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Optimal Urban Development Density along A Multi-Modal Linear Travel Corridor with Time-distance Toll Scheme

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Abstract

Consider a travel corridor with a multi-modal transport system (i.e., highway and railway) that connects a continuum of residential locations to a point of CBD. Both highway and railway are subject to congestion effects. All commuters travel along the corridor from home to work in the morning peak hour. The travel costs include travel time, schedule delay and monetary cost. The spatial dynamics of the traffic congestion on both transportation systems are determined by the trip-timing condition, that no traveler will experience a lower travel cost by departing at a different time or switching to a different mode. The flow dynamics on the highway will be considered by applying basic LWR model, while crowdedness (i.e., passenger density on the train) is used to describe the congestion on the railway. The simultaneous temporal and spatial dynamics of commute traffic pattern will be modeled by applying a second-order partial differential complementarity system approach. A time-distance road pricing scheme is applied to achieve the system optimal condition. The urban population is assumed to be located continuously along the corridor. However, the spatial population density distribution is regarded as variant. As is well known that the urban planning issue of population density distribution affects the transportation system significantly, this study aims to find the optimal urban population density distribution in a linear continuous travel corridor leading to optimal transportation system performance, with basic assumptions that it follows some given distribution pattern like negative exponential distribution. The problem is eventually formulated into a mathematical program with complementarity constraints and efficient solution algorithm is developed. Finally, numerical examples are conducted to test the model formulation validity and efficiency.

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Keywords: travel corridor; time-distance toll scheme; urban density; MPCC.

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1. Introduction

Urban traffic congestion in the morning commute is becoming more and more severe in many large cities. Due to limited land resources and prohibitively huge financial investments, we cannot afford to build up more new transportation infrastructures. The ideal way is to best manage the interactions between urban planning and transportation system. Urban form and urban density distribution are the key issues of urban planning and have significant effects on achieving sustainable transportation system. Many big cities, like Singapore, are expecting rapid population growth in a near future. It is then necessary for the urban planners to understand how to plan and allocate the new population growth in a closed city while making it possible to ensure transport systems with optimal performance. On the other hand, to alleviate peak hour commute traffic congestion, many management and control measures are applied. In cities like Singapore, electronic road pricing (ERP) is in use and it is going to be upgraded to the next generation ERP, enabling time-distance toll scheme based on GPS technology. It is imperative for urban and transportation planners to understand how urban density distribution should be planned to optimize the transportation system performance in a multimodal travel corridor, with the aid of time-distance road pricing scheme.

Many previous research works have examined the relationship between land use and transportation system. The Lowry model was the initial transportation - land use model using market simulation approach, which was developed in 1964 for the Pittsburgh region (Lowry, 1964). After that, a vast body of studies have been done and comprehensive literature reviews are available (Wilson, 1971; Meyer and Miller, 1984; Wilson, 1998; Wong et al., 1998; Wegener, 2004). For urban population density distribution, empirical studies have confirmed that population density declines as distance to the urban center increases (Mills, 1970), and the negative exponential distribution function is the most common used function to depict the population distribution. Marc (1978) proposed a combinatorial programming model of joint optimization of land use and transportation with different exact and heuristic methods for comparison. Based on game theory, a bid-rent network equilibrium model is generated to formulate the relationship between transportation and residential location as an n -player non-cooperative game, and a path-based heuristic algorithm is proposed to solve the bi-level programming (BLP) model (Chang and Mackett, 2006). As an extended work of Briceño et al. (2008), Bravo et al. (2010) put forward an integrated model to describe the combination of land use and transportation system, which could be formulated as a fixed-point problem. Yim et al. (2011) proposed a bi-level reliability-based land use and transportation combination model with origin-destination (OD) demand following a certain distribution. This model was solved by a genetic algorithm (GA) with a simple numerical example as illustration.

On the other side, as a demand management strategy, road pricing has been proven to be efficient in alleviating traffic congestion. Early works on first best toll pricing (FBTP) mainly focused on the application of the economic principle of marginal social cost pricing (MSCP). Pigou (1920) and Knight (1924) proposed the road pricing idea with MSCP application first, and then it was further studied by Beckmann et al. (1956); Walters (1961); Beckmann (1965); Vickrey (1969). Arnott et al. (1990a) studied various toll patterns for a simple network with parallel routes, and compared uniform and step tolls. The uniform toll only diverted auto drivers from one route to another without cutting their number, and step toll resulted in better efficiency by altering the departure times. Arnott et al. (1990b) presented a comprehensive economic analysis of a bottleneck model with road congestion in the morning peak hour to determine the coarse toll with optimal capacities. Johansson (1997) applied MSCP to maximize the social benefit with optimal road charges, and presented the optimal road charges as a function of speed not traffic flow. Yang and Huang (1998) explored the network equilibrium problem with queue and delay based on the economic principle of MSCP and presented some new properties of the marginal cost pricing. Yang et al. (2004) developed a BLP model to formulate the optimal entry-exit based toll design problem, which was transformed to an equivalent single level model and solved by an augmented Lagrangian algorithm. Compared to FBTP, second best toll pricing (SBTP) is more reasonable and applicable in reality, especially for time-varying and distance-based toll schemes. Chen and Bernstein (2004) proposed a bi-level SBTP design model with heterogeneous users, which could be transformed to a single level nonlinear programming (NLP) model based on several assumptions. Friesz et al. (2004) proposed a theory of dynamic congestion pricing as a continuous-time optimal problem, which showed an analysis of the necessary conditions for the optimal congestion pricing. Friesz et al. (2007) proposed a dynamic toll design problem with equilibrium constraints and showed a direct solution method for a small problem. Particularly, they gave detailed analysis of the user equilibrium (UE) with dynamic tolls and characteristics of efficient tolls. Zhang et al.

(2008) applied a two-stage method to investigate the joint morning and evening commuting problem, and optimized the time-varying road toll to improve the transportation system. Ban and Liu (2009) formulated the dynamic SBTP as a BLP model in discrete-time scheme, and solved it iteratively by converting it to a single level model using nonlinear complementarity formulation. Based on a discrete time scheme, Doan et al. (2011) developed a linear complementarity problem (LCP) model to formulate the UE constraints with optimal pricing strategy and provided analytical discussion on the existence of the optimal toll. Meng et al. (2012) focused on the optimal distance-based toll, which was determined by a positive and non-decreasing function of travel distance. This toll design problem was formulated as mixed integer linear program (MILP) model and solved by a hybrid genetic algorithm-cost averaging method. To handle a congestion pricing problem combined with dynamic user equilibrium (DUE) flow, Chung et al. (2012) adopted a robust optimization method to handle this SBTP problem and compared it with the other two alternative methods to show its efficiency. Ban et al. (2013) discussed the possibility of non-uniqueness of the UE solution by a SBTP model with a risk-neutral scheme and found that the optimal toll solutions were quite different correspondent to the different design schemes under the situation of the non-unique UE solutions.

This study unifies the two issues into one single framework. Basically, we address the problem that what is the optimal urban development density distribution so as to achieve best transportation system performance in a multimodal travel corridor. Based on this problem, the time-distance toll scheme is considered and the model is supposed to capture the spatial and dynamic travel equilibrium fully. Consider a travel corridor with a multi-modal transport system (i.e., highway and railway) that connects a continuum of residential locations to a point of CBD. Both highway and railway systems are subject to flow congestion. All commuters travel along the corridor from home to work in the morning peak hour. The travel costs include travel time, schedule delay and monetary cost. The congestion flow dynamics on the highway is considered by applying basic LWR model. Crowding cost is incorporated for rail transit users. Basically, travel time depends on the flow congestion dynamics on the transport systems, which in turn, is determined by travelers' departure time and modal choices at any location along the corridor. Complementarity conditions along with partial differential equations are used to capture the spatial and dynamic equilibrium travel pattern. Therefore, for any given population density distribution along the corridor, the spatial equilibrium travel dynamics can be solved, and thus, an optimal time-distance based toll scheme minimizing the total system travel time could be obtained. Noting that different population density distribution profiles will lead to different system performance, we relax the fixed population density distribution assumption to figure out what density distribution is the best to optimize the transportation system performance in the travel corridor. For illustration purpose, negative exponential function based population distribution patterns are applied. A continuous mathematical programming with complementarity constraints is formulated to describe the problem, which is firstly discretized and then solved through a smooth relaxation based nonlinear programming reformulation method.

This study contributes to the literature in the following aspects: firstly, the spatial and dynamic equilibrium travel pattern on the multimodal transportation system is explicitly modelled and solved, which makes it possible to consider the time and distance based road toll scheme. In doing so, we apply partial differential complementarity system approach to describe the flow dynamics and the equilibrium travel pattern as well, wherein complementarity formulation describes the trip-timing conditions that assumes travelers minimize individual travel cost through modal and departure time choices, and partial differential equations capture the traffic flow dynamics and the travel time interactions. Secondly, we examine how the density profile will affect the multimodal transportation system performance. A mathematical programming problem is formulated to find the optimal urban development density distribution with associated time-distance toll scheme. In all, this study applies rigorous mathematical programming approach to investigate the important integrated urban development and transportation planning problem that what should be the optimal urban population distribution so as to achieve best transportation system performance. This study also successfully incorporates the time and distance based toll scheme in the model formulation to consider the optimal pricing strategy for the transportation system. The developed model formulation framework is supposed to be a fundamental building block to model and solve many related problems in urban and transportation planning.

The reminder of this paper is organized as follows. In section 2, we present the mathematical model formulation. In section 3, the continuous model is approximated through a discretization scheme, and a relaxation based iterative solution procedure is illustrated to solve the model. Section 4 provides a set of numerical examples. Finally, section 5 concludes with a discussion.

2. Model Formulation

In this study, we consider a linear travel corridor with both highway and railway transportation systems. Commuters are located along the corridor continuously and travel to the central business district (CBD) for work. They choose their travel modes and departure times to minimize their travel costs. The study horizon of the peak hours and the length of corridor are assumed to be $\bar{T}$ and L respectively.

2.1. Spatial and dynamic travel equilibrium in the multimodal corridor

With given population distribution profile and road toll scheme along the corridor, the achieved spatial and dynamic equilibrium travel pattern is determined through the equilibrium conditions, which can be written in a form of complementarity problem (CP) constraints as follows:

$$\int_0^{\bar{T}} (q_{h,x}^w + q_{r,x}^w) dw = D_x \quad (1)$$

$$0 \leq q_{h,x}^t \perp \alpha TT_{h,x}^t + \beta \max(0, t^* - t - TT_{h,x}^t) + \gamma \max(0, t + TT_{h,x}^t - t^*) + p_{h,x}^t - \pi_x^* \geq 0 \quad (2)$$

$$0 \leq q_{r,x}^t \perp \alpha TT_{r,x}^t + \beta \max(0, t^* - t - TT_{r,x}^t) + \gamma \max(0, t + TT_{r,x}^t - t^*) + s_{r,x} + c_{r,x}^t - \pi_x^* \geq 0 \quad (3)$$

$$\forall x \in [0, L], t \in [0, \bar{T}] \quad (4)$$

Eq. (1) ensures that the total number of trips across the study horizon at any location amounts to the given travel demand, wherein $q_{h,x}^t$ and $q_{r,x}^t$ denote the endogenous departure rates of auto and railway users, respectively, at location x and time t . It should be noted that x here represents the distance between the location to the city boundary. D_x represents the travel demand at location x . In this study, for illustration purpose, we assume D_x equals the product of trip generation rate g_x and the population rate Q_x at location x . Trip generation rate is regarded as a given parameter, while the negative exponential function $Q_x = Q_L e^{-\varphi(L-x)}$ is used to describe the population density distribution, which is determined by two parameters Q_L and φ . Q_L denotes the population density at CBD and φ reflects the changing rate of exponent value. Thus we can also derive the travel demand distribution function $D_x = g_x Q_L e^{-\varphi(L-x)}$. Many other population distribution functions can also be applied here with no theoretical obstacles. CP conditions (2) and (3) entails the UE conditions, i.e., if there is a positive departure rate, the travel cost is minimized with the equilibrium cost π_x^* for commuters departing from location x ; otherwise, the travel cost must be larger than or equal to the minimum cost π_x^* . The total travel cost includes the travel time, scheduling delay and monetary cost for auto users, while crowding effect is also considered for rail transit users. In (2) and (3), t^* denotes the preferred arrival time, $TT_{h,x}^t$ represents the travel time for auto users departing from location x at time t to CBD. $p_{h,x}^t$ denotes the toll for auto users departing at time t from location x to CBD, which varies with time and travel distance. Specifically, $p_{h,x}^t$ can be expressed through an integral function, $p_{h,x}^t = \int_x^L \bar{p}_{h,w}^t dw$, where $\bar{p}_{h,w}^t$ denotes the toll charging rate per unit distance varying with time t and location x . $TT_{r,x}^t$ represents the travel time for railway users departing from location x at time t to CBD, which can be calculated as a linear function $TT_{r,x}^t = (L-x)/V_r$, where V_r is the fixed average train speed. $s_{r,x}$ denotes the fare by railway from location x to CBD, which is assumed to be a linear function with two parts: $s_{r,x} = s_r^0 + \kappa(L-x)$, where s_r^0 is the fixed part, and κ is the fare per unit distance. $c_{r,x}^t$ represents the crowding cost for railway users departing at time t from location x to CBD. A linear crowding cost function with respect to the in-vehicle passenger traffic is applied, as was done in Huang (2000). Specifically, crowding cost for rail users departing at time

t from location x to CBD can be calculated as $c_{r,x}^t = \int_x^L (c_0 + \lambda \int_0^w q_{r,y}^t dy) dw$, where c_0 and λ are given calibrated parameters.

Moreover, $\max(0, t^* - t - TT_{h,x}^t)$ and $\max(0, t + TT_{h,x}^t - t^*)$ in (2), as well as those $\max$ functions in (3), are used to represent the penalties of early and late arrival respectively. α , β , γ denote the unit value of travel time, early arrival delay and late arrival delay respectively, and it is reasonable to assume that $\gamma > \alpha > \beta$. The $\max$ functions in (2) and (3) can also be equivalently expressed in the following complementarity conditions.

$$\begin{aligned} 0 &\leq \theta_{h,x}^t \perp \theta_{h,x}^t - (t^* - t - TT_{h,x}^t) \geq 0 \\ 0 &\leq \theta_{r,x}^t \perp \theta_{r,x}^t - (t^* - t - TT_{r,x}^t) \geq 0 \end{aligned} \quad (5)$$

Hereby, auxiliary variable $\theta_{h,x}^t$ is introduced to help represent the scheduling delays as in $\max(0, t^* - t - TT_{h,x}^t)$ and $\max(0, t + TT_{h,x}^t - t^*)$. One can easily verify the equivalences of $\theta_{h,x}^t = \max(0, t^* - t - TT_{h,x}^t)$ and $\theta_{h,x}^t - (t^* - t - TT_{h,x}^t) = \max(0, t + TT_{h,x}^t - t^*)$ and the CP conditions in (5). If early arrival is triggered, i.e., $t^* - t - TT_{h,x}^t > 0$, we must have $\theta_{h,x}^t > 0$ and $\theta_{h,x}^t = t^* - t - TT_{h,x}^t$, and thus $\theta_{h,x}^t = \max(0, t^* - t - TT_{h,x}^t)$ and $\theta_{h,x}^t - (t^* - t - TT_{h,x}^t) = \max(0, t + TT_{h,x}^t - t^*)$; if late arrival occurs, then $t^* - t - TT_{h,x}^t < 0$ and $\theta_{h,x}^t - (t^* - t - TT_{h,x}^t) > 0$, and we must have $\theta_{h,x}^t = 0$, thus $\theta_{h,x}^t = \max(0, t^* - t - TT_{h,x}^t)$ and $\theta_{h,x}^t - (t^* - t - TT_{h,x}^t) = \max(0, t + TT_{h,x}^t - t^*)$. Similarly, we can verify the equivalent CP conditions for the $\max$ functions for railway users.

2.2. Traffic flow propagation on highway

Classical flow congestion is assumed on the highway of the travel corridor. The flow dynamics is described by applying LWR traffic flow theory (Lighthill and Whitham, 1955; Richards, 1956) and the Greenshields' model (Greenshields, 1935).

$$\frac{\partial f_{h,x}^t}{\partial x} + \frac{\partial k_{h,x}^t}{\partial t} = q_{h,x}^t \quad (6)$$

$$f_{h,x}^t = k_{h,x}^t u_{h,x}^t(k_{h,x}^t) \quad (7)$$

$$u_{h,x}^t(k_{h,x}^t) = a_x^t k_{h,x}^t + b_x^t \quad (8)$$

$$\frac{dTT_{h,x}^t}{dx} = \frac{1}{u_{h,x}^t(k_{h,x}^t)} \quad (9)$$

Eq. (6) entails the flow conservation (equation of continuity) for auto traffic, with $k_{h,x}^t$ representing the traffic density, $f_{h,x}^t$ the traffic flow and $q_{h,x}^t$ the departure rate (entry flow rate into the highway) at location x and time t . Eq. (7) and Eq. (8) denote the flow-velocity-density relationship, and we employ the Greenshields' model to depict the density-velocity relation, wherein $a_x^t = -u^{\max} / k^{\max}$, $b_x^t = u^{\max}$ and $u^{\max}$, $k^{\max}$ represent the maximum velocity and density respectively. Eq. (9) reflects the relationship between the travel time of travellers departing from two neighbouring locations, i.e., $TT_{h,x}^t - TT_{h,x+dx}^t = dx / u_{h,x}^t(k_{h,x}^t)$. Eq. (8) can be substituted into Eq. (7) and Eq. (9) to simplify the model form.

2.3. The overall model

Next, we relax the assumption of given toll scheme and fixed population distribution, and consider a bi-level continuous model, wherein the upper level optimizes the urban density distribution and road toll, while the lower

level model describes the spatial and dynamic equilibrium travel pattern in the multimodal travel corridor. Such a bi-level programming model can be equivalently formulated as a mathematical programming with equilibrium constraints (MPEC) or mathematical programming with complementarity constraints (MPCC) model. In this study, vector is used to represent the variables as: $\mathbf{z} = \{Q_L, \varphi, q_{h,x}^t, q_{r,x}^t, TT_{h,x}^t, TT_{r,x}^t, f_{h,x}^t, k_{h,x}^t, \theta_{h,x}^t, \bar{p}_{h,x}^t, \pi_x^* | x \in [0, L], t \in [0, \bar{T}]\}$.

Thus, for $\forall x \in [0, L]$, $\forall t \in [0, \bar{T}]$, the overall MPCC model is given as below:

$$\begin{aligned}
 \min_{\mathbf{z}} : & \int_0^L \int_0^{\bar{T}} (q_{h,x}^t TT_{h,x}^t + q_{r,x}^t TT_{r,x}^t) dt dx \\
 \text{s.t.} : & \int_0^L Q_L e^{-\varphi(L-x)} dx = Q_{total} \\
 & \frac{\partial f_{h,x}^t}{\partial x} + \frac{\partial k_{h,x}^t}{\partial t} = q_{h,x}^t \\
 & f_{h,x}^t = a_x^t k_{h,x}^{t^2} + b_x^t k_{h,x}^t \\
 & \frac{dTT_{h,x}^t}{dx} = \frac{1}{a_x^t k_{h,x}^t + b_x^t} \\
 & TT_{r,x}^t = (L-x) / V_r \\
 & \int_0^{\bar{T}} (q_{h,x}^w + q_{r,x}^w) dw = g_x Q_L e^{-\varphi(L-x)} \\
 & 0 \leq \theta_{h,x}^t \perp \theta_{h,x}^t - (t^* - t - TT_{h,x}^t) \geq 0 \\
 & 0 \leq q_{h,x}^t \perp \alpha TT_{h,x}^t + \beta \theta_{h,x}^t + \gamma [\theta_{h,x}^t - (t^* - t - TT_{h,x}^t)] + \int_x^L \bar{p}_{h,w}^t dw - \pi_x^* \geq 0 \\
 & 0 \leq \theta_{r,x}^t \perp \theta_{r,x}^t - (t^* - t - TT_{r,x}^t) \geq 0 \\
 & 0 \leq q_{r,x}^t \perp \alpha TT_{r,x}^t + \beta \theta_{r,x}^t + \gamma [\theta_{r,x}^t - (t^* - t - TT_{r,x}^t)] + s_r^0 + \kappa(L-x) + \int_x^L (c_0 + \lambda \int_0^w q_{r,y}^t dy) dw - \pi_x^* \geq 0
 \end{aligned} \tag{10}$$

where the objective is to minimize the total travel time cost along the corridor from the perspective of transport planners. The first constraint assumes that a certain total population across the corridor is maintained, while the population density distribution varies after the parameters Q_L and φ . An optimal population density distribution profile will be derived to understand how urban density should be planned (through policies like floor area ratio regulation, spatially-variable housing subsidies (or taxes)) to ensure optimal system performance can be achieved. Q_{total} represents the total population to be accommodated along the corridor, which is given exogenously. All the other constraints are exactly the same with (1)-(9) describing the dynamic and spatial equilibrium travel pattern.

2.4. Adaptation for other population distribution patterns

Fig. 1. Polycentric population distribution profile

The model formulation shown above in (10) is based on the assumption of monocentric urban form with negative exponential population distribution function. In reality, the urban form and urban population distribution patterns could be different in urban development planning. This model can be easily adapted for more general urban form and distribution pattern. For example, a polycentric urban form is very common, with a number of population centers located in urban area, which can be schematically demonstrated in Fig. 1.

To adapt this model formulation to consider other population distribution patterns, we only need to change the first and sixth constraint in (10) accordingly with new distribution functions and parameters, while other part of model formulation remains unchanged. For example, to apply a polycentric urban form (Small and Song, 1994; Song, 1994), assume that there are K centers along the corridor with the population density Q_{N_k} at location N_k , $k=1,2,...,K$, l_{i-N_k} represents the distance from location i to center N_k and φ_i is the error term associated with location i , we can use the following conditions to replace the corresponding first and sixth constraints in (10) respectively.

$$\sum_{i=1}^N \left(\sum_{k=1}^K Q_{N_k} e^{-\varphi_k l_{i-N_k}} + \varphi_i \right) = Q_{total}$$

$$\sum_{t=0}^M (q'_{h,i} + q'_{r,i}) = g_i \left(\sum_{k=1}^K Q_{N_k} e^{-\varphi_k l_{i-N_k}} + \varphi_i \right) \quad (11)$$

3. Solution Approach

To solve the continuous-spatial-temporal MPCC model shown above, a discretization scheme is applied. The corridor is divided into several equal sections, $i=1,2,...,N$ with each section length as $\varepsilon = L/N$, and equal time intervals $t=0,1,...,M$ with each time interval length as $\omega = \bar{T}/M$, thus the variable vector $\mathbf{z}$ in discrete form is given as: $\mathbf{z} = \{Q_L, \varphi, q'_{r,i}, q'_{h,i}, TT_{r,i}^t, TT_{h,i}^t, f_{h,i}^t, k_{h,i}^t, \theta_{h,i}^t, \bar{p}_{h,i}^t, \pi_i^* | i=1,2,...,N, t=0,1,...,M\}$. Then the above continuous MPCC model can be approximated through a discretization scheme as follows:

$$\begin{aligned} \min_{\mathbf{z} \geq 0} : & \sum_{i=1}^N \sum_{t=0}^M (q'_{h,i} TT_{h,i}^t + q'_{r,i} TT_{r,i}^t) \\ \text{s.t.} & \sum_{i=1}^N Q_N e^{-\varphi(N-i)} = Q_{total} \\ & \frac{f_{h,i}^t - f_{h,i-1}^t}{\varepsilon} + \frac{k_{h,i}^{t+1} - k_{h,i}^t}{\omega} = q_{h,i}^t \\ & f_{h,i}^t = a_i^t k_{h,i}^{t^2} + b_i^t k_{h,i}^t \\ & TT_{h,i}^t - TT_{h,i+1}^t = \varepsilon / (a_i^t k_{h,i}^t + b_i^t) \\ & TT_{r,i}^t = (N-i+1)\varepsilon / V_r \\ & \sum_{t=0}^M (q'_{h,i} + q'_{r,i}) = g_i Q_N e^{-\varphi(N-i)} \\ & 0 \leq \theta_{h,i}^t \perp \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \geq 0 \\ & 0 \leq q'_{h,i} \perp \alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \geq 0 \\ & 0 \leq \theta_{r,i}^t \perp \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \geq 0 \\ & 0 \leq q'_{r,i} \perp \left(\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q'_{r,m} \right) - \pi_i^* \right) \geq 0 \end{aligned} \quad (12)$$

[MPCC]

Note that all of the variables are nonnegative in this study, therefore we have $z \geq 0$. Suppose no traffic existing on the corridor at initial time $t = 0$ and railway operates at a constant average speed. The constraints in formulation [MPCC] (12) can be divided into two groups: standard nonlinear equations conditions in (A) and complementarity conditions in (B). Due to the complementarity conditions in (B), the MPCC problem is difficult to be solved directly based on its immediate form. We will convert it to an equivalent nonlinear programming problem (NLP) model via some reformulation techniques, referring to Ferris et al. (2002).

3.1. Equivalent NLP model for MPCC

The most straightforward way to convert the MPCC problem to standard NLP model is to use a series of equivalent inequality and equality functions to represent the complementarity conditions. Therefore, the [MPCC] (12) above can be readily transformed to a NLP model precisely as follows:

$$\begin{aligned}
 \min_{z \geq 0} &: \sum_{i=1}^N \sum_{t=0}^M (q_{h,i}^t TT_{h,i}^t + q_{r,i}^t TT_{r,i}^t) \\
 \text{s.t. (A) in [MPCC] (12)} & \\
 & \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \geq 0 \\
 & \theta_{h,i}^t (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) = 0 \\
 & \alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \geq 0 \\
 & q_{h,i}^t \left[\alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \right] = 0 \\
 \text{(B)} & \left\{ \begin{aligned} & \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \geq 0 \\ & \theta_{r,i}^t (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) = 0 \\ & \alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \geq 0 \\ & q_{r,i}^t \left[\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \right] = 0 \end{aligned} \right.
 \end{aligned} \tag{13}$$

In this above formulation, the constraints in (A) of [MPCC] (12) keep unchanged, and the complementarity conditions in (B) of [MPCC] (12) are replaced with equivalent series of equations and inequalities in (B) of [NLP] (13) following the concept of CP, i.e., $0 \leq x \perp f(x) \geq 0 \Leftrightarrow x \geq 0, f(x) \geq 0, xf(x) = 0$.

This [NLP] model in (13) is non-convex with poor mathematical properties and the Mangasarian Fromovitz Constraint Qualification (MFCQ) cannot hold (Luo et al., 1996). It is still rather difficult to solve this NLP formulation directly. One commonly used method to handle this problem is to introduce an auxiliary parameter $\mu > 0$, which constructs a relaxed complementarity slackness conditions of the exact equation constraints in [NLP] (13). In this case, MFCQ can be guaranteed and it can be solved by existing NLP solution algorithms. Specifically, the equality constraints in (B) of [NLP] (13) are replaced with inequality conditions by introducing new auxiliary parameter μ in (B) of [NLP] _{μ} (14) and generate a new NLP formulation, named [NLP] _{μ} (14). By driving the value of μ to zero progressively, an iterative procedure can be designed to solve a sequence of problems [NLP] _{μ} (14) repeatedly. One can refer to Scheel and Scholtes (2000); Ferris and Kanzow (2002); Ban et al. (2006) to obtain more applications of this approach.

$$\begin{aligned}
& \min_{z \geq 0} : \sum_{i=1}^N \sum_{t=0}^M (q_{h,i}^t TT_{h,i}^t + q_{r,i}^t TT_{r,i}^t) \\
& \text{s.t. (A) in [MPCC] (12)} \\
& \quad \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \geq 0 \\
& \quad \theta_{h,i}^t (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) \leq \mu \\
& \quad \alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \geq 0 \\
& \quad q_{h,i}^t \left[\alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma (\theta_{h,i}^t + t + TT_{h,i}^t - t^*) + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \right] \leq \mu \\
& \quad \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \geq 0 \\
& \quad \theta_{r,i}^t (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) \leq \mu \\
& \quad \alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \geq 0 \\
& \quad q_{r,i}^t \left[\alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma (\theta_{r,i}^t + t + TT_{r,i}^t - t^*) + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \right] \leq \mu
\end{aligned} \tag{14}$$

(B)

3.2. A smooth relaxation scheme

[NLP _{μ}] (14) contains a number of complicated constraints. It can be further simplified through an approximation function, a smooth relaxation scheme proposed initially in Facchinei et al. (1999).

Definition 1 (Ferris and Kanzow, 2002) *A function $\Phi : \mathbb{R}^2 \rightarrow \mathbb{R}$ is called an NCP-function if it satisfies the condition*

$$\Phi(x, y) = 0 \Leftrightarrow x \geq 0, y \geq 0, xy = 0 \tag{15}$$

The most popular NCP-functions include minimum function $\Phi_M(x, y) = \min\{x, y\}$ and Fischer-Burmeister (FB) function $\Phi_{FB}(x, y) = \sqrt{x^2 + y^2} - x - y$ (Fischer, 1992). Although the FB function is convex and differentiable on $\mathbb{R}^2$, both of the above NCP-functions are nonsmooth. To get a smooth approximation of the FB function, we use an approximate smooth FB (S-FB) function

$$\Phi_{S-FB}(x, y, \delta) = \sqrt{x^2 + y^2 + \delta^2} - x - y \tag{16}$$

by introducing an auxiliary parameter $\delta > 0$. Therefore, the nonsmooth problem is transferred to a smooth problem with the auxiliary parameter δ , except when $\delta = 0$.

Fig. 2. The feasible sets of standard complementarity constraints and $\Phi_{S-FB}(x, y, \delta)$

The different feasible sets of the standard complementarity constraints and the approximate S-FB function $\Phi_{S-FB}(x, y, \delta)$ are shown schematically in Fig. 2.

Proposition 1 (Facchinei et al., 1999) *For each $\delta \in R$, $\Phi_{S-FB}(x, y, \delta) = 0$ if and only if $x \geq 0, y \geq 0, \delta = \sqrt{2xy}$ for $\delta \geq 0$, which is a parameter converging to zero.*

Proposition 2 *If $\delta > 0$, then $\Phi_{S-FB}(x, y, \delta)$ satisfies the inequality $|\Phi_{S-FB}(x, y, \delta) - \Phi_{FB}(x, y)| \leq \delta$ for all $(x, y) \in R^2$.*

Proof. In a similar way as the proof of Lemma 3.7 in Kanzow (1997), for any $(x, y) \in R^2$,

$$|\Phi_{S-FB}(x, y, \delta) - \Phi_{FB}(x, y)| = \left| \sqrt{x^2 + y^2 + \delta^2} - \sqrt{x^2 + y^2} \right| = \frac{\delta^2}{\sqrt{x^2 + y^2 + \delta^2} + \sqrt{x^2 + y^2}} \leq \delta$$

Note that this proposition still holds at $\delta = 0$. $\square$

Following the S-FB function as in (16) above, we use $\Phi_{S-FB}(\mathbf{x}, \mathbf{y}, \delta)$ to represent the vectors $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ and $\mathbf{y} = \{y_1, y_2, \dots, y_n\}$ as follows.

$$\Phi_{S-FB}(\mathbf{x}, \mathbf{y}, \delta) \equiv \begin{Bmatrix} \Phi_{S-FB}(x_1, y_1, \delta) \\ \vdots \\ \Phi_{S-FB}(x_n, y_n, \delta) \end{Bmatrix} \quad (17)$$

Then we propose a relaxation scheme as follows:

$$\|\Phi_{S-FB}(\mathbf{x}, \mathbf{y}, \delta)\|_2 \leq \mu \quad (18)$$

Consequently, an approximated smooth model based on the relaxation scheme (18) for the original formulation [MPCC] (12) is proposed as follows:

$$\begin{aligned} \min_{\mathbf{z}} : & \sum_{i=1}^N \sum_{t=0}^M (q_{h,i}^t TT_{h,i}^t + q_{r,i}^t TT_{r,i}^t) \\ \text{s.t.} \quad & \text{(A) in [MPCC] (12)} \\ \text{[R-NLP]} \quad & \left\| \left(\Phi_{S-FB} \left(\begin{Bmatrix} \theta_{h,i}^t \\ q_{h,i}^t \\ \theta_{r,i}^t \\ q_{r,i}^t \end{Bmatrix}, \begin{Bmatrix} \theta_{h,i}^t - (t^* - t - TT_{h,i}^t) \\ \alpha TT_{h,i}^t + \beta \theta_{h,i}^t + \gamma [\theta_{h,i}^t - (t^* - t - TT_{h,i}^t)] + \sum_{j=i}^N \varepsilon \bar{p}_{h,j}^t - \pi_i^* \\ \theta_{r,i}^t - (t^* - t - TT_{r,i}^t) \\ \alpha TT_{r,i}^t + \beta \theta_{r,i}^t + \gamma [\theta_{r,i}^t - (t^* - t - TT_{r,i}^t)] + s_r^0 + \kappa(N-i+1)\varepsilon + \sum_{j=i}^N \left(c_0 + \lambda \sum_{m=1}^j q_{r,m}^t \right) - \pi_i^* \end{Bmatrix} \right), \delta \right\|_2 \leq \mu \end{aligned} \quad (19)$$

One can find that when the two parameters δ and μ approach zero, the above inequality constraint in reformulated [R-NLP] (19) can be used to approximate the complementarity conditions in (12). There are two big advantages for this reformulation model. Firstly, multiple constraints in (B) of [NLP $_{\mu}$] (14) are reduced to a single nonlinear inequality constraint as (B) in [R-NLP] (19); secondly, the left-hand side nonlinear term in (19) is a smooth function, which makes it suitable for many existing efficient NLP solution methods, such as sequential quadratic programming, penalty interior point algorithm.

3.3. Iterative algorithm

Based on the proposed [R-NLP] (19), an iterative solution algorithm is designed to solve the original problem. Basically, the R-NLP model will be solved repeatedly with the values of parameters δ and μ reduced gradually toward zero. The iterative algorithm is illustrated as follows.

Step 1 Initialization

Initialize the parameters δ , μ with the iteration number as superscript, $\delta^0 > 0$, $\mu^0 > 0$; set the update parameter $0 < \rho < 1$, current iteration number $k = 0$, and termination condition $\sigma_z > 0$ or $\sigma_f > 0$.

Step 2 Major iteration

Solve the current problem [R-NLP] (19) with δ^k and μ^k to obtain solution z^k and objective value f^k .

Step 3 Termination check

Check $\frac{\|z^k - z^{k-1}\|}{\|z^{k-1}\|}$ or $\frac{\|f^k - f^{k-1}\|}{\|f^{k-1}\|}$, if $\frac{\|z^k - z^{k-1}\|}{\|z^{k-1}\|} \leq \sigma_z$ or $\frac{\|f^k - f^{k-1}\|}{\|f^{k-1}\|} \leq \sigma_f$, then stop and go to Step 4;

otherwise, update the parameters $\delta^{k+1} = \rho\delta^k$, $\mu^{k+1} = \rho\mu^k$, $k = k + 1$, then go to Step 2.

Step 4 Solution report

Current solution is the optimal solution.

4. Numerical Results

In this section, we conduct case studies in the context of a travel corridor in Singapore, shown in Fig. 3, with central expressway (CTE) with length of 15.8km and a major rail transit line as well, which connects the city centre of Singapore with its northern part of residential area.

In the following numerical example, the corridor with length of 15.8km is divided into 30 sections (cell 1 to cell 30 is named following the direction from boundary (northern part) of the corridor to CBD (southern part)). The total population along the corridor is given as 25000 persons, and trip generation rate for morning peak hour is assumed to be 0.1 at all locations along the corridor. For simplicity, we assume each vehicle only carries one passenger. The maximum auto speed is set to be 60 km/h, the maximum capacity density is 145 veh/min, the average railway speed is 36km/h, and the parameters of crowding cost are set as $c_0 = 0$, $\lambda = 0.002$.

Fig. 3. CTE in Singapore

Firstly, we show the departure rates of auto and railway users at equilibrium in Table 1 and Table 2 respectively. From the tables, one can find the information about travellers' modal choice and departure time choice. It is straightforward to observe that auto mode is preferred by travellers living far from the city centre while rail is favoured close to the city centre. For the auto users living further toward city boundary, they need to depart earlier but have more flexible departure time choices, i.e., there is a larger departure time window interval.

Time	Location (1-15)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7:01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:02	0.70	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:03	1.20	0.91	0.87	0.64	0	0	0	0	0	0	0	0	0	0	0
7:04	1.20	0.64	0.49	0.27	0	0	0	0	0	0	0	0	0	0	0
7:05	1.19	1.16	0.85	0.51	0.19	0	0	0	0	0	0	0	0	0	0
7:06	1.07	0.91	0.87	0.65	0.38	0	0	0	0	0	0	0	0	0	0
7:07	0.71	0.73	0.62	0.49	0.43	0	0	0	0	0	0	0	0	0	0
7:08	1.25	1.90	2.84	3.36	3.64	3.65	3.08	2.76	1.98	0	0	0	0	0	0
7:09	1.90	1.99	2.07	2.18	2.12	1.98	2.10	2.35	1.36	0	0	0	0	0	0
7:10	1.83	2.18	2.33	2.28	2.10	1.88	1.80	1.79	1.91	1.41	0	0	0	0	0
7:11	1.83	2.14	2.37	2.45	2.36	2.19	1.91	1.89	2.00	1.52	4.28	1.48	0	0	0
7:12	1.83	2.14	2.34	2.43	2.43	2.50	2.36	1.86	6.35	1.55	2.37	5.69	0	0	0
7:13	1.82	2.14	2.35	2.42	2.39	2.47	2.58	2.26	1.22	1.31	1.61	0.00	0	0	0
7:14	1.82	2.11	2.33	2.43	2.40	2.40	2.51	2.38	1.19	1.14	1.65	1.93	0	0	0
7:15	1.80	2.07	2.25	2.33	2.35	2.44	2.52	2.28	1.21	1.13	1.48	1.94	0	0	0
7:16	1.79	2.03	2.16	2.18	2.15	2.24	2.53	2.27	1.13	1.13	1.42	1.91	0.48	0	0
7:17	1.77	1.96	2.06	2.04	1.95	1.92	2.00	1.98	1.20	1.08	1.31	1.71	0.59	0	0
7:18	1.76	1.88	1.92	1.87	1.76	1.70	1.71	1.73	1.16	1.05	1.24	1.53	0.93	0	0
7:19	1.76	1.85	1.83	1.72	1.59	1.51	1.49	1.53	6.49	1.10	1.21	1.32	0.89	0	0
7:20	1.76	1.85	1.81	1.67	1.50	1.41	1.37	1.31	1.21	1.15	1.18	1.08	1.23	0.61	0.59
7:21	1.75	1.84	1.80	1.66	1.49	1.40	1.40	1.41	1.28	1.05	1.16	1.01	1.31	0.54	0.46
7:22	1.73	1.80	1.77	1.63	1.47	1.39	1.43	1.56	1.62	1.25	1.14	0.85	1.61	0.42	0.22
7:23	1.70	1.74	1.69	1.56	1.40	1.31	1.36	1.51	1.75	9.24	10.89	3.60	1.21	0.28	0.03
7:24	1.66	1.66	1.59	1.45	1.29	1.20	1.23	1.37	1.64	9.48	8.26	3.57	1.76	1.04	0.24
7:25	1.56	1.52	1.43	1.28	1.13	1.04	1.03	1.09	1.30	5.08	2.95	3.84	2.21	1.93	1.01
7:26	1.23	1.18	1.13	1.01	0.88	1.01	0.79	0.79	0.86	1.16	1.87	8.32	15.61	10.88	26.01
7:27	0.53	0.42	0.48	0.43	0.75	0.38	0.49	0.38	0.42	0.47	0.85	1.38	1.69	0.88	0.44
7:28	0	0.27	0.32	0.26	0.63	0.24	0.44	0.51	0.58	0.63	0.72	0.96	0.92	2.13	0.40
7:29	0	0	0	0	0	0.38	0.30	0.34	0.77	1.04	1.19	1.05	0.88	1.92	3.76
7:30	0	0	0	0	0	0.58	0.19	0.26	0.22	1.16	0.32	0.38	0.60	1.51	1.01
7:31	0	0	0	0	0	0	0	0.78	0.23	0.18	0	0	0	0	0.36
7:32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time	Location (16-30)														
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
7:20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:21	0	1.73	0	0	0	0	0	0	0	0	0	0	0	0	0
7:22	0.39	1.30	0	0	0	0	0	0	0	0	0	0	0	0	0
7:23	0.52	1.22	0	0	0	0	0	0	0	0	0	0	0	0	0
7:24	0.93	2.19	0	0	0	0	0	0	0	0	0	0	0	0	0
7:25	1.01	1.13	1.90	0	0	0	0	0	0	0	0	0	0	0	0
7:26	27.36	2.71	2.44	0	0	0	0	0	0	0	0	0	0	0	0
7:27	0	0.73	2.35	0	0	0	0	0	0	0	0	0	0	0	0
7:28	0	0.13	0.69	1.70	0.79	0	0	0	0	0	0	0	0	0	0
7:29	2.78	3.20	1.81	3.01	3.64	0	0	0	0	0	0	0	0	0	0

7:30	2.33	3.38	2.37	1.11	2.24	1.88	2.99	0	0	0	0	0	0	0	0
7:31	0.32	2.97	15.15	1.11	1.03	2.30	3.82	0	0	0	0	0	0	0	0
7:32	1.27	0.22	1.74	0.23	0.18	0.58	2.83	0	0	0	0	0	0	0	0
7:33	0.15	0.04	0.68	0.29	0.79	0.84	4.98	0.58	9.40	0	0	0	0	0	0
7:34	0	0.11	0.17	0.49	1.48	2.85	2.40	0.60	2.53	20.47	0	0	0	0	0
7:35	0	0.23	0.21	0.42	2.04	0.09	2.24	0	0	0.71	6.64	0	0	0	0
7:36	0	0	1.25	1.13	0.41	0.10	3.10	0	0	0	0	11.59	0	0	0
7:37	0	0	0	0.65	0	1.29	0.34	0	0	0	0	0	5.25	1.41	0
7:38	0	0	0	0	0	0	0.74	0	0	0	0	0	0	8.47	0
7:39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.40
7:40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.73
7:41	0	0	0	0	0	0	0	0	0	2.12	0.12	0	0	0.12	9.09
7:42	0	0	0	0	0	0	0	0	0	0.71	1.20	3.30	0.14	0.28	4.52
7:43	0	0	0	0	0	0	0	0	0	0	0	3.43	2.19	0.55	0.79
7:44	0	0	0	0	0	0	0	0	0	0	0	0	0	12.48	3.77
7:45	0	0	0	0	0	0	0	0	0	0	0	0	0	0.23	5.02
7:46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2 Departure rate of railway users

Time	Location (1-15)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7:18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:19	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:20	0	0.03	0.43	0.29	0	0	0	0	0	0	0	0	0	0	0
7:21	0	0	0	3.55	2.29	0.50	0	0	0	0	0	0	0	0	0
7:22	0	0	0	0	6.09	4.00	0.86	0	0	0	0	0	0	0	0
7:23	0	0	0	0	0	7.72	5.41	1.66	0	0	0	0	0	0	0
7:24	0	0	0	0	0	0	8.91	6.85	2.95	0	0	0	0	0	0
7:25	0	0	0	0	0	0	0	9.38	8.32	4.35	0	0	0	0	0
7:26	0	0	0	0	0	0	0	0	6.49	8.69	5.80	1.67	0	0	0
7:27	0	0	0	0	0	0	0	0	0	2.19	9.50	8.24	4.55	0	0
7:28	0	0	0	0	0	0	0	0	0	0	10.88	10.88	10.88	7.72	3.49
7:29	0	0	0	0	0	0	0	0	0	0	0	21.12	10.71	6.62	6.62
7:30	0	0	0	0	0	0	0	0	0	0	0	0	31.18	9.53	9.53
7:31	0	0	0	0	0	0	0	0	0	0	0	0	0	20.99	20.99
7:32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time	Location (16-30)														
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
7:28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:29	1.90	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:30	5.23	2.88	0	0	0	0	0	0	0	0	0	0	0	0	0
7:31	20.41	10.37	6.51	0	0	0	0	0	0	0	0	0	0	0	0
7:32	14.15	31.81	13.25	8.28	0	0	0	0	0	0	0	0	0	0	0
7:33	0	16.15	30.55	16.73	10.30	0	0	0	0	0	0	0	0	0	0
7:34	0	0	5.35	54.06	10.57	5.37	1.78	0	0	0	0	0	0	0	0
7:35	0	0	0	1.33	61.38	11.96	9.86	0	0	0	0	0	0	0	0
7:36	0	0	0	0	0	70.95	18.81	8.58	1.39	0	0	0	0	0	0
7:37	0	0	0	0	0	1.19	50.22	18.58	15.74	5.58	0	0	0	0	0
7:38	0	0	0	0	0	0	0	80.74	23.58	15.24	3.80	0	0	0	0
7:39	0	0	0	0	0	0	0	0	61.64	44.64	19.21	9.16	0	0	0
7:40	0	0	0	0	0	0	0	0	0	30.26	74.26	18.25	5.91	0	0
7:41	0	0	0	0	0	0	0	0	0	0	20.20	55.43	16.54	13.40	7.12
7:42	0	0	0	0	0	0	0	0	0	0	0	30.23	88.70	24.56	15.98
7:43	0	0	0	0	0	0	0	0	0	0	0	0	18.95	73.50	19.34
7:44	0	0	0	0	0	0	0	0	0	0	0	0	0	9.24	72.34
7:45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The total travel costs incurred for auto users departing at different time and different locations along the corridor are shown in Fig. 4. It can be found that for users living close to city boundary, minimum travel cost occurs in a longer time interval, which means auto users departing from this location have more flexible departure time choices, for example, auto users departing from location 5 could depart between 7:05am and 7:28am coming with the same travel cost. However, the auto users living closer to CBD have a rather narrow departure time window, for example, at location 30, auto users can only depart between 7:39am and 7:45am to ensure minimum travel cost.

Fig. 4. Travel cost of auto mode along the corridor

Similarly, the different travel costs for railway users departing at different time and different locations along the corridor are shown in Fig. 5. For the railway users along the corridor, travellers depart earlier for those living close to city boundary, and multiple departure choices exist along the corridor.

Fig. 5. Travel cost of rail mode along the corridor

The optimum toll scheme changing after time and distance along the corridor are shown in Table 3. One can find that the road toll scheme is determined by the congestion flow dynamics evolving on the highway. At locations far from the city centre, the toll is raised at early time intervals to affect travellers' modal and departure time choices. With evolving congestion dynamics, tolls are imposed at locations toward the city centre. It also can be found that at some locations, different toll rates are derived within the time horizon to avoid congestion incurred by travellers' departure time choices.

Table 3 Time-location based toll scheme

Time	Location (1-15)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7:01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:02	0.843	0.880	0.429	0	0	0	0	0	0	0	0	0	0	0	0
7:03	0.840	0.877	0.488	0.493	0.107	0	0	0	0	0	0	0	0	0	0
7:04	0.840	0.876	0.486	0.513	0.466	0.266	0	0	0	0	0	0	0	0	0
7:05	0.840	0.875	0.485	0.511	0.839	0.602	0	0	0	0	0	0	0	0	0
7:06	0.840	0.876	0.485	0.511	0.161	0.151	0.147	0.144	0	0	0	0	0	0	0
7:07	0.257	0.247	0.244	0.241	0.239	0.237	0.235	0.233	0	0	0	0	0	0	0
7:08	0.842	0.879	0.490	0.517	0.878	0.878	0.889	0.899	0	0	0	0	0	0	0
7:09	0.838	0.872	0.480	0.505	0.867	0.867	0.880	0.898	0	0	0	0	0	0	0
7:10	0.839	0.871	0.478	0.501	0.861	0.861	0.869	0.891	0.116	0.098	0	0	0	0	0
7:11	0.839	0.872	0.478	0.499	0.858	0.856	0.870	0.899	0.900	0.900	0	0	0	0	0
7:12	0.839	0.872	0.478	0.500	0.857	0.852	0.859	0.893	0.900	0.900	0	0	0	0	0
7:13	0.839	0.872	0.478	0.500	0.857	0.852	0.855	0.870	0.896	0.900	0.153	0.122	0	0	0
7:14	0.839	0.872	0.478	0.500	0.858	0.853	0.857	0.872	0.897	0.900	0.886	0.899	0.069	0.053	0.048
7:15	0.839	0.872	0.478	0.500	0.857	0.853	0.858	0.875	0.898	0.900	0.884	0.200	0.176	0.158	0.144
7:16	0.839	0.872	0.478	0.500	0.858	0.852	0.857	0.877	0.899	0.900	0.890	0.253	0.225	0.204	0.188
7:17	0.839	0.872	0.479	0.501	0.860	0.855	0.855	0.877	0.899	0.900	0.893	0.312	0.275	0.250	0.231
7:18	0.839	0.872	0.479	0.502	0.860	0.858	0.864	0.874	0.899	0.900	0.893	0.378	0.324	0.294	0.273
7:19	0.839	0.872	0.480	0.503	0.862	0.858	0.865	0.890	0.898	0.900	0.890	0.437	0.369	0.335	0.313
7:20	0.839	0.872	0.480	0.504	0.864	0.861	0.866	0.883	0.899	0.900	0.893	0.899	0.837	0.709	0.831
7:21	0.839	0.872	0.480	0.504	0.864	0.863	0.872	0.897	0.900	0.900	0.898	0.899	0.832	0.706	0.830
7:22	0.839	0.873	0.480	0.504	0.864	0.862	0.885	0.900	0.900	0.900	0.900	0.900	0.845	0.701	0.826
7:23	0.839	0.873	0.480	0.504	0.864	0.862	0.889	0.900	0.900	0.900	0.900	0.900	0.871	0.700	0.824
7:24	0.839	0.873	0.481	0.505	0.865	0.863	0.871	0.889	0.897	0.900	0.866	0.899	0.860	0.525	0.502
7:25	0.839	0.873	0.481	0.505	0.866	0.864	0.873	0.898	0.900	0.900	0.814	0.864	0.819	0.735	0.835
7:26	0.839	0.874	0.482	0.506	0.867	0.866	0.884	0.900	0.900	0.900	0.900	0.814	0.758	0.658	0.869
7:27	0.840	0.875	0.484	0.508	0.869	0.872	0.899	0.900	0.900	0.900	0.900	0.900	0.597	0.553	0.467
7:28	0.152	0.032	0.316	0.682	0.875	0.892	0.900	0.900	0.900	0.900	0.900	0.900	0.899	0.476	0.614
7:29	0	0	0.061	0.064	0.816	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.757	0.600
7:30	0	0	0	0	0.061	0.221	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.775	0.851
7:31	0	0	0	0	0	0.062	0.060	0.347	0.900	0.896	0.896	0.896	0.896	0.854	0.863
7:32	0	0	0	0	0	0	0	0.070	0.067	0.046	0.870	0.866	0.864	0.863	0.883
7:33	0	0	0	0	0	0	0	0	0	0	0.061	0.621	0.843	0.840	0.898
7:34	0	0	0	0	0	0	0	0	0	0	0	0.061	0.034	0.566	0.900
7:35	0	0	0	0	0	0	0	0	0	0	0	0	0	0.116	0.112
7:36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Time	Location (16-30)														
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
7:13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7:14	0.046	0.045	0.044	0	0	0	0	0	0	0	0	0	0	0	0
7:15	0.133	0.124	0.118	0.114	0.112	0	0	0	0	0	0	0	0	0	0
7:16	0.176	0.162	0.153	0.146	0.142	0	0	0	0	0	0	0	0	0	0
7:17	0.216	0.201	0.189	0.180	0.173	0.169	0.166	0	0	0	0	0	0	0	0

7:18	0.256	0.238	0.224	0.214	0.206	0.199	0.194	0.191	0.189	0	0	0	0	0	0
7:19	0.296	0.276	0.260	0.249	0.239	0.232	0.225	0.221	0.218	0.043	0.056	0	0	0	0
7:20	0.867	0.153	0.125	0.114	0.107	0.102	0.099	0.097	0.095	0.095	0.102	0.094	0.087	0	0
7:21	0.866	0.226	0.187	0.170	0.158	0.150	0.143	0.139	0.136	0.134	0.133	0.132	0.132	0.132	0
7:22	0.863	0.274	0.243	0.226	0.213	0.203	0.195	0.189	0.185	0.181	0.179	0.178	0.177	0.147	0.177
7:23	0.859	0.346	0.302	0.282	0.266	0.254	0.244	0.238	0.232	0.228	0.225	0.223	0.222	0.201	0.221
7:24	0.491	0.454	0.421	0.400	0.384	0.371	0.358	0.350	0.344	0.339	0.335	0.333	0.331	0.330	0.329
7:25	0.866	0.899	0.497	0.419	0.351	0.321	0.289	0.277	0.267	0.261	0.258	0.255	0.253	0.265	0.252
7:26	0.876	0.899	0.474	0.456	0.437	0.412	0.378	0.362	0.347	0.339	0.334	0.331	0.329	0.331	0.327
7:27	0.802	0.899	0.519	0.502	0.493	0.486	0.449	0.434	0.415	0.402	0.396	0.392	0.389	0.388	0.387
7:28	0.692	0.900	0.900	0.900	0.900	0.900	0.779	0.778	0.172	0.151	0.144	0.141	0.139	0.139	0.138
7:29	0.632	0.791	0.900	0.900	0.900	0.900	0.414	0.410	0.376	0.354	0.343	0.335	0.332	0.330	0.329
7:30	0.606	0.653	0.827	0.900	0.900	0.900	0.854	0.854	0.742	0.257	0.237	0.227	0.222	0.220	0.219
7:31	0.894	0.622	0.702	0.900	0.815	0.900	0.862	0.862	0.798	0.362	0.336	0.320	0.314	0.310	0.308
7:32	0.900	0.900	0.523	0.900	0.729	0.886	0.894	0.882	0.821	0.419	0.412	0.396	0.390	0.386	0.384
7:33	0.900	0.900	0.900	0.900	0.825	0.900	0.900	0.900	0.867	0.768	0.768	0.294	0.286	0.280	0.278
7:34	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.859	0.819	0.815	0.373	0.371	0.365	0.363
7:35	0	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.805	0.432	0.427	0.423	0.419
7:36	0	0.122	0.060	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.473	0.461	0.456	0.453
7:37	0	0.065	0	0	0.646	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.148	0.144
7:38	0	0	0	0	0.115	0.089	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.300	0.299
7:39	0	0	0	0	0	0	0.081	0.093	0.651	0.900	0.900	0.900	0.900	0.900	0.312
7:40	0	0	0	0	0	0	0	0	0.093	0.791	0.900	0.900	0.900	0.900	0.442
7:41	0	0	0	0	0	0	0	0	0	0	0	0.263	0.900	0.900	0.553
7:42	0	0	0	0	0	0	0	0	0	0	0	0	0	0.900	0.900
7:43	0	0	0	0	0	0	0	0	0	0	0	0	0	0.172	0.900
7:44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.620
7:45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.356
7:46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.042
7:47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

To demonstrate the effects of toll scheme and urban density distribution on the multimodal transportation system performance, the previous equilibrium results based on both optimal toll scheme and urban density distribution is compared with the other two cases, wherein toll is charged with fixed rate at fixed location (toll collected by physical gantries) across the time horizon, and a certain uniform urban density distribution is assumed. The comparison results are shown in Table 4. For illustration purpose, the case with both toll and negative exponential density distribution is labelled as case (I), and the cases with uniform density and fixed toll are labelled as case (II) and (III) respectively. Here the fixed toll is assumed to be paid at the fixed location 20.

Table 4 Comparison with no toll equilibrium

Location	(I) Toll and exponential density			(II) Uniform density			(III) Fixed toll		
	Auto	Railway	Equilibrium cost	Auto	Railway	Equilibrium cost	Auto	Railway	Equilibrium cost
1	39.14	0.03	27.81	69.90	13.43	28.97	35.20	0.44	23.67
2	41.01	0.03	27.05	63.58	19.75	28.33	37.10	0.44	23.34
3	42.56	0.43	26.27	63.00	20.33	27.53	39.09	0.44	23.06
4	41.20	3.84	25.70	58.29	25.05	26.74	41.29	0.35	22.83
5	38.80	8.39	25.11	57.18	26.15	25.93	43.48	0.37	22.55
6	37.22	12.22	24.33	49.85	33.48	25.13	45.80	0.37	21.88
7	36.62	15.17	23.54	52.02	31.31	24.33	48.29	0.34	21.61
8	36.38	17.88	22.76	56.39	26.95	23.51	50.85	0.36	21.11
9	39.09	17.76	21.96	59.71	23.63	22.71	39.13	14.81	20.92
10	44.32	15.24	21.15	51.86	31.47	21.86	36.13	20.67	20.58
11	47.09	15.30	20.31	44.85	38.48	21.02	30.44	29.38	19.91
12	44.57	20.79	19.50	28.51	54.82	20.24	28.57	34.43	19.23

13	31.93	36.55	18.68	22.05	61.28	19.47	30.96	35.39	18.70
14	22.13	49.61	17.89	18.06	65.28	18.58	2.99	66.88	18.03
15	34.54	40.62	17.17	10.15	73.18	17.81	73.08	0.50	17.53
16	37.06	41.68	16.38	7.51	75.82	16.92	0.24	77.25	16.25
17	21.30	61.20	15.58	4.61	78.73	16.04	2.69	78.93	15.64
18	30.77	55.66	14.75	3.85	79.48	15.11	0.09	85.87	14.72
19	10.15	80.40	13.91	9.56	73.77	14.02	32.04	58.48	14.33
20	12.61	82.25	12.92	17.85	65.49	13.13	0.06	95.27	12.70
21	9.93	89.46	12.07	15.29	68.04	11.95	99.76	0.64	11.58
22	23.44	80.68	11.13	11.35	71.98	10.86	41.99	63.75	10.42
23	1.18	107.90	10.02	0.04	83.29	9.31	0.06	111.29	9.48
24	11.93	102.35	9.14	3.63	79.71	8.59	7.53	109.74	8.98
25	24.01	95.72	8.20	15.25	68.08	7.72	28.84	94.67	8.20
26	7.96	117.47	7.01	0.22	83.11	6.43	30.66	99.41	7.35
27	18.33	113.08	5.90	15.25	68.08	5.48	11.28	125.70	6.46
28	7.58	130.10	4.66	11.68	71.65	4.36	33.09	111.17	5.39
29	23.54	120.70	3.64	20.21	63.13	3.34	29.45	122.48	3.98
30	36.33	114.78	2.46	36.00	47.34	2.23	56.51	103.49	2.84
Sum	852.72	1647.28		877.72	1622.28		956.70	1543.30	
Total time			2.18×10^4			2.85×10^4			2.29×10^4
Total cost			3.32×10^4			4.15×10^4			3.16×10^4

In this study, we seek to minimize the total travel time along the corridor for all the travellers. The total time values are demonstrated in the second row from the bottom of Table 4. One can observe that case (I) results in lowest total travel time while uniform density case brings in highest total travel time. The gap between case (I) and (II) is more prominent than that between case (I) and (III), which implies that urban density distribution, as a urban planning factor, affect the total travel time of the transportation system more significantly.

Next, we make a simple sensitivity analysis with respect to the population density distribution to further illustrate how it affects the transportation system performance in the corridor.

Fig. 6. Comparison of total travel time based on different density distributions

Assuming the total population and trip generation rate along the corridor are fixed at 25000 persons and 0.1 respectively as stated previously, and the population density still follows the negative exponential distribution with highest density at the last section Q_{30} at CBD. Therefore, the total travel demand along the corridor is 2500 veh. Different Q_{30} densities are considered for numerical experiments to check how urban density affects the transportation system performance. The numerical results show that, when the city centre density increases, i.e., more population resides near CBD, more travellers choose rail transit and the total travel time reduces for the entire system. However, high city centre density raises the rail transit congestion effects and brings more auto users. The traffic congestion propagates on the highway and imposes the auto users along the corridor longer travel time despite departure time choices. The obtained optimal urban population density would provide urban planners a guideline for integrating the planning of urban development with best urban transport system performance, which in practice can be realized by implementing urban policies, like floor area ratio regulation and spatially-variable excise subsidies (or taxes) on housing. Fig. 6 shows the total travel times along the corridor with different density distributions, and it is obvious to observe that the optimal density occurs when $Q_{30} = 1511.1$.

5. Conclusion

This study investigates the problem on how the urban density distribution may affect the transportation system performance in a multimodal travel corridor and what is the best density distribution profile if optimal transportation system performance is desired. We also consider the time-distance based road toll scheme. A mathematical programming approach is applied, wherein the flow congestion dynamics and trip timing equilibrium conditions are fully captured. The model formulation framework developed in this study is assumed to be able to contribute on modeling and solving more related research problems on integrated urban development and transportation planning. Despite this, there are several limitations in this study. For example, only exponential urban density distribution pattern is analyzed, while more other realistic distribution patterns would be considered in the future study. Moreover, an LWR model along with specific Greenshields' model is used to describe the flow dynamics. More realistic and sophisticated traffic flow theory could be demonstrated in the future research work.

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References

- Arnott, R., de Palma, A., Lindsey, R., 1990a. Departure time and route choice for the morning commute. *Transportation Research Part B: Methodological* 24(3), 209-228.
- Arnott, R., de Palma, A., Lindsey, R., 1990b. Economics of a bottleneck. *Journal of Urban Economics* 27(1), 111-130.
- Ban, X.G., Ferris, M.C., Tang, L., Lu, S., 2013. Risk-neutral second best toll pricing. *Transportation Research Part B: Methodological* 48, 67-87.
- Ban, X.G. and Liu, H.X., 2009. A link-node discrete-time dynamic second best toll pricing model with a relaxation solution algorithm. *Networks and Spatial Economics* 9(2), 243-267.
- Ban, X.G., Liu, H.X., Ferris, M.C., Ran, B., 2006. A general MPCC model and its solution algorithm for continuous network design problem. *Mathematical and Computer Modelling* 43(5-6), 493-505.
- Beckmann, M.J., 1965. On optimal tolls for highways, tunnels and bridges. In *vehicular traffic science, Proceedings of the Third International Symposium on the Theory of Traffic Flow*, New York, pp. 331-341.
- Beckmann, M.J., McGuire, C.B., Winsten, C.B., 1956. *Studies in the Economics of Transportation*. Yale University Press, New Haven, pp. 226
- Bravo, M., Briceno, L., Cominetti, R., Cores, C.E., Martinez, F., 2010. An integrated behavioral model of the land-use and transport systems with network congestion and location externalities. *Transportation Research Part B: Methodological* 44(4), 584-596.
- Briceño, L., Cominetti, R., Cortés, C., Martínez, F.J., 2008. An integrated behavioral model of land-use and transport system: an extended network equilibrium approach. *Networks and Spatial Economics* 8(2-3), 201-224.
- Chang, J.S. and Mackett, R.L., 2006. A bi-level model of the relationship between transport and residential location. *Transportation Research Part B: Methodological* 40(2), 123-146.
- Chen, M. and Bernstein, D.H., 2004. Solving the toll design problem with multiple user groups. *Transportation Research Part B: Methodological* 38(1), 61-79.
- Chung, B.D., Yao, T., Friesz, T.L., Liu, H., 2012. Dynamic congestion pricing with demand uncertainty: A robust optimization approach. *Transportation Research Part B: Methodological* 46(10), 1504-1518.
- Doan, K., Ukkusuri, S., Han, L., 2011. On the existence of pricing strategies in the discrete time heterogeneous single bottleneck model.

- Transportation Research Part B: Methodological 45(9), 1483-1500.
- Facchinei, F., Jiang, H., Qi, L., 1999. A smoothing method for mathematical programs with equilibrium constraints. *Mathematical Programming* 85(1), 107-134.
- Ferris, M.C., Dirkse, S.P., Meeraus, A., 2002. Mathematical programs with equilibrium constraints: automatic reformulation and solution via constrained optimization, in: Kehoe, T., Srinivasan, T., Whalley, J. (Eds.), *Frontiers in Applied General Equilibrium Modeling*. Cambridge University Press, USA, pp. 67-93.
- Ferris, M.C. and Kanzow, C., 2002. Complementarity and related problems: a survey, *Handbook of Applied Optimization*, pp. 514-530.
- Fischer, A., 1992. A special newton-type optimization method. *Optimization* 24(3-4), 269-284.
- Friesz, T., Bernstein, D., Kydes, N., 2004. Dynamic congestion pricing in disequilibrium. *Networks and Spatial Economics* 4(2), 181-202.
- Friesz, T., Kwon, C., Mookherjee, R., 2007. A computable theory of dynamic congestion pricing, in: Allsop, R., Bell, M., Benjamin, H. (Eds.), *Transportation and Traffic Theory 2007*. Elsevier, pp. 1-26.
- Greenshields, B.D., 1935. A study of traffic capacity, *Proceedings Highway Research Record*, Washington, pp. 448-477.
- Huang, H.J., 2000. Fares and tolls in a competitive system with transit and highway: the case with two groups of commuters. *Transportation Research Part E: Logistics and Transportation Review* 36(4), 267-284.
- Johansson, O., 1997. Optimal road-pricing: Simultaneous treatment of time losses, increased fuel consumption, and emissions. *Transportation Research Part D: Transport and Environment* 2(2), 77-87.
- Kanzow, C., 1997. A new approach to continuation methods for complementarity problems with uniform P-functions. *Operations Research Letters* 20(2), 85-92.
- Knight, F.H., 1924. Some fallacies in the interpretation of social cost. *The Quarterly Journal of Economics* 38(4), 582-606.
- Lighthill, M.J. and Whitham, G.B., 1955. On kinematic waves. II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 229(1178), 317-345.
- Lowry, I.S., 1964. *A Model of Metropolis*. Rand Corporation
- Luo, Z.Q., Pang, J.S., Ralph, D., 1996. *Mathematical Programs with Equilibrium Constraints*. Cambridge University Press
- Marc, L., 1978. Simultaneous optimization of land use and transportation: A synthesis of the quadratic assignment problem and the optimal network problem. *Regional Science and Urban Economics* 8(1), 21-42.
- Meng, Q., Liu, Z., Wang, S., 2012. Optimal distance tolls under congestion pricing and continuously distributed value of time. *Transportation Research Part E: Logistics and Transportation Review* 48(5), 937-957.
- Meyer, M.D. and Miller, E.J., 1984. *Urban Transportation Planning: A Decision-Oriented Approach*. McGraw-Hill, New York
- Mills, E.S., 1970. Urban density functions. *Urban Studies* 7(1), 5-20.
- Pigou, A.C., 1920. *The Economics of Welfare*. Mac Millan, New York
- Richards, P.I., 1956. Shock waves on the highway. *Operations Research* 4(1), 42-51.
- Scheel, H. and Scholtes, S., 2000. Mathematical programs with complementarity constraints: stationarity, optimality, and sensitivity. *Mathematics of Operations Research* 25(1), 1-22.
- Small, K.A. and Song, S., 1994. Population and employment densities: structure and change. *Journal of Urban Economics* 36, 292-313.
- Song, S., 1994. Does generalizing density functions better explain urban commuting? Some evidence from the Los Angeles region. *Applied Economics Letters* 2(5), 148-150.
- Vickrey, W.S., 1969. Congestion theory and transport investment. *The American Economic Review* 59(2), 251-260.
- Walters, A.A., 1961. The theory and measurement of private and social cost of highway congestion. *Econometrica* 29(4), 676-699.
- Wegener, M., 2004. Overview of land use transport models, *Handbook of Transport Geography and Spatial*.
- Wilson, A.G., 1971. A family of spatial interaction models, and associated developments. *Environment and Planning* 3, 1-32.
- Wilson, A.G., 1998. Land-use/transport interaction models: past and future. *Journal of Transport Economics and Policy* 32(1), 3-26.
- Wong, C.K., Wong, S.C., Tong, C.O., 1998. A new methodology for calibrating the Lowry model. *Journal of Urban Planning and Development* 124, 72-91.
- Yang, H. and Huang, H.J., 1998. Principle of marginal-cost pricing: how does it work in a general road network? *Transportation Research Part A: Policy and Practice* 32(1), 45-54.
- Yang, H., Zhang, X., Meng, Q., 2004. Modeling private highways in networks with entry-exit based toll charges. *Transportation Research Part B: Methodological* 38(3), 191-213.
- Yim, K.K.W., Wong, S.C., Chen, A., Wong, C.K., Lam, W.H.K., 2011. A reliability-based land use and transportation optimization model. *Transportation Research Part C: Emerging Technologies* 19(2), 351-362.
- Zhang, X.N., Huang, H.J., Zhang, H.M., 2008. Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city. *Transportation Research Part B: Methodological* 42(1), 38-56.