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Parsimonious Random Vector Functional Link Network for Data Streams

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Abstract

The majority of the existing work on random vector functional link networks (RVFLNs) is not scalable for data stream analytics because they work under a batch learning scenario and lack a self-organizing property. A novel RVFLN, namely the parsimonious random vector functional link network (pRVFLN), is proposed in this paper. pRVFLN adopts a fully flexible and adaptive working principle where its network structure can be configured from scratch and can be automatically generated, pruned and recalled from data streams. pRVFLN is capable of selecting and deselecting input attributes on the fly as well as capable of extracting important training samples for model updates. In addition, pRVFLN introduces a non-parametric type of hidden node which completely reflects the real data distribution and is not constrained by a specific shape of the cluster. All learning procedures of pRVFLN follow a strictly single-pass learning mode, which is applicable for online time-critical applications. The advantage of pRVFLN is verified through numerous simulations with real-world data streams. It was benchmarked against recently published algorithms where it demonstrated comparable and even higher predictive accuracies while imposing the lowest complexities.

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1. Introduction

Significant growth of the problem space has led to a scalability issue for conventional machine learning approaches, which require iterating entire batches of data over multiple epochs. This phenomenon results in a strong demand for a simple, fast machine learning algorithm to be well-suited for deployment in numerous data-rich applications. This provides a strong case for research in the area of randomness in neural networks [5, 25], which was very popular in the late 80s and early 90s. This concept offers an algorithmic framework, which allows them to generate most of the network parameters randomly while still retaining reasonable performance [5]. One of the most prominent examples of randomness in neural networks is the random vector functional link network (RVFLN) which features solid universal approximation theory under strict conditions [7].

Due to its simple but sound working principle, randomness in neural networks has regained its popularity in the current literature [1, 26]. Nonetheless, the vast majority of works in the literature suffers from the issue of complexity which makes their computational complexity and memory burden prohibitive for data stream analytics since their complexities are manually determined and rely heavily on expert domain knowledge. The random selection of network parameters often causes the network complexity to go beyond what is necessary due to the existence of superfluous hidden nodes which contribute little to the generalization performance. Although the universal approximation capability of such an approach is assured only when sufficient complexity is selected, choosing a suitable complexity for a given problem entails expert-domain knowledge and is problem-dependent.

A novel RVFLN, namely the parsimonious random vector functional link network (pRVFLN), is proposed. pRVFLN combines the simple and fast working principles of RVFLN where all network parameters but the output weights are randomly generated with no tuning mechanism for hidden nodes. Since it characterises the online and adaptive nature of evolving intelligent systems, pRVFLN is capable of tracking any variations of data streams no matter how slow, rapid, gradual, sudden or temporal the drifts in data streams. It can initiate its learning structure from scratch with no initial

structure and its structure is self-evolved from data streams in the one-pass learning mode by automatically adding, pruning and recalling its hidden nodes [24]. Furthermore, it is compatible for online real-time deployment because data streams are handled without revisiting previously seen samples. pRVFLN is equipped with a hidden node pruning mechanism which guarantees a low structural burden and the rule recall mechanism which aims to address cyclic concept drift. pRVFLN incorporates a dynamic input selection scenario which makes possible the activation and deactivation of input attributes on the fly and an online active learning scenario which rules out inconsequential samples from the training process. pRVFLN is a plug-and-play learner where a single training process encompasses all learning scenarios in a sample-wise manner without pre-and/or post-processing steps.

pRVFLN offers at least four novelties: 1) it introduces the interval-valued data cloud paradigm which is an extension of the data cloud in [4]. This modification aims to induce robustness in dealing with data uncertainty caused by noisy measurement, noisy data, etc. Unlike conventional hidden nodes, the interval-valued data cloud is parameter-free and requires no parametrization. It evolves naturally following the real data distribution; 2) an online active learning scenario based on the sequential entropy method (SEM) is proposed. The SEM is derived from the concept of neighbourhood probability [35] but here the concept of the data cloud is integrated. The data cloud concept simplifies the sample selection process because the neighbourhood probability is inferred with ease from the activation degree of the data cloud; 3) pRVFLN is capable of automatically generating its hidden nodes on the fly with the help of a type-2 self-constructing clustering (T2SCC) mechanism [36]. This rule growing process differs from existing approaches because the hidden nodes are created from the rule growing condition, which considers the locations of the data samples in the input space; 4) pRVFLN is capable of carrying out an online feature selection process, borrowing several concepts of online feature selection (OFS) [30]. The original version [30] is generalized here since it is originally devised for linear regression and calls for some modification to be a perfect fit for pRVFLN. The prominent trait of this method lies in a flexible online feature selection scenario, which makes it possible to select or deselect input attributes on demand by assigning crisp weights (0 or 1) to input features.

The effectiveness of pRVFLN was thoroughly evaluated using numerous real-world data streams and was benchmarked against recently published algorithms in the literature, with pRVFLN demonstrating a highly scalable

approach for data stream analytics while retaining acceptable generalization performance. An analysis of the robustness of random intervals was performed. It is concluded that random regions should be carefully selected and should be chosen close to the true operating regions of a system being modelled. Moreover, we also present a sensitivity analysis of the predefined threshold and study the effect of learning components. Key mathematical notations are listed in Table 1.

The rest of this paper is structured as follows: related work is reviewed in Section 2; Section 3 elaborates basic concepts of pRVFLN, encompassing the principle of RVFLN and data cloud; network architecture of pRVFLN is discussed in Section 4; Section 5 explains the learning policy of pRVFLN; Numerical examples are presented in Section 6; conclusions are drawn in the last section of this paper.

2. Related Work

The concept of randomness in neural networks was initiated by Broomhead and Lowe in their work on radial basis function networks (RBFNs) [5]. A closed pseudo-inverse solution can be formulated to obtain the output weights of the RBFN and the centres of RBF units can be randomly sampled from data samples. This work later was generalized in [14], where the centres of the RBF neurons can be sampled from an independent distribution of the training data. The randomness in neural networks was substantiated by the findings of White [26], who developed a statistical test on hidden nodes. It was found that some nonlinear structures in the mapping function can be neglected without substantial loss of accuracy. In [26], the input weights of the hidden layers are randomly chosen. It is shown that the input weights are not sensitive to the overall learning performance.

A prominent contribution was made by Pao et al. with the random vector functional link network (RVFLN) [19]. This work presents a specific case of the functional link neural network [20], which embraces the concept of randomness in the functional link network. The universal approximation capability of the RVFLN is proven in [7] by formalising the Monte Carlo method approximating a limit-integral representation of a function. To attain the universal approximation capability, the hidden node should be chosen as either absolutely integrable or differentiable function. In practise, the region of random parameters should also be chosen carefully and the number of hidden nodes should be sufficiently large. There also exists another research

direction in this area, namely reservoir computing (RC), which puts forward a recurrent network architecture in order to take into account temporal dependencies between subsequent patterns and in order to avoid dependencies on time-delayed input attributes [16]. Recent advances in the area of randomness in neural network are found in the seminal work by Wang and Li, Stochastic Configuration Networks (SCNs) [29]. This work presents theoretical contribution of random selection of neural network parameters under selective and constructive manner using a supervisory mechanism. This work starts from the fact that random sampling of neural network parameters highly influence the stability and convergence of neural network training. Improper scope settings for the random parameters may cause a neural network to lose its learning power. It is confirmed in analysis of robustness in Section 6.4 of this paper. Comprehensive survey of randomness in neural network can be found in [25].

Table 1: Key Mathematical Notations

Symbol	Description
$A_t \in \mathbb{R}^n$	The input weight vector
β_t	The output of expansion layer
$X_t \in \mathbb{R}^n$	The input attribute
$T_t \in \mathbb{R}^m$	The target attribute
$x_e \in \mathbb{R}^{(2n+1) \times 1}$	The expanded input vector
$w_i \in \mathbb{R}^{(2n+1) \times 1}$	The output weight vector
$\tilde{G}_{i,temporal}$	The interval-valued temporal firing strength
$q \in \mathbb{R}^m$	The design factor
$\lambda \in \mathbb{R}^R$	The recurrent weight vector
$\tilde{\mu}_i \in \mathbb{R}^n$	The interval-valued local mean
$\tilde{\Sigma}_i \in \mathbb{R}^n$	The interval-valued mean square length
$\delta_i \in \mathbb{R}^n$	The uncertainty factor
$H(N X_n)$	The entropy of neighborhood probability
$I_c(\tilde{\mu}_i, X_t)$	The input coherence
$O_c(\tilde{\mu}_i, X_t)$	The output coherence
$\zeta()$	The correlation measure
$\zeta(\tilde{G}_{i,temp}, T_t)$	The mutual information between i -th rule and the target concept
$\Psi_i \in \mathbb{R}^{(2n+1) \times (2n+1)}$	The output covariance matrix

121 The vast majority of RVFLNs in the literature are not compatible with
 122 online real-time learning situations. This issue led to the development of
 123 online learning in RVFLNs, which follows a single-pass learning concept [34].
 124 The original version of RVFL is also applicable for online learning setting be-
 125 cause it makes use of the conjugate gradient algorithm. Some modification
 126 need to be implemented and involve the use of stochastic gradient principle
 127 where the gradient is obtained for every sample and iteration over a num-
 128 ber of epoch is not permitted. Nevertheless, this work is still built upon a
 129 fixed network structure which cannot evolve in accordance with up-to-date
 130 data trends. A concept of dynamic structure was offered in [12] by putting
 131 forward the notion of a growing structure. Notwithstanding their dynamic
 132 natures, concept drift remains an uncharted territory in these works because
 133 all parameters are chosen at random without paying close attention to the
 134 true data distribution. RC aims to address temporal system dynamics [16]
 135 but still does not consider a possible dramatic change of system behaviour.

136 **3. Basic Concepts**

137 This section outlines the foundations of pRVFLN encompassing the basic
 138 concept of RVFLN [19], the use of the Chebyshev polynomial as the func-
 139 tional expansion block [21] and the concept of data clouds [4].

140 *3.1. Random Vector Functional Link Network*

141 The idea of RVFLN was studied by Pao, Park and Sobajic in [19] and is
 142 one of the forms of the functional link network combined with the random
 143 vector approach [20]. It features the enhancement node performing the non-
 144 linear transformation of input attributes as well as the direct connection of
 145 input attributes to the output node. The activation degree of the enhance-
 146 ment node along with the input attributes is combined with a set of output
 147 weights to generate the final network output. The RVFLN only leaves the
 148 weight vector to be fine-tuned during the training process while the other
 149 parameters are randomly sampled from a carefully selected scope. Suppose
 150 that there are R enhancement nodes and n input attributes, the size of the
 151 output weight vector is $W \in \mathbb{R}^{(R+n)}$. The quadratic optimization problem is
 152 then formulated as follows:

$$E = \frac{1}{2N} \sum_{p=1}^N (t^{(p)} - Wd^{(p)})^2 \quad (1)$$

153 where $W \in \Re^{(R+n)}$ is the output weight vector. $d^{(p)}$ is the output of the
 154 enhancement node and N is the number of samples. The RVFLN is sim-
 155 ilar to a single hidden layer feedforward network except for the fact that
 156 the hidden nodes function as an enhancement of the input feature and there
 157 exists direct connection from the input layer to the output layer. The steep-
 158 est descent approach can be used to fine-tune the output weight vector. If
 159 matrix inversion using pseudo-inverse is feasible, a closed-form solution can
 160 be formulated. The generalization performance of RVFLN was examined in
 161 [19] and the RVFLNs convergence is also guaranteed to be attained within a
 162 number of iterations.

163 The RVFLN is a derivation of the functional link network [21]. That is,
 164 the hidden node or the enhancement node can be replaced by the functional
 165 expansion block generating a set of linearly independent functions of the
 166 entire input pattern. The functional expansion block can be formulated
 167 as trigonometric expansion [13], Chebyshev expansion, Legendre expansion,
 168 etc. [21] but our scope of discussion is limited to the Chebyshev expansion
 169 only due to its relevance to pRVFLN. Given the n -dimensional input vector
 170 $X = [x_1, x_2, \dots, x_n] \in \Re^{1 \times n}$ and its corresponding target variable y , the output
 171 of RVFLN with the Chebyshev functional expansion block is expressed as
 172 follows:

$$y = \sum_{j=1}^{2n+1} W_j \nu_j(A_n^T X_n + b_n) \quad (2)$$

173 where W_j is the output weight and $\nu_j()$ is the Chebyshev functional expansion
 174 mapping the n -dimensional input attribute and the input weight vector to
 175 the higher $2n + 1$ expansion space. As with the original RVFLN, the output
 176 weight vector W_j can be learned using any optimization method while other
 177 parameters, A_n and b_n , are randomly generated. The $2n + 1$ here results
 178 from the utilisation of the Chebyshev series up to the second order. The
 179 Chebyshev series is mathematically written as follows:

$$\nu_{order+1}(x) = 2(x)\nu_{order}(x) - \nu_{order-1}(x) \quad (3)$$

180 Because we are only interested in the Chebyshev series up to the second
 181 order, this results in $\nu_0(x) = 1, \nu_1(x) = x, \nu_2(x) = 2x^2 - 1$. Suppose that we
 182 deal with two dimensional input vector $x = [x_1, x_2]$, the Chebyshev function
 183 expansion leads to $\nu = [1, \nu_1(x_1), \nu_2(x_1), \nu_1(x_2), \nu_2(x_2)]$. The advantage of the
 184 Chebyshev functional link compared to other popular functional links such as
 185 trigonometric [13], Legendre, power function, etc. [21] lies in its simplicity of

Figure 1: Network Architecture of pRVFLN

computation. The Chebyshev function scatters fewer parameters to be stored into memory than the trigonometric function, while the Chebyshev function has a better mapping capability than the other polynomial functions of the same order. In addition, the polynomial power function is not robust against an extrapolation case. The functional expansion block can be also formed by using the Wavelet function [24] but it must be noted that the Wavelet function is sensitive to its initial values. It also requires a reliable tuning strategy to produce a good mapping of original input space.

3.2. Data Cloud

The concept of the data cloud offers an alternative to the traditional cluster concept where the data cloud is not shape-specific and evolves naturally

197 in accordance with the true data distribution. It is also easy to use because
 198 it is non-parametric and does not require any parameterization. This strat-
 199 egy is desirable because parameterization per scalar variable often calls for
 200 complex high-level approximation and/or optimization. This approach was
 201 inspired by the idea of RDE and was integrated in the context of the TSK
 202 fuzzy system [4]. Unlike a conventional fuzzy system where a degree of mem-
 203 bership is defined by a point-to-point distance, the data cloud computes an
 204 accumulated distance of the point of interest to all other points in the data
 205 cloud without physically keeping all data samples in the memory similar to
 206 the local data density. This notion has a positive impact on the memory and
 207 space complexity because the number of network parameters significantly
 208 reduces. The data cloud concept is formally written as:

$$\gamma_t^i = \frac{1}{1 + \|x_t - \mu_t^L\|^2 + \Sigma_t^L - \|\mu_t^L\|^2} \quad (4)$$

209 where γ_t^i denotes the i -th data cloud at the t -th observation. The data
 210 cloud evolves by updating the local mean μ_t^L and square length of i -th local
 211 region Σ_t^L as follows:

$$\mu_t^L = \frac{N_t^i - 1}{N_t^i} \mu_{t-1}^L + \frac{x_{t,N_i}}{N_t^i}, \mu_1^L = x_1 \quad (5)$$

$$\Sigma_t^L = \frac{N_t^i - 1}{N_t^i} \Sigma_{t-1}^L + \frac{\|x_{t,N_i}\|^2}{N_t^i}, \Sigma_1^L = \|x_1\|^2 \quad (6)$$

212
 213 where N_t^i denotes the number of samples associated to i -th cluster at the
 214 t -th observation. It is worth noting that these two parameters correspond to
 215 statistics of the i -th data cloud and are computed recursively with ease using
 216 standard recursive formulas. They do not impose a specific optimization or
 217 a specific setting to be performed to adjust their values.

218 4. Network Architecture of pRVFLN

219 pRVFLN utilises a local recurrent connection at the hidden node which
 220 generates the spatiotemporal activation degree. This recurrent connection
 221 is realized by a self-feedback loop of the hidden node which memorizes the
 222 previous activation degree and outputs a weighted combination between pre-
 223 vious and current activation degrees spatiotemporal firing strength. In the
 224 literature, there exist at least three types of recurrent network structures

Figure 2: Interval Valued Data Cloud

referring to its recurrent connections: global [9], interactive [13], and local [10], but the local recurrent connection is deemed to be the most compatible recurrent type in our case because it does not harm the local property, which assures stability when adding, pruning and fine-tuning hidden nodes. pRVFLN utilises the notion of the functional-link neural network where the expansion block is created by the Chebyshev polynomial up to the second order. Furthermore, the hidden layer of pRVFLN is built upon an interval-valued data cloud [4] where we integrate the idea of an interval-valued local mean into the data cloud.

The input coherence explores the similarity between new data and existing data clouds directly, while the output coherence focusses on their dissimilarity indirectly through a target vector as a reference. The input and output coherence formulates a test that determines the degree of confidence in the current hypothesis:

$$I_c(\tilde{\mu}_i, X_t) \leq \alpha_1, O_c(\tilde{\mu}_i, X_t) \geq \alpha_2 \quad (7)$$

Suppose that a pair of data points (X_t, T_t) is received at t -th time instant where $X_t \in \mathbb{R}^n$ is an input vector and $T_t \in \mathbb{R}^m$ is a target vector, while n

and m are respectively the number of input and output variables. Because pRVFLN works in a strictly online learning environment, it has no access to previously seen samples, and a data point is simply discarded after being learned. Due to the pre-requisite of an online learner, the total number of data N is assumed to be unknown. The output of pRVFLN is defined as follows:

$$y_o = \sum_{i=1}^R \beta_i \tilde{G}_{i,temporal}(A_t^T X_t + B_t), \tilde{G}_{temporal} = [\underline{G}, \overline{G}] \quad (8)$$

where R denotes the number of hidden nodes and β_i stands for the i -th output of the functional expansion layer, produced by weighting the weight vector with an extended input vector $\beta_i = x_e^T w_i$. $x_e \in \Re^{(2n+1) \times 1}$ is an extended input vector resulting from the functional link neural network based on the Chebyshev function up to the second order [21] as shown in (3) and $w_i \in \Re^{(2n+1) \times 1}$ is a connective weight of the i -th output node. The definition of β_i is rather different from its common definition in the literature because it adopts the concept of the expansion block, mapping a lower dimensional space to a higher dimensional space with the use of certain polynomials. This paradigm produces the extended input vector x_e and here the Chebyshev polynomial expansion block up to the second order is used to produce the extended input vector as aforementioned in Section 3.1. Suppose that three input attributes are given $X = [x_1, x_2, x_3]$, the extended input vector is expressed as the Chebyshev polynomial up to the second order $x_e = [1, x_1, \nu_2(x_1), x_2, \nu_2(x_2), x_3, \nu(x_3)]$. Note that the term 1 here represents an intercept of the output node to avoid going through the origin, which may risk an untypical gradient. $A_t \in \Re^n$ is an input weight vector randomly generated from a certain range. The bias B_t is removed for simplicity. $\tilde{G}_{i,temporal}$ is the i -th interval-valued data cloud, triggered by the upper and lower data cloud $\underline{G}_{i,temporal}, \overline{G}_{i,temporal}$. Note that recurrence is not seen in (8) because pRVFLN makes use of local recurrent layers at the hidden node. By expanding the interval-valued data cloud, the following is obtained:

$$y_o = \sum_{i=1}^R (1 - q_o) \beta_i \overline{G}_{i,temporal} + \sum_{i=1}^R q_o \beta_i \underline{G}_{i,temporal} \quad (9)$$

where $q \in \Re^m$ is a design factor to reduce an interval-valued function to a crisp one. It is worth noting that the upper and lower activation functions $\underline{G}_{i,temporal}, \overline{G}_{i,temporal}$ deliver spatiotemporal characteristics as a result of a

local recurrent connection at the i -th hidden node, which combines the spatial and temporal firing strength of the i -th hidden node. These temporal activation functions output the following.

$$\begin{aligned}\underline{G}_{i,temporal}^t &= \lambda_i \underline{G}_{i,spatial}^t + (1 - \lambda_i) \underline{G}_{i,temporal}^{t-1}, \\ \overline{G}_{i,temporal}^t &= \lambda_i \overline{G}_{i,spatial}^t + (1 - \lambda_i) \overline{G}_{i,temporal}^{t-1}\end{aligned}\quad (10)$$

where $\lambda \in \mathbb{R}^R$ is a weight vector of the recurrent link. The local feedback connection here feeds the spatiotemporal firing strength at the previous time step $\tilde{G}_{i,temporal}^{t-1}$ back to itself and is consistent with the local learning principle. This trait happens to be very useful in coping with the temporal system dynamic because it functions as an internal memory component which memorizes a previously generated spatiotemporal activation function at $t - 1$. Also, the recurrent network is capable of overcoming over-dependency on time-delayed input features and lessens strong temporal dependencies of subsequent patterns. This trait is desired in practise since it may lower the input dimension, because prediction is done based on the most recent measurement only. Conversely, the feedforward network often relies on time-lagged input attributes to arrive at a reliable predictive performance due to the absence of an internal memory component. This strategy at least entails expert knowledge for system order to determine the suitable number of delayed components.

The hidden node of the pRVFLN is an extension of the cloud-based hidden node, where it embeds an interval-valued concept to address the problem of uncertainty. Instead of computing an activation degree of a hidden node to a sample, the cloud-based hidden node enumerates the activation degree of a sample to all intervals in a local region on-the-fly. This results in local density information, which fully reflects real data distributions. This concept was defined in AnYa [4]. This concept is also the underlying component of TEDA-Class [11], all of which come from Angelov sound work of RDE [3]. This paper aims to modify these prominent works to the interval-valued case. Suppose that N_i denotes the support of the i -th data cloud, an activation degree of i -th cloud-based hidden node refers to its local density estimated recursively using the Cauchy function:

$$\tilde{G}_{i,spatial} = \frac{1}{1 + \sum_{k=1}^{N_i} \left(\frac{\tilde{x}_k - x_t}{N_i} \right)}, \quad \tilde{x}_k = [\underline{x}_{k,i}, \overline{x}_{k,i}], \quad \tilde{G}_{i,spatial} = [\underline{G}_{i,spatial}, \overline{G}_{i,spatial}] \quad (11)$$

where $\tilde{x}_k$ is k -th interval in the i -th data cloud and x_t is t -th data sample. It is observed that (11) requires the presence of all data points seen so far. Its recursive form is formalised in [4] and is generalized here to the interval-valued case:

$$\begin{aligned}\overline{G}_{i,spatial} &= \frac{1}{1 + \|A_t^T x_t - \overline{\mu}_{i,N_i}\|^2 + \overline{\Sigma}_{i,N_i} - \|\overline{\mu}_{i,N_i}\|^2}, \\ \underline{G}_{i,spatial} &= \frac{1}{1 + \|A_t^T x_t - \underline{\mu}_{i,N_i}\|^2 + \underline{\Sigma}_{i,N_i} - \|\underline{\mu}_{i,N_i}\|^2}\end{aligned}\quad (12)$$

where $\underline{\mu}_i, \overline{\mu}_i$ signify the upper and lower local means of the i -th cloud:

$$\begin{aligned}\underline{\mu}_{i,N_i} &= \left(\frac{N_i - 1}{N_i}\right)\underline{\mu}_{i,N_i-1} + \frac{x_{k,N_i} - \Delta_i}{\|N_i\|}, \quad \underline{\mu}_{i,1} = x_{1,N_i} - \Delta_i, \\ \overline{\mu}_{i,N_i} &= \left(\frac{N_i - 1}{N_i}\right)\overline{\mu}_{i,N_i-1} + \frac{x_{k,N_i} + \Delta_i}{\|N_i\|}, \quad \overline{\mu}_{i,1} = x_{1,N_i} + \Delta_i\end{aligned}\quad (13)$$

where Δ_i is an uncertainty factor of the i -th cloud, which determines the degree of tolerance against uncertainty. The uncertainty factor creates an interval of the data cloud, which controls the degree of tolerance for uncertainty. It is worth noting that a data sample is considered as a population of the i -th cloud when resulting in the highest density. Moreover, $\overline{\Sigma}_{i,N_i}, \underline{\Sigma}_{i,N_i}$ are the upper and lower mean square lengths of the data vector in the i -th cloud as follows:

$$\begin{aligned}\underline{\Sigma}_{i,N_i} &= \left(\frac{N_i - 1}{N_i}\right)\underline{\Sigma}_{i,N_i-1} + \frac{\|x_{k,N_i}\|^2 - \Delta_i}{\|N_i\|}, \quad \underline{\Sigma}_{i,1} = \|x_{1,N_i}\|^2 - \Delta_i, \\ \overline{\Sigma}_{i,N_i} &= \left(\frac{N_i - 1}{N_i}\right)\overline{\Sigma}_{i,N_i-1} + \frac{\|x_{k,N_i}\|^2 + \Delta_i}{\|N_i\|}, \quad \overline{\Sigma}_{i,1} = \|x_{1,N_i}\|^2 + \Delta_i\end{aligned}\quad (14)$$

296 Although the concept of the cloud-based hidden node was generalized in
297 TeDaClass [11] by introducing the eccentricity and typicality criteria, the
298 interval-valued idea is uncharted in [11]. Note that the Cauchy function is
299 asymptotically a Gaussian-like function, satisfying the activation function
300 requirement of the RVFLN to be a universal approximator.

301 Unlike conventional RVFLNs, pRVFLN puts into perspective a nonlinear
302 mapping of the input vector through the Chebyshev polynomial up to the
303 second order. Note that recently developed RVFLNs in the literature mostly
304 are designed with a zero-order output node [1]. The functional expansion

Figure 3: Fundamental working principle of pRVFLN

305 block expands the output node to a higher degree of freedom, which aims
 306 to improve the local mapping aptitude of the output node. pRVFLN imple-
 307 ments the random learning concept of the RVFLN, in which all parameters,
 308 namely the input weight A , design factor q , recurrent link weight λ , and
 309 uncertainty factor Δ , are randomly generated. Only the weight vector is
 310 left for parameter learning scenario w_i . Since the hidden node is parameter-
 311 free, no randomization takes place for hidden node parameters. This trait
 312 helps to improve consistency of random network in which bad random val-
 313 ues lead to poor performance. The network structure of pRVFLN and the
 314 interval-valued data cloud are depicted in Figs. 1 and 2 respectively.

315 **5. Learning Policy of pRVFLN**

316 This section discusses the learning policy of pRVFLN structured as fol-
 317 lows: Section 5.1 outlines the online active learning strategy, which actively
 318 samples relevant training samples for model updates; Section 5.2 deliberates
 319 the hidden node growing strategy of pRVFLN; Section 5.3 elaborates the hid-
 320 den node pruning and recall strategy; Section 5.4 details the online feature

321 selection mechanism; Section 5.5 explains the parameter learning scenario of
 322 pRVFLN; Section 5.6 discusses the effect of ranges of random parameters in
 323 RVFLN. Algorithm 1 shows the pRVFLN learning procedure.

324 5.1. Online Active Learning Strategy

325 The active learning component of the pRVFLN is built on the extended
 326 sequential entropy (ESEM) method, which is derived from the SEM method
 327 [35]. The ESEM method makes use of the entropy of the neighborhood prob-
 328 ability to estimate the sample contribution. The underlying difference from
 329 its predecessor [35] lies in the integration of the data cloud paradigm, which
 330 greatly relieves the effort in finding the neighborhood probability because the
 331 data cloud itself is inherent with the local data density, taking into account
 332 the influence of all samples in a local region. Furthermore, it handles the
 333 regression problem which happens to be more challenging than the classifi-
 334 cation problem because the sample contribution is estimated in the absence
 335 of a decision boundary. The concept of neighborhood probability refers to
 336 the probability of an incoming data stream sitting in the existing data clouds:

$$P(X_i \in R_i) = \frac{\sum_{k=1}^{N_i} \frac{M(X_t, x_k)}{N_i}}{\sum_{i=1}^R \sum_{k=1}^{N_i} \frac{M(X_t, x_k)}{N_i}} \quad (15)$$

337 where X_T is a newly arriving data point and x_n is a data sample, associated
 338 with the i -th data cloud and R_i is the number of data clouds. $M(X_T, x_k)$
 339 stands for a similarity measure, which can be defined as any similarity mea-
 340 sure. The bottleneck is however caused by the requirement to revisit already
 341 seen samples. This issue can be tackled by formulating the recursive expres-
 342 sion of (15). we would like to clarify that in [24], recursive update as usually
 343 done in realm of EIS [3, 2] is formed to compute (15) but the recursive up-
 344 date must be calculated per rule or locally. In the context of the data cloud,
 345 this issue becomes even simpler, because it is derived from the idea of local
 346 density and is computed based on the local mean [4]. (15) is then written as
 347 follows:

$$P(X_i \in R_i) = \frac{\Lambda_i}{\sum_{i=1}^R \Lambda_i} \quad (16)$$

348 Algorithm 1. Learning Architecture of pRVFLN

Algorithm 1: Parsimonious Random Vector Functional Link Network

Given a data tuple at t -th time instant $(X_t, T_t) = (x_1, \dots, x_n, t_1, \dots, t_m)$, $X_t \in \mathbb{R}^n$, $T_t \in \mathbb{R}^m$;
 set predefined parameters α_1, α_2

*/*Step 1: Online Active Learning Strategy/**

For $i=1$ **to** R **do**

 Calculate the neighborhood probability (16) with spatial firing strength (12)

End For

 Calculate the entropy of neighborhood probability (17)

IF (18) **Then**

*/*Step 2: Online Feature Selection/**

IF Partial=Yes **Then**

 Execute Algorithm 3

Else IF

 Execute Algorithm 2

End IF

*/*Step 3: Data Cloud Growing Mechanism/**

For $j=1$ **to** n **do**

 Compute $\xi(x_j, T_0)$

End For

For $i=1$ **to** R **do**

 Calculate input coherence and output coherence (19),(20)

For $o=1$ **to** m **do**

 Calculate $\xi(\tilde{\mu}_i, T_0)$ (21)

End For

IF (23) **Then**

 Assign a new sample to the winning data cloud, with the highest input coherence i^*

Else IF

 Create a new data cloud based on a new sample (24)

End IF

End For

*/*Step 4: Data Cloud Pruning and Recall Mechanism/**

For $i=1$ **to** R **do**

For $o=1$ **to** m **do**

 Calculate $\xi(\tilde{G}_{i,temp}, T_0)$

End For

IF (26) **Then**

 Discard i -th data cloud

End IF

End For

IF (27) **Then**

 Recall previously pruned rule i^* (28)

End IF

*/*Step 5: Adaptation of Output Weight/**

For $i=1$ **to** R **do**

 Update output weights using FWGRLS

End For

where Λ_i is a type-reduced activation degree $\Lambda_i = (1 - q)\overline{G}_{i,spatial} + q\underline{G}_{i,spatial}$. Once the neighbourhood probability is determined, its entropy is formulated as follows:

$$H(N|X_i) = - \sum_{i=1}^R P(X_i \in R_i) \log P(X_i \in N_i) \quad (17)$$

The entropy of the neighbourhood probability measures the uncertainty induced by a training pattern. A sample with high uncertainty should be admitted for the model update, because it cannot be well-covered by an existing network structure and learning such a sample minimises uncertainty. A sample is to be accepted for model updates, provided that the following condition is met:

$$H \geq thres \quad (18)$$

where *thres* is an uncertainty threshold. The higher the value of this paper the higher the number of training samples are to be discarded and vice versa. This parameter can be made adaptive rather than constant by dynamically adjusting its value to suit the learning context as done in [24]. Nevertheless, this scenario has to integrate a budget determining the maximum number of training samples. Otherwise, it often overspends and is very sensitive to the step size.

5.2. Hidden Node Growing Strategy

pRVFLN relies on the T2SCC method to grow interval-valued data clouds on demand. This notion is extended from the so-called SCC method [36] to adapt to the type-2 hidden node working framework. The significance of the hidden nodes in pRVFLN is evaluated by checking its input and output coherence through an analysis of its correlation to existing data clouds and the target concept. Let $\tilde{\mu}_i = [\underline{\mu}_i, \overline{\mu}_i] \in \mathbb{R}^{1 \times n}$ be a local mean of the *i*-th interval-valued data cloud (5), $X_t \in \mathbb{R}^n$ is an input vector and $T_t \in \mathbb{R}^n$ is a target vector, the input and output coherence are written as follows:

$$I_c(\tilde{\mu}_i, X_t) = (1 - q)\zeta(\overline{\mu}_i, X_t) + q\zeta(\underline{\mu}_i, X_t) \quad (19)$$

$$O_c(\tilde{\mu}_i, X_t) = (\zeta(X_t, T_t) - \zeta(\tilde{\mu}_i, T_t)), \quad \zeta(\tilde{\mu}_i, T_t) = (1 - q)\zeta(\overline{\mu}_i, T_t) + q\zeta(\underline{\mu}_i, T_t) \quad (20)$$

where $\zeta()$ is the correlation measure. Both linear and non-linear correlation measures are applicable here. However, the non-linear correlation measure is rather hard to deploy in the online environment, because it usually calls for the Discretization or Parzen Window method. The Pearson correlation measure is a widely used correlation measure but it is insensitive to the scaling and translation of variables as well as being sensitive to rotation [17]. The maximal information compression index (MCI) is one attempt to tackle these problems and it is used in the T2SCC to perform the correlation measure $\zeta()$ [17]:

$$\zeta(X_1, X_2) = \frac{1}{2}(\text{var}(X_1) + \text{var}(X_2) - \sqrt{(\text{var}(X_1) + \text{var}(X_2))^2 - 4\text{var}(X_1)\text{var}(X_2)(1 - \rho(X_1, X_2)^2)}) \quad (21)$$

$$\rho(X_1, X_2) = \frac{\text{cov}(X_1, X_2)}{\sqrt{\text{var}(X_1)\text{var}(X_2)}} \quad (22)$$

where (X_1, X_2) are substituted with $(\bar{\mu}_i, X_t), (\underline{\mu}_t, X_t), (\bar{\mu}_i, T_t), (\underline{\mu}_t, T_t), (X_t, T_t)$ to calculate the input and output correlation (19), (20). respectively stand for the variance of X , covariance of X_1 and X_2 , and Pearson correlation index of X_1 and X_2 . The local mean of the interval-valued data cloud represents a data cloud because it represents a point with the highest density. In essence, the MCI method indicates the amount of information compression when ignoring a newly observed sample. The MCI method features the following properties: 1) $0 \leq \zeta(X_1, X_2) \leq 0.5(\text{var}(X_1) + \text{var}(X_2))$, 2) a maximum correlation is given by $\zeta(X_1, X_2) = 0$, 3) a symmetric property $\zeta(X_1, X_2) = \zeta(X_2, X_1)$, 4) it is invariant against the translation of the dataset, and 5) it is also robust against rotation.

The input coherence explores the similarity between new data and existing data clouds directly, while the output coherence focusses on their dissimilarity indirectly through a target vector as a reference. The input and output coherence formulates a test that determines the degree of confidence in the current hypothesis:

$$I_c(\tilde{\mu}_i^*, X_t) \leq \alpha_1, O_c(\tilde{\mu}_i^*, X_t) > \alpha_2 \quad (23)$$

where $\alpha_1 \in [0.001, 0.01], \alpha_2 \in [0.01, 0.1]$ are predefined thresholds. If a hypothesis meets both conditions, a new training sample is assigned to a data

cloud with the highest input coherence i^* . Accordingly, the number of intervals N_{i^*} , local mean and square length $\tilde{\mu}_{i^*}, \tilde{\Sigma}_{i^*}$ are updated respectively with (21) and (22) as well as $N_{i^*} = N_{i^*} + 1$. A new data cloud is introduced, provided that the existing hypotheses do not pass either condition (7), that is, one of the conditions is violated. This situation reflects the fact that a new training pattern conveys significant novelty, which has to be incorporated to enrich the scope of the current hypotheses. Note that if a larger α_1 is specified, fewer data clouds are generated and vice versa, whereas if a larger α_2 is specified, larger data clouds are added and vice versa. The sensitivity of these two parameters is studied in the section V.E of this paper. Because a data cloud is non-parametric, no parameterization is committed when adding a new data cloud. The output node of a new data cloud is initialised:

$$W_{R+1} = W_{i^*}, \Psi_{R+1} = \bar{\omega}I \quad (24)$$

where $\bar{\omega} = 10^5$ is a large positive constant. The output node is set as the data cloud with the highest input coherence because this data cloud is the closest one to the new data cloud. Furthermore, the setting of covariance matrix Ψ_{R+1} leads to a good approximation of the global minimum solution of batched learning.

5.3. Hidden Node Pruning and Recall Strategy

pRVFLN incorporates a data cloud pruning scenario, termed the type-2 relative mutual information (T2RMI) method. This method was firstly developed in [6] for the type-1 fuzzy system. This method is convenient to apply here because it estimates mutual information between a data cloud and a target concept by analysing their correlation. Hence, the MCI method (21), (22) is valid to measure the correlation between two variables. Although this method has been well-established [6], to date, its effectiveness in handling data clouds and a recurrent structure as implemented in pRVFLN is an open question. Unlike both the RMI method that applies the classic symmetrical uncertainty method, the T2RMI method is formalised using the MCI method as follows:

$$\zeta(\tilde{G}_{i,temp}, T_t) = q\zeta(\underline{G}_{i,temp}, T_t) + (1 - q)\zeta(\overline{G}_{i,temp}, T_t) \quad (25)$$

where $\underline{G}_{i,temp}, \overline{G}_{i,temp}$ are respectively the lower and upper temporal activation functions of the i -th rule. The temporal activation function is included in (25) rather than the spatial activation function in order to account for the

inter-temporal dependency of subsequent training samples. The MCI method is chosen here because it possesses a significantly lower computational burden than the symmetrical uncertainty method but it is still more robust than a linear Pearson correlation index. A data cloud is deemed inconsequential, if the following is met:

$$\zeta_i > \text{mean}(\zeta) + 2\text{std}(\zeta) \quad (26)$$

where $\text{mean}(\zeta)$, $\text{std}(\zeta)$ are respectively the mean and standard deviation of the MCI during its lifespan. This criterion aims to capture an obsolete data cloud which does not keep up with current data distribution due to possible concept drift, because it computes the downtrend of the MCI values during its lifespan. It is worth mentioning that mutual information between hidden nodes and the target variable is a reliable indicator for changing data distributions because it monitors significance of a local region with respect to the recent data context.

The T2RMI method also functions as a rule recall mechanism to cope with cyclic concept drift. Cyclic concept drifts frequently happen in relation to the weather, customer preferences, electricity power consumption problems, etc. all of which are related to seasonal change. This points to a situation where a previous data distribution reappears in the current training step. Once pruned by the T2RMI, a data cloud is not forgotten permanently and is inserted into a list of pruned data clouds $R^* = R^* + 1$. In this case, its local mean, square length, population, an output node, and output covariance matrix $\tilde{\mu}_{R^*}, \tilde{\Sigma}_{R^*}, N_{R^*}, \beta_{R^*}, \Psi_{R^*}$, are retained in memory. Such data clouds can be reactivated in the future, whenever their validity is confirmed by an up-to-date data trend. It is worth noting that adding a completely new data cloud when observing a previously learned concept catastrophically erases the learning history. A data cloud is recalled subject to the following condition:

$$\max_{i^*=1, \dots, R^*}(\zeta_{i^*}) < \max_{i=1, \dots, R}(\zeta_i) \quad (27)$$

This situation reveals that a previously pruned data cloud is more relevant than any existing ones. This condition pinpoints that a previously learned concept reappears again. A previously pruned data cloud is then regenerated as follows:

$$\tilde{\mu}_{R+1} = \tilde{\mu}_{R^*}, \tilde{\Sigma}_{R+1} = \tilde{\Sigma}_{R^*}, N_{R+1} = N_{R^*}, \beta_{R+1} = \beta_{R^*}, \Psi_{R+1} = \Psi_{R^*} \quad (28)$$

Although previously pruned data clouds are stored in memory, all previously pruned data clouds are excluded from any training scenarios except (18).

459 Unlike its predecessors, this rule recall scenario is completely independent
 460 from the growing process (please refer to Algorithm 1).

461 5.4. Online Feature Selection Strategy

462 A prominent work, namely online feature selection (OFS), was developed
 463 in [30]. The appealing trait of OFS lies in its aptitude for flexible feature
 464 selection, as it enables the provision of different combinations of input at-
 465 tributes in each episode by activating or deactivating input features (1 or 0)
 466 in accordance to the up-to-date data trend. Furthermore, this technique is
 467 also capable of handling partial input attributes which are fruitful when the
 468 cost of feature extraction is too expensive. OFS is generalized here to fit the
 469 context of pRVFLN and to address the regression problem.

470 We start our discussion from a condition where a learner is provided with
 471 full input variables. Suppose that B input attributes are to be selected in
 472 the training process and $B < n$, the simplest approach is to discard the input
 473 features with marginal accumulated output weights $\sum_{i=1}^R \sum_{j=1}^2 \beta_{i,j}$ and maintain
 474 only B input features with the largest output weights. Note that the second
 475 term $\sum_{j=1}^2$ is required because of the extended input vector $x_e \in \mathbb{R}^{(2n+1)}$. The
 476 rule consequent informs a tendency or orientation of a rule in the target space
 477 which can be used as an alternative to gradient information. Although it is
 478 straightforward to use, it cannot ensure the stability of the pruning process
 479 due to a lack of sensitivity analysis of the feature contribution. To correct
 480 this problem, a sparsity property of the L1 norm can be analyzed to exam-
 481 ine whether the values of n input features are concentrated in the L1 ball.
 482 This allows the distribution of the input values to be checked to determine
 483 whether they are concentrated in the largest elements and that pruning the
 484 smallest elements wont harm the models accuracy. This concept is actualized
 485 by first inspecting the accuracy of pRVFLN. The input pruning process is
 486 carried out when the system error is large enough $T_t - y_t > \kappa$. Nevertheless,
 487 the system error is not only large in the case of underfitting, but also in
 488 the case of overfitting. We modify this condition by taking into account the
 489 evolution of system error $|\bar{e}_t + \sigma_t| > \kappa|\bar{e}_{t-1} + \sigma_{t-1}|$ which corresponds to the
 490 global error mean and standard deviation. The constant κ is a predefined
 491 parameter and fixed at 1.1. The output nodes are updated using the gradient
 492 descent approach and then projected to the L2 ball to guarantee a bounded

493 norm. Algorithm 2 details the algorithmic development of pRVFLN.

494

Algorithm 2. GOFS using full input attributes *Input:* α learning rate, χ regularization factor, B the number of features to be retained
Output: selected input features $X_{t,selected} \in \mathbb{R}^{1 \times B}$
For $t=1, \dots, T$
 */*Step 1: Check the reliability of model/**
 Make a prediction y_t
 IF $|\bar{e}_t + \sigma_t| > 1.1|\bar{e}_{t-1} + \sigma_{t-1}|$ // for regression case, check global system error $\hat{o} = \max_{o=1, \dots, m} (y_o) \neq T_t$ or // for classification, check whether a sample is correctly classified
 */*Step 2: Adapt the output weight vector and apply L2 projection/**
 $\beta_i = \beta_i - \chi \alpha \beta_i - \alpha \chi \frac{\partial E}{\partial \beta_i}, \beta_i = \min(1, \frac{1/\sqrt{\chi}}{\|\beta_i\|_2}) \beta_i$
 */*Step 3: Prune inconsequential input attribute/**
 Prune input attributes X_t except those of B largest $\sum_{i=1}^R \sum_{j=1}^2 \beta_{i,j}$
 Else
 $\beta_i = \beta_{i,t-1}$
 End IF
End For

496 where α, χ are respectively the learning rate and regularization factor. We
497 assign $\alpha = 0.2, \chi = 0.01$ following the same setting [30]. The optimization
498 procedure relies on the standard mean square error (MSE) as the objective
499 function and utilises the conventional gradient descent scenario:

$$\frac{\partial E}{\partial \beta_i} = (T_t - y_t) \left\{ \sum_{i=1}^R (1 - q) \bar{G}_{i,temporal} + \sum_{i=1}^R q \underline{G}_{i,temporal} \right\} \quad (29)$$

500 Furthermore, the predictive error has been theoretically proven to be bounded
501 in [30] and the upper bound is also found. One can also notice that the GOFS
502 enables different feature subsets to be elicited in each training observation t .

503 A relatively unexplored area of existing online feature selection is a situa-
504 tion where a limited number of features is accessible for the training process.
505 To actualise this scenario, we assume that at most B input variables can
506 be extracted during the training process. This strategy, however, cannot be
507 done by simply acquiring any B input features, because this scenario risks
508 having the same subset of input features during the training process. This

509 problem is addressed using the Bernoulli distribution with confidence level ϵ
 510 to sample B input attributes from n input attributes $B < n$. Algorithm 3
 511 provides an overview of feature selection procedure.

512

Algorithm 3. GOFs using partial input attributes *Input:* α learning rate, χ regularization factor, B the number of features to be retained, ϵ confidence level
Output: selected input features $X_{t,selected} \in \mathbb{R}^{1 \times B}$
For $t=1, \dots, T$
 */*Step 1: Generate partial input information/**
 Sample γ from Bernoulli distribution with confidence level ϵ
 IF $\gamma_t = 1$
 Randomly select B out of n input attributes $\tilde{X}_t \in \mathbb{R}^{1 \times B}$
 End IF
 */*Step 2: Check reliability of the model/**
 Make a prediction y_t
 IF $|\bar{e}_t + \sigma_t| > 1.1|\bar{e}_{t-1} + \sigma_{t-1}|$ // for regression, check the global system
 513 error $\hat{o} = \max_{o=1, \dots, m} (y_o) \neq T_t$ or // for classification, check whether a sample
 is correctly classified
 */*Step 3: Adapt the output weight vector and apply L2
 projection/**
 $\hat{X}_t = \tilde{X}_t / (B/n\epsilon) + (1 - \epsilon)$
 $\beta_i = \beta_i - \chi\alpha \beta_i - \alpha\chi \frac{\partial E}{\partial \beta_i}, \beta_i = \min(1, \frac{1/\sqrt{\chi}}{\|\beta_i\|_2})\beta_i$
 */*Step 4: Prune inconsequential input attribute/**
 Prune input attributes X_t except those of B largest $\sum_{i=1}^R \sum_{j=1}^2 \beta_{i,j}$
 Else
 $\beta_{i,t} = \beta_{i,t-1}$
 End IF
End For

514

515 As with Algorithm 2, the convergence of this scenario has been theoreti-
 516 cally proven and the upper bound is derived in [30]. One must bear in mind
 517 that the pruning process in Algorithm 2 and 3 is carried out by assigning
 518 crisp weights (0 or 1), which fully reflect activation and deactivation of input
 519 features.

5.5. Random Learning Strategy

pRVFLN adopts the random parameter learning scenario of the RVFLN, leaving only the output nodes W to be analytically tuned with an online learning scenario, whereas others, namely A_t, q, λ, Δ , can be randomly generated without any tuning process. To begin the discussion, we recall the output expression of pRVFLN as follows:

$$y_o = \sum_{i=1}^R \beta_i \tilde{G}_{i,temporal}(X_t; A_t, q, \lambda, \Delta) \quad (30)$$

Referring to the RVFLN theory, the activation function $\tilde{G}_{i,spatial}$ should satisfy the following conditions.

$$\int_R G^2(x) dx < \infty, \text{ or } \int_R [G'(x)]^2 dx < \infty \quad (31)$$

Furthermore, a large number of hidden nodes R is usually needed to ensure adequate coverage of data space because hidden node parameters are chosen at random [27]. Nevertheless, this condition can be relaxed in the pRVFLN, because the data cloud growing mechanism, namely the T2SCC method, partitions the input region in respect to real data distributions. The data cloud-based neurons are parameter-free and thus do not require any parameterization, which often calls for a high-level approximation or complicated optimization procedure. Other parameters, namely A_t, q, λ, Δ , are randomly chosen, and their region of randomisation should be carefully selected. Referring to [7], the parameters are sampled randomly from the following.

$$\begin{cases} b = -w_0 y_0 - \mu_0 \\ w_0 = \alpha c_0; \quad c_0 \in V^d; \quad V^d = [0; \Omega] \times [-\Omega; \Omega] \\ y_0 \in I^d \\ \mu_0 \in [-2\Omega, 2\Omega] \end{cases} \quad (32)$$

where μ, Ω, α are probability measures. Nevertheless, this strategy is impossible to implement in online situations because it often entails a rigorous trial-error process to determine these parameters. Furthermore, these ranges are derived to prove theoretically the universal approximation property of RVFL.

543 Assuming that a complete dataset $\Xi = [X, T] \in \Re^{N \times (n+m)}$ is observable,
 544 a closed-form solution of (7) can be defined to determine the output weights .
 545 Although the original RVFLN adjusts the output weight with the conjugate
 546 gradient (CG) method, the closed-form solution can still be utilised with
 547 ease [7]. The obstacle for the use of pseudo-inversion in the original work
 548 was the limited computational resources in 90's. Although it is easy to use
 549 and ensures a globally optimum solution, this parameter learning scenario
 550 however imposes revisiting preceding training patterns which are intractable
 551 for online learning scenarios. pRVFLN employs the FWGRLS method [22]
 552 to adjust the output weight. we also would like to clarify that FWGRLS can
 553 be seen as a derivation of FWRLS [3] where the weight decay term is added
 554 to retain the decay effect during the recursive updates. As the FWGRLS
 555 approach has been detailed in [22], it is not recounted here. The flowchart
 556 of pRVFLN is visualized in Fig. 3.

Table 2: Details of Experimental Procedure

Section	Mode	Number of Runs	Benchmark Algorithm	Pred. Parameters	NS	NI
A (Nox Emission)	Direct Partition	10 times	GENEFIS, eTS, simpeTS, DFNN, GDFNN, FAOS-PFNN, ANFIS, BARTFIS	$\alpha_1 = 0.002, \alpha_2 = 0.02$	826	170
	Cross Validation	5 times per fold	DNNE, Online RVFLN, Batch RVFLN	$\alpha_1 = 0.002, \alpha_2 = 0.02$		
B (Tool Cond. Mon.)	Direct Partition	10 times	GENEFIS, eTS, simpeTS, DFNN, GDFNN, FAOS-PFNN, ANFIS, BARTFIS	$\alpha_1 = 0.002, \alpha_2 = 0.02$	630	12
	Cross Validation	5 times per fold	DNNE, Online RVFLN, Batch RVFLN	$\alpha_1 = 0.002, \alpha_2 = 0.02$		
C (Nox E., Tool Cond. Mon.)	Cross Validation	5 times per fold	N/A	$\alpha_1 = 0.002, \alpha_2 = 0.02$	As above	As above
D (Mackey Glass)	Direct Partition	10 times	N/A	$\alpha_1 = 0.002, \alpha_2 = 0.02$	3500	4
E (BJ gas furnace)	Direct Partition	10 times	N/A	N/A	290	2

557 *5.6. Robustness of RVFLN*

558 The network parameters are usually sampled uniformly within a range of
559 $[-1,1]$ in the literature. A new finding of Li and Wang in [12] exhibits that
560 randomly generating network parameters with a fixed scope $[-\alpha, \alpha]$ does not
561 ensure a theoretically feasible solution or often the hidden node matrix is
562 not full rank. Surprisingly, the hidden node matrix was not invertible in
563 all their case studies when randomly sampling network parameters in the
564 range of $[-1,1]$ and far better numerical results were achieved by choosing
565 the scope $[-200,200]$. This trend was consistent with different numbers of
566 hidden nodes. How to properly select scopes of random parameters and
567 its corresponding distribution still require in-depth investigation [26]. In
568 practice, a pre-training process is normally required to arrive at a decent
569 scope of random parameters. Note that the range of random parameters
570 by Igel'nik and Pao [7] is still at the theoretical level and does not touch the
571 implementation issue. We study different random regions in Section 6.4 to see
572 how pRVFLN behaves under variations of the scope of random parameters.

Table 3: Prediction of Nox emissions Using Time-Series Mode

Model	RMSE	Node	Input	Runtime	Network	Samples
pRVFLN (P)	0.04±0.0009	1	5	3.4±0.14	11	596±0
pRVFLN (F)	0.04±0.009	1	5	3.46±0.25	11	596±0
eT2Class	0.045	2	170	17.98	117304	667
GENEFIS	0.1	7	18	6.59	2268	667
RIVMcSFNN	0.05	1	146	6.59	128.62	667
Simp_eTS	0.14	5	170	5.5	1876	667
BARTFIS	0.11	4	170	5.55	52	667
DFNN	0.18	548	170	4332.9	280198+NS	667
GDFNN	0.48	215	170	2144.1	109865	667
eTS	0.38	27	170	1098.4	13797	667
FAOS-PFNN	0.06	6	170	14.8	2216+NS	667
ANFIS	0.15	2	170	100.41	17178	667

Table 4: Prediction of Nox emissions Using CV Mode

Model	NRMSE	Node	Input	Runtime	Network	Samples
pRVFLN (P)	0.09±0.01	1.3±0.05	5	4.78±0.48	14.5±0.6	743.4±0.14
pRVFLN (F)	0.094±0.01	1.3±0.17	5	4.4±0.47	14.96±1.9	743.4±0.2
DNNE	0.14±0	50	170	8.74±0.05	43600+NS	744
Online RVFLN	0.52±0.02	100	170	5.13±0.52	87200	744
Batch RVFLN	0.59±0.05	100	170	6.3±0.001	87200+NS	744

573 6. Numerical Examples

574 This section presents the numerical validation of our proposed algorithm
575 using case studies and comparisons with prominent algorithms in the liter-
576 ature. Two numerical examples, namely modelling of Nox emissions from a
577 car engine and tool condition monitoring in the ball-nose end milling process,
578 are presented in Section 6.2 and 6.3 of this paper, and two other numerical
579 examples, namely modeling of S&P 500 index time series and prediction
580 of household electricity consumption, are placed in the supplemental docu-
581 ment to keep the paper compact while Section 6.1 elaborates on experimental
582 setup. We provide the analysis of robustness in Section 6.4 which offers ad-
583 ditional results with different random regions and illustrates how the scope
584 of random parameters influences the final numerical results. The influence
585 of user-defined predefined thresholds are analysed in Section 6.5. Further-
586 more, additional numerical results across different problems are provided in
587 the supplemental document.

588 6.1. Experimental Setup

589 Our numerical studies were carried out under two scenarios: the time-
590 series scenario and the cross-validation (CV) scenario. The time-series pro-
591 cedure orderly executes data streams according to their arrival and partitions
592 data streams into two parts, namely training and testing. Simulations were
593 repeated 10 times and the numerical results were averaged from 5 runs to ar-
594 rive at conclusive findings because of the random nature of pRVFLN. In the
595 time-series mode, pRVFLN was compared against 11 state-of-the-art evolv-
596 ing algorithms: eT2Class [23], RIVMcSFNN [24], BARTFIS [18], GENE-
597 FIS [22], eTS [3], simp_eTS [2], DFNN [32], GDFNN [33], FAOSPFNN [31], AN-
598 FIS [8]. The CV scenarios were implemented in our experiment in order
599 to follow the commonly adopted simulation environment of other RVFLNs

600 in the literature where each fold is repeated five times to prevent the ran-
 601 dom natures of RVFLNs affecting numerical results. The numerical results
 602 were obtained from average numerical results over all folds. pRVFLN was
 603 benchmarked against the decorrelated neural network ensemble (DNNE) [1],
 604 online and batch versions of RVFLN [26]. The MATLAB code of pRVFLN is
 605 provided in ¹ while the MATLAB codes of DNNE and RVFLN are available
 606 online ^{2,3}. Comparisons were performed against five evaluation criteria: ac-
 607 curacy, data clouds, input attribute, runtime, and network parameters. The
 608 scope of the random parameters was set in the range [0,1] but the effect of this
 609 range on numerical results is explained in Section 6.4. For all simulations,
 610 the same setting of hyper-parameters was applied $\alpha_1 = 0.002$, $\alpha_2 = 0.02$ to
 611 show that these two parameters are not case-specific. It is worth mentioning
 612 that these two values are simply picked up and are not obtained from a pre-
 613 processing step - grid search, cross validation, etc. In other words, we do not
 614 fine-tune these two parameters to arrive at presented numerical results. One
 615 can explore different values that might lead to better numerical results than
 616 those reported. All the numerical studies were carried out using the original
 617 feature space without offline feature selection to check the effectiveness of the
 618 GOFS method. Moreover, two configurations of the GOFS method, partial
 619 and full, were simulated in the numerical study. For Nox emission problem,
 620 the desired number of input attributes was set as 5 for both time-series and
 621 CV modes while, for the tool wear prediction problem, the number of input
 622 variables was selected as 8 for both time-series and CV scenarios. Normal-
 623 ization was undertaken before carrying out the simulation. To ensure a fair
 624 comparison, all the consolidated algorithms were executed using the same
 625 computational resources under the MATLAB environment. Details of the
 626 experimental procedure are given in Table 2.

627 *6.2. Modeling of Nox Emissions from a Car Engine*

628 This section demonstrates the efficacy of the pRVFLN in modeling Nox
 629 emissions from a car engine [15]. This real-world problem is relevant to vali-
 630 date the learning performance, not only because it features noisy and uncer-
 631 tain characteristics similar to the nature of a car engine, it also characterizes
 632 high dimensionality, containing 170 input attributes. That is, 17 physical

¹<http://www.ntu.edu.sg/home/mpratama/Publication.html>

²<http://homepage.cs.latrobe.edu.au/dwang/html/DNNEweb/index.html>

³<http://ispac.ing.uniroma1.it/scardapane/software/lynx/>

variables were captured in 10 consecutive measurements. Furthermore, different engine parameters were applied to induce changes to the system dynamics to simulate real driving actions across different road conditions. In the time-series procedure, 826 data points were streamed to consolidated algorithms, where 667 samples were set as training samples, and the remainder were fed for testing purposes. 10 runs were carried out to attain consistent numerical results. In the CV procedure, the experiment was run under the 10-fold CV, and each fold was repeated five times similar to the scenario adopted in [1]. This strategy checks the consistency of the RVFLNs learning performance because it adopts the random learning scenario and avoids data order dependency. Table 3 and 4 exhibit the consolidated numerical results of the benchmarked algorithms.

Table 5: Tool Wear Prediction Using Time Series Mode

Model	RMSE	Node	Input	Runtime	Network	Samples
pRVFLN (P)	0.14±0.02	1.4±0.5	8	0.14±0.04	23.8±9.3	295.6±28.4
pRVFLN (F)	0.14±0.03	1±0	8	0.07±0.02	17	206.2±83.4
eT2Class	0.16	4	12	1.1	1260	320
RIVMcSFNN	0.11	1	12	1.1	1260	315
Simp_eTS	0.22	17	12	1.29	437	320
eTS	0.15	7	12	0.56	187	320
BARTFIS	0.16	6	12	0.43	222	320
GENEFIS	0.14	14	12	0.41	2366	320
DFNN	0.27	42	12	2.41	1092+NS	320
GDFNN	0.26	7	12	2.54	259+ NS	320
FAOS-PFNN	0.38	7	12	3.76	1022+NS	320
ANFIS	0.16	8	12	0.52	296+ NS	320

It is evident that pRVFLN outperforms its counterparts in all the evaluation criteria. pRVFLN is equipped with an online active learning strategy, which discards superfluous samples. This learning module had a significant effect on predictive accuracy. Furthermore, pRVFLN utilizes the GOFs method, which is capable of coping with the curse of dimensionality. Note that the unique feature of the GOFs method is that it allows different feature subsets to be picked up in every training episode which avoids the catastrophic forgetting of obsolete input attributes, which are temporarily inactive

Table 6: Tool wear prediction using CV Mode

Model	NRMSE	Node	Input	Runtime	Network	Samples
pRVFLN (P)	0.16 $\pm$ 0.3	1.08 $\pm$ 0.23	8	0.14$\pm$0.01	25.1 $\pm$ 0.88	478.8$\pm$69.63
pRVFLN (F)	0.12$\pm$0.07	1.02$\pm$0.14	8	0.14$\pm$0.01	17.3$\pm$2.04	493.8 $\pm$ 63.8
DNNE	0.11 $\pm$ 0	50	12	0.65 $\pm$ 0.04	3310+NS	571.5
Online RVFLN	0.16 $\pm$ 0.01	100	12	0.17 $\pm$ 0.21	1400	571.5
Batch RVFLN	0.19 $\pm$ 0.04	100	12	0.2 $\pm$ 0.001	1400+NS	571.5

653 due to changing data distributions. The GOFs can handle partial input at-
 654 tributes during the training process and results in the same level of accuracy
 655 as that of the full input attributes. The use of full input attributes slowed
 656 down the execution time because it needed to deal with 170 input variables
 657 first, before reducing the input dimension. In this case study, we selected five
 658 input attributes to be kept for the training process. Our experiment shows
 659 that the number of selected input attributes is not problem-dependent and
 660 is set to the desired tradeoff between accuracy and simplicity. The fewer the
 661 number of input attributes to be selected the faster the training speed but at
 662 a cost of accuracy. We did not observe a significant performance difference
 663 when using either the full input mode or partial input mode. On the other
 664 hand, consistent numerical results were achieved by pRVFLN, although the
 665 pRVFLN is built on the random vector functional link algorithm, as observed
 666 in the CV experimental scenario. In addition, pRVFLN produced the most
 667 encouraging performance in almost all evaluation criteria. Note that the
 668 number of training samples, NS, has to be added in the network parameters
 669 for both DNNE and batch RVFLN because their learning procedures cannot
 670 be executed in a single scan rather it depends on iterating entire data samples
 671 over a number of epochs.

672 6.3. Tool Condition Monitoring of High-Speed Machining Process

673 This section presents a real-world problem from a complex manufacturing
 674 process [18]. The objective of this case study is to perform predictive ana-
 675 lytics of the tool wear in the ball-nose end milling process frequently found
 676 in the metal removal process of the aerospace industry. In total, 12 time-
 677 domain features were extracted from the force signal and 630 samples were
 678 collected during the experiment. Concept drift in this case study is evident
 679 from changing surface integrity, tool wear degradation as well as varying ma-
 680 chining configurations. For the time-series experimental procedure, the con-

Figure 4: The frequency of input features

solidated algorithms were trained using data from cutter A, while the testing phase exploited data from cutter B. This process was repeated 10 times to achieve valid numerical results. For the CV experimental procedure, the 10-fold CV process was undertaken where each fold was undertaken five times to arrive at consistent findings. Tables 5 and 6 report the average numerical results across all folds. Fig. 4 depicts how many times input attributes are selected during one fold of the CV process.

It is observed from Tables 5 and 6 that pRVFLN evolved the lowest structural complexities while retaining a high accuracy. It is worth noting that although the DNNE exceeded pRVFLN in accuracy, it imposed considerable complexity because it is an offline algorithm revisiting previously seen data samples and adopts an ensemble learning paradigm. The efficacy of the online sample selection strategy can be seen, as it leads to a significant reduction in the training samples to be learned during the experiment. Using partial input information led to subtle differences to those with the full input information. It is seen in Fig. 4 that the GOFS selected different feature subsets in every training episode. Additional numerical examples are provided in the supplemental document. It is worth mentioning that the nature

699 of RVFL-based algorithms such as pRVFLN, dnne is highly dependent on
700 the initialization step. Recently, dnne has been extended in [28] where it in-
701 corporates the concept of SCN to minimize the effect of improper parameter
702 initialization.

703 *6.4. Analysis of Robustness*

704 This section aims to numerically validate our claim in Section 5.6 that a
705 range [-1,1] does not always ensure the production of a reliable model [12].
706 Additional numerical results with different intervals of random parameters
707 are presented. Four intervals, namely [0,0.1], [0,0.5], [0,0.8], [0,3], [0,5], [0,10]
708 were tried for two case studies described in Sections 6.1 and 6.2. Our exper-
709 iments were undertaken in the 10-fold CV procedure as in previous sections.
710 Table 7 displays the numerical results.

711 For the tool wear case study, the best-performing model was generated
712 by the range [0,0.1]. The higher the range of the model, the more inferior
713 the model, to the point where a model was no longer stable under the range
714 [0,3]. On the other side, the range [0,0.5] induced the best-performing model
715 with the highest accuracy while evolving comparable network complexity
716 for the Nox emission case study. A higher scope led to a deterioration in
717 the numerical results. Moreover, the range [0,0.1] did not deliver a better
718 accuracy than the range [0,0.5] since this range did not generate diverse
719 enough random values. These numerical results are interpreted from the
720 nature of pRVFLN, a clustering-based algorithm. The success of pRVFLN
721 is mainly determined by the compatibility of the zone of influence of hidden
722 nodes on a real data distribution, and its performance worsens when the
723 scope is not representative to cover the true data distribution. That is,
724 the location of data clouds in the feature space with respect to true data
725 distribution is influential to the success of pRVFLN since the data cloud
726 will return very small or almost zero firing strength when a data sample is
727 far from its coverage. This finding is complementary to Li and Wang [12]
728 which relies on a sigmoid-based RVFLN network, and the scope of random
729 parameters can be outside the applicable operating intervals. Its predictive
730 performance is set by its approximation capability in the output space. It is
731 worth-stressing that network parameters are randomly generated in a positive
732 range since the uncertainty threshold setting the footprint of uncertainty is
733 also chosen at random. Having negative values for this parameter causes
734 invalid interval definitions and poor performance is returned as a result.

6.5. Sensitivity Analysis of Predefined Thresholds

This section examines the impact of two predefined thresholds, namely α_1, α_2 , on the overall learning performance of pRVFLN. Intuitively, one can envisage that the higher the value of α_1 , the fewer the number of data clouds are added during the training process and vice versa, whereas the higher the value of α_2 , the higher the number of data clouds that are generated. To further confirm this aspect, the sensitivity of these parameters is analysed using the box Jenkins (BJ) gas furnace problem. The BJ gas furnace problem is a popular benchmark problem in the literature, where the goal is to model the CO2 level in off gas based on two input attributes: the methane flow rate $u(n)$, and its previous one-step output $t(n-1)$. From the literature, the best input and output relationship of the regression model is known as $\hat{y}(n) = f(u(n-4), t(n-1))$. 290 data points were generated from the gas furnace, 200 of which were assigned as the training samples, and the remainder were utilised to validate the model. α_1 was varied in the range of [0.002, 0.004, 0.006, 0.008], while α_2 was assigned the values of [0.02, 0.04, 0.06, 0.08]. Two tests were carried out to test their sensitivity. That is, α_1 was fixed at 0.002, while setting different values of α_2 , whereas α_2 was set at 0.02, while varying α_1 . Moreover, our simulation followed the time-series mode with 10 repetitions as aforementioned. The learning performance of pRVFLN was evaluated against four criteria: non-dimensional error index (NDEI), number of hidden nodes, execution time, number of training samples, and number of network parameters. The results are reported in Table 8.

Referring to Table 8, it can be observed that pRVFLN can achieve satisfactory learning performance while demanding very low network, computational, and sample complexities. Allocating different values of α_1, α_2 did not cause significant performance deterioration, where the NDEI, runtime and the number of samples were stable in the range of [0.27, 0.38], [0.5, 0.79], and [10, 30] respectively. Note that the slight variation in these learning performances was also attributed to the random learning algorithm of pRVFLN. On the other hand, the number of hidden nodes and parameters remained constant at 2 and 10 respectively and were not influenced by a variation of the two predefined thresholds. It is worth mentioning that the data cloud-based hidden node of pRVFLN incurred modest network complexity because it did not have any parameters to be memorised and adapted. In all the simulations in this paper, α_1 and α_2 were fixed at 0.02 and 0.002 respectively to ensure a fair comparison with its counterparts and to avoid a laborious

773 pretraining step in finding suitable values for these two parameters.

774 **7. Conclusion**

775 A novel random vector functional link network, namely the parsimonious
776 random vector functional link network (pRVFLN), is proposed. pRVFLN
777 aims to provide a concrete solution to the issue of data streams by putting
778 into perspective a synergy between adaptive and evolving characteristics and
779 the fast and easy-to-use characteristics of RVFLN. pRVFLN is a fully evolving
780 algorithm where its hidden nodes can be automatically added, pruned
781 and recalled dynamically while all network parameters except the output
782 weights are randomly generated in the absence of any tuning mechanism.
783 pRVFLN is fitted by the online feature selection mechanism and the online
784 active learning scenario which further strengthens its aptitude in processing
785 data streams. Unlike conventional RVFLNs, the concept of interval-valued
786 data clouds is introduced. This concept simplifies the working principle of
787 pRVFLN because it neither requires any parameterization per scalar variables
788 nor follows a pre-specified cluster shape. It features an interval-valued
789 spatiotemporal firing strength, which provides the degree of tolerance for
790 uncertainty. Rigorous case studies were carried out to numerically validate
791 the efficacy of pRVFLN where pRVFLN delivered very low complexity. The
792 ensemble version of pRVFLN will be the subject of our future investigation
793 which aims to further improve the predictive performance of pRVFLN.

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Table 7: Analysis of Robustness

Scope	Criteria	Tool Wear	Nox emission
[0,0.1]	RMSE	0.13±0.008	1.9±0
	Node	1.8±0.25	1
	Input	8	5
	Runtime	0.2±0.1	0.1±0.02
	Network	30.9	11
	Samples	503.1	1
[0,0.5]	RMSE	0.14±0.02	0.1±0.01
	Node	1.92±0.2	1.98±0.14
	Input	8	5
	Runtime	0.18±0.008	5.7±0.3
	Network	32.6	21.8
	Samples	571.5	743.4
[0,0.8]	RMSE	0.47±0.42	0.18±0.3
	Node	1.4±0.05	1.96±0.19
	Input	8	5
	Runtime	0.19±0.13	5.56±0.96
	Network	23.8	21.6
	Samples	385.1	711.24
[0,3]	RMSE	Unstable	Unstable
	Node		
	Input		
	Runtime		
	Network		
	Samples		
[0,5]	RMSE	Unstable	Unstable
	Node		
	Input		
	Runtime		
	Network		
	Samples		
[0,10]	RMSE	Unstable	Unstable
	Node		
	Input		
	Runtime		
	Network		
	Samples		

Table 8: Sensitivity Analysis

PARAMETERS	NDEI	HN	RUNTIME	NP
$\alpha_1 = 0.002$	0.3	19.3	0.52	96.5
$\alpha_1 = 0.004$	0.3	19.3	0.49	96.5
$\alpha_1 = 0.006$	0.3	35.9	0.67	179.5
$\alpha_1 = 0.008$	0.3	7.3	0.4	36.5
$\alpha_2 = 0.02$	0.3	17	0.44	85
$\alpha_2 = 0.04$	0.31	143	1.41	715
$\alpha_2 = 0.06$	0.32	196.3	2.01	981.5
$\alpha_2 = 0.08$	0.32	196.3	2.01	981.5