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2015

Yan, F., Yan, H., & Asundi, A. (2015). The phase error reduction using window functions in digital holography. Proceedings of SPIE - The International Conference on Photonics and Optical Engineering (icPOE 2014), 9449, 944900-. doi:10.1117/12.2074711

<https://hdl.handle.net/10356/89558>

<https://doi.org/10.1117/12.2074711>

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The Phase Error Reduction Using Window Functions in Digital Holography

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ABSTRACT

Digital holography (DH) is a 3D imaging technique with a theoretical axial accuracy of better than 1-2 nanometers. However, practically, the axial error has been quoted to be tens of nanometers which is much larger than the theoretical value. Previous studies of the axial error mainly focused on the phase error introduced by lens. However, it is found that CCD aperture size is also an important contributors to axial error by our group. It is necessary to investigate the reduction approach of such axial error. The most possible connection between the limited CCD aperture size and the axial error is the diffraction effect. Window functions once have been applied to digital holograms for diffraction suppression and improve the lateral resolution of the intensity image. However, their impacts on phase image and the associated axial dimension measurement are still unknown. In this paper, window functions are applied to digital holograms for phase/axial error reduction. Both simulation and experiment are performed. Moreover, the relation between axial error and window functions is also illustrated by the mathematical formulas derived in the theory. And all the results validate that the window functions can reduce the axial error of digital holography.

Keywords: Holography, Resolution, Window Function, Phase Error, Axial Error

1. OBJECTIVE AND BACKGROUND

Digital holography^{1,2} (DH) is an optical technology that have the capacity of reconstructing both intensity and phase information of object field from hologram, recorded by CCD camera. Different from lateral resolution limited by the diffraction effect, axial resolution has the ability to go beyond submicrometer level and reaches an accuracy of better than 1-2 nanometers in theory, limited by the quantization effect on hologram. That's one of the advantages of DH over other 3D imaging techniques. However, in practice, the axial error is usually of tens of nanometers which is far from the theoretical value³⁻⁷. There must be some factors limiting the accuracy of DH.

However, the axial error of DH system has not been thoroughly investigated. The previous studies of the axial error mainly focused on the phase error introduced by lens. Until recently, CCD aperture size was found to be an important contributor to axial error^{8,9} by our group. Based on these findings, the reduction approaches of such axial error are necessary to be investigated. In all connections considered between the limited CCD aperture size and the axial error, diffraction effect is the most possible candidate. So it is necessary to figure out how window functions affect the reconstruction if applied on hologram.

In fact, window functions have been applied to DH for diffraction suppression^{10,11}, in order to improve the lateral resolution of the intensity image. But as far as we know, their impacts on phase image and the associated axial dimension measurement are still not investigated and unknown.

In this paper, the influence of window functions on the axial error is investigated by both simulation and experiments. The result shows that window functions help to reduce the axial error of DH and improve the reconstruction quality. Different window functions are applied on the hologram and compared for axial error reduction. The mechanisms of their impacts are also analyzed.

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The International Conference on Photonics and Optical Engineering (icPOE 2014),
edited by Ailing Tian, Anand Asundi, Weiguo Liu, Chunmin Zhang, Proc. of SPIE Vol. 9449, 94490O
© 2015 SPIE · CCC code: 0277-786X/15/\$18 · doi: 10.1117/12.2074711

Figure 1: The diffraction of object wavefield and reconstruction

2. THE ANALYSIS IN THE THEORY

Digital Holography includes two major processes: digital recording and numerical reconstruction¹². As the object wavefield can be obtained by using reference wave, we focus on the object wavefield directly.

In Fig. (1), we suppose the wavefield of a object in its initial position is $s(x)$, and it becomes $g(x)$ when reaching the CCD surface with the diffraction, and $h(x)$ is the impulse response between the two positions. With the convolution approach and Fresnel approximation¹², the frequency spectrum of an image obtained on CCD surface can be expressed as:

$$G(f) = S(f) \times H(f) \quad (1)$$

where $S(f)$ is the frequency spectrum of object wavefield $s(x)$, $H(f)$ is the frequency spectrum of the impulse response and f is frequency. On account of the separable property of Fresnel transform, we focus on one dimension case, $H(f) = e^{jkz} e^{-j\pi\lambda z f^2}$, as it can be extended to two dimension readily.

When applying window function to hologram, the product in space domain equals the convolution in frequency domain,

$$\begin{aligned} M(f) &= G(f) * W(f) \\ &= \int_{\hat{f}} G(\hat{f}) W(f - \hat{f}) d\hat{f} \end{aligned} \quad (2)$$

where $W(f)$ is the frequency spectrum of window function $w(x)$, and $f, \hat{f}$ are frequency.

Then inverse Fresnel transform is used to reconstruct the frequency spectrum of the object image:

$$R(f) = M(f) \times H^{-1}(f) \quad (3)$$

where $H^{-1}(f) = e^{-jkz} e^{-j\pi\lambda(-z)f^2}$

We can split $R(f)$ into two terms:

$$\begin{aligned} R(f) &= \int_{\hat{f}} S(\hat{f}) e^{-j\pi\lambda z(\hat{f}^2 - f^2)} W(f - \hat{f}) d\hat{f} \\ &= S(f) W(0) + \int_{\hat{f} \neq f} S(\hat{f}) e^{-j\pi\lambda z(\hat{f}^2 - f^2)} W(f - \hat{f}) d\hat{f} \end{aligned} \quad (4)$$

Finally, reconstruction wavefield is obtained with inverse Fourier transform:

$$\begin{aligned} r(x) &= \int R(f) e^{j2\pi x f} df \\ &= W(0) s(x) + \int_f \int_{\hat{f} \neq f} W(f - \hat{f}) S(\hat{f}) e^{-j\pi\lambda z(\hat{f}^2 - f^2)} e^{j2\pi x f} d\hat{f} df \end{aligned} \quad (5)$$

Figure 2: Addition of two complex numbers can be done geometrically by constructing a parallelogram.

The first term on the right side of Eq. (5) is the object wavefield $s(x)$, multiplied by a factor that's zero-frequency component $W(0)$ of the window function $w(x)$. The second term is noise part which is sum of complex numbers and need be suppressed.

In mathematics, addition of complex numbers can be regard as addition of vectors, hence it can be drawn by constructing a parallelogram, as illustrated in Fig. (2) . If the amplitude of one complex number is quite larger than the one of the other, it is safe to say the angle of the sum will approximate the bigger one.

Return to Eq. (5) , if the frequency spectrum of a window function is one at zero-frequency $W(0)$, and small enough at other frequency value, then the reconstruction wavefield will approach the object wavefield as close as possible. Take an extreme example, if the window is an infinite constant signal, in another word, its frequency spectrum is the delta function, then noise part will be zero, this means reconstruction wavefield is equal to original object wavefield, exactly. But as limited by reality, CCD surface can only be manufactured in fixed size, and a limited length of signal must have infinite frequency composition. Therefore, the ideal windows function should satisfy the conditon that its frequency spectrums decay rapidly, in order to get rid of the influence of the noise part.

3. SIMULATION INVESTIGATION OF THE INFLUENCE OF APPLYING WINDOWS ON HOLOGRAPHY

In this section, we apply different windows on the hologram. The impacts of these windows on the axial error are investigated by simulations.

We use a flat step with the optical path differences profile as object in simulation. In Fig. (3a), Its height in z-axis and length in x-axis are $0nm$ and $2.380mm$ at top area, $-100nm$ and $2.380mm$ at bottom area, respectively. In the simulation, the wavelength λ is $633nm$ and the reconstruction distance z is $100mm$. The CCD size and pixel size are $4.46mm \times 5.95mm$, $4.65\mu m \times 4.65\mu m$, respectively.

In holographic imaging, the optical axis passes through at the central vertical line of the object/image plane. In Fig. (3a), we define "step area" as the area which is between the first peak and valley from the middle point of the object/image plane, "top area" as the left top part and "bottom area" as the left bottom part. In the investigation, the reconstructed images are obtained by varying the window function.

We apply 16 practical windows one by one to the simulated hologram to check whether it improves the reconstruction quality or not. Specially, the details of 4 typical results of all are illuminated in Fig. (3) . The impacts of windows can be noticed with ease. In Fig. (3b) , when Rectangular window is used, although the edge of reconstruction is sharp, the ringing around the edge of the object is severe as seen by the intense oscillation of its error value. In Fig. (3c), applying Tukey window will result in a quite large overshoot. And in fig. (3e) - (3d) , when Barthann window or Hann window is used, the reconstructed image is much more smooth, but its edge is not that sharp like Rectangular window as well.

As seen in Fig. (4a), the peak side lobe level of the frequency spectrum of Barthann and Hann window, which have a more smooth reconstructed images, decreases quickly and sharply, while the side lobes of Rectangular window are quite high. Take a step further, let us focus on the comparison between Barthann and Tukey window. Though Barthann window performs worse in the rate of attenuation of the side lobes, it has the narrowest main lobe in all four window functions, which means the zero-frequency value $W(0)$ of Barthann is much bigger than

(a) schematic

(b) Rectangular window

(c) Tukey window

(d) Hann window

(e) Barthann window

Figure 3: 4 typical reconstructed images in simulation

the values at other frequencies. According to Eq. (5), Barthann should result in a quite smooth shape, while Tukey window will perform worse due to the width of its main lobe. Indeed, the conclusion is also exactly true in fact: Barthann does the best in Fig. (3).

The impact of window sizes, from 340 pixel to 950 pixel with a step of 20 pixel, is investigated by simulation. To evaluate concisely, the sum of absolute error, the variance in top and bottom area, and the maximum error in step area are determined as metrics, and the statistical data of all 16 windows are plotted in Fig. (5). As for the maximum error value in step area, in Fig. (5a), it is easy to summarize that Rectangular window and

Figure 4: 2048 point FFT of the 4 typical windows in Fig. (3)

Kaiser window have the least value, that means they have capacity of catching up with rapid variation. In Fig. (5b) - (5c) , Blackman, Blackmanharris, Flattop and Nuttall have the least variance in top and bottom area, that means they are suited to the signals with the relatively slow variation. In Fig. (5d) , Tukey, Hann, Hamming and Barthann perform the best in the sum of absolute error.

4. EXPERIMENT VALIDATION AND RESULTS

In this section, the impact of different windows on axial error is experimentally examined.

All 16 windows are applied to a 1280 pixel $\times$ 960 pixel hologram sample, and the reconstructed images and detailed view at group 2 element 3 (G2E3) area of the 4 windows as same as Fig. (3) are listed in Fig. (6) . As it shown, Rectangular window has the most sharp edge but oscillates intensely in boarder around. And other 3 windows have a much more smooth shape of reconstructed image. The result is same with the one in simulation and agrees with the analyses in Section (3).

Table 1: sort by the maximum error(nm) in step area and the sum of absolute error($nm \times px$)

Window	Maximum Error	Window	Total Error Sum
rectwin	46.79	taylorwin	28408.39
kaiser	47.55	hamming	29255.16
tukeywin	49.07	tukeywin	29784.33
taylorwin	51.29	barthannwin	29786.57
hamming	52.82	hann	29806.02
hann	53.25	gausswin	30071.78
barthannwin	53.88	triang	30807.57
gausswin	54.35	blackman	31888.39
blackman	55.94	kaiser	32028.46
bohmanwin	56.47	bohmanwin	32320.50
triang	56.75	rectwin	32662.35
parzenwin	57.62	parzenwin	33742.77
chebwin	57.96	chebwin	33999.71
nuttallwin	58.27	nuttallwin	34354.23
blackmanharris	58.47	blackmanharris	34602.91
flattopwin	66.85	flattopwin	80825.13

Figure 5: statistics of various window size in simulation

Moreover, images cropped at G2E3 area are compared with side views in Fig. (7). And all windows are compared by the maximum error in step area and the sum of absolute error in Table. (1). Rectangular window and Kaiser window do well in the maximum error, while Taylor, Hamming, Tukey, Barthann, Hann perform better in the sum of absolute error. As same with the simulation, those window functions, which have a narrow main lobe and high attenuation of the side lobes, perform better. The phenomenon agrees with the conclusion of Section. (2), exactly.

As Taylor will lead to large overshoot value, Hamming, Hann and Barthann are general-purpose recommendation in practical application, according to all the data from simulation and experiments. And if interested in rapid response to dynamic area, Kaiser window is also a recommended alternative to Rectangular window, due to the ringing suppression.

5. IMPACT

In this work, the influence of window functions on hologram in digital holography is investigated. In theory, with Fresnel transform, the reconstructed image is derived as two parts: original object and noise. The noise

(a) G2E3 location

(b) Rectangular window

(c) G2E3 area

(d) Tukey window

(e) G2E3 area

(f) Hann window

(g) G2E3 area

(h) Barthann window

(i) G2E3 area

Figure 6: Experiment results of the 4 windows as before, in each subfigure, left part is reconstructed image by applying corresponding window, and right part is image of the G2E3 area.

(a) reconstruction in the step area

(b) error in the step area

Figure 7: images cropped at G2E3 area of 16 windows are compared with side views

part is the sum of complex numbers multiplied by the frequency spectrum of window function. Therefore phase error can be reduced effectively by the windows that its main lobe is narrow enough and side-lobes go sufficiently rapidly toward zero, due to the feature of addition of complex numbers. And it is also demonstrated that window functions can reduce the axial error of digital holography by both simulation and experiments. It is proved that Hamming, Hann, Barthann contribute to suppress the ringing of the reconstructed image, while rectangle window and Kaiser window have better capacity of keeping up with dynamic area. Therefore, Hamming, Hann and Barthann are suggested in practical application, and Kaiser window can be an prefer alternative to Rectangular window.

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