Distributed Power Control with Robust Protection for PUs in Cognitive Radio Networks

Shimin Gong, Ping Wang, *Senior Member*, *IEEE*, Lingjie Duan, *Member*, *IEEE*

Abstract – In cognitive radio networks, it is challenging for secondary users (SUs) to estimate and control their interference at the receivers of primary users (PUs), due to incomplete or erroneous channel information between SUs and PUs. Thus, SUs need to estimate the worst-case aggregate interference at PU receivers to ensure guaranteed protection for PUs from excessive interference. As it is rare that all SU-PU channels experience the worst-case conditions simultaneously, we propose a practical model (namely, the worst-case selective robust model) for SUs to estimate their aggregate interference power. This model employs an adjustable parameter to control the number of SU-PU channels that are in the worst-case conditions. For an individual SU-PU channel, the estimation of worst-case channel gain is subject to a distribution uncertainty. Given this robust model, we study SUs' power control problem in a non-cooperative game where each SU selfishly maximizes its own throughput performance subject to coupled interference constraints at PU receivers. We study the existence and uniqueness of Nash equilibrium and propose an iterative algorithm for SUs to achieve the equilibrium in a distributed manner. Numerical results show that our algorithm provides guaranteed protection for PUs and fair throughput performance for SUs, provided with uncertain SU-PU channel information.

Index Terms – Cognitive radio, power control, game theory, distribution uncertainty, robust optimization.

I. Introduction

Power control in an underlay cognitive radio networks [1] relies on channel information between primary users (PUs) and secondary users (SUs). To harmoniously co-use PUs' licensed spectrum bands, SUs need to precisely estimate their channels to PU receivers such that their aggregate interference at PU receivers will not exceed a certain level. However, it is difficult for SUs to precisely predict their interference at PU receivers due to estimation errors in channel gain from SUs to PUs without PUs' reliable feedback [2]. The channel information is ambiguous as the channel exhibits time-varying effects induced by the change of physical channel conditions [3]. For example, a line-of-sight path between transceivers may exist for some time and disappear when the path is blocked temporarily. These imperfectness in channel estimation will inevitably cause interference violation at PU side, therefore channel uncertainty has to be taken into consideration in SUs' power control.

Recently, a growing number of works are devoted to help SUs achieve interference awareness without frequent information exchange with PUs. Some works assume a prior knowledge of PUs' locations or spatial distribution through querying a geo-location database [4], based on which SUs can estimate the interference at PU receivers based on a signal propagation model (e.g, [5], [6]). For example, the TV receivers may be densely distributed in a few separated communities, and their location information are regularly maintained at the TV base station via a geo-location database. SUs will be able to access these information if SUs are allowed to exchange information with the primary TV base station. Nevertheless, the interference estimation based on a signal propagation model cannot accurately characterize the channel fluctuations in real time. Another method is to estimate the reciprocal channel by overhearing feedback from PU receivers [2]. However, it may be unreliable as PU receivers may send back ACK packets sporadically after receiving a bulk of data streaming. Relying on out-of-date channel information will easily lead to estimation errors of channel gain and violations of PUs' interference constraints.

Since accurate channel information is not attainable, recent research proceeds in an alternate way by characterizing the channel uncertainty quantitatively. An understanding of channel uncertainty can provide a guidance in designing proactive strategies to avoid excessive interference at PU receivers. In a stochastic approach [7]–[9], the SU-PU channel gain is assumed to be composed of a deterministic component and an uncertain fluctuation component that follows a known distribution (i.e., complex normal distribution). However, exact distribution information is often unavailable as the channel is time-varying and requires a large amount of data samples for distribution estimation. The worst-case robust approach [10]–[12] reasonably restricts the uncertain channel gain to be bounded in a convex set (e.g., the norm-based or Euclidean ellipsoid-shaped set), which does not require any prior knowledge of channel gain's distribution. In fact, we are able to obtain some, though not complete, information about the uncertain channel gain, which can be used to alleviate the conservatism of the worst-case robust approach. Therefore, a new robust approach (e.g., [13]–[15]) is recently proposed to exploit partial distribution information of the uncertain channel gain. It assumes that the distribution is partially known and similar to its historical measurements. The similarity is measured in terms of moment statistics [13], [15] or a probabilistic distance [14].

Considering channel uncertainty, SUs' *robust power control* has been proposed to improve the protection for PUs and formulated in either a chance constrained non-convex problem

Manuscript received 19 April, 2014; revised 19 September, 2014; accepted 23 January, 2015. This work was supported in part by the Singapore Ministry of Education (MOE) Tier 2 Grant (ARC16/14), the SUTD-MIT International Design Centre (IDC) Grant (Project Number: IDSF1200106OH) and the SUTD-ZJU Joint Collaboration Grant (Project Number: SUTD-ZJU/RES/03/2014). The editor coordinating the review of this paper and approving it for publication was Mai Vu.

Shimin Gong and Ping Wang (the corresponding author) are with the School of Computer Engineering, Nanyang Technological University (e-mail: gong0012@e.ntu.edu.sg; wangping@ntu.edu.sg). Lingjie Duan is with the Pillar of Engineering Systems and Design, Singapore University of Technology and Design (e-mail: lingjie_duan@sutd.edu.sg).

(e.g., [7]–[9]) or a maxmin problem with a semi-definite reformulation (e.g., [11]–[13], [15]–[17]), depending on different channel uncertainty models. However, most of the existing works consider a centralized formulation that requires global knowledge of interference information at every PU receiver, suffering from serious problems in terms of scalability and signalling overheads. To facilitate distributed implementation in practice, the authors in [18] and [19] apply a dual decomposition method to solve the robust power control problem in a distributed way. Another well-applied decentralized method relies on non-cooperative game theory, in which individual users willingly achieve a stable state, namely, Nash equilibrium, by asynchronously updating their transmit power in individual's local utility maximization problem. In view of imperfect channel information, robust game theory has been recently applied to SUs' power control in [20]–[22] where the authors model the uncertain channel gain of individual link in a convex and closed elliptical uncertainty set. The authors in [20] partition every PUs interference tolerance into per-SU budgets and restrict each SU's interference less than a fixed budget. This method imposes more stringent interference constraints on SUs' power control, thus SUs' performance is compromised. While in [21] and [22], the authors relax the per-SU interference constraints and require that every PU receiver is robustly protected against the aggregate interference from all transmitting SUs, allowing more flexibility in SUs' power control and possibly improving SUs' performance.

Moreover, most of the aforementioned works focus on the uncertainty modeling of individual SU-PU channel, which however cannot be applied directly to define a worst-case aggregate interference constraint at PU receiver. In one aspect, the aggregate interference constraint is coupled with all SUs' transmit power. Some existing works (e.g., [17] and [20]) transform this global constraint to more conservative constraints, upper bounding individual SUs' worst-case interference such that their total interference is less than PUs' interference tolerance. This transformation is sufficient to ensure PUs' robust protection, but may sacrifice SUs' performance. In another aspect, it rarely happens in practice that all SU-PU channels fall in the worst-case conditions simultaneously. Thus, the estimation of worst-case aggregate interference at PU receivers is not simply the summation of worst-case interference introduced by individual SUs (e.g., [21] and [22]), as it may result in overprotection for PUs and few spectrum access opportunity for SUs. Furthermore, the worst-case robust channel model requires the understanding of channels' correlation structure to properly define the size of the uncertainty set, which is time varying and challenging to estimate in practice. For spatially distributed ad hoc SUs, their channel gains are generally independent random variables. Thus, the ellipsoid-shaped uncertainty set will be not applicable.

Our work in this paper is divided into two aspects: practically modeling the uncertainty of individual SU-PU channel gain as well as the aggregate interference at PU receivers, and presenting a distributed solution for SUs' robust power control based on the proposed uncertainty model. Compared with the existing works, our contributions are as follows:

- Firstly, for individual SU-PU channel, we model the fluctuation of channel gain in a *distribution uncertainty set*, rather than a Euclidean ellipsoid-shaped uncertainty. Based on historical channel measurements, we approximate the channel gain distribution in a closed-form, which is treated as a reference distribution. We allow the actual channel gain distribution to differ from its reference distribution, and characterize the difference by a probabilistic distance measure. As we focus on the stochastic property of channel gain fluctuation, the proposed distribution uncertainty will be resilient to very large instantaneous channel gain fluctuations that are rarely observed in channel measurements.
- Secondly, we propose a novel *worst-case selective robust model* to characterize SUs' worst-case aggregate interference at PU receivers. Considering the fact that all SU-PU channels rarely become worst simultaneously, our model will select a subset of Γ channels to be in the worst-case conditions, while the leftover channels are in normal conditions. However, we do not rely on a specific selection of channels. Instead, it can be in any combinations, i.e., any Γ out of the total N channels. For different selections of SU-PU channels, we always upper bound the aggregate interference power at a PU receiver by a prescribed threshold. The size Γ of the subset is adjustable according to PUs' robust requirements.
- Thirdly, considering SUs' selfishness, we study the robust power control problem in a *non-cooperative game model* to facilitate practical implementation. We characterize the conditions for the existence and uniqueness of Nash Equilibrium by examining the strictly diagonal convexity of SUs' cost functions. At last, we design an iterative algorithm for SUs to distributively converge to the unique Nash Equilibrium. Simulation results show that the distributed algorithm achieves fair throughput performance among SUs in terms of the Jain's fairness index [23]. Compared with centralized power control, SUs' average transmit power is slightly increased in the power control game due to SUs' uncoordinated competition.

The rest of this paper is organized as follows. In Section II, we introduce the system model, performance matrices, and distribution uncertainty for SU-PU channel gain. In Section III, we introduce the worst-case selective robust model to estimate the aggregate interference power at PU receivers that relies on the worst-case estimation of channel gains in SU-PU channels. The robust power control problem is formulated in Section IV and we present a game theoretic modeling to facilitate distributed implementation. Finally, we evaluate our algorithm in Section V and draw the conclusions in Section VI.

II. System Model

We consider a down-link cellular system consisting of K PUs spatially distributed under the coverage area of a primary base station (PBS) as shown in Fig. 1. There are N ad hoc SUs sharing the same spectrum band with the primary network without introducing intolerant interference to PUs. In an ad hoc secondary network, we assume that each SU transceiver pair is co-located compared with its distance to other SU transceiver pairs. This implies that an SU transmitter will have significantly stronger transmission efficiency to its dedicated receiver than other SU receivers. The sets of PUs and SUs are denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$ and $\mathcal{N} = \{1, 2, \ldots, N\}$, respectively. We consider a practical scenario where there is no direct information exchanged between SUs and PUs, but there exists a common control channel [24] for SUs to communicate.

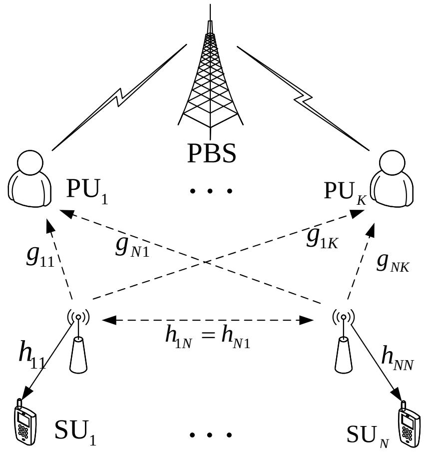


Fig. 1: System model.

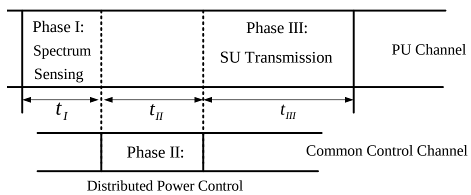


Fig. 2: SUs' channel access in time slotted structure.

A. Channel Access Structure

SUs' channel access is performed in a time slotted structure with three phases as shown in Fig. 2. Normally, the channel conditions change in a larger time scale, so we assume that the time slot is short such that the channel conditions do not change during a time slot. In the first phase T_I, the SU senses the channel and estimates the channel gain from SU transmitter to PU receivers, through overhearing the feedback packets in the reciprocal channel sent from PU receivers to PU transmitters. However, the estimation of channel gain may be unreliable due to limited observations of feedback packets. In the second phase T_{II}, SUs exchange their sensing information through the common control channel. This information exchange enables individual SU to estimate the aggregate interference power at PU receivers. Given PUs' interference constraints, each SU performs power adjustment independently to maximize its throughput performance, meanwhile avoid excessive interference to PUs. In the third phase T_{III}, SUs transmit with the power obtained in phase T_{II}.

B. Performance Metric

Let g_{nk} denote the channel gain from SU transmitter n to PU receiver k, and h_{mn} the channel gain from SU transmitter m to the SU receiver n. They are obtained in the first phase through channel sensing and estimation. After information exchange in the second phase, each SU will estimate the aggregate interference power at PU receiver $k \in \mathcal{K}$ as $\phi_k = \sum_{n=1}^{N} p_n g_{nk}$ given SU's transmit power p_n. The signal to interference and noise ratio (SINR) at SU $n \in \mathcal{N}$ is given by

$$\gamma_n = \frac{p_n h_{nn}}{\sigma_n^2 + \pi_n + \sum_{m=1,m\neq n}^{N} p_m h_{mn}}, \tag{1}$$

where σ_n^2 is the noise level received by SU n, π_n denotes the interference from primary base station, and $\sum_{m=1,m\neq n}^{N} p_m h_{mn}$ is the interference from other SUs. We define the utility of SU n as its transmission rate $r_n(\boldsymbol{p}) = \log(1+\gamma_n)$. Then we are to maximize the sum rate $\sum_{n=1}^{N} r_n(\boldsymbol{p})$, subject to a power budget constraint $0 \leq p_n \leq \bar{p}_n$ for every SU $n \in \mathcal{N}$ and an interference constraint

$$\sum_{n=1}^{N} p_n g_{nk} \leq \bar{\phi}_k, \tag{2}$$

for every PU $k \in \mathcal{K}$. Here $\boldsymbol{p} = [p_1, p_2, \ldots, p_N]$ denotes vector of SUs' transmit power, $\bar{p}_n$ is the maximum transmit power of SU n, and $\bar{\phi}_k$ is the maximum interference power tolerable by PU k. From the expression of SINR (1) and the interference constraint (2), we see that, accurate information about SU-SU channel gain h_{nm} is desirable to coordinate SUs' transmissions (e.g., avoid power races), and SUs also require accurate information about SU-PU channel gain g_{nm} to ensure reliable protection for PUs. The estimation of h_{mn} can be highly accurate as it is assisted by the information exchange between SUs through the common control channel. However, the estimation of g_{nk} is very difficult as the PU receivers are not obliged to feedback channel information to SU transmitters.

C. Channel Uncertainty Model

In fact, the channel gain g_{nm} is a composite effect of small-scale fading and large-scale path loss, and is time-varying as demonstrated in [3]. A classical modeling of its uncertainty is to assume limited error distance between the actual SU-PU channel gain g_{nk} and its sample estimate g_{nk}^0 (i.e., nominal channel gain), i.e., $|g_{nk} - g_{nk}^0| \leq \varepsilon_{nk}$ where ε_{nk} is the upper bound of estimation error. However, the wireless channel is highly dynamic and the instantaneous channel gain g_{nk} would be deviating largely from its sample estimate g_{nk}^0. In this case, it will require a large error distance ε_{nk} to restrict its fluctuating range. Nevertheless, a large error distance will make this model rather conservative. In this work, we consider a new method to estimate the uncertain channel gain, which is robust against outlier sample points in channel measurements. Through channel sensing and estimation, each SU collects a sequence of channel samples and estimates a closed-form probability distribution function of the channel gain by goodness-of-fit test. Considering estimation errors, we only treat this estimate as a reference distribution of the channel gain, while its actual distribution may be different from the reference distribution.

Let $f_{nk}(x)$ represent the actual distribution of g_{nk} through long term observations or precise estimation, while reference distribution $f_{nk}^0(x)$ is a closed-form approximation based on theoretic assumptions and simplifications. The distance D_{KL} from $f_{nk}(x)$ to its reference $f_{nk}^0(x)$ is defined by the Kullback-Leibler (KL) divergence [25], i.e., $D_{KL}\left(f_{nk}(x), f_{nk}^0(x)\right) = \mathbb{E}_{f_{nk}}\left[\ln f_{nk}(x) - \ln f_{nk}^0(x)\right]$. Then, we can define the distribution uncertainty set of g_{nk} as follows

$$\mathbb{Z}_{nk} = \left\{ f_{nk}(x) \mid D_{KL}\left(f_{nk}, f_{nk}^0\right) \leq D_{nk} \right\}. \tag{3}$$

Distance limit D_{nk} imposes an upper bound to the distance between $f_{nk}^0(x)$ and $f_{nk}(x)$. When these two distributions are similar to each other, the distance measure D_{KL} is close to zero. In practice, we can set $f_{nk}^0(x)$ as a log-normal distribution [8] and update it online according to the detection of new channel information. In each channel measurement, SUs estimate a closed-form channel gain distribution $\hat{f}_{nk}^0(x)$ and calculate its

KL divergence with respect to the reference distribution. If $D_{KL}\left(\hat{f}^0_{nk}, f^0_{nk}\right)$ exceeds the distance limit D_{nk}, the channel conditions may have significantly changed. In this case, SUs need to replace $f^0_{nk}(x)$ by the new estimation $\hat{f}^0_{nk}(x)$.

III. Worst-case Aggregate Interference Power

To provide guaranteed protection for PUs from SUs' interference, we require the worst-case aggregate interference power at PU $k \in \mathcal{K}$ to be less than $\bar{\phi}_k$. Then we simply revise the interference constraint (2) as its worst-case counterpart $\sum_{n=1}^N p_n g^w_{nk} \leq \bar{\phi}_k$ where g^w_{nk} denotes the worst-case estimation of channel gain g_{nk}. For SU-PU interference channel, worst-case means the largest estimation of g_{nk}, which will be studied later in this Section. This new interference constraint provides the highest protection for PU receivers as all SU-PU channels are assumed to experience the worst-case conditions. From a practical viewpoint, it is very conservative since hardly all channels become worst at the same time. There may be an arguably common observation that the fading effect could be correlated in different channels. For example, when PU moves from outdoors to indoors, all SU-PU channels become non-line-of-sight and have similar fading patterns. In this case, we think that the overall spectrum environment has been significantly changed and SUs have to re-build the distribution uncertainty model (i.e., update the reference distributions for each SU-PU channel).

A. *Worst-case Selective Robust Model*

Our worst-case selective robust model considers a tradeoff between conservative protection for PUs and SUs' performance improvement by introducing a *level of robustness* Γ to control the protection for PUs. This parameter allows us to select a subset of SU-PU channels in the worst-case conditions to estimate the aggregate interference power at PU receivers. However, we do not rely on a specific selection of channels. Instead, we restrict a total number of Γ channels in their worst-case conditions. But the selection of channels can be in any combinations (e.g., any Γ out of the total N SU-PU channels). Interference violation may happen when the actual number of channels in the worst-case conditions is greater than the assumed Γ (i.e., mismatch happens in this case). By setting different value of Γ, we can control the probability of interference violation, thus control the level of robustness in PUs' protection. If PUs are sensitive to interference violations, we set a larger Γ to provide enhanced protection for PUs. Otherwise, a smaller Γ is admitted if PUs are tolerable to higher interference. Note that, $\Gamma = N$ corresponds to the most robust case where all the SU-PU channels are assumed to be in the worst-case conditions, while $\Gamma = 0$ corresponds to the least robust case where all the SU-PU channel gains are taking their nominal values.

Let $S \subset \mathcal{N}$ denote the set of selected SU-PU channels, the robust PU protection requires that, for each PU receiver $k \in \mathcal{K}$, we have

$$\sum_{n\in S, n\neq l} p_n g^w_{nk} + p_l g^w_{lk}\left(\Gamma - \lfloor\Gamma\rfloor\right) + \sum_{n\in\mathcal{N}\backslash S, n\neq l} p_n g^0_{nk} \leq \bar{\phi}_k \quad (4)$$

hold for any subset $S \subset \mathcal{N}$ and $|S| = \lfloor\Gamma\rfloor$. Here $\mathcal{N}\backslash S$ denotes the set of channels in set $\mathcal{N}$, not in set S. For each PU receiver $k \in \mathcal{K}$, the estimation of aggregate interference power covers a subset S of SU-PU channels with the worst-case channel gain g^w_{nk}, while the leftover channels are in nominal conditions. For different selection $S \subset \mathcal{N}$, we always upper bound the aggregate interference power by a prescribed threshold $\bar{\phi}_k$. Note that Γ is not necessarily an integer, the cardinality of set S is bounded by $\lfloor\Gamma\rfloor$ and there are $\lfloor\Gamma\rfloor$ channels in their worst-case conditions. Besides, there is *one* additional channel from SU $l \in \mathcal{N}$ to PU k experiencing "partially" the worst-case condition, i.e., its channel gain equals to the worst-case channel gain g^w_{lk} scaled by the factional part $\Gamma - \lfloor\Gamma\rfloor$. Therefore, we have three terms in LHS of (4). The first term is the aggregate interference from SUs with the worst-case channel conditions and the third term is due to the SUs with nominal channel conditions. The second term denotes the interference from SU l with "partially" the worst-case channel gain. The combinatorial selection of set S makes the interference constraint not easy to handle. To bypass it, we have the following proposition.

Proposition 1: Introduce auxiliary variable $\mu_{nk} \geq 0$ and let $\Delta g^w_{nk} = g^w_{nk} - g^0_{nk}$, then (4) is equivalent to the following inequalities for all $k \in \mathcal{K}$ and $n \in \mathcal{N}$:

$$\sum_{n=1}^N \left(\mu_{nk} + p_n g^0_{nk}\right) \leq \bar{\phi}_k \quad (5a)$$

$$p_n\Big(\Gamma\Delta g^w_{nk} + g^0_{nk}\Big) + \Big(1-\Gamma\Big)\mu_{nk} + \sum_{m=1, m\neq n}^N \Big(\mu_{mk} + p_m g^0_{mk}\Big) \leq \bar{\phi}_k. \quad (5b)$$

Proof: We associate each channel with a variable $\omega_{nk} \in [0,1]$ to indicate whether it is selected ($\omega_{nk} = 1$), partially selected ($0 < \omega_{nk} < 1$)[1], or not selected ($\omega_{nk} = 0$) in set S. Therefore, each selection of set S in (4) will correspond to a different indicator vector $\boldsymbol{\omega}_k = [\omega_{1k}, \omega_{2k}, \ldots, \omega_{Nk}]$. Then, constraint (4) is equivalent to the following inequality:

$$\sum_{n=1}^N p_n\left(g^0_{nk} + \omega^w_{nk}\Delta g^w_{nk}\right) \leq \bar{\phi}_k, \quad \forall k \in \mathcal{K}, \quad (6)$$

where $\boldsymbol{\omega}^w_k = [\omega^w_{1k}, \omega^w_{2k}, \ldots, \omega^w_{Nk}]$ is the solution to the maximization problem as follows:

$$\max_{\boldsymbol{\omega}_k} \quad \sum_{n=1}^N p_n\Big(g^0_{nk} + \omega_{nk}\Delta g^w_{nk}\Big) \quad (7a)$$

$$s.t. \quad \sum_{n=1}^N \omega_{nk} \leq \Gamma \text{ and } 0 \leq \omega_{nk} \leq 1, \quad \forall n \in \mathcal{N}. \quad (7b)$$

It is clear that, for such a linear program, the optimal solution $\boldsymbol{\omega}^w_k$ will have $\lfloor\Gamma\rfloor$ entries at 1 and one entry at $\Gamma - \lfloor\Gamma\rfloor$, while all other entries are zeros. Introducing non-negative Lagrange multipliers λ_k, z_{nk}, and μ_{nk} associated with three inequalities in (7b), respectively, the Lagrange function is given by

$$\Lambda_1 = \sum_{n=1}^N \left(\mu_{nk} + p_n g^0_{nk}\right) + \lambda_k\Gamma + \sum_{n=1}^N \omega_{nk}\Big(p_n\Delta g^w_{nk} - \lambda_k - \mu_{nk} + z_{nk}\Big).$$

[1] For an SU-PU channel corresponding to the second term in LHS of (4), we call it partially selected in set S.

To solve the dual problem $\min_{\lambda_k,\mu_{nk},z_{nk}} \max_{\omega_{nk}} \Lambda_1$, we require $p_n \Delta g_{nk}^w - \lambda_k - \mu_{nk} + z_{nk} = 0$ for $n \in \mathcal{N}$ in the unconstrained primal problem $\max_{\omega_{nk}} \Lambda_1$, which implies $p_n \Delta g_{nk}^w - \lambda_k - \mu_{nk} = -z_{nk} \leq 0$. Therefore, we get the equivalent dual problem as follows:

$$\min_{\lambda_k,\mu_{nk}} \quad \sum_{n=1}^{N} p_n g_{nk}^0 + \lambda_k \Gamma + \sum_{n=1}^{N} \mu_{nk} \tag{8a}$$

$$s.t. \quad p_n \Delta g_{nk}^w - \lambda_k - \mu_{nk} \leq 0, \quad \forall n \in \mathcal{N} \tag{8b}$$

$$\lambda_k \geq 0, \quad \mu_{nk} \geq 0, \quad \forall n \in \mathcal{N}. \tag{8c}$$

By strong duality, the objective in (8a) achieves the same objective value with (7a). Substituting (8a) into (6), we have $\lambda_k \Gamma + \sum_{n=1}^{N} \left(\mu_{nk} + p_n g_{nk}^0\right) \leq \bar{\phi}_k$. Eliminating λ_k in (8b)-(8c), we readily obtain the equivalence in (5a)-(5b). ■

B. Estimating Worst-case Channel Gain

Inequalities (5a) and (5b) define linear constraints of SUs' transmit power p_n and μ_{nk}, which will be easy to handle. To proceed, we require the knowledge of worst-case $\boldsymbol{g}_n^w = [g_{n1}^w, g_{n2}^w, \ldots, g_{nK}^w]$ and nominal $\boldsymbol{g}_n^0 = [g_{n1}^0, g_{n2}^0, \ldots, g_{nK}^0]$ channel gains to different PU receiver $k \in \mathcal{K}$. This can be obtained distributively at each SU by overhearing the channels from PU transmitters to SU receivers. Note that, the nominal channel gain g_{nk}^0 is simply the time average of all channel samples in a sensing period, while the worst-case channel gain g_{nk}^w means the largest value of $\mathbb{E}_{f_{nk}}[g_{nk}]$ when $f_{nk}(x)$ is subject to the distribution uncertainty set $\mathbb{Z}_{nk}$ defined in (3). Thus, the worst-case channel gain g_{nk}^w can be obtained by solving the following problem,

$$\max_{f_{nk}} \quad \mathbb{E}_{f_{nk}}[x] \tag{9a}$$

$$s.t. \quad \mathbb{E}_{f_{nk}}\left[\log f_{nk}(x) - \log f_{nk}^0(x)\right] \leq D_{nk} \tag{9b}$$

$$\mathbb{E}_{f_{nk}}[1] = 1. \tag{9c}$$

Constraint (9b) requires that the distribution of g_{nk} has a limited distance to its reference distribution $f_{nk}^0(x)$. Constraint (9c) is a normalization requirement that enforces $f_{nk}(x)$ to be a valid probability distribution function. Note that objective (9a) and constraint (9c) are linear (thus convex) in terms of the uncertain distribution $f_{nk}(x)$. The constraint on KL divergence (9b) can also be verified as convex. Specifically, let $\theta \in [0, 1]$ and $f_{nk}^{(\theta)}(x) = \theta f_{nk}^{(1)}(x) + (1-\theta) f_{nk}^{(2)}(x)$, we have

$$D_{KL}\big(f_{nk}^{(\theta)}, f_{nk}^0\big) = \mathbb{E}_{f_{nk}^{(\theta)}}\left[\log\left(\frac{\theta f_{nk}^{(1)}(x) + (1-\theta) f_{nk}^{(2)}(x)}{\theta f_{nk}^0(x) + (1-\theta) f_{nk}^0(x)}\right)\right]$$
$$\leq \theta D_{KL}\big(f_{nk}^{(1)}, f_{nk}^0\big) + (1-\theta) D_{KL}\big(f_{nk}^{(2)}, f_{nk}^0\big)$$

according to the log-sum inequality [25].

The convexity of problem (9a)-(9c) implies that we can achieve the same optimum objective by solving its dual problem. Introducing Lagrange multipliers $\nu \geq 0$ and τ to constraints (9b) and (9c), respectively, we obtain its Lagrange function $\Lambda_2 = \mathbb{E}_{f_{nk}}\left[x - \nu\left(\log f_{nk}(x) - \log f_{nk}^0(x)\right) - \tau\right] + \nu D_{nk} + \tau$. The Gateaux derivative [26] of Λ_2 with respect to f_{nk} in the direction of an arbitrary function g is given by

$$\nabla_{f_{nk}} \Lambda_2 = \lim_{t \to 0} \frac{1}{t}\left[\Lambda_2\left(f_{nk} + tg\right) - \Lambda_2\left(f_{nk}\right)\right]$$
$$= \mathbb{E}_g\left[x - \nu \log f_{nk}(x) + \nu \log f_{nk}^0(x) - \tau - \nu\right].$$

Note that g is arbitrary, this implies $\nu > 0$ and $x - \nu \log f_{nk}(x) + \nu \log f_{nk}^0(x) - \tau - \nu = 0$. Therefore, the worst-case channel gain distribution is given as $f_{nk}^w(x) = f_{nk}^0(x) \frac{\exp\left(\frac{x}{\nu}\right)}{\exp\left(1+\frac{\tau}{\nu}\right)}$. Then, by the normalization requirement (9c), we have $\exp\left(1 + \frac{\tau}{\nu}\right) = \Sigma(\nu) \triangleq \int f_{nk}^0(x) \exp\left(\frac{x}{\nu}\right) dx$. Therefore, we can set $\tau = \nu\left(\log \Sigma(\nu) - 1\right)$ and the worst-case distribution $f_{nk}^w(x, \nu)$ is merely parameterized by ν, which should satisfy one of the Karush-Kuhn-Tucker (KKT) conditions as follows:

$$D_{KL}\left(f_{nk}^w, f_{nk}^0 \,|\, \nu\right) - D_{nk} = 0. \tag{10}$$

Now, we need to find a positive solution ν^* for (10). Though a direct solution is not possible, Lemma 1 in [26] ensures that the parameterized distance $D_{KL}\left(f_{nk}^w, f_{nk}^0 | \nu\right)$ is monotonically decreasing with respect to ν, which implies a bisection method to search for $\nu^* \in [\nu_{min}, \nu_{max}]$. The complexity of bisection method mainly depends on the number of iterations $\log\left(\frac{\nu_{max} - \nu_{min}}{\epsilon}\right)$.

IV. Distributed Solution via Game Modeling

Based on the relaxed interference constraints (5a)-(5b) and the estimations of channel gain $(\boldsymbol{g}_n^0, \boldsymbol{g}_n^w)$, SUs' robust power control problem is to maximize their sum throughput as follows:

$$\max_{\boldsymbol{p},\boldsymbol{\mu}} \quad \sum_{n=1}^{N} \log\left(1 + \frac{p_n h_{nn}}{\sigma_n^2 + \pi_n + \sum_{m=1,m\neq n}^{N} p_m h_{mn}}\right) \tag{11a}$$

$$s.t. \quad (5a) \text{ and } (5b)$$

$$\mu_{nk} \geq 0, \quad \forall n \in \mathcal{N}, k \in \mathcal{K} \tag{11b}$$

$$0 \leq p_n \leq \bar{p}_n, \quad \forall n \in \mathcal{N}. \tag{11c}$$

This formulation gives a flexible way to control the protection for PUs by tuning the level of robustness Γ. However, it is nonconvex and NP-hard as proved in [27] due to the non-concavity of the objective function (11a). Determination of its exact solution relies on either exhaustive or more intelligent search in the feasible space (e.g., polyblock approximation or branch and bound techniques [28]), which are computationally prohibitive and typically slow even with small problem size. Therefore, it is preferable to design efficient suboptimal algorithms. Moreover, problem (11a)-(11c) requires coordinations of the secondary network to maximize the overall throughput performance, which however may sacrifice individual SU's satisfaction.

In fact, SUs are rational and selfish, aiming to maximize their own throughput. In the following, we present an iterative solution via a game theoretic modeling, which is distributed in nature and easy to implement. To compare SUs' throughput fairness in the centralized method for (11a)-(11c) and the proposed game theoretic method, we introduce the Jain's fairness index [23] as follows:

$$\mathcal{F}(\boldsymbol{r}) = \left(\sum_{n=1}^{N} r_n\right)^2 \Bigg/ \left(N \sum_{n=1}^{N} r_n^2\right).$$

It ranges from $\frac{1}{N}$ in the worst case (i.e., most unfair) to 1 in the best case (i.e., most fair). The maximum is achieved when all SUs have the same throughput and the minimum $\frac{1}{N}$ implies that only one SU is actively transmitting.

A. Non-cooperative Game Modeling

We define the power control game as a tuple $\mathbb{C}_g = \big(\mathcal{N}, \{P_n\}_{n\in\mathcal{N}}, \Omega, \{J_n\}_{n\in\mathcal{N}}\big)$. From the optimization problem in (11a)-(11c), we find that, for each SU $n \in \mathcal{N}$, its strategy includes the transmit power p_n and $\boldsymbol{\mu}_n = [\mu_{n1}, \mu_{n2}, \ldots, \mu_{nK}]$. Let P_n denote the orthogonal set of $\boldsymbol{x}_n = [p_n, \boldsymbol{\mu}_n]^T$ defined in (11b)-(11c). From the proof of Proposition 1, the decision variable $\boldsymbol{\mu}_n$ can be viewed as a price if g^w_{nk} is not selected in set S to estimate PU's aggregate interference power. Besides, SUs' strategies are coupled in (5a)-(5b). Let $\boldsymbol{\varphi}(\boldsymbol{x}) \leq 0$ denote the constraints in (5a) and (5b) for brevity and the coupled set as $\Omega = \{\boldsymbol{x} \,|\, \boldsymbol{\varphi}(\boldsymbol{x}) \leq 0\}$, then the strategy space of SU n is given by $P_n \bigcap \Omega$. Driven by self interest, each SU $n \in \mathcal{N}$ intends to minimize a convex cost function $J_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n}) = -r_n(\boldsymbol{p}_n, \boldsymbol{p}_{-n})$ where $\boldsymbol{x}_{-n} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{n-1}, \boldsymbol{x}_{n+1}, \ldots, \boldsymbol{x}_N]$ denotes other SUs' strategy profile.

Definition 1: Given SUs' strategy profile $\boldsymbol{x}$, a new strategy $\boldsymbol{x}^+_n(\boldsymbol{x}_{-n}) \in P_n \bigcap \Omega$ is a *better response* than the current strategy $\boldsymbol{x}_n$ of SU n if $J_n(\boldsymbol{x}^+_n, \boldsymbol{x}_{-n}) \leq J_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n})$, and $\boldsymbol{x}^*_n(\boldsymbol{x}_{-n})$ is the *best response* of SU n if $J_n(\boldsymbol{x}^*_n, \boldsymbol{x}_{-n}) \leq J_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n})$ for any $\boldsymbol{x}_n \in P_n \bigcap \Omega$. The strategy profile $\boldsymbol{x}^*$ is a *Nash Equilibrium* if every SU takes its best response.

From the definition of Nash Equilibrium, every SU n will achieve its optimum in a constrained optimization problem, given other SUs' strategy profile $\boldsymbol{x}_{-n}$ and the network information $(\boldsymbol{g}^0_n\ \boldsymbol{g}^w_n)$. The local optimization problem at individual SU is given as follows:

$$\min_{\boldsymbol{x}_n} \quad J_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n}) \tag{12a}$$

$$s.t. \quad \boldsymbol{\varphi}(\boldsymbol{x}_n, \boldsymbol{x}_{-n}) \leq 0 \tag{12b}$$

$$A_n \boldsymbol{x}_n \leq \boldsymbol{b}_n, \tag{12c}$$

where coefficients $A_n = \begin{bmatrix} -1 & 1 & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{1} \end{bmatrix}^T$ and $\boldsymbol{b}_n = \begin{bmatrix} 0 & \bar{p}_n & \boldsymbol{0}^T \end{bmatrix}$ are properly defined according to (11b) and (11c). Here $\boldsymbol{0}$ denotes zero vector and $\boldsymbol{1}$ is an identity matrix of proper size. Note that, SU's cost function $J_n(x_n, x_{-n})$ is additive inverse of the transmission rate r_n. Since SUs' transmit power $\boldsymbol{p}$ is coupled in r_n, the cost function is not necessarily a convex function of $\boldsymbol{p}$. However, in the distributed game model with fixed $\boldsymbol{x}_{-n}$, the cost function $J_n(x_n, x_{-n})$ is only a function of p_n. By the definition of r_n in Section II.B, we can verify that $J_n(x_n, x_{-n})$ is convex in terms of p_n. By the KKT optimality conditions, there exist non-negative multipliers $\boldsymbol{s}_n$ and $\boldsymbol{q}_n$ such that

$$\nabla_{\boldsymbol{x}_n} J_n(\boldsymbol{x}) + A^T_n \boldsymbol{s}_n + \nabla_{\boldsymbol{x}_n} \boldsymbol{\varphi}(\boldsymbol{x})^T \boldsymbol{q}_n = 0. \tag{13}$$

The existence of Nash Equilibrium $\boldsymbol{x}^*$ that satisfies (13) for all $n \in \mathcal{N}$ can be guaranteed by [29] as the strategy space $P_n \bigcap \Omega$ defines a convex set. However, the uniqueness of Nash Equilibrium is generally not guaranteed and largely depending on the properties of $J_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n})$. In the following, we relate the uniqueness of Nash Equilibrium to the study of diagonal convexity of SUs' objective functions.

B. Uniqueness of Nash Equilibrium

Definition 2: Let $\boldsymbol{J}(\boldsymbol{x}) = [J_1(\boldsymbol{x}), \ldots, J_N(\boldsymbol{x})]^T$ and $D_{\boldsymbol{x}}\boldsymbol{J}(\boldsymbol{x}) = [\nabla_{\boldsymbol{x}_1} J_1(\boldsymbol{x}), \ldots, \nabla_{\boldsymbol{x}_N} J_N(\boldsymbol{x})]^T$. $\boldsymbol{J}(\boldsymbol{x})$ is strictly diagonal convex if

$$\left(\boldsymbol{x}^{(1)} - \boldsymbol{x}^{(2)}\right)^T \left(D_{\boldsymbol{x}}\boldsymbol{J}(\boldsymbol{x}^{(1)}) - D_{\boldsymbol{x}}\boldsymbol{J}(\boldsymbol{x}^{(2)})\right) > 0, \tag{14}$$

for any $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$ in the feasible set of $\boldsymbol{J}(\boldsymbol{x})$.

Lemma 1: If $\boldsymbol{J}(\boldsymbol{x})$ is strictly diagonal convex, game $\mathbb{C}_g$ has unique Nash Equilibrium, at which a common multiplier $\boldsymbol{q} = \boldsymbol{q}_1 = \ldots = \boldsymbol{q}_N$ exists such that (13) holds for all $n \in \mathcal{N}$.

This Lemma is directly drawn from Theorem 4 in [29]. In the following, we first identify the condition to achieve strictly diagonal convexity, and then design an iterative algorithm to achieve the unique Nash Equilibrium.

Proposition 2: Suppose we have $h_{nn} > \sum_{m=1,m\neq n}^N h_{mn}$ for any SUs $n \in \mathcal{N}$, the game $\mathbb{C}_g$ has unique Nash Equilibrium.

Proof: By Lemma 1, the proof needs to show that $\boldsymbol{J}(\boldsymbol{x})$ is strictly diagonal convex. We first simplify the condition (14) to shed some light on its implication. Let $\Delta D_{\boldsymbol{x}}\boldsymbol{J} = \left(\boldsymbol{x}^{(1)} - \boldsymbol{x}^{(2)}\right)^T \left(D_{\boldsymbol{x}}\boldsymbol{J}(\boldsymbol{x}^{(1)}) - D_{\boldsymbol{x}}\boldsymbol{J}(\boldsymbol{x}^{(2)})\right)$ for brevity. We have $\nabla_{\boldsymbol{x}_n} J_n(\boldsymbol{x}) = [\partial J_n(\boldsymbol{p})/\partial p_n, 0, \ldots, 0]^T$ and

$$\Delta D_{\boldsymbol{x}}\boldsymbol{J} = \sum_{n=1}^N \left(p^{(1)}_n - p^{(2)}_n\right) \left(\frac{\partial J_n(\boldsymbol{p}^{(1)})}{\partial p_n} - \frac{\partial J_n(\boldsymbol{p}^{(2)})}{\partial p_n}\right). \tag{15}$$

Let $\varphi_n(t) = \partial J_n(\boldsymbol{p}^t)/\partial p_n$ and $\boldsymbol{p}^t = t\boldsymbol{p}^{(1)} + (1-t)\boldsymbol{p}^{(2)}$, then $\partial J_n(\boldsymbol{p}^{(1)})/\partial p_n - \partial J_n(\boldsymbol{p}^{(2)})/\partial p_n = \varphi_n(1) - \varphi_n(0) = d\varphi_n(t)/dt$ for some $t \in [0,1]$. Note that $d\varphi_n(t)/dt = \sum_{k=1}^N \frac{\partial^2 J_n(\boldsymbol{p}^t)}{\partial p_n \partial p_k}(p^{(1)}_k - p^{(2)}_k)$, substituting it into (15), we have

$$\Delta D_{\boldsymbol{x}}\boldsymbol{J} = \sum_{n=1}^N \sum_{m=1}^N \Delta p_n \frac{\partial^2 J_n(\boldsymbol{p}^t)}{\partial p_n \partial p_m} \Delta p_m = \boldsymbol{\Delta p}^T \nabla_{\boldsymbol{p}}\boldsymbol{J} \boldsymbol{\Delta p}, \tag{16}$$

where $\Delta p_n = p^{(1)}_n - p^{(2)}_n$ and $\nabla_{\boldsymbol{p}}\boldsymbol{J}$ denotes the Jacobian matrix of $D_{\boldsymbol{x}}\boldsymbol{J}$ with respect to $\boldsymbol{p}$. Therefore, the strictly diagonal convexity (14) implies a positive definite Jacobian matrix $\nabla_{\boldsymbol{p}}\boldsymbol{J}$.

The following proof needs to show the positive definiteness of $\nabla_{\boldsymbol{p}}\boldsymbol{J}$, which has diagonal and off-diagonal elements as follows:

$$\frac{\partial^2 J_n(\boldsymbol{p})}{\partial p_n^2} = \frac{h^2_{nn}}{\left(\bar{\pi}_n + \sum_{m=1}^N h_{mn} p_m\right)^2} \tag{17}$$

$$\frac{\partial^2 J_n(\boldsymbol{p})}{\partial p_n \partial p_m} = \frac{h_{nn} h_{mn}}{\left(\bar{\pi}_n + \sum_{m=1}^N h_{mn} p_m\right)^2}, \tag{18}$$

where $\bar{\pi}_n = \sigma^2_n + \pi_n$ denotes the background noise received by SU n when all other SUs are not transmitting. Looking at the row elements of matrix $\nabla_{\boldsymbol{p}}\boldsymbol{J}$, when $h_{nn} > \sum_{m=1,m\neq n}^N h_{mn}$ for $n \in \mathcal{N}$, we have

$$\frac{\partial^2 J_n(\boldsymbol{p})}{\partial p_n^2} > \frac{h_{nn} \sum_{m=1,m\neq n}^N h_{mn}}{\left(\bar{\pi}_n + \sum_{m=1}^N h_{mn} p_m\right)^2} = \sum_{m=1,m\neq n}^N \frac{\partial^2 J_n(\boldsymbol{p})}{\partial p_n \partial p_m},$$

which implies that $\nabla_{\boldsymbol{p}}\boldsymbol{J}$ is strictly diagonally dominant. Moreover, all elements in $\nabla_{\boldsymbol{p}}\boldsymbol{J}$ are real values and the diagonal elements $\partial^2 J_n(\boldsymbol{p})/\partial p_n^2$ are positive. Therefore, we have $\nabla_{\boldsymbol{p}}\boldsymbol{J} \succ 0$ which guarantees the uniqueness of Nash Equilibrium. ■

Proposition 2 implies that Nash Equilibrium is unique if each SU transmitter has larger channel gain (i.e., dominant transmission efficiency) to its dedicated receiver than the sum of channel gains from other SU transmitters. This requirement is easily satisfied in an ad hoc secondary network where colocated SU transmitter has much shorter distance to its dedicated receiver than the distances to other SU transmitters. However, in some other cases with dense SU deployment, SUs incur strong interference to each other and the dominant transmission efficiency is not guaranteed, then SUs may reduce their maximum transmit power to ensure unique Nash Equilibrium, which we will elaborate as follows.

Proposition 3: Define a square matrix Z, its diagonal and off-diagonal elements are given by $z_{nn} = \frac{h_{nn}^2}{\left(\bar{\pi}_n + \sum_{m=1}^{N} h_{mn}\bar{p}_m\right)^2}$ and $z_{mn} = -\frac{h_{nn}h_{mn}}{\bar{\pi}_n^2}$ for $m \neq n$, respectively. If matrix Z is positive semi-definite, i.e., $Z \succeq 0$, the game $\mathbb{C}_g$ has a unique Nash Equilibrium.

Proof: The proof is straightforward by showing $\Delta D_{\boldsymbol{x}}\boldsymbol{J} > 0$. By the construction of matrix Z, we have $\partial^2 J_n(\boldsymbol{p})/\partial p_n^2 \geq z_{nn}$ and $\partial^2 J_n(\boldsymbol{p})/\partial p_n \partial p_m < -z_{mn}$, then

$$\begin{aligned}
\Delta D_{\boldsymbol{x}}\boldsymbol{J} &\geq \sum_{n=1}^{N} \left(\Delta p_n^2 \frac{\partial^2 J_n(\boldsymbol{p}^t)}{\partial p_n^2} - \sum_{m=1, m\neq n}^{N} |\Delta p_n \Delta p_m| \frac{\partial^2 J_n(\boldsymbol{p}^t)}{\partial p_n \partial p_m} \right) \\
&> \sum_{n=1}^{N} \left(|\Delta p_n|^2 z_{nn} + \sum_{m=1, m\neq n}^{N} |\Delta p_n||\Delta p_m| z_{mn} \right) \\
&= |\Delta \boldsymbol{p}|^T Z |\Delta \boldsymbol{p}| \;\geq\; 0.
\end{aligned}$$

The last inequality holds when matrix Z is positive semi-definite. ■

Note that matrix Z involves SUs' maximum transmit power $\bar{p}_n$, as well as the channel gain h_{kn} between SU transceivers and the received background noise $\bar{\pi}_n$. Since h_{mn} and $\bar{\pi}_n$ are known to SUs and can be considered as invariant. So Proposition 3 actually restricts SUs' maximum transmit power to ensure uniqueness of Nash Equilibrium. As an illustration, we consider a two-user case. The maximum transmit power are denoted by $\bar{p}_1$ and $\bar{p}_2$, respectively. We find that a set of linear inequalities as follows

$$\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} \bar{p}_1 \\ \bar{p}_2 \end{bmatrix} \leq \begin{bmatrix} \bar{\pi}_1 \left(\sqrt{\frac{h_{11}}{h_{21}}} - 1 \right) \\ \bar{\pi}_2 \left(\sqrt{\frac{h_{22}}{h_{12}}} - 1 \right) \end{bmatrix},$$

gives a sufficient condition to ensure $Z \succeq 0$. Therefore, the power control game $\mathbb{C}_g$ has unique Nash Equilibrium if we set SUs' maximum transmit power as a solution to the above inequalities.

C. Distributed Algorithm Design

A non-cooperative game model is distributed in nature and can achieve Nash Equilibrium through iterative best response update, i.e., SUs take turns to update their best response strategies as the solution to (12a)-(12b), assuming other SUs' strategy profile is fixed. By the Lagrangian method, we expect that a new strategy $\boldsymbol{x}_n^{t+1}$ is a better response if it is updated as $\boldsymbol{x}_n^{t+1} = \boldsymbol{x}_n^t + \alpha^t \Delta \boldsymbol{x}_n^t$ where

$$\Delta \boldsymbol{x}_n^t = -\nabla_{\boldsymbol{x}_n} J_n(\boldsymbol{x}_n^t, \boldsymbol{x}_{-n}^t) - A_n^T \boldsymbol{s}_n^t - \nabla_{\boldsymbol{x}_n} \boldsymbol{\varphi}(\boldsymbol{x})^T \boldsymbol{q}^t \quad (19)$$

and α^t is the step size. Here we set $\boldsymbol{q}^t$ the same for all SUs to achieve the unique Nash Equilibrium as dictated in Proposition 1. The choice of $\boldsymbol{s}_n$ and $\boldsymbol{q}$ firstly needs to satisfy the complementary slackness conditions. That is, $s_{n,i} = 0$ if $\boldsymbol{a}_n^i \boldsymbol{x}_n < b_n^i$ and $s_{n,i} \geq 0$ if $\boldsymbol{a}_n^i \boldsymbol{x}_n \geq b_n^i$, where $\boldsymbol{a}_n^i$ and b_n^i denote the i-th row of A_n and $\boldsymbol{b}_n$, respectively. Similarly, $q_i = 0$ if $\varphi_i(\boldsymbol{x}) < 0$ and $q_i \geq 0$ if $\varphi_i(\boldsymbol{x}) \geq 0$, where $\varphi_i(\boldsymbol{x})$ denotes the i-th coupled constraint in (12b). Secondly, the choice of $\boldsymbol{s}_n$ and $\boldsymbol{q}$ needs to minimize the norm of $\Delta \boldsymbol{x}^t = [\Delta \boldsymbol{x}_1^t, \Delta \boldsymbol{x}_2^t, \ldots, \Delta \boldsymbol{x}_N^t]$, which is a least squares problem. The iterative process is formally outline in Algorithm 1. The algorithm first keeps the common multiplier $\boldsymbol{q}^t$ unchanged while updates $\boldsymbol{s}_n^t$ to minimize the norm of $\Delta \boldsymbol{x}_n^t$. When the process runs over all SUs, there will be an SU updating the common multiplier $\boldsymbol{q}^t$ after collecting all the information from other SUs. The update of $\boldsymbol{q}^t$ is still to minimize the norm of $\Delta \boldsymbol{x}^t$. So we see the diminishing of $||\Delta \boldsymbol{x}^t||$ to zero as t approaches infinity. Since Nash Equilibrium $\boldsymbol{x}^*$ is unique, SUs' strategy $\boldsymbol{x}$ will converge to $\boldsymbol{x}^*$. Compare with NP-hardness of the maximization problem (11a)-(11c), the complexity of Algorithm 1 is significantly reduced as each SU in the iteration solves a local convex optimization problem in polynomial time. The convergence to Nash equilibrium is guaranteed and we practically observe finite number of iterations. Besides, the game modeling naturally bears a distributed implementation, which is more preferable than a centralized search method for problem (14a)-(14c).

Algorithm 1 Distributed Power Control Game

1: Set $t = 0$ and fix an initial value for multiplier $\boldsymbol{q}^t$
2: Start from any SU $n \in \mathcal{N}$, acquire $\boldsymbol{x}_{-n}$ and $(\boldsymbol{g}_n^0, \boldsymbol{g}_n^w)$
3: Update $\boldsymbol{s}_n^t = \arg\min_{\boldsymbol{s} \geq 0} \; ||\Delta \boldsymbol{x}_n^t(\boldsymbol{x}, \boldsymbol{s}, \boldsymbol{q}^t)||$
4: Update $\boldsymbol{x}_n^{t+1} = \boldsymbol{x}_n^t + \alpha^t \Delta \boldsymbol{x}_n^t$ where $\Delta \boldsymbol{x}_n^t$ is given in (19)
5: SU n broadcasts $\Delta \boldsymbol{x}_n^t$ and $(\boldsymbol{g}_n^0, \boldsymbol{g}_n^w)$ to its neighbors
6: If SU n collects all $\Delta \boldsymbol{x}_n^t$ from its neighbors, then updates $\boldsymbol{q}^t = \arg\min_{\boldsymbol{q} \geq 0} \; ||\Delta \boldsymbol{x}^t||$
7: Stop if Nash equilibrium is achieved, otherwise
8: Set $t = t + 1$ and go to step 2

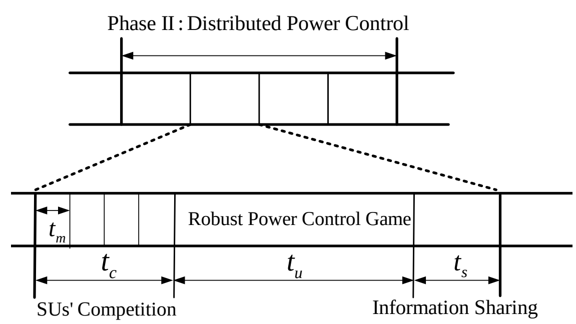


Fig. 3: Implementation in a time slotted frame structure.

The implementation of Algorithm 1 is scheduled in the second phase of a time slot, where we have to coordinate SUs' actions such that there is at most one SU updating its strategy at any iteration. This coordination is even complicated when SUs randomly join or leave the power control game. So it is impractical to assign each SU a predefined sequence number to update its strategy. Instead, we design a contention-based mechanism in the common control channel to allocate time slices for participating SUs to update their strategies. As shown in Fig. 3, a time slice contains three functional parts. In the first part t_c, SUs compete for the time slice by initiating a random back-off timer (i.e., multiples of the mini-slot t_m shown in Fig. 3). When an SU's back-off timer expires firstly and the channel is still sensed free, the winning SU will notify other SUs by broadcasting a Ready-to-Update (RTU) packet to reserve the time slice for its strategy update. If collision happens in channel competition, the conflicting SUs will reset their back-off timers and delay their attempts to update strategies. In the second

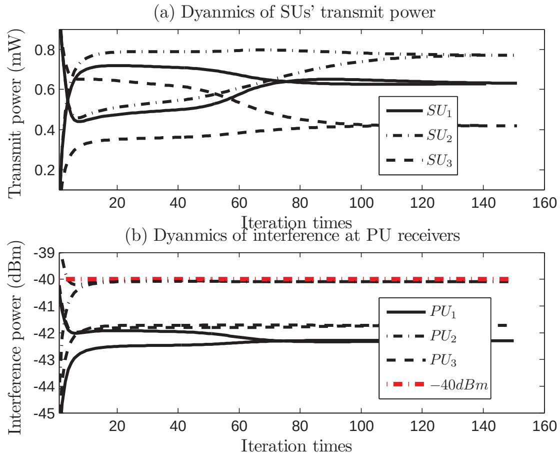


Fig. 4: Dynamics of Algorithm 1 with $N = 3$ SUs and $K = 3$ PUs.

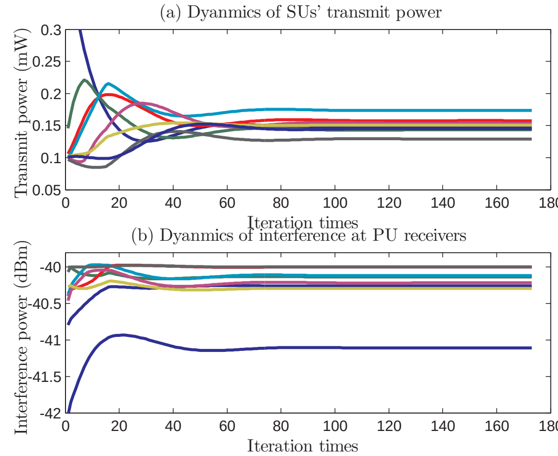


Fig. 5: Dynamics of Algorithm 1 with $N = 8$ SUs and $K = 8$ PUs.

part t_u, the winning SU updates Lagrange multipliers $(\boldsymbol{s}^t, \boldsymbol{p}^t)$ and its strategy $\boldsymbol{x}_n^{t+1}$ in Algorithm 1. In the third part t_s, the SU will broadcast a Complete-Strategy-Update (CSU) packet to share its new strategy and sensed network information with its neighboring SUs. Once receiving the CSU packet, SUs become active again to compete for the next time slice. An SU newly joining the game also needs to listen the CSU packets for some periods. This will help the new SU synchronize information with the existing secondary network.

V. Numerical Results

In this section, we evaluate the proposed robust power control algorithm through numerical experiments. We consider a cognitive radio network with 3 SUs randomly distributed in the coverage area of a primary base station providing down-link data streaming to 3 PU receivers. The maximum transmit power of SUs is set to 5 mW. Each PU has the same interference threshold $\bar{\phi} = \bar{\phi}_1 = \ldots = \bar{\phi}_K = -40$ dBm. The mean path loss is given as cd^{-e} where d is the distance between transceivers, c and e denote the propagation constant and propagation exponent, respectively. Without loss of generality, the primary and the secondary networks are assumed to have the same signal propagation models. We set $c = 2$ and $e = 3.5$ the same for each link, and the channel gains between SUs and PUs are log-normal random variables with zero mean and a standard derivation of 5dB [14]. In practice, the log-normal assumption may be not valid and we set the distance limit as $D_{nk} = 0.03$ [30] to account for the distribution uncertainty.

A. An Example of the Distributed Algorithm

We first check the dynamics of SUs' transmit power and PUs' interference. Each line in Fig. 4 represents the dynamics of an SU's transmit power or a PU's interference power. We set $\Gamma = 1$ and test the power control algorithm with two different initial transmit power vectors (i.e., $\boldsymbol{p}_1^0 = [1, 1, 1]$ mW and $\boldsymbol{p}_2^0 = [0, 0, 0]$ mW, respectively). Every two lines of the same type in Fig. 4(a) correspond to the same SU transmitter but having different initial transmit power. Clearly we can see the convergence of SUs' transmit power to fixed power levels. Due to SUs' competition and different channel conditions, the convergent power levels vary at different SU transmitters. Correspondingly in Fig. 4(b), we depict the aggregate interference power at PU receivers when the SU-PU channel matrix $G = [g_{nk}]_{n,k}$ (in dB) is given as follows

$$G = \begin{bmatrix} -50 & -43 & -45 \\ -45 & -44 & -45 \\ -42 & -41 & -43 \end{bmatrix}. \tag{20}$$

Each element g_{nk} in G denotes the channel gain from SU n to PU k, therefore the above channel matrix implies that PU_2 is the most vulnerable node to SUs' transmissions. Note that each PU receiver is tolerable to a maximum interference power at $\bar{\phi} = -40$ dBm, which is denoted by the red dotted line in Fig. 4(b). We can observe that, when SUs' transmit power achieves the equilibrium point, no PU perceives excessive interference higher than $\bar{\phi}$. In fact, there is one PU receiver (i.e., PU_2) that receives nearly the maximum interference. As the interference constraint (4) requires every PU to be well protected against the interference threshold $\bar{\phi}$, the interference constraint will be active for the most vulnerable PU receiver (i.e., PU_2) while inactive for other PU receivers (i.e., PU_1 and PU_3). Similarly, Figs. 5(a) and 5(b) plot the dynamics of SUs' transmit power and the interference at PU receiver for $N = 8$ SUs and $K = 8$ PUs, respectively, showing the convergence of Algorithm 1. For ease of presentation, we do not distinguish individual SUs and PUs in Fig. 5. Note that the number of iterations required to achieve convergence does not increase significantly when the number of SUs/PUs increases from 3 to 8.

In a practical case, there may be multiple SUs joining or leaving the power control game randomly. To guarantee the convergence of the algorithm, we require that the dynamics of SUs' behavior and the change of spectrum environment are slower than the speed to achieve a Nash Equilibrium through Algorithm 1. As shown in Fig. 6, we record the power trace of 4 SUs when they join and leave the game at different simulation time (e.g., SU_1 and SU_2 are initially active in the power control game. SU_3 and SU_4 join in the game at simulation time 200, then SU_1 and SU_3 leave the service area at simulation time 400). The granularity of the simulation time is defined as the time period required to calculate and update an SU's strategy, i.e., a time slice $t_c + t_u + t_s$ as illustrated in Fig. 3.

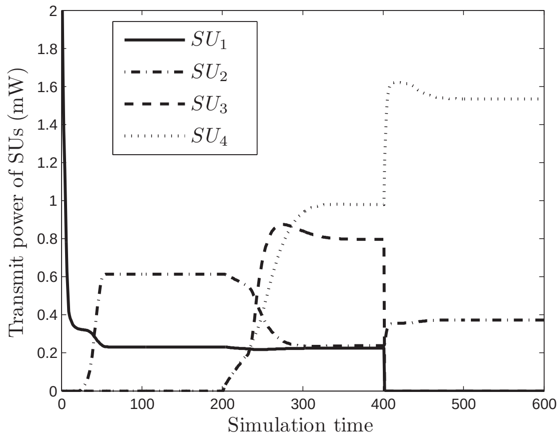


Fig. 6: SUs' power control with random user incoming and leaving.

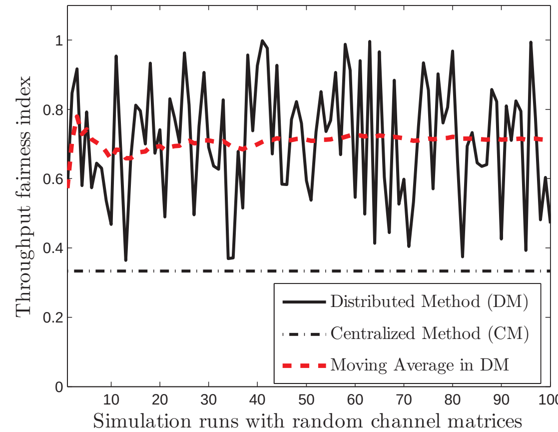


Fig. 8: Throughput fairness index is higher in the distributed method.

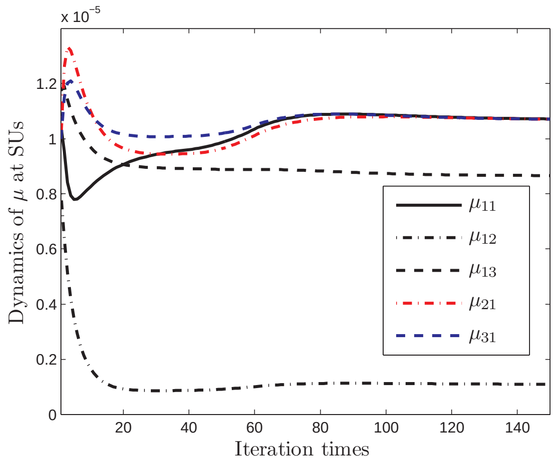


Fig. 7: Dynamics of dual variable μ_{nk} ($N = 3$, $K = 3$).

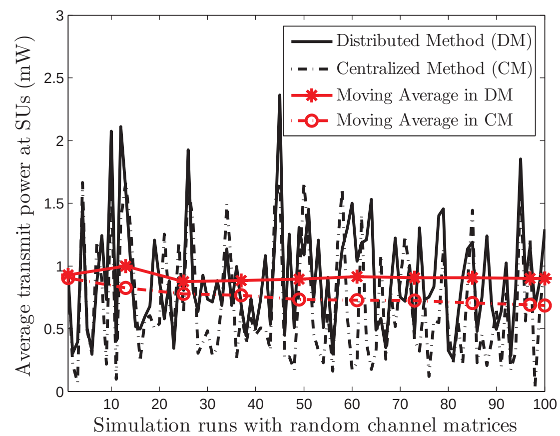


Fig. 9: Average transmit power is higher in the distributed method.

B. Compare with a Centralized Solution for (11a)-(11c)

Besides the transmit power p_n, each user n also updates a vector of dual variables $\boldsymbol{\mu}_n = [\mu_{n1}, \mu_{n2}, \ldots, \mu_{nK}]$. We plot the dynamics of $\boldsymbol{\mu}_1$ for SU_1 in Fig. 7. Note that the dual variable μ_{nk} corresponding to one PU k will stabilize at the same level for different SUs $n \in \mathcal{N}$. However, the proof of Proposition 1 implies that the convergent point of this game-theoretic method is not necessarily an optimal solution to the original problem (11a)-(11c). Looking at the problem (7a)-(7b) and its dual (8a)-(8c), we have $\mu_{nk} = 0$ whenever $\omega_{nk} < 1$ by complementary slackness, i.e., $\mu_{nk}(1 - \omega_{nk}) = 0$. Furthermore, it is easy to check from the proof of Proposition 1 that, the optimal weight ω^*_{nk} is either 0 or 1 for some integer $0 \leq \Gamma \leq N$. In our case, we set $\Gamma = 1$ and $K = 3$, therefore we will only have one positive weight ω_{nk} and consequently two zero dual variables μ_{nk}. However, this is obviously not the case in Fig. 7. That means, the centralized problem (11a)-(11c) will have a higher sum throughput than the proposed game-theoretic method.

In a decentralized network, we may be more concerned about the fairness in allocating transmission opportunity among all SUs. Note that problem (11a)-(11c) is non-convex, we find its solution by exhaustive search in the numerical evaluation and compare it with the solution obtained in Algorithm 1. In Fig. 8, we plot SUs' throughput fairness index in these two methods, respectively. We run the simulation for 100 times with randomly generated channel matrices $H = [h_{mn}]_{m,n}$ and $G = [g_{nk}]_{n,k}$. The randomness of channel matrices can be viewed as an emulation of different network topology, e.g., a larger channel gain h_{mn} implies a closer distance between SU m and n. We observe that, the fairness index of the centralized method equals to $1/3$ in all the simulation runs which implies only one SU actively transmitting at the optimum of problem (11a)-(11c), while the average fairness index of the distributed method is high up to 0.7 which means that SUs achieved a well balanced throughput performance.

In Fig. 9, we show SUs' average transmit power in the 100 simulation runs. The black (solid and dotted, respectively) curves in Fig. 9 denote SUs' average transmit power in the centralized and distributed methods, respectively. While the red (solid and dotted, respectively) curves are the moving average of SUs' average transmit power over different network topology. From the red curves, we observe that SUs' average transmit power in the distributed method is slightly increased compared to that in the centralized method. This increase can be viewed as the cost incurred by SUs' competition due to lack of a centralized controller.

C. Level of Robustness

In the robust power control game, there is a controllable parameter Γ to adjust the level of protection for PUs. It sets a limit on the total number of worst-case SU-PU channels in the estimation of PUs' aggregate interference power. When Γ equals to the number of SUs, the estimation of worst-case aggregate interference is the sum of worst-case interference introduced

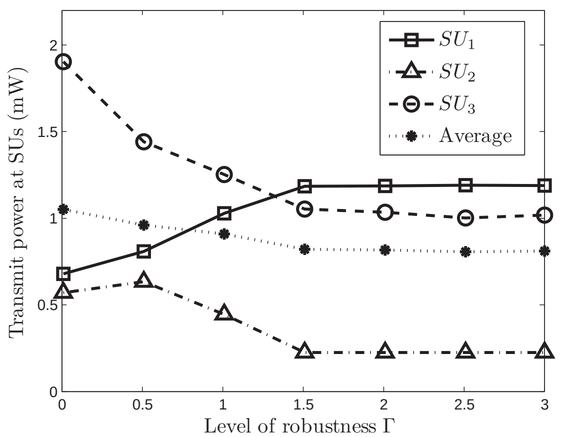


Fig. 10: Average transmit power decreases with the increase of Γ.

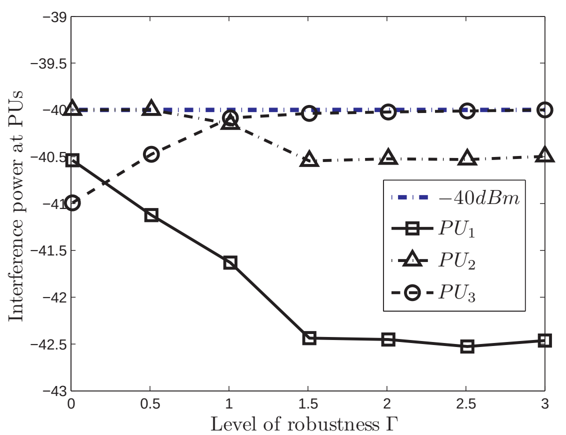


Fig. 11: Interference power at PU receivers.

by every active SU transmitter. It becomes a similar version of the worst-case robust model in [21] and [22]. When Γ is approaching zero, the robust power control problem (11a)-(11c) is actually degenerated to the nominal power control problem, in which the channel gain g_{nk}^0 is deterministic and fixed. In Fig. 10, we show SUs' transmit power with different level of robustness Γ. We observe that, with a larger Γ, the SUs need to reduce their transmit power in average to maintain the same level of protection for PUs. Therefore, our method can potentially increase SUs' performance compared with the robust model in [21] and [22]. When transmit power changes at different SU transmitters, the interference at PU receivers also experiences a turnover as shown in Fig. 11. Initially, PU_2 receives the maximum interference power $\bar{\phi}$. But when $\Gamma > 1$, PU_3 becomes the most vulnerable node and receives the maximum interference power.

To illustrate how the size of uncertainty set affects the dynamics of SUs' transmit power, we test the algorithm for different values of distance limit D_{nk}. Larger distance limit means that we have less confidence in channel estimation, i.e., the channel gain distribution may be very different from its reference distribution in terms of KL divergence. A typical value of D_{nk} is investigated in [14]. Without lose of generality, we assume all SU-PU channels bearing the same level of channel fluctuations, thus the distance limit D_{nk} is the same for all $n \in \mathcal{N}$ and $k \in \mathcal{K}$. By varying D_{nk} from 0.01 to 0.06, we study its effect on SUs' transmit power as shown in Fig. 12. We

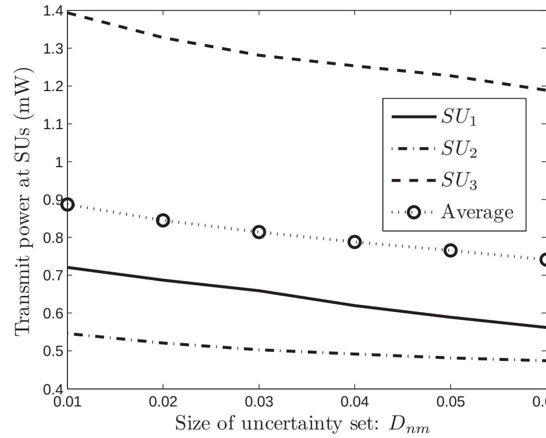


Fig. 12: Average transmit power decreases with the increase of D_{nk}.

observe that, in order to guarantee the same level of protection for PU receivers, SUs have to lower the transmit power when the channel bears larger uncertainty as D_{nk} increases.

VI. Conclusions

In this paper, we study SUs' power control problem with a robust consideration for PUs' protection. We propose a robust model to deal with two kinds of practical difficulties. Firstly, the channel gain from SUs to PUs is subject to estimation errors and we characterize it in a distribution uncertainty model. Secondly, the robust protection for PUs needs to strike a balance between conservatism and performance. We introduce the level of robustness to enable a more flexible control of the aggregate interference power at PU receivers. To facilitate a distributed implementation of the power control algorithm, we propose a game-theoretic modeling with coupled interference constraints. We show the existence and uniqueness of Nash Equilibrium under mild conditions. We also design a distributed algorithm that is proved to achieve the equilibrium, and shed some light on the design of frame structure to implement the proposed power control algorithm.

The proposed robust model and the game-theoretic approach can be extended to the multi-antenna case, but there are some interesting issues worth mentioning for the extension. We can still use the worst-case selective robust model to estimate the aggregate interference at PU receivers, but the selection of channels now becomes the selection of SUs in multi-antenna case. To estimate an SU's worst-case interference, we can employ an elliptical uncertainty set similar to [20]–[22] since multiple antennas of the same SU are close-by and correlated. Therefore, we can combine the worst-case selective robust model (which accounts for SUs' independence at different spatial locations) and the elliptical uncertainty model (which accounts for correlated antennas of the same SU) in multi-antenna case.

References

[1] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 5306–5315, Dec. 2008.

[2] S. Huang, X. Liu, and Z. Ding, "Decentralized cognitive radio control based on inference from primary link control information," *IEEE J. Sel. Area. Commun.*, vol. 29, no. 2, pp. 394–406, 2011.

[3] A. Konrad, B. Y. Zhao, A. D. Joseph, and R. Ludwig, "A Markov-based channel model algorithm for wireless networks," *Wireless Networks*, vol. 9, pp. 189–199, May 2003.

[4] X. Chen and J. Huang, "Database-assisted distributed spectrum sharing," *IEEE J. Sel. Area. Comm.*, vol. 31, pp. 2349–2361, Nov. 2013.

[5] W. Ren, Q. Zhao, and A. Swami, "Power control in cognitive radio networks: how to cross a multi-lane highway," *IEEE J. Sel. Area. Commun.*, vol. 27, pp. 1283–1296, Sep. 2009.

[6] S. Gong, P. Wang, and D. Niyato, "Optimal power control in interference-limited cognitive radio networks," in *Proc. IEEE Int. Conf. Commun. Syst. (ICCS)*, pp. 82–86, Nov. 2010.

[7] Z. Xiong, K. Cumanan, and S. Lambotharan, "Robust SINR balancing technique for a cognitive radio network using probability based interference constraints," in *Proc. IEEE Int. Symp. Dynamic Spectrum Access Networks (DySPAN)*, pp. 1–4, Apr. 2010.

[8] E. Dall'Anese, S.-J. Kim, G. Giannakis, and S. Pupolin, "Power control for cognitive radio networks under channel uncertainty," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3541–3551, 2011.

[9] S. Singh, P. Teal, P. Dmochowski, and A. Coulson, "Statistically robust cooperative beamforming for cognitive radio networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, pp. 2727–2732, Jun. 2013.

[10] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for cognitive radios using worst-case performance optimization," in *Proc. IEEE Int. Symp. Inform. Theory Appl.(ISITA)*, pp. 1–5, Dec. 2008.

[11] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Trans. Signal Process.*, vol. 57, pp. 4871–4881, Dec. 2009.

[12] U. Wijewardhana, M. Codreanu, and M. Latva-aho, "Robust beamformer design for underlay cognitive radio network using worst case optimization," in *Proc. IEEE Int. Symp. Modeling Optimization in Mobile and Ad Hoc Wireless Networks (WiOpt)*, pp. 404–411, May 2013.

[13] S. Gong, P. Wang, and J. Huang, "Robust performance of spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2217–2227, 2013.

[14] S. Gong, P. Wang, Y. Liu, and W. Zhuang, "Robust power control with distribution uncertainty in cognitive radio networks," *IEEE J. Sel. Area. Commun.*, Nov. 2013.

[15] Q. Li, A.-C. So, and W.-K. Ma, "Distributionally robust chance-constrained transmit beamforming for multiuser MISO downlink," in *Proc. IEEE Acoustics, Speech and Signal Process. (ICASSP)*, pp. 3479–3483, May 2014.

[16] E. Gharavol, Y.-C. Liang, and K. Mouthaan, "Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 2852–2860, Jul. 2010.

[17] Y. Zhang, E. DallAnese, and G. Giannakis, "Distributed optimal beamformers for cognitive radios robust to channel uncertainties," *IEEE Trans. Signal Process.*, vol. 60, pp. 6495–6508, Dec. 2012.

[18] S. Sun, W. Ni, and Y. Zhu, "Robust power control in cognitive radio networks: A distributed way," in *Proc. IEEE Int. Conf. Commun. (ICC)*, pp. 1–6, June 2011.

[19] S. Parsaeefard and A. Sharafat, "Robust distributed power control in cognitive radio networks," *IEEE Trans. Mobile Comput.*, vol. 12, pp. 609–620, Apr. 2013.

[20] J. Wang, G. Scutari, and D. Palomar, "Robust MIMO cognitive radio via game theory," *IEEE Trans. Signal Process.*, vol. 59, pp. 1183–1201, Mar. 2011.

[21] Y. Yang, G. Scutari, P. Song, and D. Palomar, "Robust MIMO cognitive radio systems under interference temperature constraints," *IEEE J. Sel. Area. Commun.*, vol. 31, pp. 2465–2482, Nov. 2013.

[22] J. Wang, M. Peng, S. Jin, and C. Zhao, "A generalized Nash equilibrium approach for robust cognitive radio networks via generalized variational inequalities," *IEEE Trans. Wireless Commun.*, vol. 13, pp. 3701–3714, Jul. 2014.

[23] T. Lan, D. Kao, M. Chiang, and A. Sabharwal, "An axiomatic theory of fairness in network resource allocation," in *Proc. IEEE Int. Conf. Computer Commun. (INFOCOM)*, pp. 1–9, Mar. 2010.

[24] K. Chowdhury and I. Akyldiz, "OFDM-based common control channel design for cognitive radio ad hoc networks," *IEEE Trans. Mobile Comput.*, vol. 10, no. 2, pp. 228–238, 2011.

[25] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York, NY, USA: Wiley-Interscience, 1991.

[26] B. Levy and R. Nikoukhah, "Robust least-squares estimation with a relative entropy constraint," *IEEE Trans. Inform. Theory*, vol. 50, no. 1, pp. 89–104, 2004.

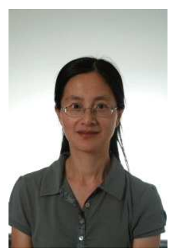
[27] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE J. Sel. Topics Signal Process.*, vol. 2, pp. 57–73, Feb. 2008.

[28] P. C. Weeraddana, M. Codreanu, M. Latva-aho, A. Ephremides, and C. Fischione, "Weighted sum-rate maximization in wireless networks: A review," *Foundations and Trends in Networking*, vol. 6, no. 1–2, pp. 1–163, 2012.

[29] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[30] S. Gong, P. Wang, W. Liu, and W. Zhuang, "Performance bounds of energy detection with signal uncertainty in cognitive radio networks," in *Proc. IEEE Int. Conf. Computer Commun. (INFOCOM)*, pp. 2286–2294, Apr. 2013.

Shimin Gong received his Bachelor degree in Electronics and Information Engineering from Huazhong University of Science and Technology, China, in 2008, and the PhD degree in School of Computer Engineering from Nanyang Technological University, Singapore, in 2014. Since 2013, he had been research associate/scientist in Singapore University of Technology and Design, and Nanyang Technological University. He was a visiting scholar at The Chinese University of Hong Kong from September to November, 2011, and a visiting scholar at University of Waterloo from June to September, 2012. His research interests are in the area of energy harvesting, relaying network, and resource allocation in wireless communications, with a special focus on uncertainty modeling and robust optimization.



Ping Wang (M'08-SM'15) received the PhD degree in electrical engineering from University of Waterloo, Canada, in 2008. Since June 2008, she has been an assistant professor in the School of Computer Engineering, Nanyang Technological University, Singapore. Her current research interests include QoS provisioning and resource allocation in multimedia wireless communications. She was a corecipient of the Best Paper Award from IEEE Wireless Communications and Networking Conference (WCNC) 2012 and IEEE International Conference on Communications (ICC) 2007. She is a member of the IEEE. She is an Editor of IEEE Transactions on Wireless Communications, EURASIP Journal on Wireless Communications and Networking, and International Journal of Ultra Wideband Communications and Systems.

Lingjie Duan (S'09-M'12) received the PhD degree from The Chinese University of Hong Kong in 2012. He is an assistant professor in the Engineering Systems and Design Pillar, Singapore University of Technology and Design (SUTD). During 2011, he was a visiting scholar in the Department of EECS at the University of California at Berkeley. His research interests include network economics, resource allocation, and energy harvesting. He is the Finalist of Hong Kong Young Scientist Award 2014, and is the awardee of CUHK Global Scholarship for Research Excellence in 2011. He is now leading the Network Economics and Optimization Lab at SUTD. He serves as a TPC Co-Chair of INFOCOM'2014 Workshop on GCCCN, Program Co-Chair of the ICCS'2014 special issue on Economic Theory and Communication Networks, and the Wireless Communication Systems (WCS) Symposium Co-Chair of IEEE ICCC 2015. He is also a TPC member for many top-tier conferences.