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A comparison study between two simulation approaches for safety assessment: Cellular Automata and SSAM

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Abstract: A comparison study is conducted between two simulation methods to estimate conflicts between road users. An improved Cellular Automata (CA) model is proposed to estimate occurrences and severity of traffic conflicts (both vehicle-vehicle and vehicle-pedestrian) at signalized intersections. The proposed CA model is compared with a calibrated method of Surrogate Safety Assessment Model (SSAM) based on VISSIM. Simulated conflicts from both methods are compared with observed vehicle conflicts from automated vehicle tracking for both occurrences and severity. Simulation results show CA approach is able to replicate realistic conflicts. However, SSAM tends to over-estimate occurrences and under-estimate severity of rear-end and lane-changing conflicts. SSAM is also found to over-estimate the severity of crossing conflicts. Furthermore, the proposed CA model is able to estimate conflicts between vehicles and pedestrians.

CE Database subject headings: Traffic safety; Simulation models; Intersections

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1. Introduction

Signalized intersections are hot spots of road safety, especially for an urban environment Singapore that has more than 1,400 signalized intersections island-wide. Over the three years from 2009 to 2011, 112 fatal accidents occurred at signalized intersections. In the year 2009, about one in five (21%) road traffic accidents occurred at signalized intersections in Singapore (Hau et al. 2010). This number increased to 23.4% in 2011 (Singapore Police Force 2012). For pedestrian safety at signalized pedestrian cross-walks, from 2007 to 2012, there has been an average of 82 accidents each year involving pedestrians, cyclists or vehicles. On average, 3 pedestrians are killed and 55 are injured in these accidents at signalized cross-walks per year (Iswaran 2013).

1.1 Previous studies of road safety assessment

The majority of traditional approaches for safety assessment at signalized intersections are based on statistical analysis of crash occurrences. A series of studies were developed to estimate road safety performance of signalized intersections for different intersection types (Persaud et al. 2002; Oh et al., 2003; Oh et al. 2010a). Kusumawati (2008) has developed an accident prediction model based on time series analysis of crash occurrences in Singapore's context. However, a successful statistical analysis is subjected to several conditions including large sample size, sufficient and accurate details for each accident, and availability of before-after data if a certain action is studied (Tarko et al. 2009).

1.2 Surrogate safety measurements and SSAM

Given the constraint on the availability and information quality of accident crash occurrences, surrogate safety measurement (SSM), are used for safety assessment. Typical SSM includes traffic conflicts, known as Traffic Conflict Technique (TCT) (Zhang and Prevedouros 2003; Kirk and Stamatiadis 2012; El-Basyouny and Sayed 2013), trajectories (Weng and Meng 2014; Weng et al. 2014), speed profile (Zheng et al. 2010; Moreno and Garcia 2013), deceleration rate (Cunto and Saccomanno 2008), and stopping distance index (Oh et al. 2010b). These studies estimates vehicle movements and identify risk and likelihood of crashes.

Frequency of surrogate measurement should be strongly associated with frequency of crashes to make sure the safety assessment based on SSM will not depart from result from accident records. To support such assumption, a significant relationship is found between conflicts and accidents according to a regression analysis at 92 signalized intersections (Sayed and Zein 1999). Furthermore, conditional crash probability given a surrogate event is estimated by Wu and Jovanise (2012). However, the SSM approach is subjected to large sample size and accuracy of movement observations. To overcome technical difficulty and large cost of field observations, simulation models are developed for simulating conflicts and assessing safety performance.

To estimate conflicts based on microscopic simulation, a software package called SSAM developed by Federal Highway Administration (FHWA). SSAM identifies critical safety indicators, such as Time-To-Collision (TTC) through trajectory files based on several algorithms (Gettman et al. 2008). In essence, SSAM relies on simulation packages (such as VISSIM), which require additional calibrations. Several researchers have evaluated the feasibility of SSAM. Ciro and Maurizio (2012) applied

this microscopic simulation approach to predict crash occurrences at un-signalized intersections. Huang et al. (2013) developed a linear regression model to compare simulated and observed conflicts to evaluate accuracy of SSAM and VISSIM simulation. It is found that SSAM is able to provide acceptable results for rear-end conflicts. However, simulation results of lane-changing and crossing conflicts are only moderately acceptable with higher Mean Average Percentage Error (MAPE) values.

1.3 Microscopic simulation based on CA

Cellular Automata (CA) models are becoming popular in recent years based on its simplicity and computation efficiency. Improved CA models have been developed to be applied to more complex traffic conditions, such as signalized intersections (Tian 2012; Jin et al. 2014). Based on flexible transition rules, it is becoming easier to use CA models to simulate microscopic traffic behavior while leveraging on parallel CA computation. Position, speed and other microscopic attributes of vehicles as well as time are discretized in the CA models (Wolfram 1984). Compared to commercial simulation packages, CA models are found to be more flexible to model microscopic vehicle movements and interactions (Chai and Wong 2013a; Kerner et al. 2011; Meng and Weng 2011). However, notwithstanding the simplicity of updating rules, several limitations of conventional CA models are identified. First, due to discrete model settings, simulation outputs are relatively coarser than continuous simulation models. Moreover, conventional CA models have not been widely used in safety aspects as

velocity change rules in basic CA models are set to ensure all vehicles are travelling with enough gaps and without collisions.

This study overcomes afore-mentioned limitations and develops an alternative tool for surrogate safety measurement through traffic simulation. Firstly, conventional CA model are improved to simulate complex movements at signalized intersections. Moreover, by incorporating safety indicators such as TTC, conventional CA model is modified for safety assessment. A comparison study is conducted between the proposed CA approach with an existing approach by SSAM and VISSIM simulation. Both models are calibrated and validated based on field observations of vehicle and pedestrian movements. Simulated conflicts from CA and SSAM are compared with observed conflicts in both occurrence and severity aspects.

2. Proposed CA model for safety assessment

Several improvements have been made to conventional CA models to improve simulation accuracy. First, to simulate mixed traffic flow, a smaller cell size is chosen. Second, accuracy of simulation is increased through usage of movement characteristics from field observations, such as maximum vehicle velocity, acceleration/ deceleration rates, and stopping propensity. Last but not least, by computation of safety indicators, occurrences and severity of vehicle conflicts are estimated.

2.1 Cell space and movement rules

According to typical lane width and vehicle sizes, in this study, each cell is selected as a square space of 0.9 m width and 0.9 m length. Lane width is assumed to be 3.6m, and each lane contains four rows of cells. Car, heavy vehicle and motorcycle, are simulated as a mixed traffic flow to approximate real traffic in Singapore.

According to the actual sizes of different vehicle types, a car in this study occupies 5×2 cells, a heavy vehicle takes 13×3 cells, and a motorcycle takes 3×1 cells. Moreover, a leading cell for each vehicle is defined (shown as the black cells in Figure 1) to record the exact position of each vehicle.

In the proposed CA model, forwarding rules for vehicle movements are modified from a multi-lane NaSch model (Spyropoulou 2007). To simulate vehicle movements at signalized intersections, several modifications, such as gap tolerance and stop-go decisions at the onset of amber are added. Lane-changing is allowed in approach and departure areas subject to enough gaps in current and target lanes. Moreover, for cars and heavy vehicles, lateral drift within the same lane is allowed due to the intent of lane-changing or giving way to following motorcycles.

Sometimes the motorcycles will follow the leading vehicle, but more often, motorcycles tend to filter into the lateral gap between two moving streams of vehicles. When approaching the stop-line, motorcycles always move into the gap alongside queuing cars and heavy vehicles where there is a wide lateral displacement, and continue to move forward to the stop-line. Such kind of lateral drift is simulated

subject to sufficient lateral gaps.

To determine the cell size of a crossing pedestrian, firstly, the average width of pedestrian is found as 0.45 m in Asian countries. According to Highway Capacity Manual (HCM), the average thickness of pedestrian is 0.3 m and the dynamic space is 0.6-0.8 m (Transportation Research Board 2000). According to a field observation conducted in Singapore, a pedestrian tends to follow the front person closely and the minimum gap is found as 0.29 m. As a result, a cell size of 0.4 m×0.4 m is chosen for a crossing pedestrian. According to proposed cell size, in vertical direction (moving direction), the average walking velocity is 3 cells/s. In horizontal (lateral) direction, the maximum velocity is 2 cells/s. A gap tolerance rule is also included in which forward (moving direction) minimum gap is chosen as 1 cell and the lateral minimum gap is 0.

Geometric layout of the proposed CA model (similar with all studied intersections) is defined as one exclusive left-turn lane, one exclusive straight-through lane, one shared lane for straight-through and right-turn, and one exclusive right-turn lane at each intersection approach.

2.2 Vehicle movement rules

As each approach and departure lane has four rows of cells in the CA model, x and y are defined to classify vehicle movements into forward and lateral directions. x stands for vehicle position along forwarding direction and y stands for vehicle position along lateral direction.

150 **(i) Forwarding rules**

151 The velocity v of each vehicle can take one of the $v_{max} + 1$ allowed integer values
152 $=0,1,\dots, v_{max}$. Suppose, x_n and v_n denote the position in x direction and velocity
153 of the leading cell, respectively, of the n^{th} vehicle. Then, $d_n = x_{n+1} - x_n$ is the space
154 headway in between the n^{th} vehicle and the $(n + 1)^{th}$ vehicle in front of it at time t .
155 Front gap is therefore defined as $g_n = d_n - b - g_t$, where b is number of cells
156 occupied by front vehicle in x direction and g_t is the applied gap tolerance. At each
157 time step $t \rightarrow t + \Delta t$, where Δt is selected as 1s the arrangement of each vehicle is
158 updated in parallel according to the following rules.

159 **Rule 1: Acceleration.**

160 If $v_n < v_{max}$, the velocity of the n^{th} vehicle is increased by φ_a , but v_n
161 remains unaltered if $v_n = v_{max}$ i.e.
162
$$v_n \rightarrow \min(v_n + \varphi_a \times \Delta t, v_{max})$$

163 **Rule 2: Deceleration (due to other vehicle).**

164 At green or amber phase:

165 If $g_n/\Delta t \leq v_n$, which means if subject vehicle continues moving, it will
166 exceed the front car at next time step. Therefore, the velocity will be reduced by φ_d
167 to $g_n/\Delta t - \varphi_d \times \Delta t$;

168 At red phase: Assume s_n is the distance between vehicle and stop-line.

169 If $\min(g_n, s_n)/\Delta t \leq v_n$, which means if subject vehicle continues moving, it
170 will exceed the front car or stop-line at next time step, the velocity of the n^{th} vehicle

171 is reduced to

172 $\min((g_n/\Delta t - \varphi_d \times \Delta t), (s_n/\Delta t)): v_n \rightarrow \min (v_n, (g_n/\Delta t - \varphi_d \times \Delta t), (s_n/\Delta t))$

173 **Rule 3:** Randomization.

174 If $v_n > 0$, the velocity of the n^{th} vehicle is decreased randomly by $\varphi_r = 1$
175 with probability p_r but v_n does not change if $v_n = 0$, whereby p_r is selected as
176 0.2 according to field observations.

177 $v_n \rightarrow \max((v_n - \varphi_r \times \Delta t), 0)$ with probability p_r

178 **Rule 4:** Vehicle movement.

179 Each vehicle is moved forward according to its new velocity determined in
180 Steps 1-3, i.e.,

$$x_n \rightarrow x_n + v_n \times \Delta t$$

181 **(ii) Lane-changing rules**

182 Along approach and departure lanes, vehicles change their lane very often. The
183 vehicles change lane in two main types. One is to move to the corresponding
184 straight-through or turning lane of the intersection; the other is to move to a less
185 congested convenient lane. According to field observation, the first type of
186 lane-changing occurs for all vehicle types while the second type of lane-changing
187 occurs mainly for cars and heavy vehicles. In the proposed CA model, one time step is
188 divided into two sub-steps: in the first sub-step, the vehicles may change lane
189 following the lane-changing rules, and in the second sub-step each vehicle may move
190 forward electively as in the single-lane model.

191 If the following three requirements are satisfied, vehicles will make a lane change in
192 two time steps (2 lateral cells per time step) with a probability of p_c^1 (first type) and
193 p_c^2 (second type).

194 (1) front gap is smaller than C_1^{car} cells (cars) or C_1^{hv} cells (heavy vehicles);

195 (2) front gap in target lane is larger than C_2^{car} cells (cars) or C_2^{hv} cells (heavy
196 vehicles); and

197 (3) rear gap in target lane is larger than C_3^{car} cells (cars) or C_3^{hv} cells (heavy
198 vehicle).

199 **2.3 Pedestrian movement rules**

200 Transition rules for pedestrians are modified from a CA model for bi-directional
201 walkways proposed by Blue and Adler (2001). Each time step is sub-divided as two
202 sub-steps, as forwarding and lateral movement. At each time step, the following rules
203 are applied in parallel updating the position and velocity of each pedestrian. Assume
204 all pedestrians are moving along y direction. Positions of each pedestrian in
205 forwarding and lateral directions are defined as y_p and x_p .

206 **i) Forwarding rules:**

207 **Rule 1:** Front gap (g_{p1}) is computed for each subject pedestrian (p_1) as:

208 $g_{p1} = v_{max}^p$ cells if front pedestrian (p_2) is moving in the same direction;

209 $g_{p1} = INT(0.5 * |y_{p1} - y_{p2}|)$ if front pedestrian is moving in opposite direction;

210 **Rule 2:** Update velocity: $v_{p1} = \min(g_{p1}/\Delta t, v_{max}^p)$

211 **Rule 3:** Overtaking: If $g_{p1}=0$ or 1, p_1 will pass p_2 in the next time step, therefore

212 $v_{p1} = \min(g_{p1}/\Delta t + 1, v_{max}^p)$

213 **Rule 4:** Update position $y_1 = y_1 + v_{p1} \times \Delta t$

214 **ii) Lateral rules:**

215 p_1 and p_2 (moving in opposing directions) in the same or neighboring lateral lanes
 216 are required to side step to avoid conflicting with each other. According to Blue and
 217 Adler (2001), pedestrian p_1 will side step if the front gap (g_{p1}) to an opposing
 218 pedestrian p_2 is within 3.2m (8 cells). Both p_1 and p_2 has a 0.5 possibility of side
 219 stepping with a lateral velocity, $v_{p1}^l=1\text{cell/s}$. If additional conflict is detected after
 220 side stepping, pedestrian(s) will sidestep again to make sure no further conflict exists.
 221 If conflict with other pedestrian occurs after the subject pedestrian has already side
 222 stepped 2 cells within one time step, the subject vehicle will not side step and the
 223 opposing pedestrian will side step to avoid conflict with 100% probability.

224 ***2.4 Classification of conflicts***

225 Observed vehicle conflicts are grouped as vehicle-vehicle (V2V) conflicts and
 226 vehicle-pedestrian (V2P) conflicts and further classified as different types, according
 227 to movement directions.

228 **(i) V2V conflicts**

229 In this study, vehicle-vehicle conflicts are classified into 8 types based on their
 230 position and vehicle movements involved, as summarized in Figure 2.

231 In order to be consistent with SSAM, the conflict types in Figure 2 are combined as
 232 rear-end, lane-changing and crossing conflicts according to the angle of conflict

vehicles' movements. Types (1)-(3) Conflicts are classified as rear-end conflicts; Types (4), (5) Conflicts are classified as lane-change conflicts, and Types (6)-(8) Conflicts are classified as crossing conflicts.

(ii) V2P conflicts at signalized crosswalk

In this study, conflicts between vehicles and pedestrians at signalized pedestrian crosswalk are classified into 4 types as shown in Figure 3. During green-man phase, pedestrians conflict with left-turn vehicles, as shown in Figures 3a and 3b. For large intersections with slip roads, such conflicts occur along slip road. During red-man phase, conflicts occur between pedestrians and straight-through or right-turn vehicles, as shown in Figures 3c and 3d.

2.5 Computation of conflict occurrences and severity

Occurrence of vehicle conflicts (between subject vehicle with another priority vehicle or pedestrian) is recorded by distinguishing occurrence of subject vehicle deceleration. In the proposed CA model, there are three possible causes of vehicle deceleration as follows:

- 1) To avoid collision with the front vehicle/pedestrian;
- 2) To avoid collision with neighboring (along-side or opposing) vehicle(s); and
- 3) To come to a stop in front of the stop-line.

The first two types of deceleration are recorded as the occurrence of vehicle conflicts. Deceleration occurrences (excluding random deceleration) are recorded together, with its time, position, velocity and deceleration rate of subject vehicle for each simulation

scenario in each time step. A new indicator entitled “Deceleration Occurrence caused by Conflicts (DOC)” is developed and incorporated into the CA model. A higher DOC indicates more frequent conflict occurrences.

Severity of conflicts is estimated by computation of two safety indicators: Time-to-Collision (TTC), which is defined as the gap distance between a subject vehicle and conflict vehicle /pedestrian divided by their velocity difference (Lee, 1976); and Post-encroachment Time (PET) which is defined as time duration when vehicle/ pedestrian reach, occupy, and leave the conflict zone (Cooper, 1984). A lower TCC or PET value means there is a very short time for a vehicle to avoid collision. The two indicators are calculated for each detected conflict.

3. Observation study

3.1 Field observation

Field observations are conducted at six intersections in Singapore with similar geometrical layouts and same signal sequences. Intersections No. 1-4 are used to calibrate model parameters, and Intersections No. 5 and 6 are used for model validation. The field observations were made during weekdays for 10:00-11:00 am as off-peak hour and 6:00-7:00 pm for peak hour during weekdays. Movement parameters for both vehicles and pedestrians, such as arrival distribution, maximum velocity, acceleration and deceleration rates, are calibrated based on these field observations.

3.2 Automatic vehicle detection and tracking

Automatic vehicle classification and tracking technologies are used to record vehicle's velocity, position, front gap, as well as vehicle type (Chai and Wong 2013b). Recorded video images are transformed as orthographic frames to create a global coordinate system for vehicle tracking. The applied classification and tracking method is validated through comparison with computer-aided manual vehicle tracking by labeling vehicles' position at each video frame, as shown in Figure 4. Figures 4c and 4d compares trajectory and velocity profiles of tracked vehicle. Overall accuracies of automatic tracking is found to be 99% for vehicle trajectory, 97% for speed profile and acceleration rates, and 94% for vehicle classification over a sample of 13,492 vehicles at 3 signalized intersections.

3.2 Detection of vehicle conflicts

Traffic conflicts are defined as "observational situation(s) in which two or more road users approach each other in space and time to such an extent that a collision is imminent if their movements remain unchanged" (Amundsen and Hydén 1977). For vehicle movements, a conflict occurs as a break-down in the interaction between road users, including vehicles and non-motorized traffic (bicyclists and pedestrians). In this study, V2V and V2P conflicts are detected based on vehicle detection and tracking.

First, thresholds of safety indicators to detect conflicts affect surrogate safety assessment. In most previous studies, a conflict with TTC smaller than 1.5s or PET smaller than 1s is considered as critical (van der Horst 1991; Wang and Stamatiadis

2013; van Nes et al. 2013). The critical TTC for unintentionally dangerous situation is selected as 3s to 4s (van der Horst and Hogema 1994; Hirst and Graham 1997). According to field observations, when TTC is at 3s, over 90% conflicts have PET values lower than 3.5s and deceleration rates larger than 2.0m/s^2 (vehicles) and 1.0m/s^2 (pedestrians). Therefore, critical TTC and PET are selected at 3s and 3.5s, critical deceleration rates are selected as 2.0m/s^2 (vehicles) and 1.0m/s^2 (pedestrians).

The critical value for each vehicle type is selected from observed change of declaration for detected vehicles. For example, subject and conflicting vehicles are detected and tracked, and changes in deceleration are profiled as shown in Figures 5a and 5b. According to Figure 5a, the maximum deceleration is 2.0 m/s^2 at $t=9\text{s}$ and 2.5m/s^2 at $t= 20\text{s}$. Moreover, to remove random deceleration or normal deceleration in front of stop-line, additional criteria, TTC and PET, are estimated at each time step based on moving velocities and relative distance between subject and conflicting vehicles, as plotted in Figures 5c and 5d. A conflict is recorded only if TTC between subject (decelerated) vehicle and conflicting vehicle is smaller than 3s or PET is smaller than 3.5s. The threshold (TTC=3s and PET=3.5s) is calibrated according to microscopic simulations, as described in Section 4. For each detected conflict, position, speed, acceleration and deceleration rates of the two vehicles (pedestrians), as well as gap between the two vehicles (pedestrians), are recorded. A total of 1,002 V2V conflicts were observed at studied intersections, including 401 rear-end conflicts,

119 lane-changing conflicts and 482 crossing conflicts. A total number of 318 V2P conflicts are also observed at signalized crosswalks.

4. Calibration and Validation of CA model

4.1 Sensitivity analysis of model parameters

A sensitivity analysis based on Elementary Effect (EE) method was conducted to test which modeling parameters will affect simulation outputs significantly (Morris 1991). EE method has been successfully applied in sensitivity analysis of simulation models with large numbers of inputs (Ge and Menendez 2012). Traffic characteristics that are tested include average stand-still front gap, maximum velocity of different vehicle types, initial velocity, average acceleration/deceleration rates, maximum acceleration/deceleration rates. Other parameters in CA model, such as random deceleration ratio, are also tested. For each input parameter, Total Sensitivity Index (TSI) is computed ($TSI(X_i) = \overline{EE(X_i)} / \bar{X}_i$). TSI value represents the percent change of simulation results related to change (at 10%-step in this study) of the input parameters; a TSI value of 0.1 (or higher) is equivalent to 1% (or larger) change in the simulation results. Most of the parameters are sensitive (pegged at $TSI > 0.1$), except for initial velocity. The most sensitive parameters are found to be maximum velocity and maximum acceleration and deceleration rates. Random deceleration ratio in NaSch model is found to be 0.2 through comparing simulation results with field observations. Other sensitive parameters are calibrated according to field observations as described in the following.

4.2 Calibration of model parameters

1) Movement characteristics

Trajectories and velocity profiles are computed for each tracked vehicle and pedestrian (sample size = 13,492) during both peak and off-peak periods. Sensitive movement characteristics for movements are calibrated and summarized in Table 1.

Multiple acceleration and deceleration rates (from 0.1 to maximum) are computed at each time step according to current velocity and front gap. Factors affecting acceleration/deceleration rates include the moving velocity (v), front gap (g) and distance to stop-line (d). Regression equations from observation study are calibrated with a minimum $R^2 = 0.892$. In the proposed CA model, acceleration or deceleration rates are computed for each vehicle at each time step.

Front gap tolerances for different types of vehicle are defined according to field observations as summarized in Table 1. A total of 450 standstill vehicles from 4 different intersection approaches are observed and the applied gap tolerance is selected as minimum observed front gap (5th percentile). Moreover, if the motorcycle is moving alongside cars or heavy vehicles, the front gap is calculated as between subject motorcycle and the front vehicle that occupies exactly along the same row of cells.

Along-side gap tolerance is not applied at approach and departure lanes and defined as 1 cell (0.9m) at intersection-box. In certain conditions, target cell is defined as cannot be occupied due to gap tolerance. In such situations, the vehicle will occupy

the next available cell which could be its present cell. Lateral gap tolerance is set for different situations based on observed traffic flow. A minimum lateral gap of 1 cell is set between two vehicles or two motorcycles. As motorcycles often ride in between two vehicles, a gap tolerance is not set between a motorcycle and a vehicle.

2) Stopping propensity

Drivers approaching a signalized intersection at the onset of amber have to decide whether to stop or cross the stop-line. According to maximum deceleration rate calibrated in Table 1, some vehicles will not be able to fully stop before the stop-line. A regression model of the probability to stop at amber onset has been calibrated for the first vehicles before stop-line at all 10 studied approaches. If the first vehicle decides to stop, following vehicles will also stop. However, if the first vehicle decides to cross the stop-line, the following vehicle will become the first vehicle. Stopping propensity of the following vehicle will thus be calculated independently. The variables include whether there is a Red Light Camera (RLC) (A, correlation coefficient $r = 0.48$), distance to stop-line (B, $r = -0.75$) at amber onset, velocity (C, $r = 0.83$), and whether it is peak hour (D, $r = 0.37$). According to the correlation analysis, the presence of RLC is a significant factor that affects the stopping propensity of individual vehicle. The stopping propensity for different vehicle types is modeled by Eqns. (1-3):

$$p_s(car) = [1 + \exp\{-(-1.02 - 1.32A + 0.129B - 0.54C - 0.03D)\}]^{-1} \quad (1)$$

$$p_s(heavy\ vehicle) = [1 + \exp\{-(-2.61 - 0.85A + 0.17B - 0.31C - 0.01D)\}]^{-1} \quad (2)$$

$$p_s (motorcycle) = [1 + \exp\{-(2.59 - 0.52A + 0.08B - 0.72C - 0.01D)\}]^{-1} \quad (3)$$

3) Probabilities of lane-changing

Probabilities in the proposed lane-changing model, p_c^1 (first type) and p_c^2 vehicles (second type), are calibrated based on field observations. For example, relationship between probability of the subject car to change lane, frequency of deceleration, and front gap is calibrated as shown in Figure 6.

4.3 Model validation

The proposed CA model is validated on microscopic level by a comparison of simulated vehicle trajectories against field data obtained by automatic vehicle detection and tracking at Intersections No. 5 and 6. In order to generate the same initial headway, observed arrival distribution and initial vehicle density are used to generate vehicles in the simulation. A total number of 13,492 vehicles and 952 pedestrians are compared between simulation and observation. Mean Percentage Error (MPE) and Root Mean Square Error (RMSE) are computed for each vehicle/pedestrian. Relatively small and acceptable errors (around 5%) in comparison between trajectories present evidence that the CA model can well describe traffic dynamics at the microscopic level. Cumulative distributions of vehicle velocities and acceleration/deceleration rates from simulation are compared with field observation, as shown in Figure 7. The results confirm that with smaller cell size and more sophisticated transition rules, the proposed CA model is able to simulate microscopic

vehicle dynamics.

Figure 7 Cumulative distributions of movement characteristics

Figure 8 shows an example of comparing simulated and observed trajectories and velocity profiles. However, even though trajectories from CA simulation are consistent with field observations, velocity profiles of pedestrians (in Figure 8d) have larger errors due to large cell sizes and discrete nature of CA model. It is found that, after a 3-term moving average, simulated velocity profiles is much more consistent with field observation (with maximum MPE smaller than 5%) for both vehicles and pedestrians. Therefore, pedestrian velocities after 3-term moving average are used for safety assessment in this study.

Moreover, for safety assessment, we compared trajectories for simulated and detected vehicle conflicts. Statistics, including minimum, maximum, mean and variance, of movement characteristics and safety indicators are computed for each type of conflicts, as shown in Table 2. Error between CA and field observation is denoted in

The results provide evidence for CA's good performance on safety analysis.

417

Table 2. Safety assessment performance of CA approach

		Range	Mean	Standard deviation
Rear-end	TTC (s)	0.41~3.00 (0.3%)*	1.18 (1.3%)	0.59 (-2.5%)
	PET (s)	0.98~3.50 (-0.2%)	1.57 (0.9%)	0.82 (-2.1%)
	v_{max} (km/h)	0~19.44 (1.2%)	12.36 (-1.9%)	14.38 (3.6%)
	$(v_1 - v_2)_{max}$ (km/h)	0~12.96 (-3.6%)	6.32 (1.7%)	7.30 (-2.1%)
	$(\varphi_d)_{max}$ (m/s ²)	-4.5~0.9 (3.0%)	-2.02 (5.6%)	2.75 (1.9%)
Lane-change	TTC (s)	0.35~3.00 (0%)	1.06 (-3.2%)	0.75 (2.3%)
	PET (s)	1.03~3.50 (-1.4%)	1.74 (3.0%)	0.68 (-4.6%)
	v_{max} (km/h)	0~38.88 (-2.3%)	16.21 (5.7%)	7.83 (-2.9%)
	$(v_1 - v_2)_{max}$ (km/h)	0~19.44 (4.6%)	8.23 (2.0%)	7.99 (-4.5%)
	$(\varphi_d)_{max}$ (m/s ²)	-4.5~0.9 (5.2%)	-1.84 (-7.3%)	2.90 (1.4%)
Crossing	TTC (s)	0.25~3.00 (0.3%)	0.97 (2.5%)	0.45 (-6.9%)
	PET (s)	0.29~3.50 (1.8%)	1.08 (0.7%)	0.49 (1.3%)
	v_{max} (km/h)	0~35.64 (4.7%)	20.95 (-1.3%)	13.14 (6.1%)
	$(v_1 - v_2)_{max}$ (km/h)	0~53.36 (5.9%)	35.64 (-3.7%)	29.42 (2.6%)
	$(\varphi_d)_{max}$ (m/s ²)	-4.5~0 (0%)	-3.26 (4.5%)	2.28 (-7.6%)

418 *: Numbers in the parenthesis indicate errors between CA and field observation

419

420 **5. Development and calibration of VISSIM and SSAM**421 **5.1 Development of VISSIM models**

422 In this study, PTV VISSIM is used to conduct microscopic simulation and produce

trajectory files for safety assessment based on SSAM. A typical cross-intersection, with the same geometric layout, signal setting and vehicle inputs as that used in the proposed CA model, was built in PTV VISSIM. For different types of vehicles, physical size and desired velocity for different types of vehicles in VISSIM are also the same as that modeled in CA. As motorcycles cannot be simulated in VISSIM, they are simulated as bicycles with motorcycles' movement characteristics. Wiedemann 74 model was selected to model car-following behavior of vehicles (PTV 2006). Reduced speed area for turning vehicles at intersection-box and priority rules between right-turn vehicles and opposite straight-through vehicles are defined according to field observations. As SSAM is not able to analyze vehicle-pedestrian conflicts (FHWA 2011), signalized pedestrian cross-walks are not simulated in PTV VISSIM.

5.2 Calibration and validation of VISSIM and SSAM

A two-stage calibration of VISSIM and SSAM is carried out based on a similar previous study (Huang et al. 2013). In the first-stage (initial stage), traffic parameters, such as traffic volume, headway and velocity, are calibrated. Sensitivity analysis, similar as described in Section 4 to calibrate the proposed CA model, is conducted to identify sensitive parameters. It is found that sensitive parameters include traffic volume, vehicle composition, average velocity, desired velocity, velocity reduction at intersection-box area, acceleration and deceleration rates, minimum gap, and total number of priority rules. These parameters are calibrated based on field observations at Intersections No. 1 to 4 as introduced in Section 3.

After setting observed traffic parameters in VISSIM, simulation results are validated through comparing simulation outputs and field observations at Intersections No. 5 and 6. Average travel time per vehicle, maximum queue length and total number of stops at each intersection are compared as shown in Table 3.

The second-stage calibration is to improve the consistency between simulated and observed vehicle conflicts. To reduce number of simulated crashes ($TTC=0$), priority rules are assigned at intersection-box area between right-turning and opposing straight-through vehicles and at departure lanes between left-turn vehicles and straight-through vehicles. Vehicles are simulated with all drivers keeping adequate lateral distance to vehicles on next lane with cooperative lane-changing. Moreover, parameters in priority rules are calibrated as minimum gap time=1.8s, and minimum headway=3.2m, to get the minimum MAPE between number of simulated and observed crossing conflicts.

6. Comparison of simulated and observed conflicts

Simulated vehicle conflicts from CA and SSAM are compared with observed vehicle conflicts at intersections with similar geometric layouts and signal settings. In CA, the simulation runs for 30 signal cycles, approximately 1 hour. In VISSIM, the simulation runs also for one hour. Outputs are calculated according to average results of 5 runs as suggested by Zheng et al. (2012). Occurrences as well as severity of each simulated conflict based on computation of safety indicators are recorded.

6.1 Conflict occurrences

Tables 4 and 5 summarize simulated conflicts from CA, SSAM and field observations. Observed conflict occurrences are averaged over the six survey sites. Conflict occurrences are classified into different severity level based on TTC and PET. A

468 smaller TTC or PET value suggests more severe conflict. Table 4 summarizes conflict
469 occurrences based on TTC values while Table 5 is based on PET values. For each
470 severity group, proportions of conflicts are calculated.

471 From Tables 4 and 5, both CA and SSAM are found to generate reasonable number of
472 conflicts compared to field observations. Several conclusions can be observed on
473 simulation results from CA and SSAM.

474 1) Compared to field observations, SSAM tends to generate more rear-end and
475 lane-change conflicts. The main reason is that route-decision module in VISSIM,
476 upon which SSAM relies in this study, may cause the over-estimation of rear-end and
477 lane-changing conflicts at intersection approaches. In VISSIM, route decisions are
478 made at a certain user-defined point. However, in CA model, drivers can dynamically
479 make route decision at any time step. This shows that the CA model is more flexible
480 in simulating vehicle movements and interactions.

481 2) Even though VISSIM is calibrated to remove simulated crashes (with 0-value TTC
482 and PET), around 50 crashes, mostly crossing conflicts, are detected in the SSAM
483 approach. It indicates that the simulation has generated some unrealistic conflicts
484 (Huang et al. 2008). Moreover, the proportion of severe crossing conflicts ($TTC \leq 0.5s$
485 or $PET \leq 1s$) in SSAM is much larger than field observations and CA.

486 3) On the contrary, compared to CA, the SSAM approach is found to under-estimate
487 the severity of rear-end and lane-changing conflicts. The most possible reason is, in
488 the context of this study, all simulated drivers in VISSIM keep relatively large lateral

distance to vehicles on adjacent lane with cooperative lane-changing.

6.2 Distribution of safety indicators

For each conflict type, Figure 9 summarizes cumulative distributions of conflicts (observation, CA and SSAM) in a range of TTC or PET values. According to Figure 9, distributions of safety indicators (both TTC and PET) from CA and field observations are very similar. However, a large number of 0 indicators are generated from SSAM and thus deviates from the other distributions of safety indicators.

From Figure 9, even though SSAM approach is calibrated to eliminate simulated crashes, the difference between SSAM and CA/observations is very obvious and vary among different conflict types. It is found that, for rear-end and lane-change conflicts, PET values from SSAM are much closer to CA and field observations than for TTC. Moreover, even though SSAM is found to over-estimate the occurrences of rear-end and lane-change conflicts, it is found to under-estimate the severity.

It is also found that although the conflicts simulated by CA and field observations show very good agreement, the severity of lane-change conflicts are slightly under-estimated by CA model according to Figure 9. This may be caused by discretization or inadequate lane-changing rules in the proposed CA model.

6.3 Vehicle movements and conflict severity

In this study, V2V conflicts in Figure 2 are classified as 3 different groups to compare with CA and SSAM. Types (1)-(3) Conflicts are classified as rear-end conflicts; Types (4) and (5) Conflicts are classified as lane-change conflicts, and Types (6)-(8)

Conflicts are classified as crossing conflicts. However, distributions of safety indicators for different conflict types within the same group may not be the same. Comparisons between conflict types in each group are computed as shown in Figure 10.

According to Figure 10, distributions of safety indicators of conflicts within the same group are very different. Among rear-end conflicts, even though the three conflict types have similar distributions of TTC values, according to PET values, Type (3) Conflict (involving U-turn vehicles) is more severe. For lane-change conflicts, Type (8) Conflict (over-taking and lane-changing) is slightly less severe than the other conflict types (downstream merging). Moreover, Type (4) Conflict (right-turn against) is found to be more severe than Type (5) Conflict (involving U-turn vehicles). It is noted conflicts are analyzed as three major groups within SSAM hence SSAM cannot differentiate conflicts within each group.

Higher conflicts reflect higher potential of actual crashes (Oh and Kim, 2010). Svensson (1998) derives distribution of traffic interactions and severity level. An empirical study is conducted to evaluate and confirms a positive relationship between near-crash events and crashes (FHWA, 2010). Both CA and SSAM are built on vehicle trajectories. Through simulation of microscopic vehicle behavior and conflicts, interactions between vehicles are estimated. Even though safety indicators are not directly comparable to conventional conflict techniques based on rating, more frequent and severe vehicle conflicts and higher possibility of collisions. Moreover,

according to accident records in Singapore from 2009 to 2011, crossing crashes constitute around 50% of total occurrences and also the highest fatality proportion (Singapore Police Force 2012). This corroborates with simulation results from both CA and SSAM that crossing conflicts, especially Conflict Type (4) has the highest severity (lower TTC and PET values) compared to other types. The accident occurrences also show that accident occurrences (especially severe injury and fatal) among all conflict types is crossing> rear-end> lane-change. Over all, observed accident data validates the proposed surrogate safety assessment based on conflicts in being able to provide reasonable assessment of safety level.

6.4 Conflicts between vehicles and pedestrians

Conflict occurrences simulated by the CA model with TTC smaller than 2.0s are summarized in Figure 11 while noting that V2P conflicts cannot be analyzed in SSAM (FHWA 2011). Based on relative movement directions, each conflict type is further divided into ipsilateral and opposite. From the simulation results, the proposed CA model allows users to estimate conflict occurrences between vehicles and pedestrians based on different severity levels.

Several conclusions can be observed in term of occurrence and severity of vehicle-pedestrian conflicts. First, conflicts during green-man phase (Conflict types (a) and (b)) occur more often than conflicts during red-man phase. As occurrence of vehicle-pedestrian conflicts is subject to number of pedestrians at cross-walk, the smaller occurrences of conflicts are due to fewer pedestrians crossing the intersection

during red-man phase. Also, in terms of severity, opposite conflicts are found to be more severe than ipsilateral ones. Most conflicts with smaller TTC values are opposite conflicts. Similar results can be observed in all other scenarios. The same is found in several studies that are based on observation of road traffic (Ren et al., 2012). Opposite movement directions will result in higher relative speed and less time for reaction to avoid collision. Moreover, opposite conflicts occur with vehicles about to enter the departure lanes. Such vehicles have higher velocity compared to vehicles that have just left the stop-line.

7. Conclusions

The study compares two safety assessment approaches based on microscopic simulation, CA and SSAM. Firstly, conventional CA model has been improved to simulate complex vehicle and pedestrian movements. Safety indicators, by way of DOC, TTC and PET, are used to record conflict occurrences and severity for safety assessment based on simulated vehicle movements.

A VISSIM-SSAM approach is conducted for a comparative study with the CA approach. Both VISSIM and CA models are calibrated and validated with observed vehicle movements and conflicts. Furthermore, VISSIM model is calibrated in various aspects such as priority rules and car-following behavior to increase the consistency with field observation and reduce simulated crash occurrences.

According to simulation results, both SSAM and CA are able to produce reasonable conflict occurrences of different severity levels. However, even though most

573 simulated crashes are eliminated in VISSIM, crash occurrences, especially for
574 crossing conflicts, are still over-estimated in SSAM compared to field observations.
575 These simulated crashes lead to a larger error when analyzing severity levels.
576 Moreover, compared to CA and field observations, SSAM is found to over-estimate
577 occurrences of rear-end and lane-change conflicts due to less realistic lane-changing
578 behavior in VISSIM. Also, as there are only 3 groups of conflicts in SSAM, it is not
579 feasible to study differences between different conflict types within the same group.

580 Compared to SSAM, the CA approach has several advantages. First, CA models allow
581 modification and local calibration of movement rules. Especially, at signalized
582 intersections, behavior and decisions of road users can also be simulated and adjusted
583 based on observation. Moreover, compared to SSAM, the CA approach is able to
584 simulate and analyze conflicts between vehicles and pedestrians.

585 However, through the comparative study, several limitations of the proposed CA
586 approach are identified. Even though the proposed model is shown to be able to
587 simulate complex movements at signalized intersections, simulated velocity profiles
588 for pedestrians are not well consistent with field observation due to large cell size and
589 discrete nature of CA model. Therefore, in future studies, smaller cell sizes for
590 pedestrian movements are suggested. Furthermore, in recent years, several continuous
591 pedestrian simulation models are developed, such as social force models. A more
592 realistic safety assessment of pedestrian movements could be achieved through
593 incorporating such pedestrian-centric models with attendant safety indicators into

594 hybrid CA models.

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599 **Notation**

600 *The following symbols are used in this paper.*

601 b = Number of cells occupied by front vehicle in x direction

602 C_1^{car} = Front gap threshold in subject lane for cars

603 C_1^{hv} = Front gap threshold in subject lane for heavy vehicles

604 C_2^{car} = Front gap threshold in target lane for cars

605 C_2^{hv} = Front gap threshold in target lane for heavy vehicles

606 C_3^{car} = Rear gap threshold in target lane for cars

607 C_3^{hv} = Rear gap threshold in target lane for heavy vehicles

608 d_n = Space headway in between the n^{th} vehicle and the $(n + 1)^{th}$ vehicle

609 g_n = Front gap in between the n^{th} vehicle and the $(n + 1)^{th}$ vehicle

610 g_{pn} = Front gap for the n^{th} subject pedestrian

611 p_c^1 = Probability of first type lane-changing

612 p_c^2 = Probability of second type lane-changing

613 p_r = Random deceleration probability

614 p_s = Stopping propensity for the n^{th} vehicle at the onset of amber

615 s_n = Distance between vehicle and stop-line

616 t = Time step

617 v_{max} = Maximum velocity of vehicles

618 v_{max}^p = Maximum velocity for pedestrian

619 v_n = Velocity in x direction of the n^{th} vehicle

620 v_{pn} = Forward velocity for the n^{th} subject pedestrian

621 v_{pn}^l = Lateral velocity for the n^{th} subject pedestrian

622 x_n = Position in x direction of the n^{th} vehicle

623 x_{pn} = Pedestrian's position along x direction for the n^{th} subject pedestrian

624 y_{pn} = Pedestrian's position along y direction for the n^{th} subject pedestrian

625 Δt = Time interval of neighboring time steps

626 φ_a = Acceleration rate

627 φ_d = Deceleration rate

628 φ_r = Random deceleration rate

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