore, the performance of the formation depends largely on the leader, which is also a single point of failure such that breaking down of the leader would cause failure of the entire formation. In virtual-structure approaches, the relative positions with respect to each other are pre-designated for robots such that the formation is rigid, which is easy to be implemented and preferred in some applications like satellite formation flying, robot show, etc. Notwithstanding, such configuration seriously restricts the motion capability of the formation. In particular, when working in obstacle environments, avoidance behaviors of such formation would be rather clumsy. Behavior based approaches have obvious physical meanings and are easy to implement, but most of the existing approaches lack rigorous mathematical analysis. As a result, it is difficult to guarantee that the formation can converge to the desired configuration eventually.

2.2 Collision Avoidance

During autonomous operations, safety is the foremost issue that should be taken into consideration. For multi-robot applications, especially those in obstacle environments, collision avoidance with obstacles and among robots is an important research topic, which has attracted attentions from researchers for many years. Interested readers can refer to [56] for an in-depth survey. According to the amount of environmental information which is available for the robot, existing collision avoidance approaches can be classified into two categories, *global motion planning approaches* and *reactive approaches*. The global motion planning approaches are under the assumption that accurate information of the whole environment is available, while the reactive ones only have access to the current sensory feedback of the local environment around the robot.

In practice, to implement a global motion planning algorithm, a map of the workspace must be built beforehand, based on which, techniques such as A* [57], D* [58],

PRM [59], RRT [60], PRM* [61], RRT* [61], etc. are employed to generate a feasible path for the robot to follow. Even though motion planning methods can provide a complete trajectory from the starting point to the destination with guaranteed collision avoidance, computational efficiency is a significant problem for such approaches. To build a map of the environment, necessary sensors like stereo cameras or laser range finders should be installed on the robot, based on which SLAM techniques are applied. As mapping algorithms generally take quite a lot of time, it is difficult to realize real time motion planning on robots with limited processing power. However, in uncertain environments which require timely replanning operations to guarantee robot safety, motion planning approaches maybe not qualified. In this case, reactive approaches which acts directly at the control level is preferred in order to achieve real-time collision avoidance. In the following, reactive approaches are introduced in detail.

Reactive approaches make use of the local information of the environment collected by the on-board sensors to compute the motion commands at every sampling time interval, which can also be treated as a reactive feedback controller, as it maps the current observation into the current control. Up to now, reactive collision avoidance has been an attractive research topic due to its simplicity and practicality in real robot implementations. As a result, many techniques have been proposed, which are introduced briefly as follows.

1. *APF approaches.* In these approaches, a potential function is defined to formulate collision constraints, which attracts the robot to a target and repulse it away from obstacles. This kind of methods are computational efficient with satisfactory performance, making it the most popular approach among the existing results. The main drawback of APF approaches is that they are easy to fall into local minima when attractive forces and repulsive forces are canceled with each other such that the robot get stuck at some places. In [62], a cooperative control law is presented for multiagent systems to guarantee collision avoidance using value functions that resemble barrier functions in static opti-

mization theory. Reference [63] proposes a switching control method that can ensure bounded trajectory tracking and collision avoidance for multi-vehicle systems, but the control input might go to infinity when avoiding collisions, which is unreasonable in practice. Reference [27] proposes a novel collision and obstacle avoidance strategy using multi-layer region control concept and combines potential functions with simple PD controller to avoid potential collisions. Reference [64] deals with combined trajectory tracking and collision avoidance problem under input saturation constraints, but they could only solve collision avoidance problem between a single vehicle and one static obstacle or the cooperative collision avoidance problem between two vehicles. Recently, another kind of APF, navigation function, has caught attentions of researchers for its capability to deal with multiple constraints and boundedness of its gradient which is critical for systems with actuation limitations. Actually, navigation function provides a way to combine goal and constraint potential functions together. References [65] and [66] deal with decentralized navigation and formation stabilization problem while avoiding inter-vehicle collisions, respectively. References [67], [40] and [68] design controllers to achieve desired objectives while avoiding obstacles and maintaining connectivity at the same time. In [69], a formation stabilization control law is proposed where constraints including workspace boundary avoidance, obstacle avoidance, inter-vehicle avoidance and connectivity maintenance are incorporated into the constraint function of a decentralized navigation function. Reference [70] considers the situation where agents may not have global environmental information and they may have to avoid collisions with unexpected spatial constraints. By introducing a Dirac delta function to deal with multiple spatial constraints, a control law is proposed to solve the task of formation tracking and obstacle avoidance in a bounded environment. Loosely speaking, vector field approach, which is to control the robot to align with and flow along the desired force field, can also be categorized as a special case of APF approaches. Some typical examples of the vector field method are [71–74].

2. *Boundary following approaches* [56]. This kind of approaches is primarily designed for static unknown obstacle avoidance, whose basic idea is following the boundary of the obstacle temporarily in order to bypass it and continue the main control objective. In detail, the robot pursues the main control objective initially until a potential collision with an obstacle is detected. The boundary following objective is thereupon activated, while the main objective is ignored temporarily. Once the leaving condition is satisfied, the main objective holds the highest priority again. In [75], a sliding mode control law is proposed which drives a Dubins-like mobile robot to move along a path with a predesignated distance from the boundary of an object. Reference [76] proposes an integrated guidance-control strategy for a unicycle-type wheeled mobile robot such that it can patrol the border of a region in a close range with a constant linear velocity. Then the proposed strategy are applied to the safe guidance problem of a ground vehicle towards a target while bypassing the enroute obstacles with a predefined safety margin. The problem of navigating a Dubins-car like robot to a target through a maze-like environment is discussed in [77], where a bio-inspired control law is proposed with mathematically rigorous analysis. It can be found that the above discussed results are along the same idea. Additionally, there are also some other attempts to deal with obstacle avoidance problems using boundary following behaviors. In [78], the DVZ principle is proposed to deal with obstacle avoidance problems, where a safety zone is defined around the vehicle in which the presence of an obstacle induces an intrusion of information that tries to drive the vehicle reaction. The boundary following behavior is achieved by maintaining a constant contact with the obstacle, thus to bypass it. Reference [79] presents a safe maneuvering zone based control scheme such that a unicycle-type robot can follow the desired path asymptotically, when possible, and contour the unpredicted obstacles.
3. *Predictive approaches*. This kind of approaches makes use of the kinematic model of the robot to predict its motion in the near future, based on which

an optimal control command is generated for the current time instant. An attractive property of this kind of method is that it is not near-sighted as it plans ahead for a given amount of time when obstacles are approaching. There are two famous examples in this category, DWA and MPC-based approaches. DWA is first proposed in [80], which searches for next velocity and steering commands in the two-dimensional space of circular trajectories after removing the commands that would lead to collisions. Then the dynamic window is introduced to further restrict admissible commands to those that can be reached within a sampling time interval by taking the acceleration constraints into account. Finally, an optimal trajectory is selected to maximize the designed objective function. In [81], a convergent DWA is proposed, which combining the traditional DWA and navigation function proposed in [82] together in a MPC and CLF framework. Reference [83] extends the original DWA, which is primarily designed for first-order nonholonomic mobile robots, to AUV applications with second-order nonholonomic constraints. Recently, MPC-based approaches have become more and more popular in collision avoidance problems, as they seem to show great potential in providing efficient navigation. In [84], a nonlinear MPC-based approach is proposed for the one-step trajectory generation of a helicopter, then a gradient-descent method is designed to provide online solutions of the optimization problem. Reference [85] designs nonlinear MPC laws for controlling multiple USVs in arbitrary formations with obstacle avoidance taken into consideration. In [86], control input saturation and collision avoidance constraints are incorporated as inequality constraints using KKT conditions in the nonlinear model predictive control framework, based on which control inputs for formation flight are computed. Coordinated standoff tracking by a pair of unmanned aerial vehicles is studied in [87], where inequality constraints are considered for collision avoidance between unmanned aerial vehicles and control input saturations using penalty functions in the model-predictive control framework. Reference [28] addresses the distributed optimal formation trajectory generation problem for multiple

nonholonomic vehicles, where several objectives are taken into account including formation achievement, inter-group collision avoidance, obstacle avoidance and dynamic formation. By employing a local-minima-free navigation function as the objective function to measure the cost-to-goal over the robot trajectory, reference [88] proposes a MPC-based algorithm for convergent navigation of a differential drive mobile robot. To complement, a widely used method dealing with moving obstacle avoidance, velocity obstacle approaches, can also be classified as a kind of predictive approach, which is to generate robot velocities outside of the velocity obstacles, the set of robot velocities that would result in a collision with a given obstacle that moves at a given velocity at some future time. It should be noted that velocity obstacle approaches are among the state-of-art collision avoidance methods at this stage [89]. Some attempts along this idea to solve collision avoidance problems are [90–93].

Main approaches for collision avoidance have been briefly introduced and categorized above. Nevertheless, there are still some other approaches, such as the vector field histogram [94], optimal avoidance [95] and so on, that do not fit into the above three categories. Due to space limitation, these results will not be illustrated detailedly in this thesis.

2.3 Connectivity and Visibility Maintenance

Connectivity of the communication network is a critical requirement for multi-robot systems to share information and reach consensus. Existing approaches proposed to preserve connectivity for multi-robot systems can be divided into two categories, *local approaches* and *global approaches*.

1. *Local approaches.* Local connectivity maintenance is to design a controller such that if a communication link exists at the initial time instant, it will

exist for all the time. Examples of local connectivity maintenance can be found in [40, 67–69, 96–98]. Reference [96] addresses the connectedness issue in multi-agent coordination by adding appropriate weights to the edges in the graph, which are along the gradient of a potential function. In [97], a distributed control law that guarantees connectivity maintenance in a network of multiple mobile agents is proposed using decentralized navigation function. Connectivity maintenance problem has also been studied in applications of consensus [67], rendezvous [68], formation stabilization [40, 69] and formation tracking [98]. In the study of [98], a multi-objective problem for multi-vehicle formations is addressed which takes trajectory tracking, inter-vehicle collision avoidance and connectivity maintenance into account simultaneously in an obstacle-free environment. However, it is a centralized method and cannot be scaled to formations with a large number of robots. Besides, it uses an unbounded potential function to deal with collision avoidance problem which may result in an infinitely large control input.

2. *Global approaches.* Obviously, preserving each communication link is quite conservative, which seriously limits the capability and flexibility of a multi-robot system. As a matter of fact, in order to ensure the information exchange among all robots, global connectivity of the communication graph is enough. In particular, some links might be broken while some others could be established as long as the overall connectivity of the graph is maintained. Reference [99] proposes a decentralized estimation procedure that allows each agent to track the algebraic connectivity of a time-varying graph, based on which a decentralized gradient controller for each agent is designed to maintain global connectivity during motion. Reference [100] presents a completely decentralized control strategy for global connectivity maintenance of the communication graph with mathematical analysis. The proposed strategy is actually a gradient-based control law which exploits decentralized estimation of the algebraic connectivity. Reference [101] introduces a control algorithm to

ensure connectivity maintenance for multi-robot systems in the presence of a bounded additional control term which drives the robots along the negative gradient of an appropriately defined function of the algebraic connectivity. Reference [102] proposes a continuous feedback control method to preserve network connectivity based on maximization of the algebraic connectivity using only local information. The proposed control law allows the connectivity to decrease and can be used in conjunction with artificial potential-based formation controllers.

Similar to connectivity maintenance, visibility maintenance is another attractive research topic in recent years. Unlike connectivity maintenance that preserves connectivity to guarantee reliable wireless communications, visibility maintenance is under the assumption that a stereo camera is mounted each robot to detect its neighbors nearby. In general, visibility maintenance is often required in leader-follower robot formations, due to the fact that stereo cameras are generally subject to limited depth range and FOV, resulting in the maximum and minimum distance requirements between the follower and the leader as well as the bearing angle constraints of the leader with respect to the follower. Violation of such constraints will lead to loss of detection. Moreover, in unknown obstacle environments, maintaining visibility is of much greater difficulty since occlusion of LOS visibility by obstacles is inevitable. Besides, camera detection may temporarily malfunction in operation, attributed to change of lighting condition, motion blurring, etc. Even though it is considered critical, few researchers have paid attention to this issue. Sensory limitations are taken into account in [103], however, they are merely used to determine the neighbors of a vehicle. In [38], time-varying *tan*-type barrier Lyapunov functions are introduced to characterize the LOS range and angle constraints. However, the LOS range and angle are defined with respect to the global coordinate frame and cannot be used to describe stereo camera limitations. Reference [104] proposes a generalized *tan*-type barrier Lyapunov function to formulate the detection range and FOV limitations of the sensor.

However, the value of the barrier function employed approaches infinity when the constraints are close to being violated, resulting in unbounded control inputs. In addition, there are also a few literature on LOS/visibility maintenance for leader-follower formations like [73, 105–107]. However, they are under the assumption that the visual detection is reliable. Besides, they either do not consider the presence of obstacles or assume that the environment information is known *a priori*.

Chapter 3

Virtual-Structure Based Formation Tracking Control of Multi-Robot Systems in Obstacle Environments

This chapter addresses the virtual-structure formation tracking control problem for a group of nonholonomic mobile robots while avoiding collisions in unknown obstacle environments and preserving connectivity amongst robots. To solve this multi-objective control problem, a multi-region control scheme is proposed by prioritizing different control objectives in different regions. The proposed control scheme defines three regions, safe region, dangerous region and transition region. In safe region where formation tracking holds the highest priority, the proposed control scheme can achieve globally asymptotic convergence to a narrow boundary layer around the individual reference trajectory, while in dangerous region where priority is given to avoidance control, collision and connectivity constraints are formulated by a new bounded potential function, based on which a bounded controller is derived. In transition region, a series of transition functions are introduced to smooth the switching

between trajectory tracking and avoidance control. Also, the boundedness of the proposed controller is analyzed. Finally, simulation and real robot experiment have been performed to validate the efficacy of the proposed method.

The contributions of this chapter are twofolds. On the one hand, this chapter considers a challenging but practical constraint that robots work in an unknown obstacle environment, while most of the existing approaches assume that map information of the environment is available. On the other hand, a series of transition regions and the corresponding transition functions are introduced to smooth the switching between formation trajectory and avoidance control.

3.1 Problem Formulation

This chapter considers the formation tracking control problem with guaranteed collision avoidance and connectivity maintenance in the presence of obstacles for n differential-drive mobile robots. It is assumed that each robot is equipped with a laser range finder and wireless communication devices. Besides, communication links among robots follow a homogeneous protocol model, in which two robots can communicate if they are within a specified maximum communication range and cannot communicate if they are outside of that range. Therefore, considering the limited sensing and communication capability, each robot can detect the obstacles within a distance R_s and two robots can communicate with each other if they are within a distance R . For simplicity and without loss of generality, assume that all robots have equal communication and actuation capabilities.

In the sequel, robot formation and communication graph will be modeled. Then the constraints of obstacle collision avoidance, inter-robot collision avoidance and connectivity will be discussed.

Figure 3.1: Diagram of multi-robot formation tracking

3.1.1 Modeling of Robot Formation

Consider n differential-drive robots in a workspace $\mathbb{W} \in \mathbb{R}^2$. The kinematic model of the i th robot is given as follows.

$$\begin{aligned}\dot{x}_i &= v_i \cos \theta_i \\ \dot{y}_i &= v_i \sin \theta_i, \\ \dot{\theta}_i &= \omega_i\end{aligned}\tag{3.1}$$

where $i \in \{1, 2, \dots, n\}$, $\mathbf{p}_i = [x_i, y_i]^T$ is the position of the robot, θ_i is the heading angle, v_i and ω_i are the linear velocity and angular velocity, respectively. In addition, to facilitate control design, the following definition is introduced

$$\dot{\mathbf{p}}_i = \mathbf{u}_i.\tag{3.2}$$

Let $\mathbf{p}_i^d \in \mathbb{R}^2$ be the individual reference trajectory for robot i . The following part illustrates how to derive the individual reference trajectory $\mathbf{p}_i^d$ for robot i , given the

formation reference trajectory, $\mathbf{x}^d = [x^d, y^d, \theta^d]^T$, which is also the trajectory of the virtual leader of the formation. If robot i has the information of $\mathbf{x}^d$ and the predesignated formation configuration, it can compute $\mathbf{p}_i^d$ via coordinate transformation. In the following, the procedure of how to get $\mathbf{p}_i^d$ from $\mathbf{x}^d$ is presented.

Definition 3.1. (*Coordinate Systems*). As is shown in fig. 3.1, $\{g\} = \{\vec{x}_g, \vec{y}_g\}$ is the global coordinate system, which is attached on the ground and $\{l\} = \{\vec{x}_l, \vec{y}_l\}$ is called local coordinate system, which is attached on the virtual leader of the formation and moves as time evolves. The direction of $\vec{x}_l$ is coincident with the tangent unit vector of the reference trajectory $\mathbf{x}^d$.

As the formation configuration is given, robot i knows its local reference coordinate with respect to $\{l\}$, $\mathbf{p}_{il}$. Consequently, $\mathbf{p}_i^d$ can be obtained from $\mathbf{x}^d$ using the following formula.

$$\bar{\mathbf{p}}_i^d = T \bar{\mathbf{p}}_{il},$$

where $\bar{\mathbf{p}}_i^d$ and $\bar{\mathbf{p}}_{il}$ are the corresponding homogeneous coordinates, and have the following expressions, $\bar{\mathbf{p}}_i^d = [\mathbf{p}_i^{dT}, 1]^T$ and $\bar{\mathbf{p}}_{il} = [\mathbf{p}_{il}^T, 1]^T$. Besides,

$$T = \begin{bmatrix} R(\theta^d) \mathbf{p}_l \\ \mathbf{0} & 1 \end{bmatrix},$$

where

$$R(\theta^d) = \begin{bmatrix} \cos \theta^d & -\sin \theta^d \\ \sin \theta^d & \cos \theta^d \end{bmatrix}$$

and $\mathbf{p}_l = [x^d, y^d]^T$.

In this way, given reference trajectory $\mathbf{x}^d$ of the formation, robot i can compute its individual reference trajectory $\mathbf{p}_i^d$ using the algorithm stated above.

Remark 3.1. In this chapter, it is assumed that each robot has the information of the reference trajectory $\mathbf{x}^d$. However, in practice, $\mathbf{x}^d$ maybe not available to all the robots, thus a consensus estimator is needed. For more details about the consensus

estimator, authors can refer to [10, 11, 108–110]. Then the estimation of $\mathbf{x}^d$ can be utilized to replace $\mathbf{x}^d$ in control design. In this chapter, for simplicity, it is assumed that each robot has the knowledge of $\mathbf{x}^d$.

3.1.2 Modeling of Communication Graph

Consider the communication graph of a n -robot system, which can be modeled by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, n\}$ and $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} | d_{ij} \leq R\}$ represent, respectively, the robot set and the link set, where $d_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|$. The set of neighbors of robot i is defined as $\mathcal{N}(i) = \{j | (i, j) \in \mathcal{E}\}$. In $\mathcal{G}$, link (i, j) implies that the distance between robot i and robot j is smaller than R and can communicate with each other.

3.1.3 Obstacle Avoidance

To formulate obstacle avoidance, a model is designed to describe the obstacle encountering process. Inspired by the virtual obstacle concept utilized in [22, 40], a nearest-distance based method is employed in this chapter to characterize the current encountered obstacle, that is, the information of the obstacle point with the nearest distance to the robot is used to construct the virtual obstacle, as shown in fig. 3.2. In fig. 3.2, obstacle ς is represented by an arbitrary shape with a bold chain line. The virtual obstacle $v(\varsigma)$ is represented by a circle with a bold solid line centered at $\mathbf{p}_o$, the point with the nearest distance from the robot i to the boundary of ς , where radius r_o is the safe distance between robot and obstacle. The FOV of robot i is represented by a sector $A\mathbf{p}_iB$ using a bold dotted line with radius R_s . Intuitively, $R_s > r_o$. When obstacle ς is located outside the FOV of robot i , it is assumed that a virtual obstacle $v(\varsigma)$ is located on the boundary of the FOV of robot i . To facilitate controller design, the obstacle collision region, $\Omega_i^O \subseteq \mathbb{R}^2$, is defined for robot i and virtual obstacle $v(\varsigma)$ as $\Omega_i^O \triangleq \{\mathbf{p}_i | r_o \leq d_{io} \leq r_o + \delta_o\}$, where δ_o is a buffer

Figure 3.2: Diagram for obstacle avoidance

distance and $d_{io} = \|\mathbf{p}_i - \mathbf{p}_o\|$, such that any robot i inside this region is considered as a potential collision with obstacle $v(\zeta)$. In addition, a critical region, $\Psi_i^O \subseteq \mathbb{R}^2$, and a transition region, $\Lambda_i^O \subseteq \mathbb{R}^2$, are also defined as $\Psi_i^O \triangleq \{\mathbf{p}_i | r_o \leq d_{io} \leq r_o + \xi\}$ and $\Lambda_i^O \triangleq \{\mathbf{p}_i | r_o + \delta_o - \mu_o \leq d_{io} \leq r_o + \delta_o\}$, where ξ is a small positive constant and μ_o is a transition distance.

Based on the definitions given above, a potential function $G_o(\mathbf{p}_i)$ and a transition function $W_o(\mathbf{p}_i)$ for obstacle avoidance are designed as follows.

$$G_o(\mathbf{p}_i) = \begin{cases} 0 & d_{io} < r_o \\ -\frac{1}{\delta_o^2}(d_{io} - r_o)^2 & r_o \leq d_{io} < r_o + \delta_o \\ +\frac{2}{\delta_o}(d_{io} - r_o) & \\ 1 & d_{io} \geq r_o + \delta_o \end{cases} \quad (3.3)$$

$$W_o(\mathbf{p}_i) = \begin{cases} 0 & d_{io} < r_o + \delta_o - \mu_o \\ -\frac{1}{\mu_o^2}(d_{io} - r_o - \delta_o + \mu_o)^2 & r_o + \delta_o - \mu_o \leq d_{io} < r_o + \delta_o \\ +\frac{2}{\mu_o}(d_{io} - r_o - \delta_o + \mu_o) & \\ 1 & d_{io} \geq r_o + \delta_o \end{cases} \quad (3.4)$$

Figure 3.3: Diagram for collision avoidance and connectivity maintenance

3.1.4 Inter-Robot Collision Avoidance

Inter-robot collision avoidance can be described in a way similar to obstacle avoidance. Consequently, the inter-robot collision region between robot i and k , which is depicted in fig. 3.3, is defined as $\Omega_{ik}^N \triangleq \{\mathbf{p}_i | r_n \leq d_{ik} \leq r_n + \delta_n\}$, where r_n is the safe distance amongst robots, δ_n is a buffer distance and $d_{ik} = \|\mathbf{p}_i - \mathbf{p}_k\|$, $k \in \mathcal{N}(i)$. In addition, a critical region and a transition region are also defined as $\Psi_{ik}^N \triangleq \{\mathbf{p}_i | r_n \leq d_{ik} \leq r_n + \xi\}$ and $\Lambda_{ik}^N \triangleq \{\mathbf{p}_i | r_n + \delta_n - \mu_n \leq d_{ik} \leq r_n + \delta_n\}$, where μ_n is a transition distance.

Thereupon, a potential function $G_n(\mathbf{p}_i)$ and a transition function $W_n(\mathbf{p}_i)$ for inter-robot collision avoidance can be expressed as

$$G_n(\mathbf{p}_i) = \prod_{k \in \mathcal{N}(i)} n_{ik}, \quad (3.5)$$

where

$$n_{ik} = \begin{cases} 0 & d_{ik} < r_n \\ -\frac{1}{\delta_n^2}(d_{ik} - r_n)^2 & r_n \leq d_{ik} \leq r_n + \delta_n \\ +\frac{2}{\delta_n}(d_{ik} - r_n) & \\ 1 & d_{ik} > r_n + \delta_n \end{cases} .$$

$$W_n(\mathbf{p}_i) = \prod_{k \in \mathcal{N}(i)} w_{ik}^n, \quad (3.6)$$

where

$$w_{ik}^n = \begin{cases} 0 & d_{ik} < r_n + \delta_n - \mu_n \\ -\frac{1}{\mu_n^2}(d_{ik} - r_n - \delta_n + \mu_n)^2 & r_n + \delta_n - \mu_n \leq d_{ik} \leq r_n + \delta_n \\ +\frac{2}{\mu_n}(d_{ik} - r_n - \delta_n + \mu_n) & \\ 1 & d_{ik} > r_n + \delta_n \end{cases} .$$

It can be observed that the functions above are similar to those of obstacle avoidance.

3.1.5 Connectivity Maintenance

Another control objective is to preserve the connectivity of the communication graph for all the time. Define $\mathcal{N}^f(i) \subseteq \mathcal{N}(i)$, which determines the desired neighbor set for robot i which can guarantee the connectivity of the communication graph. Similarly, the escape region between robot i and robot j is defined as $\Omega_{ij}^C \triangleq \{\mathbf{p}_i | R - \delta_c \leq d_{ij} \leq R\}$, where δ_c is a buffer distance and $d_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|$, $j \in \mathcal{N}^f(i)$. Links with the length belonging to the escape region have a risk of being broken. In addition, a critical region and a transition region between robot i and robot j are defined as $\Psi_{ij}^C \triangleq \{\mathbf{p}_i | R - \xi \leq d_{ij} \leq R\}$ and $\Lambda_{ij}^C \triangleq \{\mathbf{p}_i | R - \delta_c \leq d_{ij} \leq R - \delta_c + \mu_c\}$, where μ_c is a transition distance. The above defined regions are also depicted in fig. 3.3.

Thus, a potential function and a transition function of connectivity maintenance

can be designed as follows.

$$G_c(\mathbf{p}_i) = \prod_{j \in \mathcal{N}^f(i)} c_{ij}, \quad (3.7)$$

where

$$c_{ij} = \begin{cases} 1 & d_{ij} < R - \delta_c \\ -\frac{1}{\delta_c^2}(d_{ij} + 2\delta_c - R)^2 & R - \delta_c \leq d_{ij} \leq R \\ +\frac{2}{\delta_c}(d_{ij} + 2\delta_c - R) & \\ 0 & d_{ij} > R \end{cases}$$

$$W_c(\mathbf{p}_i) = \prod_{j \in \mathcal{N}^f(i)} w_{ij}^c, \quad (3.8)$$

where

$$w_{ij}^c = \begin{cases} 1 & d_{ij} < R - \delta_c \\ -\frac{1}{\mu_c^2}(d_{ij} + \delta_c + \mu_c - R)^2 & R - \delta_c \leq d_{ij} \leq R - \delta_c + \mu_c \\ +\frac{2}{\mu_c}(d_{ij} + \delta_c + \mu_c - R) & \\ 0 & d_{ij} > R - \delta_c + \mu_c \end{cases}$$

3.1.6 Control Objectives

To facilitate further discussions, the following definitions of robot i are given. First, the concept of feasible workspace is defined, which is a subset of $\mathbf{p}_i$ satisfying constraints including obstacle avoidance, inter-robot collision avoidance and connectivity preservation.

Definition 3.2. (*Feasible Workspace*). The feasible workspace of robot i , $\mathbb{W}_{fi}$, can be defined as

$$\mathbb{W}_{fi} = \{\mathbf{p}_i | G(\mathbf{p}_i) = G_o(\mathbf{p}_i)G_n(\mathbf{p}_i)G_c(\mathbf{p}_i) > 0\}. \quad (3.9)$$

From the above constraints and definitions, it can be concluded that robot i can avoid collisions with obstacles and neighboring robots with guaranteed connectivity when located in its feasible workspace $\mathbb{W}_{fi}$.

Before proceeding further, some definitions are given as follows. $\Theta_i^O \triangleq \{\mathbf{p}_i | d_{io} \geq r_o + \delta_o\}$, $\Theta_{ik}^N \triangleq \{\mathbf{p}_i | d_{ik} \geq r_n + \delta_n\}$ and $\Theta_{ij}^C \triangleq \{\mathbf{p}_i | d_{ij} \leq R - \delta_c\}$, where $k \in \mathcal{N}(i)$, $j \in \mathcal{N}^f(i)$, Θ_i^O is the obstacle collision-free region for robot i , Θ_{ik}^N is the inter-robot collision-free region between robot i and robot k , and Θ_{ij}^C is the connectivity region between robot i and robot j . In this way, inter-robot collision-free region and connectivity region for robot i can be given as $\Theta_i^N = \bigcup_{k \in \mathcal{N}(i)} \Theta_{ik}^N$ and $\Theta_i^C = \bigcup_{j \in \mathcal{N}^f(i)} \Theta_{ij}^C$, respectively. In addition, inter-robot collision region and escape region for robot i can be given as $\Omega_i^N = \bigcup_{k \in \mathcal{N}(i)} \Omega_{ik}^N$ and $\Omega_i^C = \bigcup_{j \in \mathcal{N}^f(i)} \Omega_{ij}^C$, respectively. The critical regions introduced by inter-robot collision avoidance and connectivity preservation for robot i can also be defined as $\Psi_i^N = \bigcup_{k \in \mathcal{N}(i)} \Psi_{ik}^N$ and $\Psi_i^C = \bigcup_{j \in \mathcal{N}^f(i)} \Psi_{ij}^C$, respectively. The transition regions introduced by inter-robot collision avoidance and connectivity maintenance for robot i are defined as $\Lambda_i^N = \bigcup_{k \in \mathcal{N}(i)} \Lambda_{ik}^N$ and $\Lambda_i^C = \bigcup_{j \in \mathcal{N}^f(i)} \Lambda_{ij}^C$, respectively. Thereupon, critical region, safe region and dangerous region for robot i can be defined as follows.

Definition 3.3. (*Critical Region, Safe Region, Transition Region and Dangerous Region*). Critical region for robot i can be defined as $\Psi_i \triangleq \{\mathbf{p}_i | \mathbf{p}_i \in \Psi_i^O \cup \Psi_i^N \cup \Psi_i^C\}$. Safe region for robot i can be defined as $\Gamma_i \triangleq \{\mathbf{p}_i | \mathbf{p}_i \in \Theta_i^O \cap \Theta_i^N \cap \Theta_i^C\}$. Transition region for robot i can be defined as $\Lambda_i \triangleq \{\mathbf{p}_i | \mathbf{p}_i \in \Lambda_i^O \cup \Lambda_i^N \cup \Lambda_i^C\}$. Dangerous region for robot i can be defined as $\Xi_i \triangleq \{\mathbf{p}_i | \mathbf{p}_i \in \Omega_i^O \cup \Omega_i^N \cup \Omega_i^C, \mathbf{p}_i \notin \Lambda_i\}$.

Based on the definitions given above, the control problem can be formulated as follows.

Problem 3.1. (*Multi-Objective Control*). Consider reference trajectory described by the vector $\mathbf{x}^d = [x^d, y^d, \theta^d]^T$, where $[x^d, y^d]^T \in \mathbb{R}^2$ and $\theta^d \in \mathbb{R}$ represent the reference position and heading angle, respectively. Set the desired position of robot i in the formation configuration as $\mathbf{p}_{il}$, $i \in \{1, 2, \dots, n\}$. Multi-objective control problem can be divided into two sub-problems as follows.

1. (*Trajectory Tracking*). Design a bounded control law such that the tracking

error of robot i , $\tilde{\mathbf{p}}_i = \mathbf{p}_i^d - \mathbf{p}_i$, is bounded in safe region Γ_i .

2. (Avoidance Control). Assume that the initial position of robot i is located in its feasible workspace, i.e., $\mathbf{p}_i(0) \in F_i, \forall i \in \{1, 2, \dots, n\}$. The proposed control law can guarantee that $\mathbf{p}_i \in F_i, \forall i \in \{1, 2, \dots, n\}$, for all the time.

Remark 3.2. Avoidance control defined above refers to avoidance of constraints violation. In detail, it implies avoidance of obstacles, avoidance of collisions amongst neighboring robots and avoidance of losing connectivity.

3.2 Main Results

In this section, a multi-region control scheme is presented to address the multi-objective control problem. Inspired by the idea of the work in [68, 69], a new bounded potential function $\varphi_i \in [0, 1]$ is proposed for avoidance control, which can be expressed as

$$\varphi_i = \frac{\gamma}{(\gamma^\alpha + G(\mathbf{p}_i))^{\frac{1}{\alpha}}}, \quad (3.10)$$

where γ and α are positive parameters. As the proposed potential function eq. (3.10) is bounded, the gradient of φ_i with respect to $\mathbf{p}_i$, $\frac{\partial \varphi_i}{\partial \mathbf{p}_i}$, is bounded as well.

To deal with the multi-objective control problem, the proposed control law consists of two parts, trajectory tracking and avoidance control. The trajectory tracking part is to drive robot i to track its individual reference trajectory, while the avoidance control part is to avoid collisions and loss of connectivity. Mathematically, the control law can be given as

$$\begin{aligned} v_i &= [\cos \theta_i, \sin \theta_i] \mathbf{u}_i, \\ \omega_i &= [-\sin \theta_i, \cos \theta_i] \mathbf{u}_i. \end{aligned} \quad (3.11)$$

where $\mathbf{u}_i = \mathbf{u}_i^t + \mathbf{u}_i^a$, $\mathbf{u}_i^t$ and $\mathbf{u}_i^a$ are the tracking control part and the avoidance

control part, respectively. In detail,

$$\mathbf{u}_i^t = \hat{\mathbf{p}}_i^d + g(\tilde{\mathbf{p}}_i), \quad (3.12)$$

$$\mathbf{u}_i^a = -(1 - W(\mathbf{p}_i))\mathbf{u}_i^t - k_i \frac{\partial \varphi_i}{\partial \mathbf{p}_i}, \quad (3.13)$$

where $\hat{\mathbf{p}}_i^d$ is the estimate of $\dot{\mathbf{p}}_i^d$ as $\hat{\mathbf{p}}_i^d(t) = \frac{\mathbf{p}_i^d(t+T_s) - \mathbf{p}_i^d(t)}{T_s}$, $W(\mathbf{p}_i) = W_o(\mathbf{p}_i)W_n(\mathbf{p}_i)W_c(\mathbf{p}_i)$,
 $g(\tilde{\mathbf{p}}_i) = \begin{cases} \kappa \tilde{\mathbf{p}}_i & \|\tilde{\mathbf{p}}_i\| \leq \frac{\zeta}{\kappa} \\ \zeta \frac{\tilde{\mathbf{p}}_i}{\|\tilde{\mathbf{p}}_i\|} & \text{otherwise} \end{cases}$, $\kappa, \zeta > 0$ are designed parameters.

The following theorem proves that the control laws eqs. (3.11) to (3.13) can guarantee bounded tracking of the individual reference trajectory for robot i in safe region, while collision avoidance is guaranteed and connectivity is maintained if the initial position for each robot is located in its own feasible workspace.

Theorem 3.1. *Consider robot i , $\forall i \in \{1, 2, \dots, n\}$, described as eq. (3.1) and its individual reference trajectory $\mathbf{p}_i^d$. Assume that the initial position of robot i is located in its own feasible workspace, i.e., $\mathbf{p}_i(0) \in \mathbb{W}_{fi}$. When the multi-objective control law eqs. (3.11) to (3.13) is applied, robot i can track its individual reference trajectory with bounded tracking error inside Γ_i , while collision avoidance can be guaranteed and connectivity can be maintained all the time, i.e., $\mathbf{p}_i(t) \in \mathbb{W}_{fi}$, $\forall t > 0$. In addition, tracking error can be reduced by increasing gain parameter κ .*

Proof. First, consider trajectory tracking part. Give the following Lyapunov-like function

$$V = \frac{1}{2} \tilde{\mathbf{p}}_i^T \tilde{\mathbf{p}}_i. \quad (3.14)$$

By taking the derivative of V with respect to time, it can be obtained that

$$\dot{V} = \frac{1}{2} \tilde{\mathbf{p}}_i^T (\dot{\mathbf{p}}_i^d - \mathbf{u}_i). \quad (3.15)$$

In safe region Γ_i , according to the definitions of constraint functions eqs. (3.3)

to (3.8) and eqs. (3.10) to (3.13), $G(\mathbf{p}_i) = 1$ and $\varphi_i = \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}$. Thus eq. (3.15) can be simplified as

$$\begin{aligned}\dot{V} &= \frac{1}{2} \tilde{\mathbf{p}}_i^T (\dot{\mathbf{p}}_i^d - \mathbf{u}_i^t) \\ &= \frac{1}{2} \tilde{\mathbf{p}}_i^T (\dot{\mathbf{p}}_i^d - \hat{\dot{\mathbf{p}}}_i^d - g(\tilde{\mathbf{p}}_i)), \\ &= \frac{1}{2} \tilde{\mathbf{p}}_i^T (\tilde{\dot{\mathbf{p}}}_i^d - g(\tilde{\mathbf{p}}_i))\end{aligned}$$

where $\tilde{\dot{\mathbf{p}}}_i^d = \dot{\mathbf{p}}_i^d - \hat{\dot{\mathbf{p}}}_i^d$ is the estimation error. As $g(\tilde{\mathbf{p}}_i)$ is a piecewise continuous function, the stability of the system can be discussed in a piecewise manner.

When $\|\tilde{\mathbf{p}}_i\| \leq \frac{\zeta}{\kappa}$,

$$\begin{aligned}\dot{V} &= \frac{1}{2} \tilde{\mathbf{p}}_i^T (\tilde{\dot{\mathbf{p}}}_i^d - \kappa \tilde{\mathbf{p}}_i) \\ &\leq -\frac{\kappa}{2} \|\tilde{\mathbf{p}}_i\|^2 + \frac{1}{2} \|\tilde{\mathbf{p}}_i\| \|\tilde{\dot{\mathbf{p}}}_i^d\| \\ &= -\frac{1}{2} \|\tilde{\mathbf{p}}_i\| (\kappa \|\tilde{\mathbf{p}}_i\| - \|\tilde{\dot{\mathbf{p}}}_i^d\|)\end{aligned}$$

Hence, $\dot{V} < 0$ whenever $\|\tilde{\mathbf{p}}_i\| > \frac{\|\tilde{\dot{\mathbf{p}}}_i^d\|}{\kappa}$. Therefore, the parameter ζ should be defined such that $\zeta > \|\tilde{\dot{\mathbf{p}}}_i^d\|$.

When $\|\tilde{\mathbf{p}}_i\| > \frac{\zeta}{\kappa}$,

$$\begin{aligned}\dot{V} &= \frac{1}{2} \tilde{\mathbf{p}}_i^T (\tilde{\dot{\mathbf{p}}}_i^d - \zeta \frac{\tilde{\mathbf{p}}_i}{\|\tilde{\mathbf{p}}_i\|}) \\ &\leq -\frac{\zeta}{2} \|\tilde{\mathbf{p}}_i\| + \frac{1}{2} \|\tilde{\mathbf{p}}_i\| \|\tilde{\dot{\mathbf{p}}}_i^d\| \\ &= -\frac{1}{2} \|\tilde{\mathbf{p}}_i\| (\zeta - \|\tilde{\dot{\mathbf{p}}}_i^d\|)\end{aligned}$$

As mentioned above, $\zeta > \|\tilde{\dot{\mathbf{p}}}_i^d\|$, $\dot{V} < 0$ holds. It is obvious that the Lyapunov-like function eq. (3.14) is a piecewise continuous function with respect to $\tilde{\mathbf{p}}_i$, therefore, inside safe region Γ_i , robot i can track its individual reference trajectory with a bounded error $\frac{\|\tilde{\dot{\mathbf{p}}}_i^d\|}{\kappa}$, which can be reduced by increasing gain parameter κ .

Next, avoidance control is discussed. Inside the dangerous region Ξ_i , according to eqs. (3.3) to (3.8) and eqs. (3.10) to (3.13), the control law can be rewritten as

$$\mathbf{u}_i = -k_i \frac{\partial \varphi_i}{\partial \mathbf{p}_i}.$$

According to eq. (3.10), the partial derivative of φ_i with respect to $\mathbf{p}_i$ can be expressed as

$$\frac{\partial \varphi_i}{\partial \mathbf{p}_i} = -\frac{\gamma \nabla G(\mathbf{p}_i)}{\alpha(\gamma^\alpha + G(\mathbf{p}_i))^{\frac{1}{\alpha}+1}},$$

where $\nabla G(\mathbf{p}_i) = G(\mathbf{p}_i)(\frac{\nabla G_o(\mathbf{p}_i)}{G_o(\mathbf{p}_i)} + \sum_{k \in \mathcal{N}(i)} \frac{\nabla n_{ik}}{n_{ik}} + \sum_{j \in \mathcal{N}^f(i)} \frac{\nabla c_{ij}}{c_{ij}})$. Thus the control law can be simplified as

$$\mathbf{u}_i = \beta_i G(\mathbf{p}_i) \left(\frac{\nabla G_o(\mathbf{p}_i)}{G_o(\mathbf{p}_i)} + \sum_{k \in \mathcal{N}(i)} \frac{\nabla n_{ik}}{n_{ik}} + \sum_{j \in \mathcal{N}^f(i)} \frac{\nabla c_{ij}}{c_{ij}} \right), \quad (3.16)$$

where $\beta_i = \frac{k_i \gamma}{\alpha(\gamma^\alpha + G(\mathbf{p}_i))^{\frac{1}{\alpha}+1}}$. From eq. (3.16), it can be found that the control law contains three parts, obstacle avoidance, inter-robot collision avoidance and connectivity maintenance, respectively.

Considering the critical region defined in definition 3.3, it consists of sub-regions of four types relating to the following four cases.

Case 1: Robot i almost hits obstacle $v(\varsigma)$. Mathematically, $\mathbf{p}_i \in \Psi_i^O$.

Case 2: Robot i almost hits robot k , $k \in \mathcal{N}(i)$. Mathematically, $\mathbf{p}_i \in \Psi_{ik}^N$.

Case 3: Robot i almost locates outside the communication range of robot j , $j \in \mathcal{N}^f(i)$. Mathematically, $\mathbf{p}_i \in \Psi_{ij}^C$.

Case 4: Robot i almost locates outside the communication range of more than one robot or hits more than one objects including obstacles and neighboring robots. Mathematically, robot i locates inside a region which is an interaction region formed by at least two regions among the following regions, Ψ_i^O , Ψ_{ik}^N and Ψ_{ij}^C , $\forall j \in \mathcal{N}^f(i)$ and $k \in \mathcal{N}(i)$. In this work, for simplicity and without loss of generality, consider only the simple case where robot i almost locates outside the communication range of robot j and hits obstacle simultaneously. The result in this case is easy to be extended to a more general case.

Next, it will be shown that robot i will stay inside its feasible workspace $\mathbb{W}_{fi}$ using

the proposed controller.

In case 1, $\lim_{\xi \rightarrow 0} G_o(\mathbf{p}_i) = 0$, $n_{ik} > 0$ for $\forall k \in \mathcal{N}(i)$ and $c_{ij} > 0$ for $\forall j \in \mathcal{N}^f(i)$. Thus, $\mathbf{u}_i \cong \beta_i G_n(\mathbf{p}_i) G_c(\mathbf{p}_i) \nabla G_o(\mathbf{p}_i)$, where $\nabla G_o(\mathbf{p}_i)$ can be given as

$$\nabla G_o(\mathbf{p}_i) = \begin{cases} \mathbf{0} & d_{io} < r_o \text{ or } d_{io} > r_o + \delta_o \\ -\frac{2(d_{io}-\delta_o-r_o)}{\delta_o^2 d_{io}}(\mathbf{p}_i - \mathbf{p}_o) & r_o \leq d_{io} \leq r_o + \delta_o \end{cases}. \quad (3.17)$$

Since β_i , $G_n(\mathbf{p}_i)$ and $G_c(\mathbf{p}_i)$ are all positive, it can be found that the controller is along the direction of $\mathbf{p}_i - \mathbf{p}_o$ when robot i is about to hit an obstacle, which implies that robot i is forced to move away from the obstacle in order to avoid collision. Therefore, robot i will never reach Ψ_i^O .

In case 2, there is only one robot $k \in \mathcal{N}(i)$ such that $\lim_{\xi \rightarrow 0} n_{ik} = 0$ and $n_{il} > 0$ for $\forall l \in \mathcal{N}(i)$, $l \neq k$. In addition, $c_{ij} > 0$ for $\forall j \in \mathcal{N}(i)$ and $G_o(\mathbf{p}_i) > 0$. Thus $\mathbf{u}_i \cong \beta_i \bar{n}_{ik} G_o(\mathbf{p}_i) G_c(\mathbf{p}_i) \nabla n_{ik}$, where $\bar{n}_{ik} = \prod_{l \in \mathcal{N}(i), l \neq k} n_{il}$ and ∇n_{ik} can be computed as

$$\nabla n_{ik} = \begin{cases} \mathbf{0} & d_{ik} < r_n \text{ or } d_{ik} > r_n + \delta_n \\ -\frac{2(d_{ik}-\delta_n-r_n)}{\delta_n^2 d_{ik}}(\mathbf{p}_i - \mathbf{p}_k) & r_n \leq d_{ik} \leq r_n + \delta_n \end{cases}. \quad (3.18)$$

Similarly, it can be derived that the controller is along the direction of $\mathbf{p}_i - \mathbf{p}_k$ when robot k is about to hit robot k , which implies that robot i is forced to move away from robot k in order to avoid collision. Hence, robot i never reach Ψ_{ik}^N .

In case 3, there is only one robot $j \in \mathcal{N}^f(i)$ such that $\lim_{\xi \rightarrow 0} c_{ij} = 0$ and $c_{il} > 0$ for $\forall l \in \mathcal{N}^f(i)$, $l \neq j$. Besides, $n_{ik} > 0$ for $\forall k \in \mathcal{N}(i)$ and $G_o(\mathbf{p}_i) > 0$. Thus $\mathbf{u}_i \cong \beta_i \bar{c}_{ij} G_o(\mathbf{p}_i) G_n(\mathbf{p}_i) \nabla c_{ij}$, where $\bar{c}_{ij} = \prod_{l \in \mathcal{N}^f(i), l \neq j} c_{il}$ and ∇c_{ij} is presented as

$$\nabla c_{ij} = \begin{cases} \mathbf{0} & d_{ij} < R - \delta_c \text{ or } d_{ij} > R \\ -\frac{2(d_{ij}+\delta_c-R)}{\delta_c^2 d_{ij}}(\mathbf{p}_i - \mathbf{p}_j) & R - \delta_c \leq d_{ij} \leq R \end{cases}. \quad (3.19)$$

It can also be found that the controller is along the direction of $\mathbf{p}_j - \mathbf{p}_i$ when robot i is about to lose connection with robot j , which implies that robot i is forced to

move close to robot j in order to maintain connectivity. Thus robot i never reach Ψ_{ij}^C .

In case 4, there is only one robot $j \in \mathcal{N}^f(i)$ such that $\lim_{\xi \rightarrow 0} c_{ij} = 0$ and $\lim_{\xi \rightarrow 0} G_o(\mathbf{p}_i) = 0$. Thus, $G(\mathbf{p}_i) \cong 0$ and $\nabla G(\mathbf{p}_i) \cong \mathbf{0}$. According to eq. (3.16), $\mathbf{p}_i$ is a critical point of φ_i , i.e., $\nabla \varphi_i \cong \mathbf{0}$, and φ_i achieves its maximum value at the critical point, i.e., $\varphi_i \cong 1$. Since φ_i is maximized at $\mathbf{p}_i$, no open set of initial conditions can be attracted to $\mathbf{p}_i$ along its negative gradient. Therefore, case 4 never happens under the proposed controller.

In addition, according to Theorem 2 in [111], it can be concluded that control law eqs. (3.11) to (3.13) designed for eq. (3.1) is convergent with the same attraction region as eq. (3.2).

Based on the discussions above, the proposed control law can guarantee that in safe region Γ_i , robot i can track its individual reference trajectory with bounded error while collisions are avoided and connectivity is maintained for all the time.

This completes the proof. $\square$

Remark 3.3. *In the proposed multi-region control scheme, the control objectives in different regions are different according to their priorities. In safe region, trajectory tracking has the highest priority. From the proposed controller eqs. (3.11) to (3.13), it can be found that only the trajectory tracking part works in safe region. Therefore, robots would track their individual reference trajectories with bounded error. In dangerous region, priority is given to collision avoidance and connectivity maintenance, while trajectory tracking objective is temporarily ignored. In the proposed controller, by introducing an attenuation term $-(1 - W(\mathbf{p}_i))\mathbf{u}_i^t$, the trajectory tracking part is neutralized in dangerous region, thus control effort only focuses on collision avoidance and connectivity maintenance. In transition region, the control law is actually a behavior based controller, in which the control input is a weighted combination of all the control objectives.*

In the following, boundedness of the proposed controller is discussed. When robot i locates in the safe region Γ_i , the controller can be rewritten as follows.

$$\mathbf{u}_i = \hat{\mathbf{p}}_i^d + g(\tilde{\mathbf{p}}_i).$$

Thus, the bound of the control law can be give as

$$\begin{aligned} \|\mathbf{u}_i\| &= \|\hat{\mathbf{p}}_i^d + g(\tilde{\mathbf{p}}_i)\| \\ &\leq \|\hat{\mathbf{p}}_i^d\| + \|g(\tilde{\mathbf{p}}_i)\|. \\ &\leq \|\hat{\mathbf{p}}_i^d\| + \zeta \end{aligned}$$

When robot i locates in the dangerous region Ξ_i , the controller can be given as eq. (3.16). The bound of the controller can be derived as follows. According to theorem 3.1 and definition 3.2, $G(\mathbf{p}_i) > 0$, $\forall t > 0$. Combined with eqs. (3.3) to (3.8), the following inequality can be obtained.

$$\begin{aligned} \|\mathbf{u}_i\| &\leq \frac{k_i}{\alpha\gamma^\alpha} \left\| G(\mathbf{p}_i) \left(\frac{\nabla G_o(\mathbf{p}_i)}{G_o(\mathbf{p}_i)} + \sum_{k \in \mathcal{N}(i)} \frac{\nabla n_{ik}}{n_{ik}} + \sum_{j \in \mathcal{N}^f(i)} \frac{\nabla c_{ij}}{c_{ij}} \right) \right\| \\ &\leq \frac{k_i}{\alpha\gamma^\alpha} (\|\nabla G_o(\mathbf{p}_i)\| + \sum_{k \in \mathcal{N}(i)} \|\nabla n_{ik}\| + \sum_{j \in \mathcal{N}^f(i)} \|\nabla c_{ij}\|) \end{aligned}$$

From eqs. (3.17) to (3.19), it can be obtained that in dangerous region Ξ_i , $\|\nabla G_o(\mathbf{p}_i)\| \leq \frac{2}{\delta_o}$, $\|\nabla n_{ik}\| \leq \frac{2}{\delta_n}$ and $\|\nabla c_{ij}\| \leq \frac{2}{\delta_c}$. Consequently,

$$\begin{aligned} \|\mathbf{u}_i\| &\leq \frac{k_i}{\alpha\gamma^\alpha} \left(\frac{2}{\delta_o} + \sum_{k \in \mathcal{N}(i)} \frac{2}{\delta_n} + \sum_{j \in \mathcal{N}^f(i)} \frac{2}{\delta_c} \right) \\ &\leq \frac{2k_i}{\alpha\gamma^\alpha} \left(\frac{1}{\delta_o} + \frac{|\mathcal{N}(i)|}{\delta_n} + \frac{|\mathcal{N}^f(i)|}{\delta_c} \right). \end{aligned}$$

If the parameters are chosen as $\delta_o = \delta_n = \delta_c = \delta$, the bound of the controller is given as

$$\|\mathbf{u}_i\| \leq \frac{2k_i}{\alpha\delta\gamma^\alpha} (1 + |\mathcal{N}(i)| + |\mathcal{N}^f(i)|).$$

When robot i locates in the transition region Λ_i , the controller is actually a weighted combination of trajectory tracking and avoidance. The bound of the controller can be derived as follows.

$$\begin{aligned}
\|\mathbf{u}_i\| &= \|W(\mathbf{p}_i)\mathbf{u}_i^t - k_i \frac{\partial \varphi_i}{\partial \mathbf{p}_i}\| \\
&\leq \|\mathbf{u}_i^t\| + \|k_i \frac{\partial \varphi_i}{\partial \mathbf{p}_i}\| \\
&\leq \|\hat{\mathbf{p}}_i^d(t)\| + \zeta + \frac{k_i}{\alpha\gamma^\alpha} \left\| G(\mathbf{p}_i) \left(\frac{\nabla G_o(\mathbf{p}_i)}{G_o(\mathbf{p}_i)} + \sum_{k \in \mathcal{N}(i)} \frac{\nabla n_{ik}}{n_{ik}} + \sum_{j \in \mathcal{N}^f(i)} \frac{\nabla c_{ij}}{c_{ij}} \right) \right\| \\
&\leq \|\hat{\mathbf{p}}_i^d(t)\| + \zeta + \frac{k_i}{\alpha\gamma^\alpha} (\|\nabla G_o(\mathbf{p}_i)\| + \sum_{k \in \mathcal{N}(i)} \|\nabla n_{ik}\| + \sum_{j \in \mathcal{N}^f(i)} \|\nabla c_{ij}\|)
\end{aligned}$$

From eqs. (3.17) to (3.19), it can be obtained that in transition region Λ_i , $\|\nabla G_o(\mathbf{p}_i)\| \leq \frac{2\mu_o}{\delta_o^2}$, $\|\nabla n_{ik}\| \leq \frac{2\mu_n}{\delta_n^2}$ and $\|\nabla c_{ij}\| \leq \frac{2\mu_c}{\delta_c^2}$. Therefore,

$$\begin{aligned}
\|\mathbf{u}_i\| &\leq \|\hat{\mathbf{p}}_i^d\| + \zeta + \frac{k_i}{\alpha\gamma^\alpha} \left(\frac{2\mu_o}{\delta_o^2} + \sum_{k \in \mathcal{N}(i)} \frac{2\mu_n}{\delta_n^2} + \sum_{j \in \mathcal{N}^f(i)} \frac{2\mu_c}{\delta_c^2} \right) \\
&\leq \|\hat{\mathbf{p}}_i^d\| + \zeta + \frac{2k_i}{\alpha\gamma^\alpha} \left(\frac{\mu_o}{\delta_o^2} + \frac{|\mathcal{N}(i)|\mu_n}{\delta_n^2} + \frac{|\mathcal{N}^f(i)|\mu_c}{\delta_c^2} \right)
\end{aligned}$$

If the parameters are chosen as $\delta_o = \delta_n = \delta_c = \delta$ and $\mu_o = \mu_n = \mu_c = \mu$, the bound of the controller can be given as

$$\|\mathbf{u}_i\| \leq \|\hat{\mathbf{p}}_i^d\| + \zeta + \frac{2k_i\mu}{\alpha\delta^2\gamma^\alpha} (1 + |\mathcal{N}(i)| + |\mathcal{N}^f(i)|).$$

Thus, the boundedness of the controller has been proven and the bound has been derived. Therefore, it can be concluded that by properly tuning parameters, the proposed control scheme can satisfy control input constraints.

Figure 3.4: Formation configuration in the simulation

3.3 Simulation and Experimental Results

3.3.1 Simulation Results

Consider five robots and four obstacles with circular or rectangular shapes in the simulation. The desired configuration of the robots forms an arrow shape as shown in fig. 3.4. The parameters in the simulation are presented in table 3.1. Assume that the virtual leader of the formation also satisfies the kinematic model eq. (3.1) with its linear velocity and angular velocity assigned as $v_l = 0.5 + 0.1 \sin(0.1t)$ and $\omega_l = 0.01 \cos(0.1t)$ respectively.

The simulation results using the proposed method are given in fig. 3.5. Figure 3.5(a) depicts the map of the environment and the paths of the five robots using the proposed controller. The black disks and boxes represent the obstacles. The dashed curves are the individual reference paths for the robots. The initial positions of the robots are given in table 3.1. The solid bold curves are the real paths of the robots. From this figure, it can be observed that all the robots avoided the obstacles successfully. Figure 3.5(b) shows the trajectory tracking errors of the robots. For comparison, a new simulation has also been conducted. The only difference is that

Table 3.1: Simulation parameters for virtual-structure formation tracking

Parameter	Value
Total time duration	60s
Sampling time interval	33.3ms
$\mathbf{p}_{1l}$	$[2, 0]\text{m}$
$\mathbf{p}_{2l}$	$[1, 1]\text{m}$
$\mathbf{p}_{3l}$	$[1, -1]\text{m}$
$\mathbf{p}_{4l}$	$[0, 2]\text{m}$
$\mathbf{p}_{5l}$	$[0, -2]\text{m}$
$\mathbf{p}_1(0)$	$[0, 0]\text{m}$
$\mathbf{p}_2(0)$	$[-1, 0]\text{m}$
$\mathbf{p}_3(0)$	$[1, 0]\text{m}$
$\mathbf{p}_4(0)$	$[-2, 0]\text{m}$
$\mathbf{p}_5(0)$	$[2, 0]\text{m}$
R_s	3m
R	3m
r_o	0.5m
r_n	0.5m
δ_o	0.3m
δ_n	0.3m
δ_c	0.3m
μ_o	0.2m
μ_n	0.2m
μ_c	0.2m
γ	1
α	1
k_i	0.5
κ	5
ζ	0.5

all the obstacles were removed. The trajectory tracking errors of the robots in the comparative simulation are shown in fig. 3.6(b). Combining with fig. 3.5(a), it can be found that robots tracked the corresponding individual reference trajectories with bounded error in safe region. The increases of the tracking errors were caused by obstacle avoidance reactions. Figure 3.5(c) describes the distances from each robot to the nearest obstacles, which indicates that robot did not collide with obstacles. Figure 3.5(d) shows the distances among all the robots, from which it can be found that all the distances are larger than the predesignated safe distance for collision avoidance, which implies that inter-robot collisions did not occur. Figure 3.5(e) illustrates the connectivity maintenance performance, where the second smallest eigenvalue of the Laplacian matrix in the communication graph is utilized as the connectivity indicator. From fig. 3.5(e), it can be observed that the connectivity indicator is always larger than zero, which indicates that the communication graph of the robot group remained connected during operation. Figure 3.5(f) depicts the control input $\mathbf{u}_i$ for each robot, while the linear velocity and angular velocity for each robot are shown in fig. 3.5(g).

To show the effect of the transition regions introduced in the proposed method, a comparative simulation has also been performed, where the transition regions were removed. The corresponding results are shown in fig. 3.7, where fig. 3.7(a) depicts the robot paths, fig. 3.7(b) shows $\mathbf{u}_i$, while fig. 3.7(c) depicts v_i and ω_i for each robot. From these results, it can be easily found that even though the formation tracking objective can be achieved with guaranteed safety, the control inputs chattered severely, making such controller not suitable for real applications. The comparison results indicate the efficacy of the transition regions introduced in the proposed method.

(a) Robot paths

(b) Trajectory tracking errors

(c) Distances to the nearest obstacles

(d) Distances among robots

(e) Connectivity indicator

(f) u_i

Figure 3.5: Simulation results of virtual-structure formation tracking with transition regions

Figure 3.6: Simulation results of virtual-structure formation tracking without obstacles

Figure 3.7: Simulation results of virtual-structure formation tracking without transition regions

Figure 3.8: Experimental setup for virtual-structure formation tracking

3.3.2 Experimental Results

To demonstrate the efficacy of the proposed control scheme, an experiment has been performed in Robotics I Laboratory, NTU, Singapore. The proposed algorithm was tested by a multi-robot system composed of three Pioneer 3-AT mobile robot platforms which is shown in fig. 3.8. The leading robot moved autonomously in the workspace acting as the virtual leader, while the proposed control scheme was implemented on the other two robots. The leading robot was equipped with a Dell Precision M2800 Mobile Workstation, while each of the other two robots was both equipped with a ZOTAC ZBOX-VR7N70 mobile PC. Each of the three robots was equipped with a Hokuyo-UTM-30LX Scanning Laser Rangefinder. The algorithm was programmed using C++ language under ROS, Kinetic release. In the experiment, a map was generated in advance and each robot used the AMCL package for localization. It should be noted that the map information was only used for providing global pose feedback in the experiment. The parameters in the experiment is presented in table 3.2.

In the experiment, the leading robot which played the role of the virtual leader moved from one room to another room through a narrow door, while the other two robots tried to form a desired formation with guaranteed collision avoidance

Table 3.2: Experimental parameters for virtual-structure formation tracking

Parameter	Value
Sampling time interval	20ms
$\mathbf{p}_{1l}$	$[-1.0, -0.5]\text{m}$
$\mathbf{p}_{2l}$	$[-2.0, 0.5]\text{m}$
R	3m
R_s	3m
r_o	0.4m
r_n	0.4m
δ_o	0.3m
δ_n	0.3m
δ_c	0.3m
μ_o	0.2m
μ_n	0.2m
μ_c	0.2m
γ	1
α	1
k_i	0.5
κ	0.8
ζ	0.5

and connectivity maintenance simultaneously. The map of the environment and the paths of the three robots are depicted in fig. 3.9(a). Figure 3.9(b) shows the trajectory tracking performance of the two robots, from which and combined with fig. 3.9(a) it can be observed that when there are no obstacles nearby, both robots can track their individual reference trajectories with bounded error, and when potential collisions with obstacles occurred, trajectory tracking was ignored by them, resulting in the increasing of tracking error. Figure 3.9(c) depicts the distances of both robots to their corresponding nearest obstacles, respectively, from which it can be observed that both curves are above 0.4m, the safe distance, indicating that obstacle avoidance was achieved. It should be noted that there are some discontinuous points in fig. 3.9(c), which were caused by the malfunction of the laser rangefinder at these time instants. The faulty detection points were removed in this figure. Besides, the jumps in fig. 3.9(c) were caused by detection of the thin frames on the robots, which can be observed in fig. 3.8. Inter-robot distances are shown in fig. 3.9(d), from which it can be observed that all of the distances are larger than the safe distance and

Figure 3.9: Experimental results of virtual-structure formation tracking

smaller than the communication range. Therefore, inter-robot collision avoidance and connectivity maintenance were both guaranteed.

3.4 Conclusions

This chapter addresses the virtual-structure formation tracking control problem for a group of nonholonomic mobile robots, while obstacle and inter-robot collision avoidance as well as connectivity maintenance are guaranteed. To deal with such a multi-objective control problem for multi-robot systems with conflict control objectives, a multi-region control scheme is proposed where priority is given to different objectives in different regions. In addition, a series of transition functions are intro-

duced to smooth the switching between trajectory tracking and avoidance control in transition region. A new bounded potential function is introduced to formulate multiple constraints including obstacle avoidance, inter-robot collision avoidance and connectivity preservation, based on which a bounded controller is derived. The boundedness of the proposed controller is analyzed and the bounds in different regions are derived. Further simulation and real robot experiment have been performed to demonstrate the effectiveness of the proposed method.

Chapter 4

Vision-Based Leader-Follower Formation Tracking Control of Multi-Robot Systems in Obstacle Environments

In this chapter, a practical leader-follower formation tracking control scheme is proposed for multiple nonholonomic mobile robots moving in unknown obstacle environments. Sensor limitations, including the depth range and FOV constraints, are formulated by a class of bounded barrier functions, which are then incorporated into the control law. Collision avoidance between the leader and the follower is also described as a minimum range constraint and solved simultaneously. For the safety consideration of the robots in unknown environments, a multi-region obstacle avoidance algorithm is proposed where different control objectives are given the uppermost priority in different regions. This chapter also considers the leader-loss case in which the visibility is broken due to obstacles, change of lighting condition or motion blurring. To tackle this situation, a fault-tolerant strategy is designed such that the follower will move to the place where the leader was lost as soon as

possible. The proposed approach uses only local state measurements obtained by stereo camera mounted on the follower in a communication-free environment, and takes into account control input constraints explicitly. Finally, simulation and real robot experimental results are presented to validate the efficacy of the proposed method.

The contributions of this chapter are in three aspects. First of all, a leader-tracking control law is proposed which takes sensor constraints explicitly into account. However, few researchers have paid attention to sensor limitations when designing leader-follower formation control algorithms. Secondly, a multi-region obstacle avoidance algorithm is designed to guarantee robot safety in unknown environments. Thirdly, the leader-loss situation is pointed out, while the corresponding analysis and a fault-tolerant solution are provided.

4.1 Problem Formulation

This chapter considers a typical application where two differential-drive wheeled mobile robots move in a 2D workspace with unknown obstacles, using vision-based leader-follower techniques. It is assumed that one of the robots, defined as the leader, moves autonomously in the workspace without communication between the robots, while the other one acts as the follower tracking the leader and performing obstacle and inter-robot collision avoidance subject to sensor limitations and control input constraints. The problem can be easily generalized to a snakelike multi-robot formation with $N > 2$ members.

This section consists of three parts. The mathematical model of the leader-follower pair is presented first, while the second part models the system constraints. Finally, the leader-loss situation is analyzed.

4.1.1 Modeling of Leader-Follower Pair

The kinematic model of the robot is given as follows.

$$\begin{aligned}\dot{x}_i &= v_i \cos \theta_i, \\ \dot{y}_i &= v_i \sin \theta_i, \\ \dot{\theta}_i &= \omega_i,\end{aligned}\tag{4.1}$$

where $i \in \{l, f\}$, $\mathbf{p}_i = [x_i, y_i]^T \in \mathbb{R}^2$ and θ_i are the position and heading angle of robot i with respect to the global coordinate frame $\{g\} = \{\vec{x}_g, \vec{y}_g\}$, respectively, while v_i and ω_i are the linear velocity and angular velocity of robot i , respectively, in its corresponding body-fixed local frame $\{i\} = \{\vec{x}_i, \vec{y}_i\}$.

It is assumed that the follower is equipped with a stereo camera which can detect the relative position including distance and bearing angle of the leader with respect to $\{f\}$. The relative position relationship between the leader and the follower is depicted in fig. 4.1. To facilitate further analyses, polar coordinates are used to describe the relative position relationship.

The distance ρ between the leader and the follower as well as the bearing angle α of the leader with respect to $\{f\}$ are defined as

$$\begin{aligned}\rho &= \|\mathbf{p}_l - \mathbf{p}_f\|, \\ \alpha &= \arctan 2(\tilde{y}_l, \tilde{x}_l),\end{aligned}\tag{4.2}$$

where $\arctan 2(y, x)$ is the arctangent function with two arguments returning the appropriate quadrant of the angle of point (x, y) as a numeric value in the range $(-\pi, \pi]$. Moreover, $(\tilde{x}_l, \tilde{y}_l)$ is the Cartesian coordinates of the leader with respect to $\{f\}$, i.e.,

$$\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} = \begin{bmatrix} \cos \theta_f & \sin \theta_f \\ -\sin \theta_f & \cos \theta_f \end{bmatrix} \begin{bmatrix} x_l - x_f \\ y_l - y_f \end{bmatrix}.$$

Figure 4.1: Diagram of the leader-follower pair with obstacles

As is assumed above, ρ and α can be measured by the stereo camera. Following some straightforward calculations, the dynamics of ρ and α can be given as

$$\begin{aligned}\dot{\rho} &= -v_f \cos \alpha + v_l \cos(\theta_l - \theta_f - \alpha), \\ \dot{\alpha} &= -\omega_f + \frac{v_f}{\rho} \sin \alpha + \frac{v_l}{\rho} \sin(\theta_l - \theta_f - \alpha).\end{aligned}\tag{4.3}$$

According to the physical meanings of ρ and α , $\rho \in (0, +\infty)$ and $\alpha \in (-\pi, \pi]$.

4.1.2 Modeling of System Constraints

Considering the limited depth range and FOV of the stereo camera which are described in fig. 4.2, to avoid loss of detection, ρ must be smaller than the maximum detection range $d_{\max}$ and larger than the minimum detection range $d_{\min}$, besides, α should be restricted in a specific range $[\alpha_{\min}, \alpha_{\max}]$. Furthermore, to guarantee collision-free between the leader and the follower, ρ should be larger than the safe

Figure 4.2: Diagram of the leader-loss example

distance d_{safe} . Therefore, the constraints on ρ and α can be given as follows.

$$\rho \in [\rho_{\min}, \rho_{\max}], \quad \alpha \in [\alpha_{\min}, \alpha_{\max}], \quad (4.4)$$

where $\rho_{\min} = \max\{d_{\min}, d_{\text{safe}}\}$ and $\rho_{\max} = d_{\max}$. From the discussions above, it can be found that $\rho_{\min}$, $\rho_{\max}$, $\alpha_{\min}$, $\alpha_{\max}$ are constant parameters determined by sensor limitations and safety requirement.

For the purpose of obstacle avoidance, the local position of the obstacle measured in $\{f\}$ is also defined. Similarly, the distance between the obstacle and the follower is defined as ρ_o and the bearing angle of the obstacle measured in $\{f\}$ is defined as α_o . It is assumed that a sufficient condition for obstacle avoidance is given as

$$\rho_o \geq \rho_{o,\text{safe}}, \quad (4.5)$$

where $\rho_o = \|\mathbf{p}_f - \mathbf{p}_o\|$, $\forall \mathbf{p}_o \in \mathcal{O}$, $\mathcal{O}$ is a set of all the points on obstacles in the workspace, $\rho_{o,\text{safe}}$ is the safe distance which should be determined by taking into account the robot size. In this brief, the robot can get ρ_o using a stereo camera or

a laser range finder. The relative position relationship between the robots and the obstacles is also shown in fig. 4.1.

Finally, for real robots, the linear velocity and angular velocity are bounded due to electromechanical limitations. Therefore, the leader and the follower are subject to the following control input constraints.

$$|v_l| \leq \bar{v}_l, \quad |\omega_l| \leq \bar{\omega}_l, \quad |v_f| \leq \bar{v}_f, \quad |\omega_f| \leq \bar{\omega}_f, \quad (4.6)$$

where $\bar{v}_l$ and $\bar{\omega}_l$ are the bounds of v_l and ω_l , respectively, while $\bar{v}_f$ and $\bar{\omega}_f$ are the bounds of v_f and ω_f , respectively.

4.1.3 Modeling of Leader-Loss Situation

As previously stated in the introduction, the obstacle avoidance behavior may break the distance and bearing angle constraints, resulting in loss of detection of the leader, while environment interference and motion blurring could also cause the leader-loss consequence. Therefore, visibility of the leader cannot be constantly maintained in practical applications. A leader-loss example is presented in fig. 4.2, where the leader is occluded by the obstacle. This subsection will formulate the leader-loss problem.

Before going on, picture a scenario where a policeman is pursuing a thief in a cluttered environment. The thief suddenly turns at a corner and disappears in front of the policeman. An intuitive reaction of the policeman is to reach the spot where the thief was last seen, as quickly as possible, in the hope of continuing the pursuit. Inspired by such a human nature reaction, a fault-tolerant strategy to the leader-loss situation should consist of two steps. The first step is to find the shortest path for the follower from the current state to the goal state, which is actually the last detected leader state. The second step is to control the follower to move along the planned path to the goal state as fast as possible.

To design the fault-tolerant strategy, it should be noted that this chapter is under the assumption that global state feedback is not available, that is, the global states of the leader and the follower at t_l , which is the time instant when the leader was lost, cannot be obtained. To design the algorithm, a temporary coordinate frame $\{t\} = \{\vec{x}_t, \vec{y}_t\}$ is defined which is coincident with the local frame $\{f\}$ at time t_l , i.e., $\{t\} = \{f\}|_{t=t_l}$. As the movement range for the leader-loss reaction is relatively small, the accuracy of the odometry mounted on the follower should be within tolerance. Consequently, the odometry measurements can be utilized in the algorithm design as the state feedback with respect to $\{t\}$.

From the analyses stated above, the objective of the fault-tolerant strategy is to plan the shortest path ${}^t\mathbf{p}_v(l)$ from the starting state $({}^t\mathbf{p}_f(t_l), {}^t\theta_f(t_l))$ to the goal state $({}^t\mathbf{p}_l(t_l), {}^t\theta_l(t_l))$, where l is the arc length parameter, $({}^t\mathbf{p}_f(t_l), {}^t\theta_f(t_l))$ and $({}^t\mathbf{p}_l(t_l), {}^t\theta_l(t_l))$ are the states of the follower and the leader measured at t_l with respect to $\{t\}$ respectively, and design a control law such that the follower can reach the goal state along the planned path as fast as possible.

Based on the discussions above, the leader-follower tracking control problem concerned in this chapter can be formulated as follows.

Problem 4.1. (*Leader-follower tracking control*). Consider a leader-follower pair described by eq. (4.1) moving in an unknown obstacle environment. Assume that the leader moves autonomously in the workspace, the robots cannot communicate with each other and only local state measurements are available. Design a control scheme for the follower such that

1. (*Leader tracking*). When there are no obstacles nearby, ρ and α converge to the neighborhoods of the desired values ρ_d and α_d , respectively, while ensuring that the constraints eqs. (4.4) and (4.6) will not be violated during operation.
2. (*Obstacle avoidance*). The condition eq. (4.5) and the constraint eq. (4.6) will always be satisfied.

3. (*Leader-loss reaction*). When the leader cannot be detected, the follower will move from the starting state $({}^t\mathbf{p}_f(t_l), {}^t\theta_f(t_l))$ to the goal state $({}^t\mathbf{p}_l(t_l), {}^t\theta_l(t_l))$ along the planned path ${}^t\mathbf{p}_v(l)$ while eqs. (4.5) and (4.6) will always be guaranteed.

To facilitate the discussion of the main results, the following lemmas are stated.

Lemma 4.1. For any $\alpha, \beta \in \mathbb{R}$, the following inequality always holds,

$$\left| \frac{\sin \alpha - \sin \beta}{\alpha - \beta} \right| \leq 1.$$

Proof. According to the sum-to-product trigonometric identities, it can be obtained that

$$\frac{\sin \alpha - \sin \beta}{\alpha - \beta} = \sin\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right) / \frac{\alpha - \beta}{2}.$$

Before determining the upper bound of $\frac{\sin \alpha - \sin \beta}{\alpha - \beta}$, the upper bound of function $f(x) = \frac{\sin x}{x}$ should be derived. The derivative of $f(x)$ with respect to x is $f'(x) = \frac{g(x)}{x^2}$, where $g(x) = x \cos x - \sin x$. As the derivative of $g(x)$ with respect to x is $g'(x) = -x \sin x \leq 0$, it is obvious that $g(x) \leq g(0) = 0$, $\forall x \geq 0$. Consequently, $f'(x) \leq 0$, $\forall x \geq 0$, which implies that $f(x) \leq f(0) = 1$, $\forall x \geq 0$. As $f(x)$ is an even function and $f(x) > 0$, $f(x) \in (0, 1]$. Therefore, it can be concluded that

$$\left| \frac{\sin \alpha - \sin \beta}{\alpha - \beta} \right| \leq \left| \sin\left(\frac{\alpha - \beta}{2}\right) / \frac{\alpha - \beta}{2} \right| \left| \cos\left(\frac{\alpha + \beta}{2}\right) \right| \leq 1.$$

□

Lemma 4.2. For any $x \in \mathbb{R}$, the following inequality always holds,

$$|\tanh(x)[1 - \tanh^2(x)]| \leq \frac{2\sqrt{3}}{9}.$$

Proof. Let $f(x) = \tanh(x)[1 - \tanh^2(x)]$. Taking the differentiation of $f(x)$ with respect to x yields $f'(x) = [1 - \tanh^2(x)][1 - 3\tanh^2(x)]$. As $\tanh(x) \in (-1, 1)$,

it can be concluded that when $\tanh^2(x) \geq \frac{1}{3}$, $f'(x) \leq 0$, and when $\tanh^2(x) < \frac{1}{3}$, $f'(x) > 0$. As $f(x) \rightarrow 0$ when $x \rightarrow \pm\infty$, $|f(x)| \leq \frac{2\sqrt{3}}{9}$. $\square$

4.2 Main Results

This section presents the main results. First, the leader tracking problem with distance and bearing angle constraints is studied, then a control law is proposed. Next, a multi-region obstacle avoidance algorithm is presented. Finally, the leader-loss situation is discussed and a fault-tolerant strategy is designed.

4.2.1 Leader Tracking

In this subsection, a control scheme is proposed to solve the leader tracking problem under sensor limitations while collision avoidance between the leader and the follower is also guaranteed. To facilitate the discussion about the system constraints, the following bounded barrier functions are introduced to describe the constraints of ρ and α .

$$B_\rho(\rho) = \begin{cases} \beta_\rho \frac{(\rho - \rho_d)^2}{(\rho_{\max} - \rho_d)^2}, & \rho_d < \rho \leq \rho_{\max} \\ \beta_\rho \frac{(\rho - \rho_d)^2}{(\rho_d - \rho_{\min})^2}, & \rho_{\min} \leq \rho \leq \rho_d \end{cases}, \quad (4.7)$$

$$B_\alpha(\alpha) = \begin{cases} \beta_\alpha \frac{(\alpha - \alpha_d)^2}{(\alpha_{\max} - \alpha_d)^2}, & \alpha_d < \alpha \leq \alpha_{\max} \\ \beta_\alpha \frac{(\alpha - \alpha_d)^2}{(\alpha_d - \alpha_{\min})^2}, & \alpha_{\min} \leq \alpha \leq \alpha_d \end{cases}, \quad (4.8)$$

where β_ρ and β_α are design parameters whose values are equal to $B_\rho(\rho_{\max})$, $B_\rho(\rho_{\min})$ and $B_\alpha(\alpha_{\max})$, $B_\alpha(\alpha_{\min})$ respectively. It can be found that $B_\rho(\rho)$ has the minimum value 0 at ρ_d and increases monotonically to the maximum value β_ρ when ρ approaches $\rho_{\min}$ or $\rho_{\max}$. Similar results also hold for $B_\alpha(\alpha)$.

In the sequel, the barrier functions eqs. (4.7) and (4.8) will be employed to develop a control law that can drive the follower to track the leader under distance and bearing

angle constraints. In this chapter, it is assumed that $\bar{v}_l$ is known to the follower. This assumption is reasonable, because v_l must be smaller than v_f , otherwise, the follower cannot catch up with the leader. Based on the discussions above, the following control law is proposed for the follower.

$$\begin{aligned} v_f &= \frac{1}{\cos \alpha} [k_\rho \nabla B_\rho + \bar{v}_l \tanh(\frac{\bar{v}_l \nabla B_\rho}{\delta_\rho})], \\ \omega_f &= \frac{v_f}{\rho} \sin \alpha + \frac{\bar{v}_l}{\rho} \tanh(\frac{\bar{v}_l \nabla B_\alpha}{\rho \delta_\alpha}) + k_\alpha \nabla B_\alpha, \end{aligned} \quad (4.9)$$

where k_ρ , k_α , δ_ρ and δ_α are positive constant parameters, ∇B_ρ and ∇B_α are the gradients of B_ρ and B_α with respect to ρ and α , respectively. In addition, k_ρ , k_α , δ_ρ and δ_α satisfy the following conditions.

$$\begin{aligned} k_\rho &\leq \frac{\bar{v}_f \xi_\rho - \bar{v}_l}{G_\rho}, \\ k_\alpha &\leq \frac{\bar{\omega}_f \rho_{\min} - \bar{v}_l - \bar{v}_f \xi_\alpha}{G_\alpha \rho_{\min}}, \\ \rho_d - \frac{(\rho_{\min} - \rho_d)^2}{2\beta_\rho} \sqrt{\frac{k_p \delta_\rho}{k_\rho}} &> \rho_{\min}, \\ \rho_d + \frac{(\rho_{\max} - \rho_d)^2}{2\beta_\rho} \sqrt{\frac{k_p \delta_\rho}{k_\rho}} &< \rho_{\max}, \\ \alpha_d - \frac{(\alpha_{\min} - \alpha_d)^2}{2\beta_\alpha} \sqrt{\frac{k_p \delta_\alpha}{k_\alpha}} &> \alpha_{\min}, \\ \alpha_d + \frac{(\alpha_{\max} - \alpha_d)^2}{2\beta_\alpha} \sqrt{\frac{k_p \delta_\alpha}{k_\alpha}} &< \alpha_{\max}, \end{aligned} \quad (4.10)$$

where $\xi_\rho = \min\{\cos \alpha_{\min}, \cos \alpha_{\max}\}$, $G_\rho = \max\{\frac{2\beta_\rho}{\rho_{\max} - \rho_d}, \frac{2\beta_\rho}{\rho_d - \rho_{\min}}\}$, $\xi_\alpha = \min\{\sin \alpha_{\min}, \sin \alpha_{\max}\}$ and $G_\alpha = \max\{\frac{2\beta_\alpha}{\alpha_{\max} - \alpha_d}, \frac{2\beta_\alpha}{\alpha_d - \alpha_{\min}}\}$.

The above discussions lead to the following theorem which proves that the proposed control law can drive the follower to maintain a flexible distance and bearing angle with the leader while the distance and bearing angle constraints are not violated during operation. In the meantime, the control input constraints are satisfied as well.

Theorem 4.1. *Consider a leader-follower robot pair modeled by eq. (4.1). Assume that the leader moves autonomously with v_L bounded by $\bar{v}_L$ and the constraints eq. (4.4) is satisfied at the initial time. Using the proposed control law eq. (4.9) where the parameters are chosen according to eq. (4.10), the following results can be achieved.*

1. ρ will converge to $\{\rho | \rho_d - \frac{(\rho_{\min} - \rho_d)^2}{2\beta_\rho} \sqrt{\frac{k_p \delta_\rho}{k_\rho}} < \rho < \rho_d + \frac{(\rho_{\max} - \rho_d)^2}{2\beta_\rho} \sqrt{\frac{k_p \delta_\rho}{k_\rho}}\}$ asymptotically while α will asymptotically converge to $\{\alpha | \alpha_d - \frac{(\alpha_{\min} - \alpha_d)^2}{2\beta_\alpha} \sqrt{\frac{k_p \delta_\alpha}{k_\alpha}} < \alpha < \alpha_d + \frac{(\alpha_{\max} - \alpha_d)^2}{2\beta_\alpha} \sqrt{\frac{k_p \delta_\alpha}{k_\alpha}}\}$.
2. The constraints eq. (4.4) will not be violated during operation.
3. The constraints eq. (4.6) will always be satisfied.

Proof. First, consider Lyapunov function candidate $V_\rho = B_\rho$. Then, from eqs. (4.3) and (4.9), it can be obtained that

$$\dot{\rho} = -k_\rho \nabla B_\rho - \bar{v}_l \tanh\left(\frac{\bar{v}_l \nabla B_\rho}{\delta_\rho}\right) + v_l \cos(\theta_l - \theta_f - \alpha).$$

According to the above equations, it can be derived that

$$\begin{aligned} \dot{V}_\rho &= -k_\rho (\nabla B_\rho)^2 - \bar{v}_l \nabla B_\rho \tanh\left(\frac{\bar{v}_l \nabla B_\rho}{\delta_\rho}\right) + v_l \nabla B_\rho \cos(\theta_l - \theta_f - \alpha) \\ &\leq -k_\rho (\nabla B_\rho)^2 - \bar{v}_l \nabla B_\rho \tanh\left(\frac{\bar{v}_l \nabla B_\rho}{\delta_\rho}\right) + \bar{v}_l |\nabla B_\rho| \end{aligned}$$

Based on Lemma 1 of [104], it can be obtained that

$$\dot{V}_\rho \leq -k_\rho (\nabla B_\rho)^2 + k_p \delta_\rho.$$

Thus, when $(\nabla B_\rho)^2 > \frac{k_p \delta_\rho}{k_\rho}$, $\dot{V}_\rho < 0$. Therefore, ρ will converge into $\{\rho | (\nabla B_\rho)^2 > \frac{k_p \delta_\rho}{k_\rho}\}$, i.e. $\{\rho | \rho_d - \frac{(\rho_{\min} - \rho_d)^2}{2\beta_\rho} \sqrt{\frac{k_p \delta_\rho}{k_\rho}} < \rho < \rho_d + \frac{(\rho_{\max} - \rho_d)^2}{2\beta_\rho} \sqrt{\frac{k_p \delta_\rho}{k_\rho}}\}$, asymptotically. Based on condition eq. (4.10), the above set does not violate the distance constraint eq. (4.4).

Meanwhile, consider Lyapunov function candidate $V_\alpha = B_\alpha$. It can be derived from eqs. (4.3) and (4.9) that

$$\dot{\alpha} = -k_\alpha \nabla B_\alpha - \frac{\bar{v}_l}{\rho} \tanh\left(\frac{\bar{v}_l \nabla B_\alpha}{\rho \delta_\alpha}\right) + \frac{v_l}{\rho} \sin(\theta_l - \theta_f - \alpha).$$

Differentiating V_α with respect to time yields

$$\begin{aligned} \dot{V}_\alpha &= -k_\alpha (\nabla B_\alpha)^2 - \frac{\bar{v}_l \nabla B_\alpha}{\rho} \tanh\left(\frac{\bar{v}_l \nabla B_\alpha}{\rho \delta_\alpha}\right) + \frac{v_l \nabla B_\alpha}{\rho} \sin(\theta_l - \theta_f - \alpha) \\ &\leq -k_\alpha (\nabla B_\alpha)^2 - \frac{\bar{v}_l \nabla B_\alpha}{\rho} \tanh\left(\frac{\bar{v}_l \nabla B_\alpha}{\rho \delta_\alpha}\right) + \frac{\bar{v}_l |\nabla B_\alpha|}{\rho}. \end{aligned}$$

According to Lemma 1 of [104], it is easy to obtain

$$\dot{V}_\alpha \leq -k_\alpha (\nabla B_\alpha)^2 + k_p \delta_\alpha,$$

which implies that when $(\nabla B_\alpha)^2 > \frac{k_p \delta_\alpha}{k_\alpha}$, $\dot{V}_\alpha < 0$. Consequently, α will converge into $\{\alpha | (\nabla B_\alpha)^2 > \frac{k_p \delta_\alpha}{k_\alpha}\}$, i.e. $\{\alpha | \alpha_d - \frac{(\alpha_{\min} - \alpha_d)^2}{2\beta_\alpha} \sqrt{\frac{k_p \delta_\alpha}{k_\alpha}} < \alpha < \alpha_d + \frac{(\alpha_{\max} - \alpha_d)^2}{2\beta_\alpha} \sqrt{\frac{k_p \delta_\alpha}{k_\alpha}}\}$, asymptotically. It can be easily verified that the above set also satisfies the bearing angle constraints eq. (4.4).

Thus item 1 has been proven.

Next, the distance and bearing angle constraints will be analyzed. For the distance constraints, when $\rho \rightarrow \rho_{\max}^-$, $\dot{\rho} < 0$. As $\dot{\rho} = \dot{\rho} \nabla B_\rho$ and $\nabla B_\rho > 0$ when $\rho \rightarrow \rho_{\max}^-$, $\dot{\rho} < 0$. And when $\rho \rightarrow \rho_{\min}^+$, $\dot{\rho} < 0$. As $\nabla B_\rho < 0$ when $\rho \rightarrow \rho_{\min}^+$, $\dot{\rho} > 0$. This means that when ρ approaches the upper or the lower bound, the control law will drive the follower such that ρ decreases to avoid exceeding the bound. Similarly, it can also be concluded that when α is about to violate the constraints, the control law will react to keep α within the given range.

Finally, the control input constraints will be analyzed. From eq. (4.9), it is obvious that

$$|v_f| \leq \frac{1}{\xi_\rho} k_\rho |\nabla B_\rho| + \frac{1}{\xi_\rho} \bar{v}_l.$$

From eq. (4.7), it can be easily obtained that $|\nabla B_\rho| \leq G_\rho$. Thus, combined with eq. (4.10), $|v_f| \leq \bar{v}_f$ is satisfied. Similarly, from eq. (4.9),

$$|\omega_f| \leq \frac{\bar{v}_f}{\rho_{\min}} \xi_\alpha + \frac{\bar{v}_l}{\rho_{\min}} + k_\alpha |\nabla B_\alpha|.$$

From eq. (4.8), it can be found that $|\nabla B_\alpha| \leq G_\alpha$. Thus, according to eq. (4.10), $|\omega_f| \leq \bar{\omega}_f$ can also be guaranteed.

This completes the proof. $\square$

4.2.2 Unknown Obstacle Avoidance

The previous subsection addresses the leader tracking problem in an obstacle-free environment. However, in many practical applications, the obstacle information is out-of-date or even unavailable to the robots because of obstacles not detected beforehand, and/or new random obstacles in the operation. Therefore, an algorithm for unknown obstacle avoidance should be designed to guarantee the safety of the robots.

Before going on, the relative position relationship between the follower and the obstacle is analyzed and depicted in fig. 4.3. In our algorithm, the follower only considers the nearest obstacle around it and a set of regions are defined according to the relative position of the nearest obstacle with respect to $\{f\}$, which are front transition region $\mathbb{R}_{TF}$, back transition region $\mathbb{R}_{TB}$, front safe region $\mathbb{R}_{SF}$ and lastly back safe region $\mathbb{R}_{SB}$. Mathematically, these four regions are defined as

$$\begin{aligned} \mathbb{R}_{TF} &\triangleq \{\mathbf{p}' | \rho_{o,\text{safe}} < \rho' \leq \rho_{o,\text{sen}}, |\alpha'| \leq \frac{\pi}{2}\}, \\ \mathbb{R}_{TB} &\triangleq \{\mathbf{p}' | \rho_{o,\text{safe}} < \rho' \leq \rho_{o,\text{sen}}, |\alpha'| > \frac{\pi}{2}\}, \\ \mathbb{R}_{SF} &\triangleq \{\mathbf{p}' | \rho' \leq \rho_{o,\text{safe}}, |\alpha'| \leq \frac{\pi}{2}\}, \\ \mathbb{R}_{SB} &\triangleq \{\mathbf{p}' | \rho' \leq \rho_{o,\text{safe}}, |\alpha'| > \frac{\pi}{2}\}, \end{aligned}$$

Figure 4.3: Diagram of obstacle avoidance

where $\mathbf{p}' = [x', y']^T$, $\rho' = \|\mathbf{p}' - \mathbf{p}_f\|$, $\alpha' = \arctan 2(\tilde{y}', \tilde{x}')$,

$$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \end{bmatrix} = \begin{bmatrix} \cos \theta_f & \sin \theta_f \\ -\sin \theta_f & \cos \theta_f \end{bmatrix} \begin{bmatrix} x' - x_f \\ y' - y_f \end{bmatrix},$$

$\rho_{o,\text{sen}}$ and $\rho_{o,\text{safe}}$ are the sensitive and safe range for obstacle avoidance, respectively.

Figure 4.3 illustrates an example where the nearest obstacle is detected in $\mathbb{R}_{TF}$.

To avoid any potential collision with an obstacle ahead, the instinctive reaction of a human driver is to steer the car away from the obstacle. Inspired by this, our algorithm adopts a steering strategy on the basis of the relative position of the detected obstacles. To facilitate the derivation of the obstacle avoidance algorithm, a bounded barrier function is introduced as follows to describe the obstacle bearing angle constraint.

$$B_{\alpha_o} = \begin{cases} \beta_{\alpha_o} \frac{(\alpha_o - \alpha_{od})^2}{\alpha_{od}^2}, & 0 < \alpha_o \leq \alpha_{od} \\ \beta_{\alpha_o} \frac{(\alpha_o + \alpha_{od})^2}{\alpha_{od}^2}, & -\alpha_{od} \leq \alpha_o \leq 0 \end{cases},$$

where α_o is the bearing angle of the nearest obstacle with respect to $\{f\}$, β_{α_o} is a designed parameter and α_{od} represents the critical bearing angle. The introduction

of B_{α_o} is to help design an avoidance control law that will drive the follower to move away from the obstacle. Essentially, the controller should try to steer the follower such that $|\alpha_o|$ is larger than α_{od} . Generally, $\alpha_{od} = \frac{\pi}{2}$ because under this condition, the follower will move away from the obstacle with a positive v_F . In experiments, however, α_{od} should be assigned a value larger than $\frac{\pi}{2}$, due to friction and mechanical disturbances.

Based on the definitions given above, a multi-region obstacle avoidance algorithm is proposed in algorithm 4.1. In this algorithm, the control input is composed of two parts, namely the leader tracking part and the obstacle avoidance part, where $v_{f,t}$ and $v_{f,a}$ are the leader tracking and obstacle avoidance linear velocities respectively, while $\omega_{f,t}$ and $\omega_{f,a}$ are the leader tracking and obstacle avoidance angular velocities respectively. These control input parts are designed as follows.

$$\begin{aligned} v_{f,t} &= \frac{1}{\cos \alpha} [k_\rho \nabla B_\rho + \bar{v}_l \tanh(\frac{\bar{v}_l \nabla B_\rho}{\delta_\rho})], \\ \omega_{f,t} &= \frac{v_f}{\rho} \sin \alpha + \frac{\bar{v}_l}{\rho} \tanh(\frac{\bar{v}_l \nabla B_\alpha}{\rho \delta_\alpha}) + k_\alpha \nabla B_\alpha, \\ v_{f,a} &= v_{\text{avoid}}, \\ \omega_{f,a} &= k_{\alpha_o} \nabla B_{\alpha_o}, \end{aligned}$$

where v_{avoid} is a positive constant, k_{α_o} is a positive design parameter and ∇B_{α_o} is the gradient of B_{α_o} with respect to α_o . Besides, the control parameters satisfy $|v_{\text{avoid}}| \leq \bar{v}_f$, $k_{\alpha_o} \leq \frac{\bar{\omega}_f}{G_{\alpha_o}}$, where $G_{\alpha_o} = \frac{2\beta_{\alpha_o}}{\alpha_{od}}$.

It can be concluded from the algorithm that when ρ_o is larger than $\rho_{o,\text{sen}}$, which means that the obstacle does not threaten the safety of the follower, the leader tracking objective is given the top priority and obstacle avoidance is not activated. When in $\mathbb{R}_{TF}$, the obstacle locates in front of the follower with ρ_o smaller than $\rho_{o,\text{sen}}$ but larger than $\rho_{o,\text{safe}}$. In this region, v_f remains identical to $v_{f,t}$, while ω_f is the combination of $\omega_{f,t}$ and $\omega_{f,a}$. Such a reaction helps the follower try to achieve the leader tracking and obstacle avoidance objectives simultaneously. While in $\mathbb{R}_{TB}$, the obstacle locates behind the follower. Under this situation, the follower need not

Algorithm 4.1 Multi-region obstacle avoidance algorithm

Input: $\rho_o, \alpha_o.$ **Output:** $v_f, \omega_f.$

```

1: switch (Region where the obstacle is located)
2: case  $\mathbb{R}_{TF}$ :
3:    $v_f = v_{f,t}, \omega_f = \min\{\omega_{f,t} + \omega_{f,a}, \bar{\omega}_f\}.$ 
4: case  $\mathbb{R}_{TB}$ :
5:    $v_f = v_{f,t}, \omega_f = \omega_{f,t}.$ 
6: case  $\mathbb{R}_{SF}$ :
7:    $v_f = 0, \omega_f = \omega_{f,a}.$ 
8: case  $\mathbb{R}_{SB}$ :
9:    $v_f = v_{f,a}, \omega_f = 0.$ 
10: default:
11:    $v_f = v_{f,t}, \omega_f = \omega_{f,t}.$ 
12: end switch

```

consider the obstacle avoidance objective. In $\mathbb{R}_{SF}$, ρ_o is smaller than $\rho_{o,\text{safe}}$. In this region, the follower stops and rotates so that the obstacle transfers to $\mathbb{R}_{SB}$, where the follower maintains its heading angle and moves away. From the analyses given above, it can be found that using the proposed algorithm, ρ_o is kept greater than or equal to $\rho_{o,\text{safe}}$ such that obstacle avoidance can be guaranteed. In addition, it can be easily verified that the control input constraints are satisfied as well.

4.2.3 Leader-Loss Reaction

This subsection will study the leader-loss situation and design a fault-tolerant strategy to tackle this problem. The basic flow of the proposed strategy is presented in algorithm 4.2.

The first step of the proposed fault-tolerant strategy is to solve a shortest path plan problem. According to [112], the shortest path with a constraint on a maximum curvature takes the form of *CSC*, where *C* denotes a circular arc with minimum turning radius and *S* denotes a straight-line segment. As the minimum turning

Algorithm 4.2 Fault-tolerant strategy for leader-loss situation**Input:**

$({}^t\mathbf{p}_l(t_l), {}^t\theta_l(t_l)), ({}^t\mathbf{p}_f(t_l), {}^t\theta_f(t_l)), \rho_o, \alpha_o.$

Output:

$v_f, \omega_f.$

- 1: Plan the shortest path ${}^t\mathbf{p}_v(l)$ from $({}^t\mathbf{p}_f(t_l), {}^t\theta_f(t_l))$ to $({}^t\mathbf{p}_l(t_l), {}^t\theta_l(t_l))$;
- 2: Design the path following control part $v_{f,p}$ and $\omega_{f,p}$ to drive the follower moving along ${}^t\mathbf{p}_v(l)$;
- 3: Design the control law v_f and ω_f using algorithm 4.1.

radius of the robot considered in this chapter is zero, the shortest path between $({}^t\mathbf{p}_f(t_l), {}^t\theta_f(t_l))$ and $({}^t\mathbf{p}_l(t_l), {}^t\theta_l(t_l))$ is merely a straight line connecting ${}^t\mathbf{p}_f(t_l)$ and ${}^t\mathbf{p}_l(t_l)$.

Next, the control law for driving the follower to move along the planned path to the goal point should be designed, while the obstacle avoidance and control input constraints should also be guaranteed. Similar to the control scheme proposed in section 4.2.2, the overall control input is also composed of two parts, which are path following part and obstacle avoidance part. The obstacle avoidance part is the same as that in section 4.2.2. In the sequel, we mainly focus on the path following part.

The path following control method adopted in this chapter is based on the control law proposed in [113]. Similarly, a virtual target is introduced which moves along the path depicted in fig. 4.2. To facilitate further discussions, the following definitions are given. ${}^t\mathbf{p}_f = [{}^tx_f, {}^ty_f]^T$ and ${}^t\theta_f$ are the position and heading angle of the follower in $\{t\}$, respectively, while ${}^t\mathbf{p}_v = [{}^tx_v, {}^ty_v]^T$ and ${}^t\theta_v$ are the position and heading angle of the virtual target in $\{t\}$, respectively. ${}^v\mathbf{p}_f = [{}^vx_f, {}^vy_f]^T$ and ${}^v\theta_f$ are the position and heading angle of the follower in $\{v\}$, respectively, where $\{v\} = \{\vec{x}_v, \vec{y}_v\}$ is the coordinate frame attached on the virtual target as shown in fig. 4.2. Similar

with [113], the path following error dynamics can be derived as follows.

$$\begin{aligned} {}^v\dot{x}_f &= -\dot{l} + v_{f,p} \cos {}^v\theta_f, \\ {}^v\dot{y}_f &= v_{f,p} \sin {}^v\theta_f, \\ {}^v\dot{\theta}_f &= \omega_{f,p}, \end{aligned}$$

where ${}^v\theta_f = {}^t\theta_f - {}^t\theta_v$.

To drive the follower moving along the path, let the rate of progression of the virtual target along the planned path be governed by

$$\dot{l} = v_{f,p} \cos {}^t\theta_f + k_1 {}^v x_f, \quad (4.11)$$

where k_1 is a positive constant parameter. Then, the path following control law is designed as follows.

$$\begin{aligned} v_{f,p} &= v_{\text{follow}}, \\ \omega_{f,p} &= \dot{\delta} - \gamma \tanh({}^v y_f) [1 - \tanh^2({}^v y_f)] v_{f,p} \frac{\sin {}^v\theta_f - \sin \delta}{{}^v\theta_f - \delta}, \\ &\quad - k_2 \tanh({}^v\theta_f - \delta) \end{aligned} \quad (4.12)$$

where k_2 and γ are positive constant parameters, v_{follow} is a positive constant, which means that the follower moves at a constant speed along the path, and δ is the expected transient maneuver, which is given as $\delta = -\theta_\delta \tanh(\beta_\delta {}^v y_f)$, where θ_δ and β_δ are positive design parameters. In addition, the parameters of eq. (4.12) should satisfy the following conditions.

$$\begin{aligned} v_{\text{follow}} &\leq \bar{v}_f, \\ k_2 + \theta_\delta \beta_\delta v_{f,p} + \frac{2\sqrt{3}}{9} \gamma v_{f,p} &\leq \bar{\omega}_f. \end{aligned} \quad (4.13)$$

It can be proven that based on the proposed path following control law eqs. (4.11) and (4.12), ${}^v\mathbf{p}_f$ and ${}^v\theta_f$ will converge to zero asymptotically. A formal statement of

this result is presented in the following theorem.

Theorem 4.2. *Consider a robot modeled by eq. (4.1) and a desired path ${}^t\mathbf{p}_v(l)$ parameterized by l . The control law eqs. (4.11) and (4.12) ensure that ${}^v x_f$, ${}^v y_f$ and ${}^v \theta_f$ are bounded, while ${}^v \mathbf{p}_f$ and ${}^v \theta_f$ converge to zero asymptotically. In addition, control input constraints eq. (4.6) are satisfied.*

Proof. Consider the following Lyapunov function candidate.

$$V = \frac{1}{2} {}^v x_f^2 + \frac{1}{2} \tanh^2({}^v y_f) + \frac{1}{2\gamma} {}^v \theta_f^2. \quad (4.14)$$

Differentiating eq. (4.14) with respect to time and combining with eqs. (4.11) and (4.12) yields

$$\begin{aligned} \dot{V} = & -k_1 {}^v x_f^2 + \tanh({}^v y_f)[1 - \tanh^2({}^v y_f)]v_{f,p} \sin \delta \\ & - \frac{k_2}{\gamma} ({}^v \theta_f - \delta) \tanh({}^v \theta_f - \delta). \end{aligned}$$

According to the definition of δ and the fact that $\sin(\cdot)$ and $\tanh(\cdot)$ are both odd, $\tanh({}^v y_f) \sin \delta \leq 0$. Besides, as $v_{f,p} > 0$ and $\tanh(x) \in (-1, 1)$, it can be obtained that $\tanh({}^v y_f)[1 - \tanh^2({}^v y_f)]v_{f,p} \sin \delta \leq 0$. Therefore, $\dot{V} \leq 0$. From eq. (4.14), it can be concluded that ${}^v x_f$, ${}^v y_f$ and ${}^v \theta_f$ are all bounded. It can also be obtained that $\mathbb{E} = \{{}^v \mathbf{p}_f, {}^v \theta_f | \dot{V} = 0\} = \{{}^v \mathbf{p}_f = \mathbf{0}, {}^v \theta_f = 0\}$, which is an invariant set. Thus, the maximum invariant set in $\mathbb{E}$ is itself. According to the LaSalle's invariance principle [114], ${}^v \mathbf{p}_f$ and ${}^v \theta_f$ will asymptotically converge to zero.

Next, the input constraints are discussed. According to lemmas 4.1 and 4.2, it can be derived that

$$|\omega_{f,p}| \leq \left| \dot{\delta} \right| + k_2 + \frac{2\sqrt{3}}{9} \gamma v_{f,p}.$$

As $\dot{\delta} = -\theta_\delta [1 - \tanh^2(\beta_\delta {}^v y_f)] \beta_\delta {}^v \dot{y}_f$, $\left| \dot{\delta} \right| \leq \theta_\delta \beta_\delta v_{f,p}$. Therefore,

$$|\omega_{f,p}| \leq \theta_\delta \beta_\delta v_{f,p} + k_2 + \frac{2\sqrt{3}}{9} \gamma v_{f,p}.$$

According to the parameter selection condition eq. (4.13), control input constraints

Figure 4.4: Formation configuration in the simulation

are satisfied.

Thus, the proof completes. $\square$

4.3 Simulation and Experimental Results

In this section, a numerical example is designed to illustrate the performance of our proposed leader-follower tracking control scheme. Then further real robot experiment has been performed to validate the effectiveness of the proposed method.

4.3.1 Simulation Results

The proposed method was validated by a numerical simulation in MATLAB. In the simulation, a snakelike leader-follower formation consisting of three robots was considered (depicted in fig. 4.4), where the first robot acted as the leader of the whole formation. In particular, the leader robot was the leader of follower 1, which played the leader role for follower 2 at the same time. The testing parameters is given in table 4.1. In the simulation, the leader robot moved autonomously in a maze-like environment including five walls and three small rectangular obstacles, which is shown in fig. 4.5(a). In addition, to illustrate the effectiveness of the proposed leader loss reaction strategy, it was manually set in the simulation that the stereo camera malfunction during 30s $\sim$ 32s and 120s $\sim$ 122s for follower 1, while follower 2 cannot detect its leader during 50s $\sim$ 52s and 100s $\sim$ 103s.

Table 4.1: Simulation parameters for leader-follower formation tracking

Parameter	Value
Total time duration	160s
Sampling time interval	33.3ms
$\mathbf{p}_l(0)$	$[0, 1]^T \text{m}$
$\theta_l(0)$	$\frac{\pi}{2} \text{rad}$
$\mathbf{p}_{f_1}(0)$	$[-1, -1]^T \text{m}$
$\theta_{f_1}(0)$	$\frac{\pi}{2} \text{rad}$
$\mathbf{p}_{f_2}(0)$	$[-1, -3]^T \text{m}$
$\theta_{f_2}(0)$	$\frac{\pi}{2} \text{rad}$
$\rho_{\min}$	1m
$\rho_{\max}$	3m
$\alpha_{\min}$	-35°
$\alpha_{\max}$	35°
$\rho_{o,\text{safe}}$	0.5m
$\bar{v}_l$	0.5m/s
$\bar{\omega}_l$	0.5rad/s
$\bar{v}_f$	1m/s
$\bar{\omega}_f$	2.5rad/s
ρ_d of follower 1	1.5m
α_d of follower 1	0°
ρ_d of follower 2	1.5m
α_d of follower 2	0°
β_ρ	10
β_α	1
k_ρ	0.007
δ_ρ	0.1
k_α	0.7
δ_α	0.1
$\rho_{o,\text{sen}}$	0.8m
$\beta_{\alpha o}$	6
$k_{\alpha o}$	1
α_{od}	99°
v_{avoid}	0.3m/s
k_1	1
k_2	1
γ	1
v_{follow}	0.3m
θ_δ	$\frac{\pi}{3}$
β_δ	1

Figure 4.5(a) depicts the paths of the three robots and the map of the environment, from which it can be observed roughly that the two followers tracked their corresponding leader successfully, while obstacle avoidance was achieved at the same time. Figures 4.5(b) and 4.5(c) show the leader detection results, ρ and α , by follower 1 and 2 respectively. For comparison, a new simulation has also been conducted, whose results are depicted in fig. 4.6. The differences are that all the walls and obstacles were removed and the emulated camera malfunction was not imposed. The leader detection results, ρ and α , by follower 1 and 2 in the comparative simulation are given in figs. 4.6(b) and 4.6(c), respectively. Combining with fig. 4.5(a), it can be concluded that when there are no obstacles around, ρ and α both converge to the predesignated set respectively, which demonstrates the efficacy of the proposed leader tracking controller. In particular, there are several fluctuations of ρ and α during operation, which were caused by obstacle avoidance reactions and the manually designed detection interruptions. Figures 4.5(d) and 4.5(e) describe the obstacle detection results by follower 1 and 2 respectively, from which it can be observed that using the proposed multi-region obstacle avoidance algorithm, the distances from two followers to their corresponding nearest obstacles were both larger than the prespecified safe distance. Therefore, the effectiveness of the proposed obstacle avoidance can be validated. Then, we will review figs. 4.5(b) and 4.5(c). As mentioned above, there are two leader detection interruptions during operation for both two followers. It can also be observed from these two figures that during these interruptions, leader cannot be detected, resulting in increasings of leader tracking error in both ρ and α . However, once stereo cameras recover to work well, the tracking errors can converge to the desired sets again, which implies that our proposed fault-tolerant strategy is competent to handle the leader-loss situation. Figures 4.5(f) and 4.5(g) show the control inputs, velocity and angular velocity, of follower 1 and 2.

(a) Map and robot paths in the simulation

(b) ρ and α (follower 1) (c) ρ and α (follower 2) (d) ρ_o and α_o (follower 1) (e) ρ_o and α_o (follower 2) (f) v and ω (follower 1)

(g) v and ω (follower 2)

Figure 4.5: Simulation results of leader-follower formation tracking

(a) Map and robot paths in the simulation

(b) ρ and α (follower 1) (c) ρ and α (follower 2)

Figure 4.6: Simulation results of leader-follower formation tracking without obstacles

Figure 4.7: Experimental setup for leader-follower formation tracking

4.3.2 Experimental Results

The presented approach has been implemented on a multi-robot system consisting of two Pioneer 3-AT mobile robot platforms as depicted in fig. 4.7. Leader robot was equipped with a Dell Precision M2800 Mobile Workstation, while the follower robot was equipped with a ZOTAC ZBOX-VR7N70 mobile PC and a ZED stereo camera. Both two robots were equipped with a Hokuyo UTM-30LX Scanning Laser Rangefinder. The algorithms were programmed using C++ language under ROS, Kinetic release. In the experiment, leader robot moved autonomously in the workspace and our proposed method was applied on the follower robot. The testing parameters in the experiment are given in table 4.2.

In the experiment, the leader moved from a room with many unknown obstacles to another room through a door autonomously. The follower tracked the leader while performing obstacle avoidance simultaneously. Figure 4.8 depicts the map of the environment and the paths of two robots, where the green one denotes the leader path and the red one is the path of the follower. It should be noted that the map is only used to help exhibit the experiment results vividly, which was

Table 4.2: Experimental parameters for leader-follower formation tracking

Parameter	Value
Sampling time interval	33.3ms
$\rho_{\min}$	1m
$\rho_{\max}$	3m
$\alpha_{\min}$	-35°
$\alpha_{\max}$	35°
$\rho_{o,\text{safe}}$	0.5m
$\bar{v}_l$	0.3m/s
$\bar{\omega}_l$	2rad/s
$\bar{v}_f$	0.7m/s
$\bar{\omega}_f$	2rad/s
ρ_d	1.5m
α_d	0°
β_ρ	10
β_α	1
k_ρ	0.0068
δ_ρ	0.35
k_α	0.39
δ_α	0.1
$\rho_{o,\text{sen}}$	0.9m
$\beta_{\alpha o}$	1
$k_{\alpha o}$	1
α_{od}	120°
v_{avoid}	0.3m/s
k_1	1
k_2	1
γ	1
v_{follow}	0.3m
θ_δ	$\frac{\pi}{2}$
β_δ	1

generated by the leader using the SLAM technique, while the follower did not utilize the map information for control. Figure 4.9(a) shows ρ during operation while fig. 4.9(b) describes α . From these two figures and also fig. 4.8, it can be found that when the robots arrived at the second room without obstacles nearby, ρ and α both converged into the respective designed sets, which indicates that the proposed leader tracking control law is capable of controlling the follower to track the leader with a flexible formation structure. It can be revealed from figs. 4.9(a) and 4.9(b) that the follower first detected the leader at about 38s and leader-loss situation happened frequently from 50s to 70s, which may be caused by obstacle avoidance behavior, illumination disturbance or motion blurring. Even though the leader was lost frequently, the follower managed to re-detect it and recover tracking, which demonstrates the effectiveness of the proposed fault-tolerant strategy. Figures 4.9(c) and 4.9(d) describe ρ_o and α_o respectively. These two figures show that between 47s and 64s, there were obstacles located within $\rho_{o, \text{sen}}$ and thus the obstacle avoidance algorithm was activated. Based on these two figures, it can be concluded that the follower succeeded in avoiding the obstacles. At about 51s, ρ_o was a little smaller than 0.5m, which can be explained by inertia of the robot despite zero input velocity. The obstacle avoidance can also be observed directly from fig. 4.8, in which the follower successfully avoided densely distributed obstacles when moving from the first room to the other. Figures 4.9(e) and 4.9(f) depict v_f and ω_f respectively. From these two figures, it can be concluded that the control input constraints were satisfied. Before 75s, the control inputs changed dramatically, which was caused by the obstacle avoidance behavior and the leader-loss reaction. After that, the control inputs were smooth.

4.4 Conclusions

This chapter studies the leader-follower tracking control problem for multiple non-holonomic mobile robots, and a series of practical issues including sensor limitations,

Figure 4.8: Map and robot paths in the experiment

obstacle and inter-robot collision avoidance, leader-loss situation, etc., have been explicitly taken into account. The proposed control scheme is under the assumption that global pose measurements are not available and there are no communication links among robots, which makes the proposed approach more applicable for practical applications than existing methods. In this chapter, sensor limitations and leader-follower collision avoidance are modeled as distance and bearing angle constraints. To describe these constraints, a bounded barrier function is introduced and incorporated in the leader tracking control law. To guarantee the safety of the robots, a multi-region obstacle avoidance algorithm is designed which prioritizes different objectives in different regions. It is proven that when there are no obstacles nearby, the proposed approach can form a flexible formation structure while distance and bearing angle constraints are not violated. In addition, the leader-loss situation has first been considered. Then a fault-tolerant strategy is proposed when the visibility is broken. Real robot experiment was conducted to demonstrate the effectiveness of the proposed method.

Figure 4.9: Experimental results of leader-follower formation tracking

Chapter 5

Flexible Leader-Follower Formation Analysis and Tracking Control of Multi-Robot Systems Using Relative Pose Measurements

In this chapter, the flexible formation tracking control problem for multiple non-holonomic mobile robots is studied, while the previous two chapters focus on rigid formation. To facilitate the analysis on the motion capabilities of rigid and flexible formations, we introduce for the first time *formation flexibility* as a quantifying metric. A mathematically rigorous analysis has been performed from the motion planning perspective, resulting in the conclusion that flexible formation is superior to rigid formation in terms of motion capability. Due to the lack of global pose measurements, a stereo camera is installed on each follower to detect the relative position and orientation relationship with the leader. In consideration of the noise and intermittent interruption in visual observations, a particle filter based relative

pose estimation approach is proposed to estimate the instantaneous position and orientation of the leader in the local frame attached on each follower. To form a flexible formation, a trajectory reconstruction method is employed to transform the leader historical trajectory to the current local coordinate system of the follower, then the reference point is generated for the follower to track. To guarantee robot safety in unknown obstacle environments, a multi-objective control law is proposed for the follower to achieve and coordinate reference tracking, obstacle avoidance and inter-robot collision avoidance. Finally, simulation and real robot experiment are performed to validate the effectiveness of the proposed method.

The contributions of this chapter are in three aspects. First of all, the proposed algorithm works with relative pose measurements, while existing flexible formation control related work is under the assumption that global pose measurements are available. Secondly, the leader historical trajectory can be reconstructed in real time. However, most of the existing approaches assume that leader trajectory is determined in advance. Thirdly, the proposed method can guarantee robot safety in unknown obstacle environments.

5.1 Mathematical Modeling

This chapter considers n nonholonomic mobile robots moving along a reference trajectory while maintaining a desired formation configuration. To facilitate further discussions, in this section, mathematical models of leader-follower pair, rigid formation and flexible formation are presented, and then *formation flexibility* is defined to characterize the motion capability of a formation.

Before proceeding further, global and local coordinate frames are defined. Global frame, which can also be called the inertial frame, is represented as $\{g\} = \{\vec{x}_g, \vec{y}_g\}$. Local frame is attached on and moving with each robot. For example, the local frame of robot i is represented as $\{i\} = \{\vec{x}_i, \vec{y}_i\}$ with $\vec{x}_i$ pointing towards its motion

direction.

Remark 5.1. *The pre-superscript is used to describe the coordinate frame in which the corresponding variable is expressed. For example, ${}^i\mathbf{p}_j$ represents the position of robot j with respect to $\{i\}$. For a variable without pre-superscript, it means that this variable is defined with respect to $\{g\}$.*

5.1.1 Leader-Follower Pair

This chapter considers the differential-drive wheeled mobile robot, whose kinematic model is given as follows.

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i, \mathbf{u}_i^w, \mathbf{w}_i), \quad (5.1)$$

where $i \in \{l, f_j\}$, $j \in \{1, 2, \dots, n-1\}$, $\mathbf{x}_i = [x_i, y_i, \theta_i]^T$, $\mathbf{u}_i^w = [v_i^l, v_i^r]^T$ and $\mathbf{w}_i = [w_i^l, w_i^r]^T$. The function $f(\cdot)$ is defined as $f(\mathbf{x}_i, \mathbf{u}_i^w, \mathbf{w}_i) = A_i \mathbf{u}_i$, where $\mathbf{u}_i = B(\mathbf{u}_i^w + \mathbf{w}_i)$, $\mathbf{u}_i = [v_i, \omega_i]^T$,

$$A_i = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{d} & \frac{1}{d} \end{bmatrix}.$$

The physical meanings of the variables listed above are illustrated as follows, $\mathbf{p}_i = [x_i, y_i]^T$ and θ_i are the position and orientation of robot i with respect to $\{g\}$, respectively, v_i and ω_i are the linear velocity and angular velocity of robot i in $\{i\}$, respectively, d is the distance along the axle between two wheel centers, v_i^l and v_i^r are the linear velocities of the left and right wheels, respectively, w_i^l and w_i^r are the mutually independent random variables representing additive process noise to left and right wheels, respectively. It is assumed that robot l , which has the knowledge of the desired trajectory needed to complete the task, acts as the leader of the whole formation, while robot f_j , $j \in \{1, 2, \dots, n-1\}$, act as the followers. The class of nonholonomic mobile robots considered in this chapter is shown in fig. 5.1, where the notations given above are also depicted.

It can be easily found that different from chapters 3 and 4, this chapter considers a more detailed kinematic model with additive process noise to wheels in order to facilitate accurate estimation of the leader local position and orientation.

Figure 5.1: Diagram of leader-follower pair

Furthermore, the linear velocity and angular velocity of a robot should satisfy certain constraints due to electromechanical limitations. In detail, robots are subject to the following kinematic constraints.

$$v_{\min} \leq v_i \leq v_{\max}, |\omega_i| \leq \omega_{\max}, \quad (5.2)$$

where $v_{\min}$ and $v_{\max}$ are the lower and upper bounds of v_i , respectively, while $\omega_{\max}$ is the bound of ω_i . For ground mobile robots, reverse movement is generally allowed. Therefore, $v_{\min} < 0$, $v_{\max} > 0$ and $\omega_{\max} > 0$.

To obtain relative pose measurements of the leader with respect to the follower, a RGBD camera is employed which can provide accurate range, bearing and orientation observations. Thus, the measurement equation can be written as follows.

$$\mathbf{y}_{f_j} = h({}^{f_j}\mathbf{x}_l, \boldsymbol{\varepsilon}_{f_j}), \quad (5.3)$$

where $\mathbf{y}_{f_j} = [\rho_{f_j}, \alpha_{f_j}, {}^{f_j}\theta_l]^T$, $\boldsymbol{\varepsilon}_{f_j} = [\varepsilon_{\rho_{f_j}}, \varepsilon_{\alpha_{f_j}}]^T$, ${}^{f_j}\mathbf{x}_l = R(\theta_{f_j})(\mathbf{x}_l - \mathbf{x}_{f_j})$ and

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In addition, function $h(\cdot)$ is defined as $\rho_{f_j} = \|{}^{f_j}\mathbf{p}_l\| + \varepsilon_{\rho_{f_j}}$, $\alpha_{f_j} = \text{atan2}({}^{f_j}y_l, {}^{f_j}x_l) + \varepsilon_{\alpha_{f_j}}$ and ${}^{f_j}\theta_l = \theta_l - \theta_{f_j}$. ρ_{f_j} and α_{f_j} are the range and bearing measurements of the leader with respect to $\{f_j\}$, respectively, while $\varepsilon_{\rho_{f_j}}$ and $\varepsilon_{\alpha_{f_j}}$ are the independent random measurement noise.

Remark 5.2. *In practice, any robot i can serve as the reference leader of some other robots in a formation. To avoid ambiguity, the robot which has the knowledge of the desired trajectory of the whole formation is called the global leader, while the robot acting as the reference leader of some other robots is defined as the local leader. In this chapter, for convenience and without loss of generality, we only consider the case where there is only one global leader and no local leader in the formation.*

5.1.2 Rigid Formation

In general, rigid formation falls into two categories, perfect rigid formation and semi-perfect rigid formation, which are different in terms of the orientation requirement. For perfect rigid formation, it is defined that not only the positions of the robots form a desired geometric shape, but also the orientations keep consensus at the same time. But for semi-perfect rigid formation, the orientation constraint is ignored.

As rigid formation is defined in the rectangular coordinate system which is plotted in fig. 5.2, the local position of robot f_j in $\{l\}$ is assigned as ${}^l\mathbf{p}_{f_j} = [p_{rf_j}, q_{rf_j}]^T$. For perfect rigid formation, ${}^l\theta_{f_j} = 0$. Thus, the position of robot f_j can be derived as

Figure 5.2: Diagram of rigid formation

follows.

$$\mathbf{p}_{f_j} = \mathbf{p}_l + {}^gR_l {}^l\mathbf{p}_{f_j}, \quad (5.4)$$

where

$${}^gR_l = \begin{bmatrix} \cos \theta_l & -\sin \theta_l \\ \sin \theta_l & \cos \theta_l \end{bmatrix}$$

is the rotation transformation from $\{l\}$ to $\{g\}$. For perfect rigid formation, the orientation of robot f_j is

$$\theta_{f_j} = \theta_l. \quad (5.5)$$

5.1.3 Flexible Formation

In this chapter, flexible formation is defined in curvilinear coordinates rather than in rectangular coordinates as in rigid formation. A flexible formation containing three robots is shown in fig. 5.3. The local position of robot f_j with respect to $\{l\}$ in curvilinear coordinates is ${}^l\mathbf{p}_{f_j} = [p_{ff_j}, q_{ff_j}]^T$, where p_{ff_j} represents the longitudinal

curvilinear separation and q_{ff_j} represents the lateral curvilinear separation.

Figure 5.3: Diagram of flexible formation

Next, the position and orientation of robot f_j will be determined. First, define the trajectory and control inputs of robot l with respect to time t as $\mathbf{t}_l(t) = [x_l(t), y_l(t), \theta_l(t)]^T$ and $\mathbf{u}_l(t) = [v_l(t), \omega_l(t)]^T$, respectively, where $t \in [0, t_f]$. The curvature of the trajectory is denoted by $c_l(t)$. Then, the trajectory set of robot l can be given as $\Gamma = \{\mathbf{t}_l(t) | t \in [0, t_f]\}$. Similarly, with a slight abuse of notation, define the trajectory and control inputs of robot l with respect to arc length s as $\mathbf{t}_l(s) = [x_l(s), y_l(s), \theta_l(s)]^T$ and $\mathbf{u}_l(s) = [v_l(s), \omega_l(s)]^T$, respectively, where $s \in [0, s_f]$. The curvature of the trajectory defined on arc length is $c_l(s)$.

Define s_i as the curvilinear coordinate of robot i along Γ , where $s_i(t) = \int_0^t v_l(\tau) d\tau$. Obviously, $s_{f_j} = s_l + p_{ff_j}$. Consequently, the position of robot f_j can be obtained as

$$\mathbf{p}_{f_j} = \mathbf{p}_l(s_{f_j}) + {}^g R_l(\theta_l(s_{f_j}))^l \mathbf{p}'_{f_j}, \quad (5.6)$$

where ${}^l\mathbf{p}'_{f_j} = [0, q_{ff_j}]^T$. The orientation of robot f_j is

$$\theta_{f_j} = \theta_l(s_{f_j}). \quad (5.7)$$

5.1.4 Formation Flexibility

In the above two subsections, rigid and flexible formations are modeled. However, as mobile robots are subject to linear velocity and angular velocity limitations, the motion of the leader should be further constrained in order to maintain the desired formation. Therefore, the admissible motion of the leader is utilized to describe the motion capability of a formation. Thus, the following definition is given.

Definition 5.1. (*Formation flexibility*): For a multi-robot formation $\mathfrak{F}$ composed of n mobile robots described by eqs. (5.1) and (5.2), assume that the leader moves at a constant velocity v_l , $v_l > 0$, the curvature of the leader c_l , with which the kinematic constraint eq. (5.2) is satisfied for $\forall i \in \{l, f_j\}$, $j \in \{1, 2, \dots, n-1\}$, is defined as the formation flexibility f of $\mathfrak{F}$.

To compare the motion capabilities of two formations, the ranges of their *formation flexibilities* are compared. For example, consider two formations $\mathfrak{F}_1$ and $\mathfrak{F}_2$, whose leaders move at the same constant velocity. The range of the *formation flexibility* f_1 of $\mathfrak{F}_1$ is $\mathbb{F}_1$, while the range of the *formation flexibility* f_2 of $\mathfrak{F}_2$ is $\mathbb{F}_2$. If $\mathbb{F}_1 \subseteq \mathbb{F}_2$, it means that $\mathfrak{F}_2$ is more flexible than $\mathfrak{F}_1$, and vice versa.

To facilitate the discussion of the main results, the following three lemmas are given.

Lemma 5.1. For $a, b, \alpha, \beta \in \mathbb{R}$ with a and b not equal to zero simultaneously, if $\tan \alpha = \frac{a}{b}$ and $\tan \beta = \frac{b}{a}$, the following equality always holds, $\alpha + \beta = \frac{\pi}{2} + k\pi$, $k \in \mathbb{N}$.

Proof. If $ab \neq 0$, it can be obtained that $\tan \alpha = \cot \beta = \tan(\frac{\pi}{2} - \beta)$. Thus, $\alpha = \frac{\pi}{2} - \beta + k\pi$, $k \in \mathbb{N}$, that is, $\alpha + \beta = \frac{\pi}{2} + k\pi$. If $ab = 0$, as a and b are not

equal to zero simultaneously, without loss of generality, assume that $a = 0$. Thus, $\alpha = k_1\pi$ and $\beta = \frac{\pi}{2} + k_2\pi$, $k_1, k_2 \in \mathbb{N}$. Therefore, $\alpha + \beta = \frac{\pi}{2} + k\pi$. $\square$

Lemma 5.2. *Given constants $a, b \in \mathbb{R}$,*

$$g_1(x) = \frac{a + \sqrt{a^2 + b(x + a^2)}}{x + a^2}$$

is a monotonically decreasing function in its domain and

$$g_2(x) = \frac{a - \sqrt{a^2 + b(x + a^2)}}{x + a^2}$$

is a monotonically increasing function in its domain.

Proof. For $g_1(x)$, its derivative can be obtained as

$$g_1'(x) = -\frac{(a + \sqrt{a^2 + b(x + a^2)})^2}{2\sqrt{a^2 + b(x + a^2)}(x + a^2)^2}.$$

Obviously $g_1'(x) \leq 0$, therefore $g_1(x)$ is monotonically decreasing. Similarly, the derivative of $g_2(x)$ is

$$g_2'(x) = \frac{(a - \sqrt{a^2 + b(x + a^2)})^2}{2\sqrt{a^2 + b(x + a^2)}(x + a^2)^2} \geq 0,$$

which implies that $g_2(x)$ is monotonically increasing. $\square$

Lemma 5.3. *Consider a mobile robot i modeled by eq. (5.1) and assume that the linear and angular velocities, v_i and ω_i , are imposed on the robot discretely with the sampling time interval T_s . The pose variation over time interval $[t_{k-1}, t_k)$, where $t_k = kT_s$, is presented as follows*

$$\mathbf{x}_{i,k} = \mathbf{x}_{i,k-1} + \Delta \mathbf{x}_{i,k-1}, \quad (5.8)$$

where

$$\begin{aligned}\Delta \mathbf{x}_{i,k-1} &= [\Delta x_{i,k-1}, \Delta y_{i,k-1}, \Delta \theta_{i,k-1}]^T, \\ \Delta x_{i,k-1} &= \begin{cases} T_s v_{i,k-1} \cos \theta_{i,k-1} & \omega_{i,k-1} = 0 \\ \frac{v_{i,k-1}}{\omega_{i,k-1}} [\sin \theta_{i,k} - \sin \theta_{i,k-1}] & \omega_{i,k-1} \neq 0 \end{cases}, \\ \Delta y_{i,k-1} &= \begin{cases} T_s v_{i,k-1} \sin \theta_{i,k-1} & \omega_{i,k-1} = 0 \\ \frac{v_{i,k-1}}{\omega_{i,k-1}} [\cos \theta_{i,k-1} - \cos \theta_{i,k}] & \omega_{i,k-1} \neq 0 \end{cases}, \\ \Delta \theta_{i,k-1} &= T_s \omega_{i,k-1}.\end{aligned}$$

Proof. According to eq. (5.1), it can be derived that

$$\theta_{i,k} = \theta_{i,k-1} + \int_{t_{k-1}}^{t_k} \omega_i(\tau) d\tau.$$

As $\omega_i(\tau) = \omega_{i,k-1}$, $\forall \tau \in [t_{k-1}, t_k)$,

$$\theta_{i,k} = \theta_{i,k-1} + T_s \omega_{i,k-1}.$$

Furthermore, it can be easily obtained that

$$x_{i,k} = x_{i,k-1} + \int_{t_{k-1}}^{t_k} v_i(\tau) \cos(\theta_i(\tau)) d\tau. \quad (5.9)$$

As $v_i(\tau) = v_{i,k-1}$ and $\omega_i(\tau) = \omega_{i,k-1}$, $\forall \tau \in [t_{k-1}, t_k)$, eq. (5.9) can be simplified as

$$x_{i,k} = x_{i,k-1} + v_{i,k-1} \int_{t_{k-1}}^{t_k} \cos(\theta_{i,k-1} + \omega_{i,k-1}(\tau - t_{k-1})) d\tau.$$

When $\omega_{i,k-1} = 0$,

$$\int_{t_{k-1}}^{t_k} \cos(\theta_{i,k-1} + \omega_{i,k-1}(\tau - t_{k-1})) d\tau = T_s \cos \theta_{i,k-1},$$

and when $\omega_{i,k-1} \neq 0$,

$$\int_{t_{k-1}}^{t_k} \cos(\theta_{i,k-1} + \omega_{i,k-1}(\tau - t_{k-1})) d\tau = \frac{1}{\omega_{i,k-1}} [\sin \theta_{i,k} - \sin \theta_{i,k-1}].$$

Consequently, when $\omega_{i,k-1} = 0$,

$$x_{i,k} = x_{i,k-1} + T_s v_{i,k-1} \cos \theta_{i,k-1},$$

and when $\omega_{i,k-1} \neq 0$,

$$x_{i,k} = x_{i,k-1} + \frac{v_{i,k-1}}{\omega_{i,k-1}} [\sin \theta_{i,k} - \sin \theta_{i,k-1}].$$

Similarly, it can also be derived that when $\omega_{i,k-1} = 0$,

$$y_{i,k} = y_{i,k-1} + T_s v_{i,k-1} \sin \theta_{i,k-1},$$

and when $\omega_{i,k-1} \neq 0$,

$$y_{i,k} = y_{i,k-1} + \frac{v_{i,k-1}}{\omega_{i,k-1}} [\cos \theta_{i,k-1} - \cos \theta_{i,k}].$$

Thus, the proof completes. □

5.2 Comparative Analysis on Motion Capabilities of Rigid and Flexible Formations

In this section, the motion capabilities of rigid and flexible formations will be analyzed using the *formation flexibility* concept.

5.2.1 Formation Flexibility of Rigid Formation

In this part, the motion capabilities of perfect and semi-perfect rigid formations are analyzed, then the corresponding *formation flexibilities* f_{pr} and f_{spr} are calculated. According to definition 5.1, each robot in the formation should satisfy the kinematic

constraints eqs. (5.1) and (5.2). By taking the derivative on both sides of eq. (5.4), it can be obtained that

$$\begin{aligned}\dot{x}_{f_j} &= -p_{rf_j}\omega_l \sin \theta_l + (v_l - q_{rf_j}\omega_l) \cos \theta_l, \\ \dot{y}_{f_j} &= p_{rf_j}\omega_l \cos \theta_l + (v_l - q_{rf_j}\omega_l) \sin \theta_l.\end{aligned}\tag{5.10}$$

As each robot satisfies the nonholonomic constraint $\dot{x}_i \sin \theta_i = \dot{y}_i \cos \theta_i$, combining with eq. (5.10), it can be derived that

$$p_{rf_j}\omega_l \cos(\theta_{f_j} - \theta_l) = (v_l - q_{rf_j}\omega_l) \sin(\theta_{f_j} - \theta_l).\tag{5.11}$$

Consequently, the orientation of robot f_j can be obtained as follows.

$$\theta_{f_j} = \theta_l + \arctan \frac{p_{rf_j}\omega_l}{v_l - q_{rf_j}\omega_l}.\tag{5.12}$$

It should be noted that for eq. (5.12), $p_{rf_j}\omega_l$ and $v_l - q_{rf_j}\omega_l$ should not be equal to zero simultaneously. When $p_{rf_j}\omega_l = v_l - q_{rf_j}\omega_l = 0$, it is defined that $\theta_{f_j} = \theta_l$.

To form a perfect rigid formation, eq. (5.5) should be satisfied. From eq. (5.12), it can be obtained that $p_{rf_j}\omega_l = 0$, meaning that $p_{rf_j} = 0$ and/or $\omega_l = 0$, which implies that to form a perfect rigid formation, the leader must move along a straight line, otherwise, the rectangular longitudinal offset of the follower should be zero. Therefore, for a generic perfect rigid formation where p_{rf_j} is not necessarily equal to zero, the *formation flexibility* $f_{pr} = 0$, indicating that perfect rigid formation is rather clumsy. As a concession, semi-perfect rigid formation, which does not have the orientation requirement, will be analyzed in the following.

For each follower f_j satisfying dynamics eq. (5.1), its velocity v_{f_j} can be expressed as

$$v_{f_j} = \dot{x}_{f_j} \cos \theta_{f_j} + \dot{y}_{f_j} \sin \theta_{f_j}.\tag{5.13}$$

By taking eq. (5.10) into eq. (5.13) and after some straightforward manipulations,

it can be obtained that

$$v_{f_j} = v_l \sqrt{p_{rf_j}^2 c_l^2 + (1 - q_{rf_j} c_l)^2} \sin(\theta_{f_j} - \theta_l + \varphi), \quad (5.14)$$

where $\varphi = \text{atan2}(v_l - q_{rf_j} \omega_l, p_{rf_j} \omega_l)$. Furthermore, angular velocity ω_{f_j} can also be derived from eq. (5.12) as

$$\omega_{f_j} = \omega_l + \frac{p_{rf_j}(v_l \dot{\omega}_l - \dot{v}_l \omega_l)}{p_{rf_j}^2 \omega_l^2 + (v_l - q_{rf_j} \omega_l)^2}, \quad (5.15)$$

when $p_{rf_j} \omega_l$ and $v_l - q_{rf_j} \omega_l$ are not equal to zero simultaneously, and $\omega_{f_j} = \omega_l$ when $p_{rf_j} \omega_l = v_l - q_{rf_j} \omega_l = 0$.

The following theorem gives the range of the *formation flexibility* of semi-perfect rigid formation, in which the kinematic constraint eq. (5.2) are not violated for any robot.

Theorem 5.1. *Consider a semi-perfect rigid formation $\mathfrak{F}_{\text{spr}}$ containing n robots where the leader l moves at a positive and constant velocity v_l and angular velocity ω_l , both of which satisfy eq. (5.2). The velocity v_{f_j} and angular velocity ω_{f_j} of each follower f_j also satisfy eq. (5.2) if and only if the formation flexibility f_{spr} of $\mathfrak{F}_{\text{spr}}$ satisfies*

$$f_{\text{spr}} \in \mathbb{F}_{\text{spr}} = \bigcap_{j=1,2,\dots,n-1} (\mathbb{F}_{\text{spr},Aj} \cup \mathbb{F}_{\text{spr},Bj} \cup \mathbb{F}_{\text{spr},Cj}) \cap \mathbb{F}_{\text{spr},Dj}, \quad (5.16)$$

where

$$\begin{aligned}
\mathbb{F}_{\text{spr},Aj} &= \{f_{\text{spr}} \in \mathbb{R} | c_{l1j} \leq f_{\text{spr}} \leq c_{l2j}, q_{rfj} f_{\text{spr}} < 1\}, \\
\mathbb{F}_{\text{spr},Bj} &= \begin{cases} \{f_{\text{spr}} \in \mathbb{R} | c'_{l1j} \leq f_{\text{spr}} \leq c'_{l2j}, q_{rfj} f_{\text{spr}} > 1\}, & q_{rfj}^2 v_l^2 \geq (p_{rfj}^2 + q_{rfj}^2)(v_l^2 - v_{\min}^2) \\ \emptyset & \text{otherwise} \end{cases}, \\
\mathbb{F}_{\text{spr},Cj} &= \begin{cases} \{f_{\text{spr}} \in \mathbb{R} | f_{\text{spr}} = \frac{1}{q_{rfj}}\}, & q_{rfj} \neq 0, p_{rfj} = 0 \\ \{f_{\text{spr}} \in \mathbb{R} | f_{\text{spr}} = \frac{1}{q_{rfj}}\}, & q_{rfj} \neq 0, p_{rfj} \neq 0, \left| \frac{p_{rfj}}{q_{rfj}} v_l \right| \leq v_{\max} \\ \emptyset & \text{otherwise} \end{cases}, \\
\mathbb{F}_{\text{spr},Dj} &= \{f_{\text{spr}} \in \mathbb{R} | |g_{fj}(f_{\text{spr}})| \leq \omega_{\max}\}, \\
c_{l1j} &= \frac{v_l q_{rfj} - \sqrt{v_l^2 q_{rfj}^2 - (p_{rfj}^2 + q_{rfj}^2)(v_l^2 - v_{\max}^2)}}{v_l(p_{rfj}^2 + q_{rfj}^2)}, \\
c_{l2j} &= \frac{v_l q_{rfj} + \sqrt{v_l^2 q_{rfj}^2 - (p_{rfj}^2 + q_{rfj}^2)(v_l^2 - v_{\max}^2)}}{v_l(p_{rfj}^2 + q_{rfj}^2)}, \\
c'_{l1j} &= \frac{v_l q_{rfj} - \sqrt{v_l^2 q_{rfj}^2 - (p_{rfj}^2 + q_{rfj}^2)(v_l^2 - v_{\min}^2)}}{v_l(p_{rfj}^2 + q_{rfj}^2)}, \\
c'_{l2j} &= \frac{v_l q_{rfj} + \sqrt{v_l^2 q_{rfj}^2 - (p_{rfj}^2 + q_{rfj}^2)(v_l^2 - v_{\min}^2)}}{v_l(p_{rfj}^2 + q_{rfj}^2)}, \\
g_{fj}(f_{\text{spr}}) &= v_l f_{\text{spr}} + \frac{p_{rfj} \dot{f}_{\text{spr}}}{p_{rfj}^2 f_{\text{spr}}^2 + (1 - q_{rfj} f_{\text{spr}})^2}.
\end{aligned}$$

Proof. First of all, velocity v_{fj} is analyzed in the following three cases.

Case 1. $v_l - q_{rfj} \omega_l > 0$.

This case can be divided into four situations. 1) If $p_{rfj} \geq 0$ and $\omega_l \geq 0$, it can be obtained from eqs. (5.12) and (5.14) that $\theta_{fj} - \theta_l \in [0, \frac{\pi}{2})$ and $\varphi \in (0, \frac{\pi}{2}]$. Thus, $\theta_{fj} - \theta_l + \varphi \in (0, \pi)$. 2) If $p_{rfj} < 0$ and $\omega_l \geq 0$, it can be obtained from eqs. (5.12) and (5.14) that $\theta_{fj} - \theta_l \in (-\frac{\pi}{2}, 0]$ and $\varphi \in [\frac{\pi}{2}, \pi)$. Thus, $\theta_{fj} - \theta_l + \varphi \in (0, \pi)$. 3) If $p_{rfj} \geq 0$ and $\omega_l < 0$, this situation is similar with 2). Hence, it can also be obtained that $\theta_{fj} - \theta_l + \varphi \in (0, \pi)$. 4) If $p_{rfj} < 0$ and $\omega_l < 0$, this situation is similar with 1). Consequently, it can be obtained that $\theta_{fj} - \theta_l + \varphi \in (0, \pi)$. From

the discussions above and combined with lemma 5.1, $\theta_{f_j} - \theta_l + \varphi = \frac{\pi}{2}$. Therefore, $v_{f_j} = v_l \sqrt{p_{rf_j}^2 c_l^2 + (1 - q_{rf_j} c_l)^2}$. According to eq. (5.2), it can be derived that $c_{l1j} \leq c_l \leq c_{l2j}$.

Case 2. $v_l - q_{rf_j} \omega_l < 0$.

This case can also be divided into four situations. 1) If $p_{rf_j} \geq 0$ and $\omega_l \geq 0$, it can be obtained from eqs. (5.12) and (5.14) that $\theta_{f_j} - \theta_l \in (-\frac{\pi}{2}, 0]$ and $\varphi \in [-\frac{\pi}{2}, 0)$. Thus, $\theta_{f_j} - \theta_l + \varphi \in (-\pi, 0)$. 2) If $p_{rf_j} < 0$ and $\omega_l \geq 0$, it can be obtained that $\theta_{f_j} - \theta_l \in [0, \frac{\pi}{2})$ and $\varphi \in (-\pi, -\frac{\pi}{2}]$. Thus, $\theta_{f_j} - \theta_l + \varphi \in (-\pi, 0)$. 3) If $p_{rf_j} \geq 0$ and $\omega_l < 0$, this situation is similar with 2). Thus, $\theta_{f_j} - \theta_l + \varphi \in (-\pi, 0)$. 4) If $p_{rf_j} < 0$ and $\omega_l < 0$, this situation is similar with 1). Hence, $\theta_{f_j} - \theta_l + \varphi \in (-\pi, 0)$. According to the discussions above and lemma 5.1, $\theta_{f_j} - \theta_l + \varphi = -\frac{\pi}{2}$. Therefore, $v_{f_j} = -v_l \sqrt{p_{rf_j}^2 c_l^2 + (1 - q_{rf_j} c_l)^2}$. From eq. (5.2), it can be derived that $c'_{l1j} \leq c_l \leq c'_{l2j}$, if $q_{rf_j}^2 v_l^2 \geq (p_{rf_j}^2 + q_{rf_j}^2)(v_l^2 - v_{\min}^2)$. However, if $q_{rf_j}^2 v_l^2 < (p_{rf_j}^2 + q_{rf_j}^2)(v_l^2 - v_{\min}^2)$, the motion constraint eq. (5.2) cannot be satisfied.

Case 3. $v_l - q_{rf_j} \omega_l = 0$.

This is a special case. As $v_l \neq 0$, the condition implies that $q_{rf_j} \neq 0$ and $\omega_l \neq 0$. When $p_{rf_j} = 0$, $v_{f_j} = 0$ according to eq. (5.14), which satisfies eq. (5.2). When $p_{rf_j} \omega_l > 0$, $\theta_{f_j} - \theta_l = \frac{\pi}{2}$ according to eq. (5.12) and $\varphi = 0$ according to eq. (5.14). In this case, $\theta_{f_j} - \theta_l + \varphi = \frac{\pi}{2}$. When $p_{rf_j} \omega_l < 0$, $\theta_{f_j} - \theta_l = -\frac{\pi}{2}$ and $\varphi = \pi$. Thus, $\theta_{f_j} - \theta_l + \varphi = \frac{\pi}{2}$. Based on the discussions above, $v_{f_j} = v_l |p_{rf_j} c_l|$. As $c_l = \frac{1}{q_{rf_j}}$, to satisfy eq. (5.2), $\left| \frac{p_{rf_j}}{q_{rf_j}} v_l \right| \leq v_{\max}$ should hold.

Next, angular velocity ω_{f_j} will be analyzed. When $p_{rf_j} \omega_l$ and $v_l - q_{rf_j} \omega_l$ are not equal to zero simultaneously, eq. (5.15) can be rewritten as

$$\omega_{f_j} = v_l c_l + \frac{p_{rf_j} \dot{c}_l}{p_{rf_j}^2 c_l^2 + (1 - q_{rf_j} c_l)^2}.$$

To satisfy eq. (5.2), $|g_{f_j}(f_{\text{spr}})| \leq \omega_{\max}$. If $p_{rf_j} \omega_l = v_l - q_{rf_j} \omega_l = 0$, $\omega_{f_j} = \omega_l$, which

satisfies eq. (5.2).

Based on definition 5.1 and the discussions above, to guarantee that v_{f_j} and ω_{f_j} of each follower f_j satisfy eq. (5.2), f_{spr} should satisfy the condition eq. (5.16). Besides, the sufficiency of this condition is also obvious.

This completes the proof. $\square$

5.2.2 Formation Flexibility of Flexible Formation

The following theorem describes the *formation flexibility* of flexible formation.

Theorem 5.2. *Consider a flexible formation $\mathfrak{F}_f$ containing n robots where the leader l moves at a positive and constant velocity v_l and angular velocity ω_l , both of which satisfy eq. (5.2). The velocity v_{f_j} and angular velocity ω_{f_j} of each follower f_j also satisfy eq. (5.2) if and only if the formation flexibility f_f of $\mathfrak{F}_f$ satisfies*

$$f_f \in \mathbb{F}_f = \bigcap_{j=1,2,\dots,n-1} \{f_f \in \mathbb{R} | c_{l1j} \leq f_f \leq c_{l2j}, |f_f| \leq \frac{\omega_{\max}}{v_l}\}, \quad (5.17)$$

where

$$c_{l1j} = \begin{cases} \frac{v_l - v_{\max}}{v_l q_{ff_j}} q_{ff_j} \geq 0 \\ \frac{v_l - v_{\min}}{v_l q_{ff_j}} q_{ff_j} < 0 \end{cases}, c_{l2j} = \begin{cases} \frac{v_l - v_{\min}}{v_l q_{ff_j}} q_{ff_j} \geq 0 \\ \frac{v_l - v_{\max}}{v_l q_{ff_j}} q_{ff_j} < 0 \end{cases}.$$

Proof. According to the analyses of Property 2 and 3 in [44] and definition 5.1, eq. (5.17) can be easily derived. $\square$

5.2.3 Comparison on Formation Flexibilities of Rigid and Flexible Formations

Based on the analysis presented above, the formation flexibilities of rigid and flexible formations are compared in the following. Consider a perfect rigid formation $\mathfrak{F}_{\text{pr}}$,

a semi-perfect rigid formation $\mathfrak{F}_{\text{spr}}$ and a flexible formation $\mathfrak{F}_{\text{f}}$ whose leaders both move at the same positive and constant velocity v_l . The formation flexibilities of $\mathfrak{F}_{\text{pr}}$, $\mathfrak{F}_{\text{spr}}$ and $\mathfrak{F}_{\text{f}}$ are denoted by f_{pr} , f_{spr} and f_{f} , respectively, while the ranges of f_{pr} , f_{spr} and f_{f} are $\mathbb{F}_{\text{pr}}$, $\mathbb{F}_{\text{spr}}$ and $\mathbb{F}_{\text{f}}$, respectively.

According to the discussions above, $\mathbb{F}_{\text{pr}} = 0$, $\mathbb{F}_{\text{spr}}$ and $\mathbb{F}_{\text{f}}$ are given in eqs. (5.16) and (5.17) respectively. Obviously, it can be found that perfect rigid formation has the lowest motion capability among these three formations. From eqs. (5.16) and (5.17), it can be found that the motion capability of semi-perfect rigid formation depends on v_l , p_{rf_j} , q_{rf_j} , $v_{\min}$, $v_{\max}$ and $\omega_{\max}$, while the motion capability of flexible formation only depends on v_l , q_{rf_j} , $v_{\min}$ and $v_{\max}$. Besides, when $p_{rf_j} = 0$ for all j , it can be easily verified that $\mathbb{F}_{\text{spr}} = \mathbb{F}_{\text{f}}$. This is reasonable because a semi-perfect rigid formation with $p_{rf_j} = 0$ for all j is actually the same as a flexible formation with $p_{ff_j} = 0$ for all j . Furthermore, according to lemma 5.2, it can be easily proven that $\mathbb{F}_{\text{spr}} \subseteq \mathbb{F}_{\text{f}}$. Therefore, flexible formation is more flexible than semi-perfect rigid formation.

5.3 Vision-Based Flexible Formation Tracking Control

This section proposes a flexible formation tracking control scheme without global pose measurements, which is described in fig. 5.4. The proposed control scheme consists of four parts, vision-based pose estimation, leader trajectory reconstruction, reference generator and multi-objective control law, which will be illustrated in the following subsections.

Figure 5.4: Diagram of the proposed flexible formation control scheme

5.3.1 Vision-Based Pose Estimation

To implement leader-follower formation maneuvering, relative pose of the leader is of interest for estimation by the follower robot. In this work, a particle filter based relative pose estimation approach is employed to estimate the local position and orientation of the leader with respect to the follower, due to the nonlinearities and non-Gaussian noise in the process and observation models.

In practice, control inputs are imposed on the robot discretely, while measurements are also acquired periodically. Therefore, before going into further details, the discrete-time system and measurement equations should be given as follows.

$$\begin{aligned} \mathbf{x}_{i,k} &= f_{k-1}(\mathbf{x}_{i,k-1}, \mathbf{u}_{i,k-1}^w, \mathbf{w}_{i,k-1}), \\ \mathbf{y}_{f_j,k} &= h_{k-1}({}^{f_j} \mathbf{x}_{l,k}, \boldsymbol{\varepsilon}_{f_j,k}), \end{aligned} \quad (5.18)$$

where $f_{k-1}(\cdot)$ and $h_{k-1}(\cdot)$ are the discretized functions. For $f_{k-1}(\cdot)$, according to lemma 5.3, $f_{k-1}(\mathbf{x}_{i,k-1}, \mathbf{u}_{i,k-1}^w, \mathbf{w}_{i,k-1}) = \mathbf{x}_{i,k-1} + \Delta \mathbf{x}_{i,k-1}$.

Assume that the PDF of the initial state ${}^{f_j} \mathbf{x}_{l,0}$, which is the initial position and orientation of the leader l with respect to $\{f_j\}$, is already known. The relative pose

of the leader with respect to the follower can be estimated recursively by a particle filter based algorithm, which is presented in algorithm 5.1.

Algorithm 5.1 Vision-based pose estimation algorithm

Input:

$\mathbf{y}_{f_j,k}$, PDFs of $f_j \mathbf{x}_{l,0}$, $\mathbf{w}_{i,k}$ and $\varepsilon_{f_j,k}$

Output:

$f_j \hat{\mathbf{x}}_{l,k}$

- 1: Generate a set of L particles $\{f_j \mathbf{x}_{l,0}^{m,+}, q_k^m\}_{m=1}^L$ according to the PDF of $f_j \mathbf{x}_{l,0}$;
 - 2: **for** $k = 1, 2, \dots$ **do**
 - 3: Perform the time update step to obtain a set of *a priori* particles $f_j \mathbf{x}_{l,k}^{m,-}$ using eq. (5.19);
 - 4: **if** $\mathbf{y}_{f_j,k}$ exists **then**
 - 5: Update the weight q_k^m of each particle $f_j \mathbf{x}_{l,k}^{m,-}$ conditioned on $\mathbf{y}_{f_j,k}$;
 - 6: **else**
 - 7: $\{q_k^m\}_{m=1}^L \leftarrow \{q_{k-1}^m\}_{m=1}^L$;
 - 8: **end if**
 - 9: Normalization: $q_k^m = \frac{q_k^m}{\sum_{m=1}^L q_k^m}, \forall m \in \{1, 2, \dots, L\}$;
 - 10: **if** $\hat{N}_{eff} < N_{thr}$ **then**
 - 11: Resampling: generate a set of a posteriori particles $f_j \mathbf{x}_{l,k}^{m,+}$ based on $\{q_k^m\}_{m=1}^L$;
 - 12: $q_k^m = \frac{1}{L}, \forall m \in \{1, 2, \dots, L\}$;
 - 13: **end if**
 - 14: Estimation: $f_j \hat{\mathbf{x}}_{l,k} = \sum_{m=1}^L q_k^m f_j \mathbf{x}_{l,k}^{m,+}$;
 - 15: **end for**
-

To initialize, L initial particles are randomly generated on the basis of the PDF of $f_j \mathbf{x}_{l,0}$, which are approximated by a set of particles and associated weights, $\{f_j \mathbf{x}_{l,0}^{m,+}, q_k^m\}_{m=1}^L$. The proposed estimation algorithm consists of two stages, time update stage and measurement update stage. The time update stage predicts the relative pose of the leader in the local frame of the follower, which can be expressed as follows. As the pose estimation algorithm is carried out in the follower, control inputs of the leader, $\mathbf{u}_{l,k-1}^w$, is assumed to be known by the follower in real time via wireless communication.

$$\begin{aligned}
 f_j \mathbf{x}_{l,k}^{m,\sim} &= f_{k-1}(f_j \mathbf{x}_{l,k-1}^{m,+}, \mathbf{u}_{l,k-1}^w, \mathbf{w}_{l,k-1}), \\
 f_j \mathbf{x}_{f_j,k}^{m,\sim} &= f_{k-1}(\mathbf{0}, \mathbf{u}_{f_j,k-1}^w, \mathbf{w}_{f_j,k-1}), \\
 f_j \mathbf{x}_{l,k}^{m,-} &= R(f_j \theta_{f_j,k}^{m,\sim})(f_j \mathbf{x}_{l,k}^{m,\sim} - f_j \mathbf{x}_{f_j,k}^{m,\sim}).
 \end{aligned} \tag{5.19}$$

where $\forall m \in \{1, 2, \dots, L\}$ and

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In addition, $\mathbf{w}_{i,k-1}$ is randomly generated based on its PDF which is assumed to be known. In the measurement update stage, the weight q_k^m is updated for each particle $^{f_j}\mathbf{x}_{l,k}^{m,-}$ conditioned on the measurement $\mathbf{y}_{f_j,k}$, which can be obtained by evaluating the likelihood function $p(\mathbf{y}_{f_j,k} | ^{f_j}\mathbf{x}_{l,k}^{m,-})$ according to the measurement equation eq. (5.18) and the assumed known PDF of $\boldsymbol{\varepsilon}_{f_j,k}$. Resampling is employed in the algorithm when the effective particles quantity $\hat{N}_{eff} = \frac{1}{\sum_{m=1}^L (q_k^m)^2}$ is smaller than a threshold N_{thr} to prevent weight degeneracy.

5.3.2 Leader Trajectory Reconstruction

To form a flexible formation, the follower should keep the historical trajectory information of the leader. Technically, define a sliding most-recent time window $T_w = k_w T_s$, where k_w is a positive integer. The follower f_j keeps the trajectory of the leader l from a historical time step $k - k_w + 1$ to the current time step k . As there does not exist a global coordinate frame, the stored historical trajectory of l is with respect to $\{f_j\}$. Mathematically, the stored historical leader trajectory at time step k can be written as $^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k}$. It should be noted that the historical leader trajectory at time step $k - 1$, $^{f_j}\hat{\mathbf{x}}_{l,k-k_w:k-1}$, is with respect to $\{f_j\}$ at $k - 1$, however, when being updated to $^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k}$ at k , it should be transformed to $\{f_j\}$ at k as well. To perform the transformation, the pose variation of f_j should first be determined, then the coordinates of the historical trajectory of the leader are

transformed, which can be illustrated as

$$\begin{aligned} {}^{f_j}\tilde{\mathbf{x}}_{f_j,k} &= f_{k-1}(\mathbf{0}, \hat{\mathbf{u}}_{f_j,k-1}^w, \mathbf{0}), \\ {}^{f_j}\hat{\mathbf{x}}_{l,t} &= R({}^{f_j}\theta_{f_j,k}^w)({}^{f_j}\hat{\mathbf{x}}_{l,t} - {}^{f_j}\tilde{\mathbf{x}}_{f_j,k}), \end{aligned} \quad (5.20)$$

for $t = k - k_w + 1$ to k , where $\hat{\mathbf{u}}_{f_j,k-1}^w$ is the measurement of real $\mathbf{u}_{f_j,k-1}^w$ by encoders. Based on the discussions above, the leader trajectory reconstruction algorithm is presented in algorithm 5.2.

Algorithm 5.2 Leader trajectory reconstruction algorithm

Input:

$${}^{f_j}\hat{\mathbf{x}}_{l,k-k_w:k-1}, {}^{f_j}\hat{\mathbf{x}}_{l,k}, \hat{\mathbf{u}}_{f_j,k-1}^w$$

Output:

$${}^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k}$$

1: Marginalization:

$${}^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k-1} \leftarrow {}^{f_j}\hat{\mathbf{x}}_{l,k-k_w:k-1} \setminus {}^{f_j}\hat{\mathbf{x}}_{l,k-k_w};$$

2: Coordinate Transformation: eq. (5.20);

3: Augmentation:

$${}^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k} \leftarrow {}^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k-1} \cup {}^{f_j}\hat{\mathbf{x}}_{l,k};$$

5.3.3 Reference Generator

In this subsection, a reference generation algorithm is designed to find the reference state for the follower f_j from the historical trajectory of the leader, ${}^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k}$, obtained above. The proposed reference generator is presented in algorithm 5.3. In terms of the obtained historical trajectory, given a sufficiently small sampling time interval, the incremental trajectory between two points can be approximated as a straight line. Thus, the trajectory length between ${}^{f_j}\hat{\mathbf{x}}_{l,t+1}$ and ${}^{f_j}\hat{\mathbf{x}}_{l,t}$, $t \in \{t \in \mathbb{N} | k - k_w + 1 \leq t \leq k - 1\}$, is given as $\Delta d_{t,t+1} = \|{}^{f_j}\hat{\mathbf{p}}_{l,t} - {}^{f_j}\hat{\mathbf{p}}_{l,t+1}\|$.

Algorithm 5.3 Reference generation algorithm**Input:**

$${}^{f_j}\hat{\mathbf{x}}_{l,k-k_w+1:k}, p_{ff_j}, q_{ff_j}, {}^lv_{l,k}$$

Output:

$${}^{f_j}\mathbf{x}_{r,k}, {}^{f_j}v_{r,k}, {}^{f_j}\omega_{r,k}$$

- 1: Solve p according to $\arg \min_p \left| \sum_{t=p}^{k-1} \Delta d_{t,t+1} - p_{ff_j} \right|$;
- 2: Compute curvature ${}^{f_j}\hat{c}_{l,p} = \frac{{}^{f_j}\hat{\theta}_{l,t+1} - {}^{f_j}\hat{\theta}_{l,t}}{\Delta d_{t,t+1}}$;
- 3: Compute reference pose:
 ${}^{f_j}\mathbf{p}_{r,k} = {}^{f_j}\hat{\mathbf{p}}_{l,p} + {}^gR_l({}^{f_j}\hat{\theta}_{l,p}) {}^lp'_{f_j}, {}^{f_j}\theta_{r,k} = {}^{f_j}\hat{\theta}_{l,p}$;
- 4: Compute reference linear and angular velocities:
 ${}^{f_j}v_{r,k} = {}^lv_{l,k}(1 - q_{ff_j} {}^{f_j}\hat{c}_{l,p}), {}^{f_j}\omega_{r,k} = {}^{f_j}v_{r,k} {}^{f_j}\hat{c}_{l,p}$;

5.3.4 Multi-Objective Control Scheme

As the reference states of the followers have been obtained, the followers should track their own reference states such that the flexible formation can be formed. In this chapter, a linear trajectory tracking control law is employed to produce the control input $\mathbf{u}_{f_j,k}^w$ for the follower f_j at time step k , which is given as follows.

$$\mathbf{u}_{f_j,k}^w = B^{-1}\mathbf{u}_{f_j,k}, \mathbf{u}_{f_j,k} = \mathbf{u}_{Ff_j,k} + K_{s,k} {}^{f_j}\mathbf{x}_{r,k}, \quad (5.21)$$

where $\mathbf{u}_{Ff_j,k} = [{}^{f_j}v_{r,k} \cos {}^{f_j}\theta_{r,k}, {}^{f_j}\omega_{r,k}]^T$,

$$K_{s,k} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 \text{sgn}({}^{f_j}v_{r,k}) & k_3 \end{bmatrix},$$

$k_1 = k_3 = 2\zeta\sqrt{{}^{f_j}\omega_{r,k}^2 + b{}^{f_j}v_{r,k}^2}$, $k_2 = b|{}^{f_j}v_{r,k}|$, $\zeta \in (0, 1)$ is a damping coefficient, and $b > 0$ is a designed parameter. Interested readers can refer to [115] for details of this controller.

Using controller eq. (5.21), a flexible formation can be formed. However, in cluttered environments, the follower may collide with obstacles located on or close to the reference trajectory. Thus, the follower should have obstacle avoidance capability. Furthermore, obstacle avoidance reaction may lead the follower to collide with other

robots nearby. Therefore, to guarantee safety of the formation, a multi-objective control law should be designed for the follower such that when there are no potential collisions, the reference trajectory can be tracked accurately, but when there are obstacles or other robots nearby, obstacle and inter-robot collision avoidance should be performed.

To deal with obstacle avoidance, the nearest obstacle point to the robot, $\mathbf{p}_o$, is considered. To facilitate avoidance control design, some regions are defined, which are given as follows. $\Phi_{f_j}^O \triangleq \{\mathbf{p}_{f_j} | d_{f_j o} \geq r_o + \delta_o\}$, $\Omega_{f_j}^O \triangleq \{\mathbf{p}_{f_j} | r_o \leq d_{f_j o} < r_o + \delta_o\}$, $\Lambda_{f_j}^O \triangleq \{\mathbf{p}_{f_j} | r_o + \delta_o - \mu_o \leq d_{f_j o} < r_o + \delta_o\}$, where $d_{f_j o} = \|\mathbf{p}_{f_j} - \mathbf{p}_o\|$, r_o is the safe distance between robot and obstacle, δ_o is a buffer distance, and μ_o is a transition distance. Based on the regions defined above, the following two functions are designed.

$$G_o(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j o} < r_o \\ -\frac{1}{\delta_o^2}(d_{f_j o} - r_o)^2 & r_o \leq d_{f_j o} < r_o + \delta_o, \\ +\frac{2}{\delta_o}(d_{f_j o} - r_o) & \\ 1 & d_{f_j o} \geq r_o + \delta_o \end{cases}, \quad (5.22)$$

$$W_o(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j o} < r_o + \delta_o - \mu_o \\ -\frac{1}{\mu_o^2}(d_{f_j o} - r_o - \delta_o + \mu_o)^2 & r_o + \delta_o - \mu_o \leq d_{f_j o} < r_o + \delta_o \\ +\frac{2}{\mu_o}(d_{f_j o} - r_o - \delta_o + \mu_o) & \\ 1 & d_{f_j o} \geq r_o + \delta_o \end{cases}$$

where $G_o(\mathbf{p}_{f_j})$ is the potential function and $W_o(\mathbf{p}_{f_j})$ is the transition function.

To address inter-robot collision avoidance, the following regions are also defined. $\Phi_{f_j}^N \triangleq \bigcup_{n_j \in N(f_j)} \Phi_{f_j n_j}^N$, $\Omega_{f_j}^N \triangleq \bigcup_{n_j \in N(f_j)} \Omega_{f_j n_j}^N$, $\Lambda_{f_j}^N \triangleq \bigcup_{n_j \in N(f_j)} \Lambda_{f_j n_j}^N$, where $\Phi_{f_j n_j}^N \triangleq \{\mathbf{p}_{f_j} | d_{f_j n_j} \geq r_n + \delta_n\}$, $\Omega_{f_j n_j}^N \triangleq \{\mathbf{p}_{f_j} | r_n \leq d_{f_j n_j} < r_n + \delta_n\}$, $\Lambda_{f_j n_j}^N \triangleq \{\mathbf{p}_{f_j} | r_n + \delta_n - \mu_n \leq d_{f_j n_j} < r_n + \delta_n\}$. In addition, $d_{f_j n_j} = \|\mathbf{p}_{f_j} - \mathbf{p}_{n_j}\|_2$, $n_j \in N(f_j)$, $N(f_j)$ is the set of neighboring robots of f_j , r_n is the safe distance among robots, δ_n is a buffer distance, and μ_n is a transition distance. Then, the potential function $G_n(\mathbf{p}_{f_j})$ and transition

function $W_n(\mathbf{p}_{f_j})$ can be expressed as

$$G_n(\mathbf{p}_{f_j}) = \prod_{n_j \in N(f_j)} g_{n_j}^n(\mathbf{p}_{f_j}), \quad W_n(\mathbf{p}_{f_j}) = \prod_{n_j \in N(f_j)} w_{n_j}^n(\mathbf{p}_{f_j}), \quad (5.23)$$

where

$$g_{n_j}^n(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j n_j} < r_n \\ -\frac{1}{\delta_n^2}(d_{f_j n_j} - r_n)^2 & r_n \leq d_{f_j n_j} < r_n + \delta_n \\ +\frac{2}{\delta_n}(d_{f_j n_j} - r_n) & \\ 1 & d_{f_j n_j} \geq r_n + \delta_n \end{cases}$$

and

$$w_{n_j}^n(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j n_j} < r_n + \delta_n - \mu_n \\ -\frac{1}{\mu_n^2}(d_{f_j n_j} - r_n - \delta_n + \mu_n)^2 & r_n + \delta_n - \mu_n \leq d_{f_j n_j} < r_n + \delta_n \\ +\frac{2}{\mu_n}(d_{f_j n_j} - r_n - \delta_n + \mu_n) & \\ 1 & d_{f_j n_j} \geq r_n + \delta_n \end{cases}$$

From the discussions above, it can be deduced that to guarantee safety, the following constraint must be satisfied.

$$G(\mathbf{p}_{f_j}) = G_o(\mathbf{p}_{f_j})G_n(\mathbf{p}_{f_j}) > 0. \quad (5.24)$$

Then several regions are defined in the following definition based on the regions introduced above.

Definition 5.2. (*Safe Region, Dangerous Region, Transition Region, Critical Region*). Safe region of robot f_j is defined as $\Phi_{f_j} \triangleq \{\mathbf{p}_{f_j} | \mathbf{p}_{f_j} \in \Phi_{f_j}^O \cap \Phi_{f_j}^N\}$. Transition region of robot f_j is defined as $\Lambda_{f_j} \triangleq \{\mathbf{p}_{f_j} | \mathbf{p}_{f_j} \in \Lambda_{f_j}^O \cup \Lambda_{f_j}^N\}$. Dangerous region of robot f_j is defined as $\Omega_{f_j} \triangleq \{\mathbf{p}_{f_j} | \mathbf{p}_{f_j} \in \Omega_{f_j}^O \cup \Omega_{f_j}^N, \mathbf{p}_{f_j} \notin \Lambda_{f_j}\}$.

When robot f_j is located in safe region Φ_{f_j} , where there are no obstacles and neighboring robots nearby, the reference tracking controller eq. (5.21) is imposed on the

robot such that the desired flexible formation can be formed. However, when f_j is inside transition region Λ_{f_j} and dangerous region Ω_{f_j} , where some obstacles or neighboring robots are very close to it, the robot must be driven to avoid collisions with them. To facilitate the avoidance controller design, the following potential function is designed.

$$\varphi_{f_j} = \frac{\gamma}{(\gamma^\alpha + G(\mathbf{p}_{f_j}))^{\frac{1}{\alpha}}}, \quad (5.25)$$

where γ and α are positive parameters. From eqs. (5.22) to (5.24), it can be found that $\varphi_{f_j} \in [\frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}, 1]$. Based on eq. (5.25), a multi-objective controller is proposed as follows.

$$\mathbf{u}_{f_j} = R'(\theta_{f_j})(\mathbf{u}_{f_j}^t + \mathbf{u}_{f_j}^a), \quad (5.26)$$

where

$$R'(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

$\mathbf{u}_{f_j}^t = \dot{\mathbf{p}}_{f_j}^d + \rho(\mathbf{p}_{f_j}^d - \mathbf{p}_{f_j})$ and $\mathbf{u}_{f_j}^a = -(1 - W(\mathbf{p}_{f_j}))\mathbf{u}_{f_j}^t - k_{f_j}\nabla\varphi_{f_j}$ are the tracking part and avoidance part, respectively, $\mathbf{p}_{f_j}^d$ is the reference position, $W(\mathbf{p}_{f_j}) = W_o(\mathbf{p}_{f_j})W_n(\mathbf{p}_{f_j})$, ρ and k_{f_j} are positive parameters. The following theorem proves that using the proposed controller eq. (5.26), collision avoidance can be guaranteed.

Theorem 5.3. *Consider robot f_j described as eq. (5.1) and the reference position trajectory $\mathbf{p}_{f_j}^d$. Assume that the initial position of f_j satisfies eq. (5.24), i.e., $G(\mathbf{p}_{f_j}(0)) > 0$. When the controller eq. (5.26) is applied, collision avoidance can be maintained for all the time, i.e., $G(\mathbf{p}_{f_j}(t)) > 0, \forall t > 0$.*

Proof. First of all, we will prove that for $\dot{\mathbf{p}}_{f_j} = \mathbf{u}_{f_j}^t + \mathbf{u}_{f_j}^a$, $G(\mathbf{p}_{f_j}(t)) > 0$ for $\forall t > 0$. Inside Ω_{f_j} , according to eqs. (5.22), (5.23) and (5.26), it can be found that the tracking part $\mathbf{u}_{f_j}^t$ is neutralized such that

$$\dot{\mathbf{p}}_{f_j} = -k_{f_j}\nabla\varphi_{f_j}. \quad (5.27)$$

Based on eq. (5.25), it can be derived that

$$\nabla\varphi_{f_j} = -\frac{\gamma\nabla G(\mathbf{p}_{f_j})}{\alpha(\gamma^\alpha + G(\mathbf{p}_{f_j}))^{\frac{1}{\alpha}+1}}, \quad (5.28)$$

where $\nabla G(\mathbf{p}_{f_j}) = G(\mathbf{p}_{f_j})\left(\frac{\nabla G_o(\mathbf{p}_{f_j})}{G_o(\mathbf{p}_{f_j})} + \sum_{n_j \in N(f_j)} \frac{\nabla g_{n_j}^n(\mathbf{p}_{f_j})}{g_{n_j}^n(\mathbf{p}_{f_j})}\right)$.

Consider the following Lyapunov function candidate

$$V = \varphi_{f_j} - \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}. \quad (5.29)$$

By taking the derivative on both sides of eq. (5.29) with respect to time and combining with eq. (5.27), it can be obtained that

$$\dot{V} = -k_{f_j}(\nabla\varphi_{f_j})^T \nabla\varphi_{f_j} \leq 0. \quad (5.30)$$

As $G(\mathbf{p}_{f_j}) > 0$ implies that $0 \leq V < 1 - \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}$ and the fact that $V = 1 - \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}$ if and only if $G(\mathbf{p}_{f_j}) = 0$, combining with eq. (5.30) and $G(\mathbf{p}_{f_j}(0)) > 0$, it can be derived that $G(\mathbf{p}_{f_j}(t)) > 0$ for $\forall t > 0$.

According to Theorem 2 in [111], it can be concluded that inside Ω_{f_j} , control law eq. (5.26) designed for model in eq. (5.1) is convergent with the same attraction region as $\dot{\mathbf{p}}_{f_j} = \mathbf{u}_{f_j}^t + \mathbf{u}_{f_j}^a$. Therefore, using control law eq. (5.26), $G(\mathbf{p}_{f_j}(t)) > 0$ for $\forall t > 0$. $\square$

It is obvious that the tracking part $\mathbf{u}_{f_j}^t$ is actually a simple PD tracking controller, which controls the robot to track its reference position $\mathbf{p}_{f_j}^d$. From the proposed controller eq. (5.26), it can be observed that $\mathbf{u}_{f_j}^t$ works in transition region Λ_{f_j} such that the overall control is the combination of reference tracking and collision avoidance, which is beneficial in that even though potential collisions are tried to be avoided, the proposed controller can drive the robot to the reference position roughly as well. This special design is helpful to reduce the possibility of loss of

leader detection while avoiding potential collisions.

As in this chapter global pose measurements do not exist, the proposed control law eq. (5.26) should be modified such that all the variables are in the local frame $\{f_j\}$. In addition, for real robot implementations, the control law should be in discrete time domain. Therefore, the control law can be modified as follows.

$$\mathbf{u}_{f_j,k} = \mathbf{u}_{f_j,k}^t + \mathbf{u}_{f_j,k}^a, \quad (5.31)$$

where $\mathbf{u}_{f_j,k}^t = {}^{f_j}\hat{\mathbf{p}}_{r,k} + \rho^{f_j} \mathbf{p}_{r,k}$ and $\mathbf{u}_{f_j,k}^a = -(1 - W(\mathbf{p}_{f_j,k}))\mathbf{u}_{f_j,k}^t - k_{f_j} \nabla \varphi_{f_j,k}$. ${}^{f_j}\hat{\mathbf{p}}_{r,k} = ({}^{f_j}\mathbf{p}_{r,k} - {}^{f_j}\mathbf{p}_{r,k-1})/T_s$ is the estimation of ${}^{f_j}\dot{\mathbf{p}}_{r,k}$.

In summary, the proposed multi-objective control scheme can be presented as follows. When the robot f_j is located in safe region Φ_{f_j} , eq. (5.21) is employed, and when f_j is located in transition region Λ_{f_j} and dangerous region Ω_{f_j} , eq. (5.31) is applied.

5.4 Simulation and Experimental Results

In this section, the proposed flexible leader-follower tracking control method is first validated by a numerical example. Then real robot experimental results are presented to further demonstrate the efficacy of the proposed algorithm.

5.4.1 Simulation Results

To illustrate the effectiveness of the proposed method, a numerical simulation was performed in MATLAB using one leader and two followers. In the simulation, the leader followed a reference path, while two followers tracked the leader aiming at keeping a flexible formation. The parameters of the simulation platform are presented in table 5.1.

Table 5.1: Simulation parameters for flexible leader-follower formation tracking

Parameter	Value
Total time duration	160s
Sampling time interval	33.3ms
Vision detection max. range	5m
Vision detection min. range	1m
Camera horizontal FOV	100°
Kinematic process noise in each wheel	$\mathcal{N}(0, 0.05^2)\text{m/s}$
Visual observation noise in range	$\mathcal{N}(0, 0.05^2)\text{m/s}$
Visual observation noise in bearing	$\mathcal{N}(0, 0.3^\circ)\text{m/s}$
Visual observation noise in heading	$\mathcal{N}(0, 0.3^\circ)\text{m/s}$
Particle quantity L	6000
N_{thr}	3000
${}^l\mathbf{p}_{f1}$	$[1.5, 0.5]^T\text{m}$
${}^l\mathbf{p}_{f2}$	$[3.0, -0.5]^T\text{m}$
ζ	0.9
b	1
r_o	0.5m
r_n	0.5m
δ_o	0.3m
δ_n	0.3m
μ_o	0.2m
μ_n	0.2m
γ	1
α	1
ρ	2
k_a	0.8
$\mathbf{p}_l(0)$	$[0, 1]^T\text{m}$
$\theta_l(0)$	$\frac{\pi}{2}\text{rad}$
$\mathbf{p}_{f1}(0)$	$[-1, -1]^T\text{m}$
$\theta_{f1}(0)$	$\frac{\pi}{2}\text{rad}$
$\mathbf{p}_{f2}(0)$	$[1, -1]^T\text{m}$
$\theta_{f2}(0)$	$\frac{\pi}{2}\text{rad}$

Figure 5.5: Simulation results of flexible leader-follower formation tracking

Figure 5.6: Simulation results of flexible leader-follower formation tracking without obstacles

In the simulation, the leader followed a predesignated path in an obstacle environment. The followers tracked the leader in a flexible formation while performing obstacle and inter-robot collision avoidance simultaneously. Figure 5.5(a) depicts the map of the environment and the paths of the three robots. The black rectangles and disks represent the walls and obstacles respectively. The initial positions and orientations of the robots are given in table 5.1. From fig. 5.5(a), it can be observed that when there were no obstacles nearby, the robots can form the desired flexible formation successfully. When the formation were approaching the three round obstacles and at each turning, the robots would react to avoid collisions with obstacles and neighboring robots while still attempting to keep the desired formation. Figure 5.5(b) shows the formation error of follower f_1 during operation, from which it can be found that there are two major fluctuations, at 30s ~ 40s and 120s ~ 140s, respectively. A comparative simulation has also been performed under the same setting except that all the obstacles are removed, whose results are given in fig. 5.6. Figure 5.6(b) depicts the formation error of follower f_1 in this simulation. Combining figs. 5.5(b) and 5.6(b) with fig. 5.5(a), it can be inferred that these two major fluctuations of f_1 were caused by the first and the third round obstacles, respectively. As the robot should react to avoid collisions with obstacles and neighboring robots, the formation objective was temporarily less prioritized. After the avoidance reaction, the robots continued to form the desired formation. Besides the two major fluctuations, there are two slight fluctuations of f_1 at around 50s and 110s, respectively. From fig. 5.5(a), it can be deduced that these small fluctuations were caused by the turning movement at the second and the fourth walls, as follower f_1 was at the inner side at each of the two turns which was close to the walls. Similar phenomena can be found on follower f_2 during operation from figs. 5.5(c) and 5.6(c). Figures 5.5(d) and 5.5(e) describe the collision avoidance performance of two followers. Figure 5.5(d) depicts the distances from the two followers to their corresponding nearest obstacle point, respectively, while fig. 5.5(e) shows the inter-robot distances among three robots. From these two figures, it can be found that the distances are constantly kept larger than 0.5m, the designed safe distance, which demonstrates

the efficacy of the proposed multi-objective control scheme.

5.4.2 Experimental Results

To validate the proposed method in a real environment, an experiment was carried out at the Robotics I Laboratory, NTU, Singapore. The proposed algorithm was implemented on a multi-robot system composed of two Pioneer 3-AT mobile robots as shown in fig. 5.7. The leader was equipped with a Dell Precision M2800 Mobile Workstation, while the follower was equipped with a ZOTAC ZBOX-VR7N70 mobile PC and a ZED stereo camera. Both the robots were equipped with a Hokuyo UTM-30LX Scanning Laser Rangefinder. The algorithm was programmed using C++ language under ROS, Kinetic release. In the experiment, the leader moved autonomously and the proposed approach was tested on the follower. The testing parameters in the experiment are listed in table 5.2.

Figure 5.7: Experimental setup for flexible leader-follower formation tracking

In the experiment, the leader moved from a room to another one through a narrow door autonomously, while the follower tracked the leader trying to form a flexible formation with guaranteed collision avoidance at the same time. The map of the environment and the paths of two robots are both described in fig. 5.8(a). Figure 5.8(b) depicts the formation error of the follower, from which it can be observed

Table 5.2: Experimental parameters for flexible leader-follower formation tracking

Parameter	Value
Sampling time interval	33.3ms
Vision detection max. range	6m
Vision detection min. range	0.5m
Camera horizontal FOV	100°
Kinematic process noise in each wheel	$\mathcal{N}(0, 0.06^2)\text{m/s}$
Visual observation noise in range	$\mathcal{N}(0, 0.05^2)\text{m/s}$
Visual observation noise in bearing	$\mathcal{N}(0, 0.5^\circ)\text{m/s}$
Visual observation noise in heading	$\mathcal{N}(0, 0.5^\circ)\text{m/s}$
Particle quantity L	5000
N_{thr}	2500
${}^l\mathbf{p}_{f_1}$	$[1.6, -0.5]^T\text{m}$
ζ	0.9
b	3
r_o	0.4m
r_n	0.5m
δ_o	0.3m
δ_n	0.3m
μ_o	0.2m
μ_n	0.2m
γ	1
α	1
ρ	1
k_a	0.8

Figure 5.8: Experimental results of flexible leader-follower formation tracking

that although starting with a relatively large formation error, the follower achieved a satisfactory formation accuracy before 20s. It can be revealed from fig. 5.8(b) that the formation error experienced dramatic fluctuations between time points 30s and 50s, during which the follower was avoiding nearby obstacles, as illustrated in fig. 5.8(d). At around time point 50s, as revealed in figs. 5.8(b) and 5.8(c), the follower lost visual detection of the leader, which degraded the relative pose estimation accuracy, and thus increased the formation error. It should be noted that despite the detrimental effects brought by obstacle avoidance and visual measurement interruption, the amplitude of the formation error was bounded by 0.2m at most of the time, which is acceptable in practical applications. After the leader detection was recovered, the formation error converged to the neighborhood of zero again. Figure 5.8(d) depicts the distance from the follower to the nearest obstacle, measured by the onboard laser range finder. The distance is found always larger than 0.6m, which also indicates that both obstacle avoidance and inter robot collision avoidance were guaranteed.

5.5 Conclusions

This chapter has addressed the flexible leader-follower formation control problem of multiple nonholonomic mobile robots while global pose measurements are not available. First of all, the motion capabilities of rigid and flexible formations have been analyzed with the introduction of *formation flexibility* as an evaluation metric, resulting in the conclusion that flexible formation is superior to traditional rigid formation due to more flexible motions that its formation leader can perform. To achieve relative localization between the leader and the follower, a particle filter based relative pose estimation algorithm is employed aiming at estimating the position and orientation of the leader with respect to the local frame of the follower using the noise corrupted and intermittently interrupted visual observations. By using a trajectory reconstruction approach, the leader trajectory is transformed to

the current local coordinate system of the follower, from which the reference point is generated. To track the reference point, a multi-objective control law is proposed which can work in unknown obstacle environments with guaranteed collision avoidance. Simulation and real robot experiment were conducted to validate the efficacy of the proposed method.

Chapter 6

Conclusions and Future Work

This chapter summarizes the main results of this thesis and provides suggestions for future research.

6.1 Conclusions

This thesis investigates the formation control problem for multiple nonholonomic mobile robots by taking a series of practical constraints explicitly into consideration. The obtained results are summarized as follows.

1. Chapter 3 studies virtual-structure based formation tracking control problem for multiple nonholonomic mobile robots, while obstacle and inter-robot collision avoidance as well as connectivity maintenance are guaranteed at the same time. A new bounded potential function is introduced to formulate multiple constraints which is then incorporated into the proposed multi-region control scheme. Three regions, including safe region, dangerous region and transition region, are defined to facilitate controller design, which is used to prioritize different control objectives in different situations. A series of transition func-

tions are introduced to achieve smooth switching between trajectory tracking and avoidance control. The proposed control strategy can be utilized for safe virtual-structure formation tracking of multiple mobile robots in obstacle environments.

2. Chapter 4 investigates vision-based leader-follower formation tracking control problem for multiple nonholonomic mobile robots, where sensor limitations, collision avoidance and leader-loss situation, are explicitly taken into consideration. Sensor limitations and leader-follower collision avoidance requirement are modeled by distance and bearing angle constraints. A class of bounded barrier functions are introduced to formulate these constraints. To ensure robot safety in unknown obstacle environments, a multi-region obstacle avoidance algorithm is designed to balance different objectives in different regions. The leader-loss situation has been considered for the first time, and a fault-tolerant solution is proposed to recover visibility once it is broken. The proposed algorithm can achieve safe formation tracking for a snake-like multi-robot formation.
3. Chapter 5 addresses the flexible leader-follower formation tracking control problem for multiple nonholonomic mobile robots using relative pose measurements. A new evaluation metric, formation flexibility, has been introduced to analyze the motion capabilities of rigid and flexible formations. To achieve relative localization between the leader and the follower, a particle filter based relative pose estimation algorithm is employed to estimate the local position and heading angle of the leader with respect to the follower. A leader trajectory reconstruction approach is designed to transform the leader historical history to the current local frame of the follower, from which the reference point is generated for the follower to track. To guarantee robot safety in obstacle environments, a multi-objective control law is proposed for the follower to achieve and coordinate reference tracking and collision avoidance. The proposed strategy can be used in flexible leader-follower formation tracking

applications in unknown obstacle environments where global pose feedback is unavailable.

It should be noted that the proposed approaches in this thesis are all for the formation control problem of multiple differential-drive mobile robots. However, they are easy to be extended to some other types of mobile robots with similar kinematics like tricycle-type mobile robots and car-like mobile robots. These three types of robots cover most of the commonly used ground mobile robots nowadays. For robots with special mechanical design, the proposed algorithms may be modified according to the specific kinematics.

6.2 Recommendations for Future Work

Although multi-robot formation control has been investigated in depth in the literature and this thesis, there are still several interesting research problems that are worthy to be studied, which are outlined as follows.

1. Virtual-structure based formation control is significant in many practical applications such as sensitive area patrolling, multi-robot show and so forth, which has also been investigated in chapter 3. However, in almost all of the existing results, to realize virtual-structure formation control, it is assumed that global pose measurements are available, where most of them are based on GPS feedback and some others like the work in chapter 3 are based on a pre-generated map. Obviously, this assumption is not realistic in many applications which require robots to work in unknown and GPS-denied environments. Additionally, applying the relative localization method commonly used in leader-follower approaches to virtual-structure approaches seems to be limited due to range and FOV limitations of the stereo camera. Therefore, to

design a practical cooperative localization strategy for virtual-structure formation implementations is a meaningful research topic for future research.

2. Vision-based leader-follower formation tracking control has been studied in chapter 4 using polar coordinates, and the proposed approach can be easily generalized to a snake-like multi-robot formation with $N > 2$ members, which is advantageous in obstacle environments. However, for general leader-follower formations where multiple followers are included, collision avoidance amongst followers is considered to be a more challenging issue, which cannot be addressed properly with the proposed algorithm. Thus, this is another interesting research problem.
3. The proposed algorithms in this thesis aim at maintaining a fixed formation configuration for all the time, while obstacle avoidance is achieved using reactive collision avoidance approaches. However, even though reactive collision avoidance approaches are computationally efficient and easy to be implemented, a local trajectory re-planning manipulation based on the local map information should be beneficial in making the robot motion more smooth and natural. Therefore, in the future, a hierarchical approach which combines trajectory re-planning and reactive avoidance together can be investigated to achieve better obstacle avoidance performance. Furthermore, in obstacle environments, it is unnecessary to preserve a fixed formation shape for all the time. Formation reconfiguration problem can be well studied according to the surroundings to obtain more flexible and safer maneuvers in cluttered environments.
4. In many multi-robot formation applications, communication is necessary to coordinate and balance conflicts among robots. However, communication links may suffer from cyberattacks, time delay or packet drops in real implementations, which would cause performance degradation or even task failure of the whole system. In this case, secure algorithms can be investigated to deal with these issues, which as a result can make the system more robust and reliable.

5. In this thesis, algorithms are designed based on the kinematic model of the robot, which is common in practical applications as most of the off-the-shelf mobile robot platforms nowadays only provide velocity input interface with their low-level velocity controller inaccessible for the users. Such hierarchical structure is widely accepted in real control applications and can achieve satisfactory performance. However, for some other applications, like autonomous driving cars, where indexes such as robot speed, heading rate, acceleration, dexterity, etc. are critically required, dynamic model of the robot should be specifically considered and involved in the algorithm design, which is another meaningful but challenging research direction.

Author's Publications

Journal Papers

1. **Y. Wang**, M. Shan, Y. Yue, and D. Wang, "Vision-based flexible leader-follower formation tracking of multiple nonholonomic mobile robots in unknown obstacle environments," *IEEE Transactions on Control Systems Technology*, DOI: 10.1109/TCST.2019.2892031.
2. **Y. Wang**, D. Wang, and S. Zhu, "Cooperative moving path following for multiple fixed-wing unmanned aerial vehicles with speed constraints," *Automatica*, vol. 100, 82-89, 2019.
3. **Y. Wang**, D. Wang, S. Yang, and M. Shan, "A practical leader-follower tracking control scheme for multiple nonholonomic mobile robots in unknown obstacle environments," *IEEE Transactions on Control Systems Technology*, DOI: 10.1109/TCST.2018.2825943.
4. **Y. Wang**, D. Wang, and S. Zhu, "Formation tracking of multi-vehicle systems in unknown environments using a multi-region control scheme," *International Journal of Control*, vol. 90, no. 12, 2760-2771, 2017.
5. **Y. Wang**, D. Wang, and S. Zhu, "A new navigation function based decentralized control of multi-vehicle systems in unknown environments," *Journal of Intelligent & Robotic Systems*, vol. 87, no. 2, 363-377, 2017.

Conference Papers

1. **Y. Wang**, and D. Wang, “Non-singular moving path following control for an unmanned aerial vehicle under wind disturbances,” in *Proceedings of the 2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Dec. 2017, pp. 6442-6447.
2. **Y. Wang**, D. Wang, and B. C. Ng, “Finite time moving target tracking using nonholonomic vehicles with distance and bearing angle constraints,” in *Proceedings of the 2017 American Control Conference (ACC)*, May 2017, pp. 2962-2967.
3. **Y. Wang**, D. Wang, and E. Mihankhah, “Navigation of multiple mobile robots in unknown environments using a new decentralized navigation function,” in *Proceedings of the 2016 14th International Conference on Control, Automation, Robotics & Vision (ICARCV)*, Nov. 2016.

Papers in Review

1. **Y. Wang**, M. Shan, and D. Wang, “A comparative analysis on rigid and flexible formations of multiple differential-drive mobile robots: a motion capability perspective,” *submitted to the 2019 European Control Conference (ECC)*.

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