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A POD study of turbulent near wake of S805 airfoil in deep stall

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A POD analysis was performed on the turbulent flow in the near wake region of an S805 airfoil in deep stall at an angle of attack of 30° . The flow was measured using tomographic particle image velocimetry at a Reynolds number of 4600. Instantaneous turbulence structures which were significant contributors to the first two POD modes were studied. These structures included the large-scale Kármán vortex and small-scale shear layer vortices which are interacting with the Kármán vortex. An interesting correspondence was observed between the rotational vectors in POD modes 4 & 5 and the locations of the leading-edge shear layer vortices in the flow fields which are major contributors to these two modes. Similarity was found in the autospectral functions computed from the velocity fields which are significant contributors to the first five POD modes. The wavelengthes corresponding to the peak values of autospectral functions could be related to the size of the induced flows by the Kármán vortex and the

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streamwise spacing of the shear layer vortices.

Nomenclature

ϕ_n	= n^{th} POD mode
a_n	= coefficient of n^{th} POD mode
μ_n	= eigenvalue corresponding to the n^{th} POD mode
T	= time interval
Ω	= spatial domain
k	= turbulent kinetic energy
V_∞	= freestream velocity
c	= airfoil's chord length
ν	= kinematic viscosity
σ_{an}	= RMS value of the coefficient of n^{th} POD mode
$\omega_x, \omega_y, \omega_z$	= vorticity components in x , y , and z directions, respectively
λ_2	= second eigenvalue of velocity gradient tensor
G_{vv}	= autospectral density functions of fluctuating cross-stream velocity component

I. Introduction

The angle of attack (AOA) of the rotor blade of a horizontal-axis wind turbine which operates at off-design conditions can be quite large. For example, Lee & Wu [1–3] found that if a small 5KW wind turbine with a design tip-speed-ratio (TSR) of 6 was operated at TSR=3 when the speed of the incoming flow was increased, the AOA at the inboard section of the rotor blade was about 30°. Due to the rotation of the blade, the stall appears to be delayed. This interesting aerodynamic phenomenon called stall delay was first observed by Himmelskamp in 1945 [4] and has been the focus of quite a few numerical [5–13] and experimental [1–3] studies. In order to further advance the knowledge of stall delay to improve the design and engineering practice of rotating machinery, the benchmark case of a static airfoil in deep stall deserves more investigations.

Separated flows over various airfoils at low Reynolds numbers have been the subject of much

research interest since such flows exist in many places such as the inboard sections of wind turbines and helicopter rotors, high-altitude vehicles, and low speed aircraft, etc. Many of the past research studies investigated flows at pre-stall conditions with small AOAs [14–20] which were quite different from those at large AOAs. The separated flow over a deeply stalled airfoil, which is the interest of the current study, has long been expected to behave similarly to the flow over a bluff body [21, 22]. Relevant background knowledge on turbulent flow separation over a two-dimensional stalled airfoil at large AOA, and over a cylinder which is the most studied bluff body is provided below.

From smoke visualizations and hot-wire measurements on a NACA0012 airfoil, Huang and his coworkers [22, 23] identified four vortex shedding modes at various Reynolds numbers and AOAs. These modes were laminar, subcritical, transitional and supercritical. At AOA larger than 20° the flow was essentially in the supercritical regime for Reynolds numbers on the order of a few thousands. In this mode at large AOA, the turbulent boundary-layer separation occurred and turbulent organized vortices of the type of von Kármán vortex street were shed in the wake. The low-frequency shedding of the Kármán vortices was superimposed by the high-frequency shear-layer instability waves. Due to the large spacing between the separated shear layers from airfoil’s leading and trailing edges, no direct interaction was observed by Huang et al. [22, 23] between these upper and lower shear-layer instability waves. At the same AOA, the Strouhal numbers, $St = f_s d / U_\infty$ (where f_s is the frequency of the vortex shedding in the wake, d is the projection of the airfoil perpendicular to the freestream direction, and U_∞ is the freestream velocity), were approximately constant irrespective of Re in the supercritical regime. These constants ranged from 0.12 to 0.22, and the lower St corresponded to the higher AOAs [22]. Rodriguez et al. [24] performed direct numerical simulations (DNS) of separated flows over a NACA0012 airfoil at AOA= 9.25° and 12° which were in the transitional and supercritical modes, respectively. Their results at the higher AOA were consistent with the past experimental observations. They also observed that at AOA= 12° the trailing edge vortices entered the suction side to interact with the turbulent vortex packets developed from the leading edge, and eventually formed von Kármán like vortex street similar to the separated flow over a cylinder.

The shear layers separated from the cylinder were known to be turbulent when Reynolds num-

bers were larger than a few thousands [25]. The small-scale shear layer vortices amalgamated in the near wake to form the Kármán vortices [26]. Streamwise vortices had been observed by a number of studies [26–29] revealing the three-dimensionality of the turbulent vortical structures in both near and far wakes of the cylinder. In a recent combined effort of PIV measurements and DNS simulations of a cylinder’s turbulent near wake, Dong et al. [30] observed notable differences in flow structures at two different Reynolds numbers.

As noted by Yarusevych et al. [20], there were far less results of flow separations over airfoils than those over bluff-bodies. More research is therefore needed on the turbulent wake of airfoils at low Reynolds numbers. In addition, turbulence structures had been observed to exist and play very important dynamic roles in classical turbulent channel flows and boundary layers [31, 32], and they had been the focus of many recent research works (e.g. [33–39]). However, similar structural analysis appears non-existing for the turbulent wake of airfoils at low Re. Although it was known qualitatively that the wake consisted of shear layers and von Kármán like vortices, the existence of strong turbulence fluctuations would render different topologies of these vortical structures, which deserves more investigations.

Although POD analysis has been a common tool to study the coherent structures in turbulence, the link between the structures obtained from POD and those observed in the instantaneous flow fields was found quite weak. Moin & Moser [40] claimed unclear relationship between the characteristic eddies in the channel flow and the instantaneous structures. Baltzer & Adrian [41] also stated that it was only *likely* that the hairpin vortex packets in wall turbulence contributed strongly to the dominant POD modes. However, very recently, Wu [42] devised a novel application of proper orthogonal decomposition (POD) to physically relate the instantaneous energetic turbulence structures to the dominant POD modes. The objective of the current work is to apply this new method to investigate the instantaneous structures which are major turbulence contributors in the near wake of an airfoil at deep stall. Analysis on such turbulence structures in this flow regime is still lacking, and the present study is meant to contribute in this regard.

II. Proper Orthogonal Decomposition

The principle and equations of POD can be found in many places in the literature and they are only briefly summarized below. In this study, well-established snapshot POD [43, 44] was applied on the three-dimensional fluctuating velocity fields measured by tomographic PIV. In POD, any instantaneous fluctuating velocity field $u(x, t)$ can be decomposed into the form of

$$u(x, t) = \sum_{n=1}^{\infty} a_n(t) \phi_n(x), \quad (1)$$

where $\phi_n(x)$ are deterministic spatial POD modes, and $a_n(t)$ are random temporal coefficients. In snapshot POD, the coefficients $a_n(t)$ are obtained first by solving the eigenvalue problem with a positive definite Hermitian kernel of the form

$$\mu_n a_n(t) = \int_T \left(\int_{\Omega} u(x, t) u(x, t') dx \right) a_n(t') dt', \quad (2)$$

where the integrations are over the spatial domain Ω and a time interval T . For an ensemble of measured velocity fields, the spatial domain could be the whole or part of the field of view and the time domain represents the ensemble or the collection of samples of the velocity fields. The eigenvalues μ_n are real and positive and form a decreasing and convergent series. The POD modes can then be computed through the equation

$$\phi_n(x) = \frac{\int_T u a_n dt}{\int_T a_n^2 dt}. \quad (3)$$

The summation of the eigenvalues is equal to twice the turbulent kinetic energy of the flow, i.e. $\sum \mu_n = 2k = \langle \mathbf{u} \cdot \mathbf{u} \rangle$. Since the POD modes are usually normalized to be orthonormal, the POD coefficients are related to the eigenvalues through

$$\int_T [a_n(t)]^2 dt = \mu_n, \quad (4)$$

which dictates that the instantaneous fluctuating velocity field with a larger value of a_n^2 contributes more to the eigenvalue μ_n . As found by Wu [42], equation 4 provides a possible means to identify the instantaneous fluctuating flow structures that contribute significantly to the first few dominant POD modes. In addition, since the contribution to the eigenvalue μ_n comes from a_n^2 , instantaneous velocity fields with either positive a_n or negative a_n contain equivalently important information on the dominant flow structures for the n^{th} POD mode.

Fig. 1 Tomographic PIV setup.

III. Experiments

Experiments were conducted in a wind tunnel with a test section of $0.4\text{m} \times 0.4\text{m}$. The freestream turbulence intensity was about 0.4%. The side walls of the wind tunnel are made of glass and the top and bottom walls are made of acrylic for optical access in the tomographic PIV measurements. The airfoil used here was S805 designed by National Renewable Energy Laboratory (NREL) in USA for the purpose of horizontal-axis wind turbines [45]. The airfoil was mounted wall-to-wall at the vertical center of the wind tunnel's test section. The chord length of the airfoil was chosen to be 30 mm so that the Reynolds number of the flow was similar to that of the blade model studied in Lee & Wu [1–3]. The aspect ratio of the airfoil was therefore 13.3. The Reynolds number based on the chord length of the airfoil and the freestream velocity was $Re = V_\infty c / \nu \approx 4600$. A large angle of attack (AOA) of 30° was chosen in this study at which the airfoil was in deep stall with a massively separated flow.

Tomographic particle image velocimetry (Tomo-PIV) was used to measure the velocity fields of the separated flow on the suction surface of the airfoil. The experimental setup is shown in Figure 1. An 8mm-thick laser light volume illuminated the flow at the airfoil's spanwise center. The duration of the laser pulses is 7 ns. The seeding was provided by olive oil atomized by a Laskin nozzle. The seeding density was adjusted to yield the best reconstructed particle images from all four cameras.

The images of the particles were recorded by four 12-bit CCD cameras with 105mm lenses and each camera's CCD chip has 2456×2058 pixels. The pixel size is $3.45 \mu m$. The in-line configuration of the cameras was used as in Tang et al. [46] such that the backward-scattering of the laser light from the particles was captured, yielding similar particle intensities in the four recordings. In the in-line configuration, the four cameras were arranged at the same side of the laser light volume and along an arc line, as illustrated in figure 1. To reduce the laser light reflection from the airfoil's surface, the airfoil was painted with Rhodamine B and a band-pass filter ($532 \pm 10 nm$ with $>95\%$ transmission) was used for each camera. The sampling frequency is 2.5Hz. FastMART algorithm was applied in LaVision Davis software to reconstruct the three-dimensional light intensity fields of the particles. Volume self-calibration was also implemented to improve the calibration accuracy. Standard image preprocessing such as minimum subtraction, image normalization and Gaussian smoothing were applied in Davis software. Three-dimensional velocity vector fields were obtained by recursive volumetric cross-correlation method. The final interrogation volume size is $96 \times 96 \times 96$ voxels with 75% overlap. The resultant vectors were then validated and smoothed to remove high frequency noise. Each velocity field has $106 \times 92 \times 11$ ($x \times y \times z$) vectors with a vector grid spacing of 0.76mm. The total field of view was about $80 mm \times 70 mm \times 8 mm$. Five hundred statistically independent velocity fields were measured. The fluctuating velocity fields used for the present POD analysis were then computed by subtracting the ensemble mean velocities from the measured instantaneous velocity fields. The signal-to-noise ratio is above 1.5 for the measured velocities. The overall uncertainty in the measured velocities was about 0.15 voxels in the spanwise direction and was about 0.1 voxels in the other two directions. Since the freestream velocity in the present flow was about 7 voxels, the overall uncertainty of the current measurements of the instantaneous velocities was therefore about $1.5 \sim 2\%$ of the freestream velocity.

IV. Results and discussion

A convergence test for the current POD analysis was done first. Figure 2 shows the numerical values of the first five eigenvalues with different numbers of velocity fields in the POD analysis. It illustrates that for the current flow, POD analysis converges very well when the sample size is larger than 350. It also shows that higher modes converge faster than the lower modes.

Fig. 2 Numerical values of the first 5 eigenvalues with different numbers of velocity fields in the POD analysis.

Table 1 Contributions to the turbulent kinetic energy from the first 10 POD modes.

Mode n	1	2	3	4	5	6	7	8	9	10
	27.8%	16.3%	6.8%	4.2%	3.7%	3.0%	2.3%	1.9%	1.3%	1.2%

The energy contributions from the first 10 POD modes are tabulated in table 1. The first POD mode accounts for 27.8% of the total turbulent kinetic energy and the second mode contributes 16.3%. The third mode accounts for 6.8% which is less than half of the second mode. Therefore, the first two modes are the dominant modes and their contributions to the turbulent kinetic energy of the current flow are more than 44%. As such, the turbulence structures contributing significantly to these first two POD modes are indeed energetic structures and they are therefore the main focus of the current study.

The vector plots of the first two POD modes, ϕ_1 and ϕ_2 , are presented in figure 3. The main feature of the first mode is a single large-scale counter-clock-wise vortex with a strongly induced flow. It should be noted that the concept of “induced” flow originated from the application of the Biot-Savart law to determine the velocity in the irrotational fluid surrounding a vortex. However, the wake is not irrotational. Therefore, the term of “induced” flow was only loosely used here to describe

Fig. 3 Vector plots of the first two POD modes. (a) Mode 1 ϕ_1 ; (b) Mode 2 ϕ_2 . The black vectors represent the streamwise velocity component u and the cross-stream component v in the $x - y$ plane at the spanwise center of the measurement volume. The insets illustrate the three-dimensional velocity fields in the regions of interest which are squared in the $x - y$ planes. The colors of the three-dimensional vectors in the insets reflect the magnitudes of the streamwise velocity component u .

the flow around a vortex. The shear layer vortices from either leading-edge or trailing-edge of the airfoil cannot be observed in this mode. The induced flow on the right side of the vortex appears stronger than that on the left side. This vortex is located at a little farther than 1.5 chord lengths from the airfoil's leading edge in the streamwise direction. The inset of figure 3 (a) shows vectors around that vortex in the measurement volume. The colors of the vectors reflect the magnitudes of u component. It is shown that the large-scale vortex in ϕ_1 is essentially two-dimensional with little variation in the spanwise direction. The second POD mode illustrates two counter rotating vortices with strong induced flows between them. The center of the left counter-clock-wise vortex is at 1.35 chord lengths downstream of the airfoil's leading edge and the right clock-wise one is at $x = 1.85c$. The vertical separation of these two vortices is about 0.12 chord lengths which is much smaller than

Fig. 4 Histograms of normalized POD coefficients for (1) mode 1; and (b) mode 2.

their streamwise separation. Therefore the flows between these two vortices are mainly vertically upward. The three-dimensional vectors around this main structure of ϕ_2 , which is shown in the inset, reveal its two-dimensionality, again with little change in the spanwise direction. Note that the results of two-dimensional features of these first two POD modes are due to the statistically two-dimensional flow over the airfoil. It should also be noted that the wake of the airfoil is likely to be dominated by the alternative shedding of vortices from the leading and trailing edges. However, this cyclic behavior may not be captured by the POD analysis of the randomly sampled velocity fields in the present study. This possibly important dynamic phenomenon of the near wake flow of the airfoil will be examined in the future via time-resolved PIV measurements.

The instantaneous turbulence structures contributing significantly to the first two POD modes can be found from their coefficient values [42]. The histograms of the coefficients for the first two modes, a_1 and a_2 , normalized by their own root-mean-square (RMS) values, are shown in figure 4. The distributions of both coefficients are observed not to be symmetrical. There exist a little more instantaneous fluctuating velocity fields with larger positive a_1 and a_2 values. Since the contribution to the eigenvalues comes from a^2 , more fields with larger positive POD coefficients imply that first two POD modes are more influenced by instantaneous turbulence structures with positive coefficients than those with negative coefficients.

Figure 5 (a) shows a representative instantaneous fluctuating flow field with a large magnitude

of positive coefficient of the first mode, whose $a_1 = 2.06\sigma_{a1}$, where σ_{a1} is the root-mean-square (RMS) value of a_1 . This particular field illustrates a large-scale vortex (labeled D in the figure). It has the same rotational sense as the vortex in the first mode, ϕ_1 . Strong flows around it can be observed to cover a large part of the near wake region. Three small-scale vortices (labeled A, B, and C) can also be seen along the leading-edge shear layer, but they induce much weaker flows. The contour surfaces of λ_2 in figure 5 (b) show that although these vortices are mainly along the spanwise direction, they illustrate three-dimensional features. The insets present the three-dimensional flows around the shear-layer vortices A & B produced by the K-H instability, and around the large vortex D which is the first vortex in the von Kármán vortex packet in the wake. Weak spanwise variations of the velocities can be observed within the depth of the measured flow volume. They produce vorticity components in the x and y directions, rendering three-dimensional corrugations of the vortices. However, ω_x and ω_y are much weaker than ω_z . All the eight fluctuating velocity fields (1.6% of the total number of the measured fields) whose a_1 values are larger than $2\sigma_{a1}$ have been examined. The common feature is the vast region of strongly induced fluctuating flow by the large counter-clockwise Kármán vortex located at $x=1.5\sim 1.7c$.

An example of the instantaneous flow field with a large magnitude of negative POD coefficient for the first mode ($a_1 = -2.44\sigma_{a1}$) is shown in figure 6. In addition to the clockwise Kármán vortex C, there also exist one leading-edge shear layer vortex A and one trailing-edge shear layer vortex B. Although the contour surfaces of λ_2 of these three vortices appear to illustrate that they are well separated from each other, the induced fluctuating flows by them are clearly interconnected. The existence of the weak streamwise (x) and cross-flow (y) vorticity components can also be inferred from the corrugations of λ_2 , particularly for vortices B and C. These three interacting vortices are the most commonly observed features of the turbulence structures in all eight fields whose absolute values of the negative a_1 are larger than $2\sigma_{a1}$.

An instantaneous fluctuating field which is one of the major contributors to the second POD mode with a positive coefficient of $a_2=2.12\sigma_{a2}$ is presented in figure 7 (a). There is an extended region of vertically induced flow by a pair of counter-rotating vortices C and D. Vortex C is the first Kármán vortex and vortex D appears to be the shear layer vortex from the airfoil's trailing

Fig. 5 (a) An instantaneous fluctuating velocity field contributing significantly to the first POD mode with $a_1 = 2.06\sigma_{a1}$. σ_{a1} is the RMS value of the POD coefficients of the first mode. The vectors in the $x-y$ plane represent the u and v velocity components in the middle plane of the measurement volume. The insets illustrate the three-dimensional velocity fields in the two squared regions of interest in the $x-y$ plane. The contours on the surface e_i of the insets represent the vorticity component, ω_i . (b) The contour surfaces of normalized λ_2 to illustrate the vortices in the velocity field shown in (a).

edge. With the help of the shear layer vortex B from the airfoil's leading edge, the vertical flow is further extended upward to cover almost the whole height of the wake flow. These three vortices and the flow associated with them are quite similar to those shown in figure 6. It is not clear, from the current random sampling of PIV measurements, if the flow in figure 7 (a) could be produced by the downstream advection of the structure similar to that in figure 6. Time-resolved PIV will be applied in the future to explore this possibility. This pairing of a Kármán vortex and a trailing-edge shear layer vortex to induce a large region of strong vertical flow is the main turbulent structure in those flow fields whose positive a_2 coefficients are larger than $2\sigma_{a2}$.

Fig. 6 (a) An instantaneous fluctuating velocity field with $a_1 = -2.44\sigma_{a1}$. The vectors in the $x-y$ plane represent the u and v velocity components in the middle plane of the measurement volume. (b) Contour surfaces of normalized λ_2 to reveal the vortices in the field shown in (a).

Figure 7 (b) presents a field having a negative POD coefficient of $a_2 = -2.27\sigma_{a2}$. The major turbulence structure is the pair of vortices E and F inducing extended region of strong downward flow. A leading-edge shear layer vortex is merging with vortex E in this flow field. It appears that vortex E is accumulated by this small shear layer vortex to join the Kármán vortex street. There are only two fields with magnitudes of negative a_2 values larger than $2\sigma_{a2}$, but all eleven fields with $a_2 > 1.7\sigma_{a2}$ have been examined. It was found that the common features include a large-scale strong downward flow induced by a Kármán vortex, and other vortices distributed randomly.

To compare with the flow structures which are the dominant contributors to the first two POD modes, a flow field whose a_1 and a_2 are both very small is presented in figure 8. No major Kármán vortex can be observed, which indicates that, at this instant of time, the Kármán vortex street is further downstream, beyond $2c$ from the airfoil's leading edge. The three leading-edge shear layer vortices and the one trailing-edge shear layer vortex do not induce strong velocity fluctuations and thus produce much smaller turbulent kinetic energy. The vortices along the two different shear layers are well separated and do not appear to be directly interacting. This flow field fits well into the flow visualization for a NACA 0012 airfoil at large AOAs observed by Huang & Lin [22]. They found that the large spacing between the leading- and trailing-edge shear layers prevented the interaction

Fig. 7 (a) An instantaneous fluctuating velocity field contributing significantly to the second POD mode with $a_2 = 2.12\sigma_{a2}$. (b) An instantaneous fluctuating velocity field with $a_1 = -2.27\sigma_{a2}$. σ_{a2} is the RMS value of the POD coefficients of the second mode. The vectors in the $x - y$ plane represent the u and v velocity components in the middle plane of the measurement volume. The contour surfaces of normalized λ_2 were also presented for these two fields to illustrate the imbedded vortices.

of the shear-layer instability waves.

The small-scale shear layer vortices were known to be produced from roll-ups of instable shear layers from the airfoil's leading and trailing edges. They were also known to produce much less turbulent kinetic energy than the large-scale Kármán vortices. Therefore, it is not surprising that the first two POD modes do not reveal them in the vector plots. However, their existences should be abundant since the turbulent wake starts from the roll-up of the instable shear layer. They have also been observed in the above instantaneous dominant flow structures to induce flows directly interacting with that induced by the Kármán vortex. As such, it is of great interest to inquire which

Fig. 8 An instantaneous field with both small a_1 ($a_1 = -0.08\sigma_{a1}$) and a_2 ($a_2 = -0.22\sigma_{a2}$) values. Both fluctuating velocity field and λ_2 field are shown.

higher POD mode(s) will reveal these shear layer vortices. To explore this, the vector plots of POD modes 3 to 10 are presented in figure 9. Mode 3 still mainly reflects the feature of the Kármán vortices, although the lower vortex appear to be a trailing-edge shear layer vortex which advects into the turbulent wake and interacts with the Kármán vortices. The definitive appearance of the shear layer vortices, from either the leading- or trailing-edge, starts from Mode 4. While other higher POD modes include turbulence structures in the wake, it is very interesting to observe that mode 4 and 5 feature only shear layer vortices from the airfoil's leading edge, while other parts of the flow within $2c$ are almost quiescent. However, the instantaneous flow fields which are major contributors to these two POD modes do not necessarily exclude the wake flows which produce large velocity fluctuations.

The centers of the shear layer vortices along the airfoil's leading edge in the instantaneous fields with a magnitude of a_4 larger than $1.5\sigma_{a4}$ are presented in figure 10 (a). Similar to some past studies [34–36, 50, 51], the center of the vortex was determined by the location of maximum value of λ_2 . They are plotted against the vectorial plot of POD mode 4 around the leading-edge shear layer. It is clear that the shear layer vortices in those major contributors of ϕ_4 are clustered in concentrated regions corresponding to the rotational vectors of ϕ_4 . The clockwise rotational vectors in ϕ_4 are therefore mainly formed by the induced flows of vortices in the fields with large positive a_4 values, while the counter-clockwise rotational vectors are produced by the vortices in the fields with large

magnitudes of negative a_4 values. The obvious gaps between the clusters of the vortices are filled by the induced upward or downward vectors in ϕ_4 . Similar observations can also be found in figure 10 (b) for mode 5. However, the vortices in the main contributing flow fields for ϕ_5 appear to be located at the gaps of the vortices in ϕ_4 . When the center locations of the shear layer vortices in figure 10 (a) and (b) are plotted together (figure 10 (c)), it can be found that, in general, the regions clustered by the vortices in the flow fields contributing significantly to mode 4 and 5 do not overlap, except within $0.8c$ from the airfoil's leading edge. Further, these vortices are fairly concentrated in a narrow band in the y direction within $x = 1.3c$, but they are more scattered further downstream.

Figure 11 presents the one-sided autospectral density functions of fluctuating cross-stream velocity component, G_{vv} , computed from the fluctuating fields which are major contributors ($|a_i| > 1.5\sigma_{ai}$) to the first 5 POD modes. Three y locations have been chosen to compute G_{vv} : (1) $y/c = 0.07$ along the airfoil's leading edge; (2) $y/c = -0.3$ along the airfoil's chord center; and (3) $y/c = -0.6$ along the airfoil's trailing edge. The procedure outlined in Bendat & Piersol [52] was followed to compute the spectral functions. The wavenumbers for G_{vv} were computed by $k/(N\Delta x)$, where $k = 0, 1, \dots, N/2$, N is the total number of grids in x direction, and Δx is the grid spacing in the velocity field. As in Dong et al. [30], averaging in the spanwise (z) direction was also performed in computing G_{vv} . G_{vv} from the whole ensemble of fields are also shown for comparison. The dashed lines in figure 11 illustrate that the one-sided autospectral functions of streamwise velocity component, G_{uu} , only show a monotonic decrease with wavenumbers at all three y locations and therefore no interesting spectral information can be observed in the current field of view from G_{uu} . Since the numbers of the files meeting the threshold of $1.5\sigma_{ai}$ for each of the first 5 POD modes is about 70, G_{vv} computed from 70 random velocity fields is also included in figure 11 as dotted lines. The comparison to G_{vv} computed from the full ensemble indicates adequate statistical convergence of these spectral functions with a sample size of 70.

Within the field of view of the current PIV experiments, the spectral functions along the airfoil's leading edge mainly reflect the features of the shear layer vortices. The peak values of G_{vv} for the major contributors of the first five POD modes are about the same as that of G_{vv} computed from the full ensemble of velocity fields. Although the peak values of G_{vv} do not occur exactly at the same

wavenumbers, the difference is insignificant since it is within the current wavenumber resolution of 6.5 (1/m). The peaks are located at a wavenumber of approximately 60 (1/m), corresponding to a wavelength of 0.56 chord lengths. This length compares very well with the streamwise separations of the shear layer vortices shown in the instantaneous velocity fields in figures 5 to 8. Therefore, it may be considered as the average spacing of the shear layer vortices in the airfoil's leading-edge shear layer. On the other hand, small differences can be observed in the profiles of G_{vv} between these POD modes. These differences are mainly caused by the fluctuating velocities induced by the large-scale Kármán vortex. For the first POD mode, G_{vv} computed from the major contributors obtain significant power spectral from wavenumbers lower than the peak wavenumber. As can be inferred from the vector plot of ϕ_1 in figure 3 (a), energy-containing large-scale cross-stream velocities are induced by the first Kármán vortex and they extend above the airfoil's leading edge. However, the energies at lower wavenumbers for G_{vv} of POD mode 2 are much lower, indicating much weaker induced large-scale cross-stream velocities along the leading-edge shear layer. In contrast, the profiles of G_{vv} for mode 4 and 5 are narrower than the rest of the modes and the peak values are much higher, showing the dominance of the shear layer vortices in those instantaneous velocity fields contributing significantly to these two modes.

The power spectral functions along the airfoil's chord center reveal the spatial features of the large-scale Kármán vortex. Therefore the peak values of G_{vv} are all located at a much lower wavenumber of about 23 (1/m). This corresponds to a wavelength of about $1.45c$. As shown in figure 5, one large Kármán vortex induces positive and negative cross-stream velocities on each side, constituting half a wave. As such, the half wavelength of about $0.73c$ reflects the average size of the induced flow, at $y/c = -0.3$, by the Kármán vortex within the current PIV field of view. This value of the half wavelength is quite consistent with the size of the induced flow by the Kármán vortex along the chord's center in the instantaneous fields in both figure 5 and figure 6. The peaks of G_{vv} along the chord's center are also found to decrease with higher modes, simply because the instantaneous fields which are major contributors to higher POD modes possess less energies from the Kármán vortex.

The wavenumber corresponding to the peak values of G_{vv} along the airfoil's trailing edge is

about 31 (1/m), which infers a length scale of approximately 1 chord length. This wavelength is too large for the size of the induced flow by the shear layer vortex from airfoil's trailing edge, and too large for the streamwise separation between the trailing-edge shear layer vortex and the Kármán vortex. It may be associated with the extent of the induced cross-stream flow by the Kármán vortex along the airfoil's trailing edge, and therefore the half wavelength of $0.5c$ implies its size. This length scale compares well to the size of the downward and upward flows observed in figure 7 along the airfoil's trailing edge. Similar to those along the airfoil's center, the peak values of G_{vv} decrease with higher modes.

Autospectral density functions from instantaneous fields satisfying a lower threshold of $a_1 > 1\sigma_{a1}$ for the first POD mode are compared in figure 12 to G_{vv} computed with $a_1 > 1.5\sigma_{a1}$ to examine the effects of the thresholds. It can be seen that different thresholds do not change the wavenumber corresponding to the peak values of G_{vv} . However, due to the inclusion of velocity fields containing smaller turbulent kinetic energies, the values of G_{vv} are reduced, particularly at wavenumbers corresponding to the induced flows by the Kármán vortex. On the other hand, G_{vv} values reflecting the features of shear-layer vortices along airfoil's leading edge are negligibly affected.

V. Conclusions

Tomographic particle image velocimetry has been applied to measure the three-dimensional velocity fields in a volume of turbulent flow in the near wake of an S805 airfoil in deep stall at an AOA of 30° and at a Reynolds number of approximately 4600. Through a POD analysis, instantaneous turbulence structures contributing significantly to the first and second POD modes, and therefore to the turbulent kinetic energy, have been identified. Dominant flow structures for the first POD mode include a large region of induced flow by a counter-clockwise Kármán vortex and a three-vortex structure with one large-scale Kármán vortex and two small-scale shear layer vortices. Turbulent structures that are major contributors to the second POD mode include (1) an extended region of strong upward flow induced by the pairing of a Kármán vortex and a trailing-edge shear layer vortex; and (2) a large region of strong downward flow associated with several vortices including a Kármán vortex.

Although the first three POD modes are dominated by the flows associated with the first Kármán

vortex, modes 4 and 5 represent only the small-scale shear layer vortices along the airfoil's leading-edge. The vortices in the major contributing velocity fields to POD modes 4 and 5 are found to cluster in the rotational vectors of these two modes, emphasizing the contributions of the vortices in these dominant fields to form POD modes 4 and 5. In addition, the center locations of these vortices in mode 4 and 5 do not overlap beyond $x/c = 0.8$.

Similar peak wavenumbers are found in the autospectral functions G_{vv} computed from the instantaneous fluctuating velocity fields which contribute significantly to the first 5 POD modes. Along the airfoil's leading-edge shear layer, the wavelength corresponding to the peak value of G_{vv} is consistent with the streamwise separation of the shear layer vortices. Due to the large-scale induced flow by the Kármán vortex, G_{vv} computed from the major contributors to the first POD mode obtains significant low wavenumber energies. In contrast, profiles of G_{vv} obtained from main contributors to modes 4 and 5 illustrated much more concentrated energies at the peak wavenumbers. Finally, the peak wavenumbers along the airfoil's center and trailing-edge reflect the sizes of the induced flows by the Kármán vortex at these two cross-stream locations.

In the future, the dynamics of these energetic turbulence structures will be investigated using a time-resolved PIV system. In addition, studies covering a wide range of Reynolds numbers and angles of attack will be carried out.

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Fig. 9 Vector plots of higher POD modes, from mode 3 to mode 10, in the $x-y$ planes.

Fig. 10 Central locations of the vortices existed in the instantaneous fields contributing significantly to the fourth and fifth POD modes, ϕ_4 and ϕ_5 . (a) Centers of the vortices in the instantaneous fields whose magnitudes of a_4 are larger than $1.5\sigma_{a_4}$. They are plotted against the vector plot of ϕ_4 in the $x-y$ plane. (b) Centers of the vortices in the instantaneous fields whose magnitudes of a_5 are larger than $1.5\sigma_{a_5}$. They are plotted against the vector plot of ϕ_5 in the $x-y$ plane. (c) Overlay of the central locations of the vortices shown in (a) and (b).

Fig. 11 One-sided autospectral density functions of fluctuating cross-stream velocity component, G_{vv} , computed from instantaneous velocity fields whose magnitudes of POD coefficients, from a_1 to a_5 , are larger than $1.5\sigma_a$ along (a) airfoil's leading edge; (b) airfoil chord's center; and (c) airfoil's trailing edge. From top to bottom: G_{vv} from whole ensemble of fields; G_{vv} from fields with $|a_1| > 1.5\sigma_{a1}$; G_{vv} from fields with $|a_2| > 1.5\sigma_{a2}$; G_{vv} from fields with $|a_3| > 1.5\sigma_{a3}$; G_{vv} from fields with $|a_4| > 1.5\sigma_{a4}$; and G_{vv} from fields with $|a_5| > 1.5\sigma_{a5}$. The dashed lines are G_{uu} computed from the whole ensemble of fields. The dotted line is G_{vv} computed from a random selection of 70 fields to compare with G_{vv} computed by using the whole ensemble.

Fig. 12 The threshold effects on G_{vv} . Two thresholds of $1.0\sigma_{a1}$ and $1.5\sigma_{a1}$ are used to select the instantaneous fields to compute G_{vv} .