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Experimental observation of optical Weyl points

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Abstract

Weyl fermions are hypothetical two-component massless relativistic particles in three-dimensional (3D) space, proposed by Hermann Weyl in 1929. Their band-crossing points, called “Weyl points”, carry a topological charge and are therefore highly robust. There has been much excitement over recent observations of Weyl points in microwave photonic crystals and the semimetal TaAs. Here, we report on the first experimental observation of Weyl points of light at optical frequencies. These are also the first observations of “type-II” Weyl points for photons at any wavelength, which have strictly positive group velocity along one spatial direction. We use a 3D structure consisting of laser-written waveguides, and show the presence of type-II Weyl points by (1) observing conical diffraction along one axis when the frequency is tuned to the Weyl point; and (2) observing the associated Fermi arc surface states. The realization of Weyl points at optical frequencies allow these novel electromagnetic modes to be further explored in the context of linear, nonlinear, and quantum optics.

The observation of Weyl points, in microwave photonics [1] and condensed matter physics [2, 3], has attracted a great deal of attention because they constitute the simplest possible topologically-nontrivial band structure in 3D. Their topological protection guarantees the existence of “Fermi arc” surface states [4–7], and they can be associated with many interesting phenomena including chiral anomalies [8, 9], unconventional superconductivity [10], and large-volume single-mode lasing [11]. They take two distinct forms: type-I Weyl points have a Fermi surface (in the electronic context) or isofrequency surface (for photonics) that is point-like, whereas type-II Weyl points have a Fermi or isofrequency surface that is conical [12, 13]. In photonics, type-I Weyl points were predicted [14] and subsequently observed [1] in macroscopic photonic crystals at microwave frequencies. There have been significant efforts, both theoretical [14–20] and experimental [21–23], to realize Weyl points at the technologically important optical frequency regime; this is challenging, however, due to the need for 3D fabrication of intricate photonic crystal structures. Photonic type-II Weyl points have also been previously theoretically proposed [21, 24], though not experimentally observed.

Here we observe photonic type-II Weyl points, at optical frequencies, in a 3D photonic crystal structure of evanescently-coupled single-mode waveguides, fabricated using femtosecond direct laser writing [25] (see Methods section). The waveguides form an array aligned along a particular spatial axis z , as shown in Fig. 1(a). For an appropriately designed array, we show that the 3D photonic band structure is locally described by a 2×2 Weyl Hamiltonian

$$\hat{H} \approx v_{\perp} (\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y) + v_z \delta k_z (\hat{I} - |b| \hat{\sigma}_z), \quad (1)$$

whose eigenvalues $\delta\omega$ are the band frequencies relative to a chosen origin (the Weyl frequency). Here $\delta\mathbf{k}_{\perp} = (\delta k_x, \delta k_y)$ and δk_z are the wavevector components transverse and parallel to the waveguide axis respectively (relative to the Weyl point), with group velocities v_j , $\hat{\sigma}_{x,y,z}$ are the Pauli matrices, $\hat{I}$ is the identity operator, and b is a dimensionless parameter. For our system, $|b| \ll 1$. Eq. (1) is derived by taking the standard “paraxial” description of weakly-confined optical waveguide modes [26, 27], and casting the results into a 3D photonic band structure; details are provided in the Supplementary Information.

The Hamiltonian (1) describes a type-II Weyl point [13, 24]. As shown schematically in Fig. 1(b), the dispersion is strongly anisotropic, with both bands having positive group velocities in the z direction. This is because the waveguide modes move in a single direction

along z with negligible back-scattering, even as they undergo diffraction in the transverse directions x and y . The corresponding band structure has distinctive isofrequency surfaces: as shown in Fig. 1(b), the isofrequency surfaces at $\delta\omega = 0$ are cones in k space, whereas for small nonzero $\delta\omega$ the isofrequency surfaces become hyperboloids. We will use this fact in our experimental probe of the optical Weyl point.

Weyl points are topologically protected because the Pauli matrices, together with $\hat{I}$, span the space of 2×2 Hermitian matrices. As long as coupling to any other modes of the system can be neglected, within this two mode subspace the Weyl point acts as a source or sink of Berry curvature, carrying a quantized topological charge $C = \pm 1$. Under weak perturbations, a Weyl point merely moves in $\delta\omega$ and k space; it can be eliminated only via annihilation with a partner Weyl point of opposite charge. The Hamiltonian (1) with charge $C = -1$ can be modified to include a partner by replacing the $|b|\hat{\sigma}_z$ term with $|b|(1+\delta k_z/q)\hat{\sigma}_z$; the partner Weyl point with opposite charge $C = +1$ appears at $\delta k_z = -q$ and at a lower frequency, as shown in Fig. 1(c). At spatial boundaries, each such pair of Weyl points is associated with “Fermi arc” surface states, whose isofrequency dispersion curves form open arcs connecting the projections of the two Weyl points in the surface Brillouin zone [4–7] (Note that even though we refer to “Fermi” arcs, the modes in question are bosonic). Importantly, the surface states span the range of frequencies separating the type-II Weyl points, as shown in Fig. 1(d). This will provide us with another experimental probe for the optical Weyl point.

Our waveguide array design, depicted in Fig. 1(a), consists of a bipartite square lattice with two helical waveguides in each unit cell, both having clockwise helicity, radius R , and period Z in the z direction. A microscope image of a cross-sectional cut of the array at the input facet is shown in Fig. 2(a). Adjacent waveguides are out of phase by a half-cycle, so that they do not evolve in synchrony, but their nearest-neighbor distance changes through a period in z [28]; the design is closely linked to the concept of anomalous Floquet topological insulators [29]. The helical modulation breaks the inversion symmetry of the structure—a necessary condition for realizing Weyl points in time-reversal-invariant systems such as this one [14]. The time-reversal symmetry T implies the Brillouin zone must contain at least four Weyl points: for each Weyl point, T maps to an equivalent Weyl point of the same charge located at the opposite side of the Brillouin zone, corresponding to modes propagating in the opposite direction along z .

As mentioned above, waveguide arrays are commonly analyzed in the paraxial limit [26], by splitting the three spatial dimensions into the transverse (x, y) plane and the axial direction z . The Maxwell equations describing the diffraction of light through the array can be reduced to a two-dimensional (2D) Schrödinger-like equation, with z acting as a temporal direction. The operating wavelength (or frequency) enters as an adjustable parameter in the equation, while the wavevector component in the z -direction, k_z , acts as the effective “energy”. In other words, the paraxial equation describes *isofrequency surfaces* of the full 3D photonic band structure. Crucially, surfaces taken at different frequencies may correspond to topologically distinct 2D band structures of the paraxial equation. In the Supplementary Information, we show that the isofrequency surfaces for $\delta\omega > 0$ ($\delta\omega < 0$) correspond to conventionally insulating (topologically insulating) 2D paraxial band structures; the isofrequency surface at the Weyl point, $\delta\omega = 0$, corresponds to a 2D paraxial band structure poised at a topological transition. Notably, the $\delta\omega < 0$ regime is also where the Fermi arc surface states occur.

Previously, a honeycomb lattice of helical waveguides was used to realize a photonic topological insulator in Ref. [30]; however, that design does not readily support observable optical Weyl points. The reason that the present design is ideal for supporting Weyl points can be described as follows. The helical waveguides in the two sublattices come closer and farther apart from one another over the course of propagation along z . Therefore, the effect of waveguide helicity is strongest when either the lattice constant is small (so the helices get very close together and the modes are strongly coupled); or the wavelength is sufficiently long (such that the individual waveguide modes are large and therefore are strongly coupled). Therefore, the lattice is trivial if the waveguides are weakly coupled, and topological if they are strongly coupled. The transition takes place in the middle - and can be tuned to be straightforwardly accessed experimentally in the paraxial regime. The Weyl point occurs at the transition.

We analyze the presence of the Weyl point in systems of different lattice constant, a . Figure 2(b) shows a phase diagram in a and wavelength λ which indicates the lines in this parameter space for which Weyl points appear. The lines indicate the degeneracies in the bulk band structure (e.g., the 1D band structure cut in Fig. 2(c)) dividing the topological regime (which supports Fermi arcs) from the trivial regime (which does not). Note that while changes in a shift the Weyl points in frequency and wavevector, the Weyl

points exist for all a in this range. Their robustness under this smooth deformation is a signature of their topological protection. In Fig. 2(d-f), we plot three isofrequency surfaces calculated for structures of radius $R = 4\ \mu\text{m}$ and period $Z = 1\ \text{cm}$: (d) shows the $\delta\omega > 0$ case (lattice constant $a = 29\ \mu\text{m}$, wavelength $\lambda = 1450\ \text{nm}$), (e) shows the $\delta\omega = 0$ case ($a = 27\ \mu\text{m}$, $\lambda = 1525\ \text{nm}$); and (f) the $\delta\omega < 0$ case ($a = 25\ \mu\text{m}$, $\lambda = 1600\ \text{nm}$). These numerical results were obtained via the evolve-and-project method described in Ref. [28]. These calculations show that the Weyl point resides at $\mathbf{k} = (0, 0, 0.08\pi/Z)$ at $\lambda = 1525\ \text{nm}$ for $a = 27\ \mu\text{m}$, where $\mathbf{k}$ is the wavevector in the three-dimensional Brillouin zone of the photonic crystal band structure.

We prove the existence of a type-II Weyl point in two distinct experiments: (i) by observing conical diffraction associated with the isofrequency surface of a type-II Weyl point [13]; and (ii) by observing the Fermi arc surface states that emerge from the Weyl point.

The conical diffraction experiment relies on the fact that the isofrequency surface for the type-II Weyl point is a cone, as shown in Fig. 1(b) and Fig. 2(e). At this frequency, every Bloch wave has the same group velocity $v_\perp$ in the transverse plane. Thus, an initial wavepacket injected into the waveguide array, which can be decomposed into a superposition of Bloch waves, evolves into a ring as depicted in Fig. 3(a); the angle at which the ring expands is given by $v_\perp/v_z$. In the context of paraxial optics, this is precisely conical diffraction, marked by a fixed angle of diffraction relative to the axis of the waveguide array [31, 32].

We emphasize that a type-I Weyl point would not exhibit conical diffraction, because the associated isofrequency surface is a point rather than a cone. Moreover, for a given photonic band structure with a type-II Weyl point, conical diffraction occurs at a *single* probe wavelength; for $\delta\omega \neq 0$, the isofrequency surfaces are hyperboloids rather than cones, and hence an injected wavepacket produces a broad diffraction pattern rather than a ring. This wavelength selectivity is notably distinct from the conical diffraction phenomena observed for band structures lacking type-II Weyl points, such as a honeycomb lattice of straight waveguides [32]. Such a structure necessarily lacks Weyl points because it is inversion symmetric, but it exhibits a continuous family of conical isofrequency surfaces, corresponding to a Dirac line node [33]. Thus, in the honeycomb lattice, conical diffraction can be observed over a broad range of wavelengths. By contrast, conical diffraction is found to occur in our structure at a single wavelength, implying that the 3D bands meet at a single point.

We perform the conical diffraction experiment by injecting light into a single waveguide

at the input facet, which then couples through a waveguide splitter to a pair of neighboring waveguides (within one unit cell at the center of the lattice) with equal intensity and phase. The in-coupling region occupies the first 1 cm of the chip. The resulting diffraction patterns, imaged at the end of the chip, are shown in Fig. 3(b-d). For lattice constants $a = 29 \mu\text{m}$ (at wavelength 1450nm , Fig. 3(b)) and $a = 25 \mu\text{m}$ (at wavelength 1600nm , Fig. 3(d)), for which the isofrequency band structure is in a conventional and topological phase, respectively, we observe a filled-in disc-like diffraction pattern, which is characteristic of parabolic dispersion. When the lattice constant is tuned to $a = 27 \mu\text{m}$, we observe a clear ring-like conical diffraction pattern at $\lambda = 1525\text{nm}$, shown in Fig. 3(c). Deviations from a perfect ring occur because the lattice is discrete, which makes it impossible for the light to reside on a perfectly circular and zero-width ring. Additionally, the diffraction pattern is distorted by the square symmetry of the lattice, which induces a saddle point in the isofrequency surface at the Brillouin zone edge. Fig. 3(e-g) show full-wave beam-propagation simulations, which agree strongly with the experimental results of Fig. 3(b-d). Thus, the presence of conical diffraction clearly establishes the presence of the Weyl point.

To quantify the observation of conical diffraction, we define a dimensionless parameter that measures the degree to which the diffraction pattern is conical:

$$C = \frac{\int d\mathbf{r} r^2 |\psi(\mathbf{r})|^2}{\left(\int d\mathbf{r} r |\psi(\mathbf{r})|^2\right)^2}, \quad (2)$$

where $\mathbf{r} = (x, y)$ and $r = \sqrt{x^2 + y^2}$ is the distance from the origin (the origin is defined to be the center point between the two excited waveguides). The quantity C measures how “ring-like” a wavefunction is, and is reminiscent of the inertial moment of a rotating body in a mechanical system (but is dimensionless in this case). It takes the value 1 for an infinitely thin ring, and is larger than 1 for all other patterns. In Fig. 3(h-j) we plot C against wavelength for the three previous lattice constants, namely $a = 29, 27, 25 \mu\text{m}$. The monotonic behavior of Figs. 3(h) and 3(j) shows that for lattice constants $a = 29 \mu\text{m}, 25 \mu\text{m}$ the Weyl point does not occur within the wavelength range of the tunable laser (1450–1650 nm), or is very near the boundary. That said, the Weyl points do exist within the spectrum for each value of a described here. The minimum in C observed in Fig. 3(i) corresponds to conical diffraction (see Figs. 3(c) and 3(f) for experimental and numerically computed conical diffraction patterns). The conical diffraction associated with the isofrequency surface

provides direct evidence of the existence of the type-II Weyl point. As mentioned above, for every Weyl point there must be a partner Weyl point - which in this case occurs at $\mathbf{k} = (0, 0, 0.53\pi/Z)$ at wavelength $\lambda = 2113\text{nm}$ for $a = 27\mu\text{m}$, and lies outside the accessible parameter range of the experiment.

Next, we demonstrate the existence of the Fermi arc surface states associated with the type-II Weyl point. As stated earlier, these surface states span the range of frequencies between this Weyl point and its partner. Specifically, we expect them to appear below the Weyl frequency (where the 2D paraxial band structure becomes topologically non-trivial) and link the two bulk bands. To probe for surface states, we inject light via a single waveguide at the top of the lattice and observe the output facet. If a surface state is present, light should stay confined to the surface (see schematic depiction in Fig. 4(a)); otherwise it should diffract into the bulk. The positions of all Weyl points and corresponding Fermi arc states in the Brillouin zone are shown in Fig. 4(b)—note that we have identified a total of 4, but focus on the single Weyl point observable in our experiments. Fig. 4(c) shows the detailed surface state dispersion in the vicinity of the Weyl point - surface states only appear below the Weyl point frequency (i.e., above the Weyl point wavelength). Note that we plot the Fermi arc dispersion in the experimentally accessible wavelength region. Near the partner Weyl point (at $\lambda = 2113\text{nm}$), the waveguide array is close to cutoff, meaning the surface states have large penetration into the bulk of the array and surrounding material, making them impossible to compute accurately. Figure 4(d-h) shows the output wavepacket at wavelength 1550 nm, with decreasing lattice constant from left to right. Changing the lattice constant a simply shifts the wavelength at which the Weyl point occurs (larger a means it occurs at longer wavelength λ , or smaller frequency), allowing us to probe deeper into the $\delta\omega > 0$ or $\delta\omega < 0$ regimes. Fig. 4(d,e) shows that deep in the $\delta\omega > 0$ regime ($a = 29, 28\mu\text{m}$), the input light spreads into the bulk, indicating the absence of surface states. By contrast, Fig. 4(g,h) shows that deep in the $\delta\omega < 0$ regime ($a = 26, 25\mu\text{m}$), most of the light stays confined to the top and proceeds in a clockwise sense; this indicates the existence of surface states. Of course, some of the light is still coupled to bulk modes, which are present at every frequency; but the deeper we go into the $\delta\omega < 0$ regime, the stronger the surface state overlap, as shown in Fig. 4(h). Full-wave beam-propagation simulations that correspond to the cases shown in Fig. 4(d-h) are shown in Figs. 4(i-m). Corresponding isofrequency surfaces are shown in Figs. 4(n-r); these clearly show that states confined to

opposite surfaces, top and bottom (marked in red and blue), emerge for $\delta\omega < 0$, exactly as expected for the type-II Weyl points.

In conclusion, we have made the first direct experimental observations of a Weyl point at optical frequencies, which is a type-II Weyl point. We observed conical diffraction occurring at a single frequency (corresponding to the topological transition at the Weyl point), as well as Fermi arc surface states emerging from the Weyl point. The observation of Weyl points in optics can lead to a range of novel phenomena arising from the interplay of the Weyl dispersion and nonlinear [34], non-Hermitian [35], and quantum optics [36, 37].

Methods The waveguides are written in Corning Eagle XG borosilicate glass, refractive index $n_0 = 1.5078$. We employed a Titanium:Sapphire laser and amplifier system (Coherent:RegA 9000) with pulse duration 270fs, repetition rate 250kHz, and pulse energy 950nJ. The laser writing beam was sent through a beam shaping cylindrical telescope to control the shape and size of the focal volume. The beam was then focused inside the glass chip using an 80X, aberration-corrected microscope objective (NA = 0.75). A high-precision three-axis Aerotech motion stage (model ABL20020) is used to translate the sample during fabrication. Experiments are performed by butt-coupling a single mode optical fiber to waveguides at the input facet of the chip, which subsequently couples to the waveguide array. The input light is supplied by a tunable mid-infrared diode laser (Agilent 8164B), which can be tuned through the 1450 nm–1650 nm wavelength range. After a total propagation distance of 4 cm within the array, the light output from the waveguide array is observed using a 0.2NA microscope objective lens and a near-infrared InGaAs camera (ICI systems).

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FIG. 1. (a) Schematic diagram of the waveguide array structure, composed of two interpenetrating square lattices of helical waveguides, with the two sublattices out of phase from one another by a half-cycle. (b) Type-II Weyl dispersion diagram corresponding to the model Hamiltonian (1), in the δk_x - δk_z plane; isofrequency curves show a crossing at the Weyl point (corresponding to a cone in the full 3D band structure), and are hyperbolic at other frequencies. (c) Similar to (b), but plotted along $\delta k_x = \delta k_y = 0$, and including a ‘partner’ Weyl point at $\delta k_z = -q$, as well as the Fermi arc joining the two (green dashed line). (d) Similar to (b), but from a different angle, and showing the Fermi arc surface dispersion. The model parameters are $b = 0.25$ and $q = 10$.

FIG. 2. (a) Microscope image of the output facet of structure, representing a two-dimensional cut of the waveguide array for fixed z . (b) Numerically-determined phase diagram of the structure, as a function of lattice constant a and wavelength λ . Type-II Weyl points reside along the red curves, and Fermi arc surface states exist between these two curves (yellow region). (c) Bulk band structure for the two relevant bands plotted as a function of k_z (in the $k_x = k_y = 0$ -plane, using the extended-zone scheme). Type-II Weyl points arise at their intersection. (d-f) Isofrequency surfaces for the topologically trivial case (no Fermi arcs), at the Weyl point, and the topological (with Fermi arc) case, at $a = 29, 27, 25 \mu\text{m}$, at wavelengths 1450 nm, 1525 nm and 1600 nm, respectively. The open circles in the phase diagram shown in (b) correspond to the band structures in (d-f). All results in (b)–(f) are calculated numerically [28], using experimental parameters.

FIG. 3. (a) Schematic diagram of conical diffraction occurring in the waveguide array at a type-II Weyl point. (b-d) Intensity plots at the output facet, as we sweep through the Weyl point, at $a = 29, 27, 25 \mu\text{m}$ and wavelengths $1450 \text{ nm}, 1525 \text{ nm}$ and 1600 nm , respectively. The green circles indicate the position of the input waveguides. Clear conical diffraction is observed in (b), at the Weyl point. (Second row: e-g) Full-wave simulations corresponding to the parameters of (b-d). (Third row: h-j) Plot of the quantity C as a function of λ , which quantifies how ring-like the wavefunction for $a = 29, 27, 25 \mu\text{m}$, as a function of wavelength from $1450\text{--}1650 \text{ nm}$. In (h) and (j), there is no clear minimum, indicating the lack of a Weyl point within this wavelength range. However, (i) shows a clear minimum, at the wavelength where the Weyl point lies. This minimum corresponds to the wavefunction shown in (c and f).

FIG. 4. (a) Schematic diagram of light confinement to the surface of the structure as a result of a Fermi arc surface state. (b) Position of the four type-II Weyl points in the three-dimensional cubic Brillouin zone (represented by the enclosing box), with their topological charges indicated. WP1 and WP2 are partner Weyl points, and WP3 and WP4 are their time-reversed equivalents (exact values of $\mathbf{k}$ in text). (c) Plot of the Fermi arc dispersion relation in the surface Brillouin zone, as a function of k_x and k_z (note that the surface is terminated in the y -direction), calculated numerically using the method of Ref. [28]. (d-h) Output intensity plots, when light is input at the center of the top surface of the structure (indicated by green circles) at wavelength 1550 nm, for decreasing $a = 29, 28, 27, 26, 25 \mu\text{m}$. For decreasing lattice constant a , increased confinement to the surface of the structure indicates the formation and presence of Fermi arc surface states. (i-m) Corresponding full-wave beam propagations, showing strong agreement with experimental results. (n-r) Numerically calculated isofrequency contours, showing the presence of surface states forming at $a = 27 \mu\text{m}$ as the Weyl point is crossed. The red and blue curves indicate surface states on the top and bottom of the sample, respectively. The trajectories of the surface state wavepackets are indicated by red arrows.

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Supplementary Information: Experimental observation of optical Weyl points

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I. MAPPING THE 2D DIRAC HAMILTONIAN TO A 3D TYPE-2 WEYL HAMILTONIAN

As described in the main text, the steady-state diffraction of light through the waveguide array, at a given operating frequency ω , can be described by a paraxial approximation. Given the field amplitude $\mathcal{E}(x, y, z)$ with a certain fixed mode polarization (see the discussion in the main text), we define a slowly-varying envelope field $\psi(x, y, z)$ by

$$\mathcal{E}(x, y, z) = \psi(x, y, z) e^{ik_0 z}, \quad (1)$$

where $k_0 = n_0\omega/c$, and n_0 is the ambient refractive index of the medium. In the limit of negligible backscattering along the z axis, $\psi(x, y, z)$ satisfies a 2D Schrödinger equation

$$i \frac{\partial \psi}{\partial z} \approx \hat{H} \psi(x, y, z), \quad \text{where } \hat{H} = -\frac{1}{2k_0} \nabla_{\perp}^2 - \frac{k_0}{n_0} \delta n(x, y, z). \quad (2)$$

Here, x and y are the directions perpendicular to the waveguide axis, while z plays the role of time. $\nabla_{\perp}^2$ is the 2D Laplacian, and $\delta n = n - n_0$ is the refractive index shift.

For a photonic lattice in which δn is periodic in z , with period L_z , Eq. (2) turns into a Floquet problem. The Floquet eigenstates satisfy

$$\psi(x, y, z + L_z) = \psi(x, y, z) e^{i\beta z}, \quad (3)$$

$$\hat{H}_F \psi = \beta \psi, \quad (4)$$

where $\hat{H}_F$ is the Floquet Hamiltonian and its eigenvalue β is the Floquet quasi-energy. The Floquet Hamiltonian is defined using the z -evolution operator over one period (analogous to the usual time-evolution operator):

$$\exp(i\hat{H}_F L_z) = \mathcal{T} \exp \left[i \int_0^{L_z} dz \hat{H}(x, y, z) \right]. \quad (5)$$

Referring back to Eq. (1), we see that a 2D Floquet eigenstate with quasi-energy β corresponds to an electromagnetic Bloch wave of the underlying 3D photonic structure, having Bloch wave-vector component k_z , where

$$\beta = k_z - k_0. \quad (6)$$

The specific photonic lattice we are interested in (see the main text) has a Floquet Hamiltonian $\hat{H}_F$ that can be tuned to a topological transition at some frequency ω_0 . To lowest order in the detuning $\delta\omega \equiv \omega - \omega_0$, $\hat{H}_F$ is described by a Dirac equation,

$$\hat{H}_F \approx v_d (k_x \hat{\sigma}_x + k_y \hat{\sigma}_y) + b \frac{n_0 \delta\omega}{c} \hat{\sigma}_z, \quad (7)$$

for some dimensionless real constants v_d and b . The extra factor of n_0/c in the last term is for later convenience. As our experimental and simulation results show, the system can be tuned across the transition point—i.e., between negative and positive Dirac mass—by varying ω .

The Floquet eigenproblem of Eq. (7) depends on the detuning $\delta\omega$ as an implicit parameter, and yields the Floquet quasi-energy β as an eigenvalue. To make contact with the Weyl Hamiltonian, we must re-arrange it into an equation with $\delta\omega$ as the eigenvalue and β as a parameter. First, we must re-parameterize Eq. (6) in terms of the frequency detuning:

$$\beta = \delta k_z - \frac{\delta\omega}{c_0}, \quad \text{where } \begin{cases} c_0 \equiv c/n_0, \\ \delta k_z \equiv k_z - \omega_0/c_0. \end{cases} \quad (8)$$

Using this, we re-arrange the Floquet eigenproblem of Eq. (7) to obtain

$$\left[v_d (k_x \hat{\sigma}_x + k_y \hat{\sigma}_y) - \delta k_z \hat{I} \right] \psi = -(\hat{I} + b \hat{\sigma}_z) \frac{\delta\omega}{c_0} \psi, \quad (9)$$

where $\hat{I}$ denotes the identity matrix. This has the form of a generalized eigenvalue problem. To convert it into a Hamiltonian eigenvalue problem, we seek to factorize the operator on the right-hand side and rescale the state vectors:

$$\varphi = \hat{\mathcal{W}}\psi, \quad \hat{\mathcal{W}}^2 = -c_0^{-1}(\hat{I} + b\hat{\sigma}_z). \quad (10)$$

This would then satisfy

$$\hat{H}'\varphi = \delta\omega\varphi, \quad (11)$$

$$\hat{H}' \equiv \hat{\mathcal{W}}^{-1} \left[v_d(k_x\hat{\sigma}_x + k_y\hat{\sigma}_y) - \delta k_z \hat{I} \right] \hat{\mathcal{W}}^{-1}. \quad (12)$$

Assuming that $|b| < 1$ (see below for more discussion), we can directly verify that appropriate re-scaling operators, consistent with Eq. (10), are

$$\hat{\mathcal{W}} = i \left(\frac{1}{2c_0} - \sqrt{\frac{1-b^2}{4c_0^2}} \right)^{1/2} \hat{I} + i \left(\frac{1}{2c_0} + \sqrt{\frac{1-b^2}{4c_0^2}} \right)^{1/2} \hat{\sigma}_z, \quad (13)$$

$$\hat{\mathcal{W}}^{-1} = \frac{c_0}{\sqrt{1-b^2}} \left[i \left(\frac{1}{2c_0} - \sqrt{\frac{1-b^2}{4c_0^2}} \right)^{1/2} \hat{I} - i \left(\frac{1}{2c_0} + \sqrt{\frac{1-b^2}{4c_0^2}} \right)^{1/2} \hat{\sigma}_z \right]. \quad (14)$$

Due to the assumption that $|b| < 1$, all terms in square roots are all positive. Using Eq. (14) on Eq. (12) yields

$$\hat{H}' = \frac{v_d c_0}{\sqrt{1-b^2}} (k_x\hat{\sigma}_x + k_y\hat{\sigma}_y) + \frac{c_0}{1-b^2} \delta k_z (\hat{I} - |b|\hat{\sigma}_z), \quad (15)$$

We simplify this by defining

$$v_{\perp} = \frac{v_d c_0}{\sqrt{1-b^2}}, \quad (16)$$

$$v_z = \frac{c_0}{1-b^2}, \quad (17)$$

Thus, the Hamiltonian finally reduces to

$$\hat{H}' = v_{\perp} (k_x\hat{\sigma}_x + k_y\hat{\sigma}_y) + v_z \delta k_z (\hat{I} - |b|\hat{\sigma}_z). \quad (18)$$

This is a type-II Weyl Hamiltonian. Lorentz invariance is broken by the $v_z \delta k_z \hat{I}$ term, and as long as the assumption $|b| < 1$ holds the z component of the wave group velocity, $v_z(1 \pm |b|)$, is strictly positive.

We can perform an order-of-magnitude estimate for the dimensionless Lorentz invariance breaking parameter b . In the $a = 27\mu\text{m}$ sample the Weyl point occurs at $\lambda_1 \approx 1525\text{nm}$. Detuning to $\lambda_2 \approx 1550\text{nm}$ generates an effective mass of size $\approx \frac{\pi}{2Z}$, where $Z = 1\text{cm}$ is the helix pitch (the effective mass corresponds to the splitting Δk_z between the isofrequency surfaces at $k_x = k_y = 0$). Therefore $\Delta\omega n_0 b/c \approx \frac{\pi}{4Z}$. Using $k_0 = 2\pi n_0 \omega/c = 2\pi/\lambda$ and $\omega = c/(n_0 \lambda)$, $\Delta\omega = \frac{c}{n_0} (\frac{1}{\lambda_1} - \frac{1}{\lambda_2}) \approx \frac{c}{n_0 \lambda} \frac{\delta\lambda}{\lambda}$ and $b = \frac{\pi\lambda}{4Z} \frac{\lambda}{\delta\lambda} \approx 7 \times 10^{-3} \ll 1$ so we are well within validity of assumption $|b| < 1$. Note that the scale of b is set by $\delta n/n_0 \sim 10^{-3}$; in the “photonic crystal” limit when propagation is no longer paraxial and backscattering cannot be ignored, the constraint $|b| < 1$ may be violated.

II. WEYL POINT ROBUSTNESS

As noted in the main text, the topological protection of Weyl points originates from the fact that the Pauli matrices (together with the identity) form a complete basis for the space of 2×2 Hermitian matrices. In our system, this two-dimensional space corresponds to the two sublattices formed by our single mode waveguides. When there are more than two relevant (i.e. near-degenerate) modes the Pauli matrices no longer form a complete basis, and coupling between the two “Weyl modes” and the additional modes can potentially open up band gaps without requiring the annihilation of opposite charges. In our system there are two sources of additional near-degenerate modes. Firstly, in the paraxial regime the two polarization states are uncoupled, and therefore there exist two copies of each Weyl point, one for each polarization. However, their degeneracy is lifted by the intrinsic structural birefringence of the waveguides, which introduces a splitting in k_z calculated by full-wave Maxwell simulations to be $2 \times 10^{-6} \mu\text{m}^{-1}$,

large compared to the Brillouin zone size $\sim 1/Z = 10^{-4} \mu\text{m}^{-1}$. Furthermore, in condensed matter systems spin-orbit coupling between perfectly degenerate spin states does not open up a gap because the duplicate Weyl points have the same charge. Secondly, in the paraxial regime opposite propagation directions are decoupled. The helical modulation in the z direction could potentially (very weakly) couple counterpropagating waves, but for the weak index modulations used in our experiments such coupling would first manifest as photonic band gaps at the high symmetry points of the Brillouin zone, $k_z z = n\pi$. In our structure the Weyl points are displaced from these points, so this coupling can also be neglected.

III. EDGE STATES AND FERMI ARCS

Here we relate the observation of a topological transition and edge states in the main text to the existence of Fermi arcs in the type-II Weyl Hamiltonian. Consider a semi-infinite lattice with surface normal in the y direction. Looking for modes localized in the y direction, one can define a surface Bloch Hamiltonian $\hat{H}_{2D}(k_x, \omega)$ with eigenvalues k_z , and using the same procedure outlined above obtain an equivalent eigenvalue problem for ω . The Fermi arcs will form surfaces in the (k_x, k_z, ω) parameter space bounded by the bulk band edges, e.g. as shown in Fig. 4(c) of the main text.

To identify Weyl points based on the surface spectrum, instead of considering these full eigenvalue surfaces in 3D space it is more convenient to consider the surface states at a fixed frequency ω , which form lines (isofrequency contours) within the 2D Brillouin zone. As long as no bulk states are encountered, contours corresponding to topologically trivial surface states must be closed, while Fermi arcs form open contours terminating at Weyl points of opposite chirality [1, 2]. A sufficient criterion to identify a type-I Weyl point is to consider any closed loop within the Brillouin zone that avoids the bulk bands and count the number of intersecting isofrequency contours: if the number of intersecting lines is odd, there is a Fermi arc terminating at a Weyl point encircled by the loop.

Because of the tilted dispersion at a type-II Weyl point, an encircling loop at a single frequency cannot avoid bulk states and this procedure has to be modified according to Ref. [3]: instead of a loop at a single frequency, one should instead divide the loop into two segments within the bulk band gap at frequencies on either side of the Weyl point, e.g. as in Figs. 4(k,o) of the main text. When $\delta\omega > 0$ (Fig. 4k) there are no surface states that can cross the segment. When $\delta\omega < 0$ (Fig. 4o) a single surface state (per edge) will cross the segment if the projection of the Weyl point onto the $(k_x, k_z \sim \beta Z)$ plane is encircled. Equivalently, the surface states link form a single line joining the two bulk bands. Thus, the odd number of crossings induced by the topological transition of the 2D Floquet Hamiltonian signifies the type-II Weyl point.

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