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Wavefront subaperture stitching with Shack-Hartmann sensor

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ABSTRACT

A new approach for large field of view measurement using the Shack-Hartmann wavefront sensor (SHWS) is proposed. The object to be measured is divided into several subaperture units. Each unit is measured using the SHWS and a new algorithm is applied to stitch these units. Both theoretical principle and experimental verification are provided in this paper. A comparison between the present global optimization stitching scheme with the conventional sequential stitching method highlights the benefits of the new system. Experimental validation of the SHWS stitching algorithm is carried out by using a coated mirror. Comparison is made with a phase shifting interferometer and good agreement is found. Lastly, a new methodology for large aperture measurement of optical components is provided.

Keywords: Phase retrieval; Optical testing; Subaperture stitching; Shack-Hartmann wavefront sensor.

1. INTRODUCTION

In modern optical system, large aperture components are widely used in diverse fields such as astronomical optics, space optics and adaptive optics. In order to ensure the accuracy of the products, interferometry is commonly used to evaluate components [1]. The main advantage of interferometry is high precision, but on the other hand, the measuring dynamic range is small and highly vulnerable to environment. In addition, the cost of interferometric systems is generally high. Those factors limit its widespread usage.

Nowadays, most of optical surface measurement techniques are based on wavefront detection. Shack-Hartmann sensor, as one of the classic wavefront investigation instruments, has been proposed and applied in many previous studies [2]. Shack-Hartmann sensor has remarkable advantages such as anti-interference ability and shows incomparable flexibility. Wavefront can be retrieved directly without complicated adjustment process like fine manipulation and tuning generally needed when using interferometric setup. Excellent features make Shack-Hartmann sensor playing a great role in wavefront sensing.

With the increase on component aperture size, the manufacturing process has become very complex and difficult. The cost of larger aperture optical measurement equipment will increase exponentially. Furthermore, several problems arise when working with large aperture. Typically, the larger the aperture is, the lower the lateral resolution becomes. Moreover, high frequencies information is not well recovered. Hence subaperture stitching is introduced to address this problem [3-4]. The basic principle of the algorithm can be compared as a jigsaw game. The final result will be an aggregation of all wavefront pieces. It only needs to comply with some certain principles. Subaperture stitching with Shack-Hartmann sensor combines both the advantages of interferometry and SHWS [5-6]. A complete stitching algorithm and process is presented in this paper.

2. THEORY

2.1 Stitching principle

The basic idea of stitching is to scan a large aperture and ensure that each subaperture slightly overlaps each other. Then one can extract the relative shift information from the overlapping area between two adjacent subapertures. By unifying all the subapertures data based on one specified reference plane, the full aperture information can be recovered. For instance, let's consider the two subapertures as shown in Fig. 1 (a). W_1 and W_2 are the wavefronts of the two subapertures, the overall wavefront will be obtained by stitching them together.

Fig. 1 (a) two subapertures stitching; (b) stitching chain; (c) 3 by 3 subapertures parallel stitching mode

$W_1(x, y)$ and $W_2(x, y)$ denote the phases of the two subapertures separately. It can be expressed in the form of the following equation [7]:

$$\begin{cases} W_1(x, y) = P_1 + T_{x1} \cdot x + T_{y1} \cdot y + W_0(x, y) \\ W_2(x, y) = P_2 + T_{x2} \cdot x + T_{y2} \cdot y + W_0(x, y) \end{cases} \quad (1)$$

Where (x, y) is the system coordinates, P_i is the shift amount along beam axis direction, T_{xi} and T_{yi} ($i=1, 2$) are the tilt amounts along x , y axis and W_0 is the reference aperture. The overlapping area should have the same information within both W_1 and W_2 . Thus it can be used as a hint to deduce the relations between them. Then relative shifts and tilts in each subaperture can be determined. By combining both expressions, Eq. (1) can be rewritten as $\Delta W = \Delta P + \Delta T_x \cdot x + \Delta T_y \cdot y$, where $\Delta P = P_1 - P_2$, $\Delta T_x = T_{x1} - T_{x2}$, $\Delta T_y = T_{y1} - T_{y2}$. Mathematically, a minimum of three points which are not on the same line is necessary to solve this equation. Then ΔP , ΔT_x , ΔT_y can be deduced. However, it is preferable to increase the number of data point to minimize the error. As it is an overdetermined set of equations, least squares fitting algorithm can be used to calculate these three parameters.

The same principle can be applied in multiple stitching. One repeats the steps above until all the subapertures have covered the whole surface. This will achieve the purpose of using small aperture to recover the large aperture. When stitching more than two subapertures, the stitching sequence should be discussed. Generally multiple stitching scheme can be divided into two categories. One is called sequential stitching, and the other is parallel stitching. As the subapertures structure diagram in Fig. 1 (b) illustrate, one should use 1→2→3→4 as the order for sequential stitching, and 2→1, 2→3, 2→4 for parallel stitching. The stitching sequence is crucial for the final accuracy. In sequential stitching, the residual error will accumulate gradually with the increase times of stitching. Furthermore, the information between 2&4 is not used and this loss of information generally have an impact on accuracy. In most occasions, parallel stitching mode is preferred.

In order to accomplish stitching with good accuracy, a global optimization stitching algorithm is proposed. The global optimization can be seen as a parallel stitching mode. Unlike sequential stitching, this error homogenization algorithm requires the sum of residuals squares in all overlapping area to be minimum [8-9].

Assuming the total number of subapertures is M , each aperture is defined as $A_1 \dots A_M$, A_M is set as the reference aperture. In order to put the apertures in the same reference coordinate system, we introduce the following transform coefficients: $(T_{x1}, T_{y1}, P_1) \dots (T_{x(m-1)}, T_{y(m-1)}, P_{m-1}), (T_{x(m+1)}, T_{y(m+1)}, P_{m+1}) \dots (T_{xM}, T_{yM}, P_M)$. These transform coefficients should satisfy the equation:

$$\begin{aligned}
 S = & \sum_{i=1}^M \left\{ [\Phi_{i1}(x, y) + f_i] - [\Phi_{1i}(x, y) + f_1] \right\}^2 + \dots \\
 & + \sum_{i=m-1}^M \left\{ [\Phi_{i(m-1)}(x, y) + f_i] - [\Phi_{(m-1)i}(x, y) + f_{m-1}] \right\}^2 + \dots \\
 & + \sum_{i=m+1}^M \left\{ [\Phi_{i(m+1)}(x, y) + f_i] - [\Phi_{(m+1)i}(x, y) + f_{m+1}] \right\}^2 + \dots \\
 & + \sum_{i=M-1}^M \left\{ [\Phi_{i(M-1)}(x, y) + f_i] - [\Phi_{(M-1)i}(x, y) + f_{M-1}] \right\}^2 = \min
 \end{aligned} \tag{2}$$

Where S is the sum of residuals squares, Φ_{ij} is the measured phase in each subaperture (if there is no overlapping area for subaperture i and j , this should be zero), $f_n = T_{xn}x + T_{yn}y + P_n$ is a fitting polynomial (m is the reference aperture, $f_m = 0$), $\sum_{i=n}^M \{ \}^2$ gives the sum of squares of deviations between adjacent apertures in overlapping area, and $n = 1, \dots, m-1, m+1, \dots, M$. By solving the partial derivatives of transform coefficients (one can let partial derivative equals zero), the set consisting of $3(M-1)$ number of equations is shown below:

$$\left[\sum_{i=1}^M \Delta \Phi_{ij} \right]_j = \left[\sum_{i=1}^M Q_{ij} R_{ij} \right]_j \tag{3}$$

Where $j = 1, \dots, m-1, m+1, \dots, M$

$$\text{Here, } \Delta \Phi_{ij} = \begin{bmatrix} \sum_{i \cap j} [\Phi_i(x, y) - \Phi_j(x, y)] x \\ \sum_{i \cap j} [\Phi_i(x, y) - \Phi_j(x, y)] y \\ \sum_{i \cap j} [\Phi_i(x, y) - \Phi_j(x, y)] \end{bmatrix}, R_{ij} = \begin{bmatrix} T_{xj} - T_{xi} \\ T_{yj} - T_{yi} \\ P_j - P_i \end{bmatrix}, Q_{ij} = \begin{bmatrix} \sum_{i \cap j} x^2 & \sum_{i \cap j} xy & \sum_{i \cap j} x \\ \sum_{i \cap j} xy & \sum_{i \cap j} y^2 & \sum_{i \cap j} y \\ \sum_{i \cap j} x & \sum_{i \cap j} y & \sum_{i \cap j} 1 \end{bmatrix}$$

Where $T_{xm} = T_{ym} = P_m = 0$ and m is the reference subaperture. The transform coefficients (T_{xi}, T_{yi}, P_i) can be obtained by solving the equations.

2.2 Treatment on overlapping area

According to the Eq. (2)~(3), stitching coefficients for reference aperture are deduced. The data in each subaperture can be calibrated into one reference coordinate system. Averaging method is applied in the overlapping region of each related subaperture [10]:

$$W(x_j, y_j) = \frac{1}{m} \sum_{i=1}^m W(x_{ij}, y_{ij}) \quad (4)$$

Where (x_{ij}, y_{ij}) is the coordinate of overlapping region in the i^{th} subaperture. However, stitching gaps can arise when using such approach leading to a false stitching map. In order to eliminate the stitching gaps, we introduce the concept of weights which are the factors used to proceed weighted smoothing algorithm in overlapping regions. Let's consider the two subapertures as shown in Fig. 2.

Fig. 2 Weighted smoothing diagram

Where, P is one point in overlapping part of subaperture 1 and 2. The distance from P to the center of circle W_1 is d_1 , and the distance from P to the center of circle W_2 is d_2 . The radii for subaperture 1, 2 are r_1 , r_2 respectively. Then W can be expressed in the form of the following equation:

$$W = \frac{W_1(r_1 - d_1)^2 + W_2(r_2 - d_2)^2}{(r_1 - d_1)^2 + (r_2 - d_2)^2} \quad (5)$$

It can be seen that the weight is smaller when the point is closer to the edge. The sum of two subaperture weights always equals 1. When considering multiple subapertures (n), the equation above can be generalized to:

$$W = \frac{W_1(r_1 - d_1)^2 + W_2(r_2 - d_2)^2 + \dots + W_n(r_n - d_n)^2}{(r_1 - d_1)^2 + (r_2 - d_2)^2 + \dots + (r_n - d_n)^2} = \frac{\sum_{i=1}^n W_i(r_i - d_i)^2}{\sum_{i=1}^n (r_i - d_i)^2} \quad (6)$$

3. EXPERIMENT

3.1 Verification by radius of curvature

In order to verify the effectiveness of our stitching algorithm, we have developed a new method for experimental validation. In optics, laser beam propagation can be approximated by assuming that the laser beam has an ideal Gaussian profile. When refracted by a lens, Gaussian beam properties are changed depending on the focal length. Generally the beam diameter of commercial He-Ne laser is small, and the output laser is collimated. As we choose to apply the algorithm on a curved wave, we use one short-focus lens to generate a spherical wavefront.

Here the theoretical model can be built as illustrated in Fig. 3 (a) where ω_0 , ω_0' are the beam waist radii in lens object space and image space respectively, z and z' are the distances from beam waist to lens center in those two spaces and f is the focal length of the lens.

Fig. 3 (a) Gaussian beam transformation; (b) curvature experiment schematic diagram

Based on Gaussian beam lens transformation equation, the relationship between beam waist and focal length is known as:

$$z' = f + \frac{(z-f)f^2}{(z-f)^2 + \left(\frac{\pi\omega_0^2}{\lambda}\right)^2}, \quad \omega_0' = \sqrt{\frac{f^2\omega_0^2}{(z-f)^2 + \left(\frac{\pi\omega_0^2}{\lambda}\right)^2}} \quad (7)$$

When $z > f$, ω_0' decreases monotonically with increasing z . If $z \rightarrow \infty$, one can see that $z' \approx f$ and $\omega_0' \approx \frac{f}{z}\omega_0$. Here $z \gg f$, the beam waist position in image space will be much closer to the lens focal. Furthermore, based on the wavefront radius of curvature equation:

$$R(l) = l \left[1 + \left(\frac{\pi\omega^2}{\lambda l} \right)^2 \right] \quad (8)$$

Where R is the radius of curvature and l is the distance between test position and beam waist. The wavefront at any plane is spherical (except in waist) and the radius of curvature is varying with propagation. Comparing the radius of curvature along the propagation direction in different positions with theoretical values, one can test our algorithm.

A He-Ne laser is put onto an isolation platform and a short-focus lens is added in front of it. We assume the laser beam waist position is located at output window and that the beam waist radius ω_0 is 1mm. In this experiment, the distance from laser setup to lens z is 1m and focal length f is 1cm. Hence ω_0' here is 10 μ m. The setup geometry is shown in Fig. 3 (b). Please note that l is 1cm (focal length) less than the distance between lens and test position. Substituting those parameters into Eq. (8), one can calculate the theoretical results. The validation results are shown below.

Table 1. Theoretical, experimental radius of curvature and error percentages in different positions

Distance (cm)	16.5	36.5	56.5	76.5
Stitching results
Theoretical (m)	0.1650	0.3650	0.5650	0.7650
Experimental (m)	0.1538	0.3932	0.5723	0.7680
Error	6.79%	7.73%	1.29%	0.39%

3.2 Performance of sequential stitching and global optimization stitching

The comparison on stitching efficiency for those two approaches is presented below. The testing grid we use is shown in Fig. 1 (3). In this experiment, we use 9 subapertures in total. The 5th subaperture (middle) is the reference. We choose 5→1→2→3→4→9→8→7→6 as the order for sequential stitching, while 5→1, 5→2,..., 5→9 is chosen for global optimization stitching. The residuals RMS values in each overlapping area are calculated, as shown in Fig. 4. Here horizontal axis represents current overlapping sequence. For example, 1 means 5→1 and so on. Please note the unit on vertical axis is μm . The light source is a He-Ne laser, which has undergone a beam shaping to spherical or plane wave separately.

Fig. 4 Residuals RMS curves (a) spherical wavefront; (b) plane wavefront

Fig. 5 Comparison on Zernike polynomial coefficients (a) sequential & spherical; (b) sequential & global; (c) plane & spherical; (d) plane & global;

Fig. 5 is the Zernike polynomial analysis in those four combinations respectively. We can notice obviously that the higher orders of Zernike polynomial are suppressed in global stitching mode. This suggests that the residuals are averagely smaller in global mode.

3.3 Comparison between interferometer and Shack-Hartmann sensor

In this section, a coated mirror, 25mm diameter, is tested with full aperture interferometer and stitching Shack-Hartmann sensor. The actual instruments used are shown in Fig. 6 below. In the experiment, the model of interferometer is AccuFiz-E 633nm which is manufactured by 4D Technology Corporation and the calibrated precision is 0.0002λ . The Shack-Hartmann wavefront sensor is a Thorlabs WFS150-5C with $150\mu\text{m}$ pitch lenslet array and its wavefront accuracy is $\lambda/15$ RMS at 633nm.

Fig. 6 (a) AccuFiz-E interferometer (a close-up of sample in red frame); (b) WFS150-5C Shack-Hartmann sensor; (c) the shifted light spots in SHWS (displayed in reversed color); (d) measured wavefront by SHWS in one-shot

By using a square subaperture, one can extract the central circle. In this experiment, we set the SHWS lenslet number to 31 by 31. The side length is 4.65mm ($31 \times 150\mu\text{m}$). In order to fully cover the mirror surface, we choose 9 by 9 parallel grids, as shown in Fig. 7 (a). The setup schematic is illustrated in Fig. 7 (b). The plane wave from the interferometer can be used as the input light source for SHWS. Then, the same initial conditions are used for comparison. By using a beam splitter, measurement is performed with both setups simultaneously. Fig. 7 (c) is the surface profile retrieved with the interferometer and Fig. 7 (d) is one of the stitching results recovered using SHWS. The residual map is shown in Fig. 7 (e).

Fig. 7 (a) the distribution of subapertures; (b) experimental setup schematic diagram; (c) the full aperture result in interferometer GUI ; (d) the stitching result from SHWS; (e) residual map between the two results

The multiple tests results are illustrated in Table 2. The position of SHWS is reset to initial coordinates for each test. The residual features shows relatively constant values.

Table 2. The residual features from three tests

	Test 1	Test 2	Test 3
PV (λ)	0.3145	0.3984	0.3506
RMS (λ)	0.0756	0.0939	0.0826

Considering the precision of SHWS, the residual error is acceptable. The stitching result is valid and of satisfactory accuracy. The stitching errors can be influenced by different factors, such as the movement of translation stage, the alignment of optical elements and the wavefront fluctuation in subapertures etc.

4. CONCLUSION

In this paper we present a novel approach to acquire full aperture information by the use of Shack-Hartmann sensor. Through theoretical derivation and experimental proof, the stitching process is shown to be effective and reliable. It provides a new option for subaperture stitching measurement.

5. ACKNOWLEDGEMENT

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