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Dynamic Spectrum Access Networks with Heterogeneous Users: How to Price The Spectrum?

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Abstract—In a pricing-based spectrum trading market, an interesting issue is to study how secondary users make their selections in a rational manner when there are available spectrum of different qualities in the market. In this case, primary systems inherently require a precise pricing strategy to maximize their profits especially when the secondary users' selection preferences are not uniform nor predictable during the dynamic access. In this paper, we propose a novel spectrum pricing mechanism for primary systems by taking into account the selection preferences of heterogeneous secondary users and quality diversity of leased spectrum caused by different interference levels and channel fading characteristics. Our work mainly focuses on exploring the optimal pricing solution in the case that the selection preference of secondary buyers is stochastic. In the proposed system model, the spectrum to be leased is divided into a series of uniform subchannels of various qualities for spectrum trading. We analyze the impact of secondary users' preference on spectrum trading, and propose an iterative algorithm for spectrum pricing by introducing Hotelling model to describe the relationship between the spectrum diversity and the primary system's utility. Proofs of the integrability of the utility function and existence of Nash equilibrium used in the proposed algorithm are provided. Numerical results demonstrate the effectiveness of the proposed pricing method in improving the primary system's profits.

Index Terms—Dynamic spectrum access, spectrum leasing, Hotelling model, Nash equilibrium

I. INTRODUCTION

CURRENT static spectrum access mechanism cannot effectively support the explosively increasing demands for emerging multimedia services from mobile users due to the under-utilization of network resources such as licensed spectrum. Dynamic spectrum access has been considered as an efficient way to address the conflicting requirements between congested available spectrum bands for new applications and idle authorized spectrum [1]-[4].

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In dynamic spectrum access networks, designing adaptive resource allocation schemes has drawn much research attention especially on admission control, users' requirements of Quality of Service (QoS), efficiency of spectrum sensing, spectrum pricing and interference management [5]-[7]. Specifically, spectrum trading provides an efficient method for secondary users to dynamically access licensed bands where financial gains can encourage primary users to lease unused spectrum temporarily to enhance the utility efficiency autonomously with proper market mechanisms [8]-[10].

Market-based spectrum access strategies always need to consider several significant factors such as rational spectrum pricing in monopoly or competitive markets, dynamic network traffic, smooth spectrum handoff, and proper power allocation in spectrum allocation, etc. Specifically, to balance the market demand and supply and inspire spectrum owners to engage in dynamic spectrum trading, spectrum auction with fixed or varied suppliers was proposed [11][12]. Truthful auction mechanisms play key role in guaranteeing the regular implementation of auction activity and the benefits for both sides. Besides, joint channel and interference pricing has investigated to properly charge the secondary users by accurately assessing the marginal cost of the primary system's idle channel and potential influence of increasing interference caused by the incoming secondary users [13][14]. Multi-stages optimization objective is often formulated to pursue maximal social welfare and spectrum utility during the course. Moreover, some special cases are further considered for improving the fairness of spectrum dealing. For example, secondary users can provide certain relay transmission to primary users for spectrum authority where an agreement of relay service should be reached in advance [15].

Compared to the auction-based method, pricing-based spectrum access can lower interaction overhead and adapt to distributed network circumstance [16]-[18]. In [16], the authors formulated a contract design problem where a primary license holder wishes to reap profit from selling its spectrum authority to potential secondary users/buyers. In the proposed model, different kinds of secondary users with various communications requirements, uncertainty tolerance of available capacity were considered to attain a fair agreement of spectrum trading. In [17], the authors proposed a pricing-based system model where the primary idle spectrum can be rented to multiple secondary users at the same time. A nonlinear and multiobjective optimization problem with respect to sharing spectrum dynamically was investigated and intelligent algorithms were applied to solve the problem. In [18], the authors investigat-

ed the optimal spectrum procurement and pricing from the perspective of a cognitive mobile virtual network operator, considering a second market between the spectrum owner and secondary users. Differentiated pricing among heterogeneous secondary users is adopted to improve the profit of the virtual operator.

Due to the obstacle of constant interaction and lack of smooth centralized control, frequent join and quit of mobile nodes, time-varying channel characteristics as well as the uncertainty of users' preference selection remain challenging for designing spectrum access strategies [19][20]. Many existing research works are deployed with the basic assumption that the information interaction between the participants are transparent and the cost, time as well as power assumption involved in the pricing progress can be ignored. How to design an elastic strategy for spectrum allocation becomes significant to support dynamic primary and secondary users' demands [21][22].

During dynamic spectrum access, most of current research studies extend their technical roadmaps with an underlying assumption that all the leased channels are homogeneous and undergo uniform interference. However, as the available spectrum to be leased is not predictable and always appearing in non-consecutive bands, it may have various fading characteristics. Furthermore, due to secondary users' diverse positions in heterogeneous cells, even the same spectrum band does not mean the same utility for different users sometimes. Thus, heterogeneous spectrum and differential user selection on the spectrum deserve full investigation which is the main concern of this paper. Actually, this problem has attracted interest of researchers recently. Specifically, the authors in [23] studied the price of free spectrum to heterogeneous users by considering a heterogeneous pool of customers who own different trade-offs for spectrum trading. The research shows that heterogeneous customers being added to unlicensed spectrum can reduce unnecessary spending to customers and at the same time increase the profit of producers. In [24], secondary users were assumed to be heterogeneous in their valuations and demands of the spectrum. The authors used differential pricing among the heterogeneous users to improve the profits of the primary system and allowed it to perform necessary admission control. In [23] and [24], the leased spectrum was homogeneous, while the secondary users' demands were diverse. In [25], the effects of heterogeneous spectrum were studied where two kinds of channels and uniform user selection preference were consideration for simplicity. In our study, to solve the problem of spectrum pricing with heterogeneous spectrum and secondary preferences, differential and stochastic secondary preferences are intensively investigated. The emphasis and main perspectives of this paper are different to the methods above-mentioned. The key improvement of this paper lies in that we design a pricing-based solution for dynamic spectrum trading to solve the problem of differentiated spectrum pricing in case of complex heterogeneous spectrum and stochastic user demands. Comparative analyses between our proposal and the methods proposed in Refs. [24] and [25] are provided to testify the performances in the following simulation section.

In this paper, we investigate how to price the spectrum

properly when heterogeneous spectrum quality and stochastic preference of secondary users are required to be considered as essential factors. A concept of spectrum pool is introduced to the proposed system model where all the leased spectrum constitutes the pool and is divided into uniform channels for selling to secondary market. Suffering from different levels of interference, these channels have various qualities. In this case, secondary users are supposed to select channels for usage based on their preference. It can be envisioned that a secondary buyer with sufficient capital or urgent demand for ideal QoS can pick a high-quality channel. In the proposed model, the secondary users' spectrum preference is not uniform. Then, a key issue arises on how to design a proper pricing mechanism that ensures primary system's profit by accurately predicting and analyzing the customers' preference in advance. It is similar to a kind of economic issue, and we aim to address the problem by using Hotelling model which is an appropriate tool for describing the pricing problem with product diversity.

The main contributions of this paper is summarized as:

- We investigate the dynamic spectrum pricing in the case that heterogeneous spectrum and secondary users are taken into account. We seek an optimal pricing solution based on analyzing the spectrum preference of secondary users which complies with nonuniform distribution.
- A preference parameter is introduced to describe the preference degree of secondary buyers on various kinds of channels. Detailed presentation and analysis of the preference parameter on influencing channel pricing are provided. To the best of our knowledge, few research works investigate how to price the spectrum when heterogeneous spectrum is on market and the demands of secondary users are stochastic.
- An objective function is built by using Hotelling game model which is an appropriate tool for describing the pricing bargaining in the situation where product and cost diversity need to be considered. Besides, this paper also analyzes the effects of preference coefficients of secondary users on the final channel pricing and system profits.
- An iterative algorithm for spectrum pricing is obtained by achieving a Nash equilibrium. The proposed system model in this work is more practical and close to the actual trading since the consumers always intend to make a rational selection between the product quality and price.
- The proofs for the integrability of utility function and existence of Nash equilibrium are provided. Furthermore, numerical results are provided to evaluate how the pricing parameters affect the primary system's profit.

The remainder of the paper is organized as follows. We introduce the system model for dynamic spectrum pricing in Section II. Section III gives the utility function and receives a Nash equilibrium of this proposal. Essential proofs for algorithm rationality are also provided. Furthermore, numerical results are supplied to analyze the performance of pricing algorithm in Section IV. Finally, we conclude this paper in Section V.

Fig. 1. System pool

II. SYSTEM MODEL

A. Spectrum Pool

As radio spectrum authorized to licensed systems remains largely inefficient in many areas, a change in the utilization of idle spectrum to enable unlicensed service or users to operate on licensed idle bands has been taken into account [26]. In this paper, we consider a system model in which primary users and numbers of cognitive users coexist in dynamic access networks. The system model is shown in Fig. 1. In this case, the idle spectrum is scheduled to lease and the secondary users without monetary payment are not allowed to use the bands. We aim to explore the pricing-based spectrum sharing from the perspective of primary systems. As the primary systems cannot conduct detailed questionnaire surveys to collect the secondary users' purchase preferences, comprehensive analysis and precise predication on the users' behaviours need to be unfolded. We suppose one of the primary users in the networks acts as the virtual network operator who collects the leased spectrum and sells the spectrum to secondary users to maximize the overall profits. As the primary users have the initiatives to lease the spectrum by authorizing a primary agent to operate, the sensing cost is saved. Otherwise, a secondary virtual operator should spend numerous energy on sensing the spectrum which will add an additional cost on the spectrum trading unavoidably.

Besides, all the idle spectrum managed by the primary virtual operator can form a shared-use pool in a broad band for trading. The spectrum pool is shown in Fig. 2. In this paper, we introduce heterogeneous spectrum to the spectrum trading. We suppose the unused spectrum owned by different primary systems has **non-uniform spectrum** qualities as shown in Fig. 2. The qualities of the leased spectrum in the pool vary due to different interference levels and wireless channel fading characteristics. **Complex spectrum environment and random distribution of secondary users lead to the quality diversity.** Furthermore, the overall bands are divided into a number of unified channels for leasing where a secondary user can select channels for usage and the primary users price the spectrum in centralized mode. The channels in the pool are assumed to be orthogonal without severe mutual interference.

In order to facilitate the trading, the primary virtual oper-

ator should build a common control channel in advance to broadcast trading information and interact with the secondary customers. The primary virtual operator is assumed to perform interactions with the secondary users to obtain essential trading information, such as purchase intentions and service types. The primary operator can first broadcast the information including leased channel number, channel bands, reusing status and channel width, etc, to all the users in the networks. Potential secondary customers then return their purchase intentions and service types through the common channel to the primary operator in response. After gathering all useful information, the primary operator makes a decision on the channel pricing by centralized calculation, then broadcasts the prices to the secondary customers. Unlike spectrum auction, the proposed mechanism does not require pricing bargain or other special interactions among the transactors. By analyzing the secondary customers' service types and predicting their selection preferences, a pricing-based spectrum sharing solution will be achieved. Secondary users can acknowledge the primary operator which kinds of channel they wish to rent according to the prices released and their own preferences.

Due to the spectrum quality diversity, secondary users should make rational choices on the spectrum since ideal channel corresponds to high price. On the other hand, for the primary systems, it is essential to set a proper pricing mechanism to maximize the system profit according to the secondary customers' purchasing behaviors. In this paper, we formulate the spectrum pricing problem into Hotelling model which is usually applied to describe the pricing competition when product diversity and user preference in a monopoly market need to be taken into account. This economical model was first proposed by Hotelling and became known as it specializes the pricing rule when product differentiation is obvious which leads to price insensitivity [27]. Hotelling generalized and developed Bertrand's game model by further considering different manufacturers' locations in geographic space, thus this model could be used to solve the pricing problem involved in product differentiation. The model has two basic preconditions: product differentiation and market monopoly. In fact, a significant character for the mature and competitive market is that the majority of the whole market is always controlled by a few huge firms. And in our model, the only opportunity for the secondary users being authorized to access the band is to purchase the spectrum from the pool. We also assume all the participants involved in this deal are rational and have the desire to maximize their profits. Thus, secondary users make their channel selections based on the channel price, available budget, individual demand and spectrum assessment. Meanwhile, the primary systems give the customers product price based on prior market evaluation.

B. Utility Functions

We first analyze the benefits received by the secondary users involved in this spectrum trading. To formulate the pricing problem, we introduce the Hotelling game model which can properly describe the issue of pricing sensitivity caused by product diversity. During the course of applying game model,

Fig. 2. Spectrum pool

proper definitions for the benefits of both primary systems and secondary users are significant. For the secondary spectrum buyer, the main intention of its spectrum trading behavior is to achieve certain amount of transmission capacity in the leased band. In exchange, the secondary buyer has to pay essential cost within its current budget. It can be envisioned that the user with more budget fund may prefer to choose a high-quality channel for usage. On the other hand, even with similar available funds, different secondary users may choose their channels with various qualities due to differentiated service types. We also pay attention to the interference levels of the leased channels which have apparent effects on the channel capacities. Therefore, the reduction parts of the secondary user's utility contain the monetary cost and the interference level. We further suppose the secondary users involved in the spectrum trading are rational who wishes to seek a balance between the channel quality and channel price. Thus, for spectrum i , the utility function of a secondary user can be expressed as

$$U_s = \kappa \times \theta \times C_i + \alpha B - \phi p_i - \varepsilon I_i, \quad (1)$$

where C_i denotes the transmission capacity obtained by the secondary customer on the leased channel, which can be expressed as $\omega \log_2(1 + \frac{P_w}{I_{s_i}})$. For fixed bandwidth ω and power P_w , $I_{s_i} = \sum_{i \neq j} g_{ij} f_{ij} p_j + n_0$ is various which leads to the diversity of spectrum quality. Generally, the interference level suffered by the secondary user on the leased band varies based upon wireless fading, users' locations and transmit powers on the adjacent channels. For simplicity, we assume the wireless fading in surrounding environment does not change much with time, therefore the primary systems can provide steady prices for the leasing channels which facilitate the secondary users' selections and spectrum trading. Besides, p_i is the spectrum price in fixed channel band, I_i denotes the interference level caused by the secondary user on the spectrum owner, and B is the available fund of the secondary user. We also introduce a parameter θ to denote the selection preference of the secondary user. In this utility function, we introduce monetary coefficient κ and ε to transform transmission capacity and interference

Fig. 3. Layer structure of channel selection

level to uniform unit. We also add coefficient α and ϕ to the utility function. Thus, in (1), κ , α , ϕ and ε are the weighted coefficients of several variables for the secondary user. Apparently, the setting of weighted coefficients has a great impact on the user utility.

When making a decision, we consider a secondary user has four kinds of spectrum to select as depicted in Fig. 2. The secondary user has to evaluate the available spectrum by considering several significant factors, including spectrum quality, accessible fund, channel price and interference level. During the course of evaluation, a secondary user should first judge the weight of each factor defined in (1). If the user is relatively wealthy, coefficient α will be small and priority can be given to the channel quality. If the customer is sensitive to price, ϕ should be large. Secondary users can first identify a factor with first priority, then the factor with second priority. During the course, secondary users should also take their own preference into account. When numbers of the considering factors become increasing, secondary users will struggle to make a rational choice in qualitative analysis. The selection problem can be decomposed into three levels as shown in Fig. 3.

To compare the impacts of factors $C_1, C_2, \dots, C_n$ of one layer on a factor S_i of another layer, such as the importance of different choice criteria on final channel selection, it is essential to make a comparison between only two factors rather than multiple factors at the same time. Selecting two factors C_i and C_j each time, we use a_{ij} to denote the impact ratio of C_i and C_j on S_i , all the comparison results can be expressed in matrix as following

$$A = (a_{ij})_{n \times n}, \quad a_{ij} > 0, \quad a_{ij} = \frac{1}{a_{ji}}. \quad (2)$$

We can rewrite (2) as

$$A = \begin{pmatrix} \omega_1/\omega_1 & \omega_1/\omega_2 & \cdots & \omega_1/\omega_n \\ \omega_2/\omega_1 & \omega_2/\omega_2 & \cdots & \omega_2/\omega_n \\ \vdots & \vdots & \ddots & \vdots \\ \omega_n/\omega_1 & \omega_n/\omega_2 & \cdots & \omega_n/\omega_n \end{pmatrix}, \quad (3)$$

where $\omega = (\omega_1, \omega_2, \dots)^T$ is the weighted coefficient vector

satisfying $\sum_{i=1}^n \omega_i = 1$. Thus, as mentioned above, proper selection of the weighted coefficients in the secondary user's utility function is significant. In subsequent simulation tests, we will provide numerical results to testify the effects of different parameter selection in spectrum trading

Then, we give the utility function of the primary system as following

$$\begin{aligned} \pi_i &= (p_i - M_i)D_i \\ &= (p_i - \mu C_i)N \int g(\theta)d\theta, \end{aligned} \quad (4)$$

where π_i is the benefit of primary system i from spectrum trading, p_i is the channel pricing in spectrum quality C_i , N is the number of secondary users, μ is the marginal cost coefficient, D_i denotes demand function, θ is user preference and $g(\cdot)$ is the distribution function. We can obtain from primary utility function that the user selection preference plays a key role in deciding the system profit. A secondary buyer's preference diversity on different kinds of channels not only influences its own selection but also the channel's price. Besides, for primary systems, it can be imaged that the channel pricing is obviously related to the scarcity degree and market evaluation. Secondary users' selection preferences are mainly affected by channel pricing and available funds. In fact, secondary user's preference shows the relation of demand and supply in current market. In Fig. 4, we give the preference curves of different secondary users. Different nodes in same curve represent the spectrum deals in same selection preference. The curves drop down with the progress of spectrum deal, which means secondary users need to pay for the leased channels if they wish to achieve channel authority. Three curves in Fig. 4 represent the effects of different user preferences in spectrum trading. When the channel qualities are various, the trading curves change in respond as shown in Fig. 5. Secondary users need to pay more cost to obtain high-quality channels, therefore the curves representing the high-quality channels have larger gradient. Furthermore, when we take the preference curves of both primary systems and secondary users into account, the exchange path can be given as shown in Fig. 6. Furthermore, to achieve a distinct exchange pattern for both sides, we mix the two preference curves together in Fig. 6, where the coordinate system and curves marked in red line denote the performance of the secondary customers and the black line represents the primary system. In the figure, the preference curves for the primary systems and secondary users have been depicted by two coordinates origins, respectively. We link the junction points of the curve families and mark the points by a dotted line as shown in Fig. 6. It can be proved that spectrum trading occurs at these points.

III. PRICING-BASED SPECTRUM ALLOCATION

A. Pricing Game

After analyzing the relationship between secondary user's preference and channel pricing, we then discuss the pricing game for the channels with different kinds of qualities. In this work, we assume there are four kinds of systems with diverse

Fig. 4. Preference curves of secondary users

Fig. 5. Diversity curves of different kinds of channels

Fig. 6. Exchange path

Fig. 7. Preference distribution of secondary users

spectrum qualities as denoted in Fig. 2. Thus, in this case, secondary users should make rational choices between the four kinds of channels based on their preference and channel pricing. In the definition of secondary user's utility function, we introduce a parameter of secondary selection preference to analyze the users' selection behavior during the spectrum trading. Then, the demand functions for different kinds of channels can be given as

$$D_1(p_1, p_2, p_3, p_4) = N \int_{\underline{\theta}_i}^{\bar{\theta}_i} g(\theta) d\theta, \quad (5)$$

where p_i denote the pricing of spectrum i and N is the user number. $[\underline{\theta}_i, \bar{\theta}_i]$ represent the region in which secondary users will choose spectrum i when their preference parameters locate. In this case, we define $\theta_1, \theta_2, \theta_3, \theta_4$ to represent the preference values corresponding to a secondary user's channel selection on C_1, C_2, C_3, C_4 , respectively, and there is $0 < \theta_1 < \theta_2 < \theta_3 < \theta_4$. Thus, θ locates at the region $[\theta_1, \theta_4]$. Furthermore, C_i denote the quality of spectrum i and we assume $0 < C_1 < C_2 < C_3 < C_4$. Therefore, it can be obtained that a secondary user will select a channel with spectrum quality C_2 when its preference parameter locates near θ_2 .

For given preference region $[\theta_1, \theta_4]$, different probability distribution functions (PDFs) of the preference parameter will lead to obvious diversity of the final pricing scheme of primary systems. When secondary users have abundant budgets to purchase idle channels or are eager to have high-quality channels, it can be predicted that they will prefer to select a good channel even affording expensive cost. Under this circumstance, the user's preference θ will approach to θ_4 , otherwise it will be close to θ_1 . In general, the secondary user's preference will randomly locate in region $[\theta_1, \theta_4]$. We assume the total number of secondary customers is N , and the users can be divided into three types whose preferences locate in $[\theta_1, \theta_2]$, $[\theta_2, \theta_3]$ and $[\theta_3, \theta_4]$, respectively. Thus, there are three kinds of secondary buyers with high middle-and-low grade consumption capabilities. Then, the preference distribution of the secondary users in this case is shown in Fig. 7. In this figure, we assume the PDFs of user preferences comply with the normal distribution as we assume θ_i are stochastic variables. Furthermore, we consider $N = N_1 + N_2 + N_3$, and

the user numbers denoted by N_i are shown as Fig. 7. Thus, in this case, the preference distributions of all the secondary users are not uniform, and we generally classify the users into three types and then investigate the corresponding spectrum pricing in the user characteristic.

A secondary user's selection preference is impacted by many factors, such as available budget, spectrum quality and spectrum pricing denoted as (1). It is natural that a user's preference θ_i will increase when the spectrum pricing decreases or budget fund becomes abundant. In this situation, we consider when a user's preference locates at the region $[\theta_1, \theta_2]$, then the user will select a channel between spectrum I and spectrum II with qualities C_1 and C_2 , respectively. Similarly, a secondary customer could select spectrum II or spectrum III with qualities C_2 and C_3 , when its satisfies $\theta_2 \leq \theta \leq \theta_3$. Besides, we define a non-preference coefficient to describe the balancing state for a secondary buyer as shown in Fig. 7. When $\theta = \theta_b$, it means the secondary user has no preference for spectrum I and spectrum II since equal utility can be attained at this moment. Thus, there is the following equation

$$\begin{aligned} U_s^{C_1} &= U_s^{C_2} \\ \Rightarrow \kappa \times \theta_b \times C_1 + \alpha B - \phi p_1 - \varepsilon I_1 &= \kappa \times \theta_b \times C_2 + \alpha B \\ &\quad - \phi p_1 - \varepsilon I_1. \end{aligned} \quad (6)$$

Then, we have

$$\theta_b = \frac{\phi(p_1 - p_2) + \varepsilon(I_1 - I_2)}{\kappa(C_1 - C_2)}, \quad (7)$$

Therefore, in given preference region, we can obtain the profit function for spectrum I as

$$\begin{aligned} \pi_1 &= (p_1 - M_1)D_1 \\ &= (p_1 - M_1)N_1 \int_{\theta_1}^{\theta_b} g(\theta) d\theta. \end{aligned} \quad (8)$$

Then, we assume θ'_b is the balancing point in $[\theta_1, \theta_2]$, thus we have the profit functions for spectrum II as

$$\begin{aligned} \pi_2 &= (p_2 - M_2)(D_2 + D'_2) \\ &= (p_2 - M_2)(N_1 \int_{\theta_1}^{\theta'_b} g(\theta) d\theta + N_2 \int_{\theta_2}^{\theta'_b} g(\theta) d\theta). \end{aligned} \quad (9)$$

We can also obtain the profit functions for spectrum III and spectrum IV as

$$\begin{aligned} \pi_3 &= (p_3 - M_3)(D_3 + D'_3) \\ &= (p_3 - \mu C_3)(N_2 \int_{\theta'_b}^{\theta_3} g(\theta) d\theta + N_3 \int_{\theta_3}^{\theta''_b} g(\theta) d\theta), \\ \pi_4 &= (p_4 - M_4)D_4 \\ &= (p_4 - M_4)N_4 \int_{\theta''_b}^{\theta_4} g(\theta) d\theta, \end{aligned} \quad (10)$$

where θ''_b denotes the balancing point in $[\theta_3, \theta_4]$.

Theorem 1. For spectrum I, when the secondary consumer's spectrum preference θ has a normal distribution, we obtain the

profit of spectrum I can be expressed as

$$\begin{aligned}\pi_1 &= (p_1 - M_1)D_1 \\ &= \frac{N_1(p_1 - \mu C_1)}{2} \left\{ \sqrt{1 - \exp\left[-\frac{2(\theta_1 - \theta_b)^2}{\pi\sigma^2}\right]} \right\}.\end{aligned}\quad (11)$$

Please refer to Appendix A for the proof.

Likewise, we can obtain the profits of spectrum II, spectrum III and spectrum IV as

$$\begin{aligned}\pi_2 &= \frac{N_1(p_1 - \mu C_1)}{2} \left\{ \sqrt{1 - \exp\left[-\frac{2(\theta_2 - \theta_b)^2}{\pi\sigma^2}\right]} \right\} \\ &+ \frac{N_2(p_2 - \mu C_2)}{2} \left\{ \sqrt{1 - \exp\left[-\frac{2(\theta_1 - \theta'_b)^2}{\pi\sigma'^2}\right]} \right\},\end{aligned}\quad (12)$$

$$\begin{aligned}\pi_3 &= \frac{N_2(p_2 - \mu' C_2)}{2} \left\{ \sqrt{1 - \exp\left[-\frac{2(\theta_2 - \theta'_b)^2}{\pi\sigma'^2}\right]} \right\} \\ &+ \frac{N_3(p_3 - \mu'' C_3)}{2} \left\{ \sqrt{1 - \exp\left[-\frac{2(\theta_1 - \theta''_b)^2}{\pi\sigma''^2}\right]} \right\},\end{aligned}\quad (13)$$

$$\pi_4 = \frac{N_3(p_4 - \mu'' C_4)}{2} \left\{ \sqrt{1 - \exp\left[-\frac{2(\theta_4 - \theta''_b)^2}{\pi\sigma''^2}\right]} \right\}\quad (14)$$

Then, based on the necessary conditions for Nash equilibrium, the optimal channel pricing at step $k+1$ can be obtained as

$$p_i^{k+1} = p_i^k + \frac{\partial \pi_i}{\partial p_i}.\quad (15)$$

Therefore, we have the following equations

$$p_1^{k+1} = p_1^k + N_1 \left\{ \sqrt{1 - \lambda} + \frac{2\lambda(p_1 - \mu C_1)(\theta_1 - \theta_b)}{\pi\sigma^2\kappa'\sqrt{1 - \lambda}} \right\},\quad (16)$$

where $\lambda = e^{\frac{2(\theta_1 - \theta_b)^2}{\pi\sigma^2}}$, $\kappa' = \kappa(C_1 - C_2)$, $\theta_b = \frac{\phi(p_1 - p_2) + \varepsilon(I_1 - I_2)}{\kappa(C_1 - C_2)}$. And, there is

$$\begin{aligned}p_2^{k+1} &= p_2^k + \frac{4N_1\lambda'(p_1 - \mu C_1)(\theta_1 - \theta_b)}{2\pi\sigma^2\kappa'\sqrt{1 - \lambda'}} \\ &+ N_2 \left\{ \sqrt{1 - \lambda''} + \frac{2\lambda''(p_1 - \mu' C_2)(\theta'_b - \theta_2)}{\pi\sigma'^2\kappa''\sqrt{1 - \lambda''}} \right\},\end{aligned}\quad (17)$$

where $\lambda' = e^{\frac{2(\theta_1 - \theta'_b)^2}{\pi\sigma'^2}}$, $\kappa'' = \kappa(C_2 - C_3)$, $\theta'_b = \frac{\phi(p_2 - p_3) + \varepsilon(I_2 - I_3)}{\kappa(C_2 - C_3)}$, $\lambda'' = e^{\frac{2(\theta_2 - \theta'_b)^2}{\pi\sigma'^2}}$. Besides, there is

$$\begin{aligned}p_3^{k+1} &= p_3^k + \frac{4N_2\lambda''(p_2 - \mu' C_2)(\theta'_b - \theta_2)}{2\pi\sigma'^2\kappa''\sqrt{1 - \lambda''}} \\ &+ N_2 \left\{ \sqrt{1 - \lambda'''} + \frac{2\lambda'''(p_2 - \mu'' C_3)(\theta''_b - \theta_2)}{\pi\sigma''^2\kappa''' \sqrt{1 - \lambda'''}} \right\},\end{aligned}\quad (18)$$

where $\lambda''' = e^{\frac{2(\theta_2 - \theta''_b)^2}{\pi\sigma''^2}}$, $\kappa''' = \kappa(C_3 - C_4)$, $\theta''_b = \frac{\phi(p_3 - p_4) + \varepsilon(I_3 - I_4)}{\kappa(C_3 - C_4)}$. Then, we have

$$p_4^{k+1} = p_4^k + \frac{N_3}{2} \left\{ \sqrt{1 - \lambda''''} + \frac{2\lambda''''(p_4 - \mu''' C_4)(\theta_4 - \theta''_b)}{\pi\sigma''^2\kappa'''' \sqrt{1 - \lambda''''}} \right\},\quad (19)$$

where $\lambda'''' = e^{\frac{2(\theta_4 - \theta''_b)^2}{\pi\sigma''^2}}$, $\kappa'''' = \kappa(C_4 - C_3)$.

Supplemental comments for the pricing mechanism are given as **Remarks** below.

Remark 1. In this case, the marginal cost for the leased channel is defined as $M_i = \mu C_i$ which implies an assumption $M_h > M_l$ due to $\mu C_h > \mu C_l$ in condition of $h > l$. Thus, the high-quality channels mean more marginal costs for primary systems. From another aspects, primary systems desire to receive more profits from the high-quality channels rather than preserving the channels for other applications in future. As a result, the primary systems choose to afford the potential cost losing for exchanging current profit increase.

Remark 2. Similar to common deal, it is necessary for the primary systems to cover the cost increase for different kinds of channels by the pricing difference. Thus, we have the equation as

$$M_h - M_l = \mu(C_h - C_l) \leq P_h^* - P_l^*,\quad (20)$$

where $h \neq l$. In respond to the channel's quality difference, the primary systems need to keep the pricing reasonable so as to insure the secondary users who buy the high-quality channels feel it is worth affording the higher price.

Remark 3. In this work, we investigate the pricing algorithm when secondary user's selection preferences normally distribute in given region. In fact, secondary preference can also be assumed to comply with other types, such as uniformed distribution and linear distribution. When the preference parameter's distribution function is complicated, we need to consider whether the utility functions of primary systems is integrable which is a necessary condition to fix the optimal solution.

Remark 4. The iterative pricing algorithm proposed in this section is not a distributed solution which can be performed by every single primary user separately. Since it has a nested and iterative structure, the centralized primary virtual operator should collect all the essential information and parameters then run the algorithm to identify the channels' prices parallel. A separate outcome for single kind of channel cannot be attained. As shown from the equations, the items and powers of the proposal are small. The algorithm consumes very limited computation resource, therefore no special equipment is required to fulfill the solution. As shown from the numerical results in following simulation section, the iterative algorithm can converge at no more than 20 iterations generally. The convergency speed is fast and the computation time can be nearly ignored in common personal computer.

B. Nash Equilibrium

Selecting a proper utility function which can accurately describe the whole benefit of the objective player is significant in applying game model. In general, the utility function adopted should have a pure Nash equilibrium to ensure the unique and rational game solution. In this paper, we choose (8) as the utility function of licensed spectrum I. In this subsection, we discuss the rationality of the proposed utility function by testifying whether it is consistent integrable and has a pure Nash equilibrium. In our system model, we suppose the secondary user's demand is stochastic, thus the user's preference can be subject to normal distribution in given region. As shown from the utility function expressed in (8),

a key point to fix the final pricing solution is to ascertain the utility function is consistent integrable. In the appendix, Theorem 2 is provided to prove it. Furthermore, as a utility function may lead to multiple solutions, it is essential to remove the surplus solutions and find the optimal one for the rationality of the Nash equilibrium deduced. The proof for the existence of a pure Nash equilibrium is provided in appendix section. Similar conclusions can be applied to the other utility functions denoted by (9) to (11) for spectrum II, III and IV.

Theorem 2. For a given utility function ζ_i , if there is $\tau \geq 0$ satisfying $\sup_{i \in N} E|\zeta_i|^\tau < \infty$, then we conclude $\zeta_i, i \in N$ is consistent integrable.

Please refer to Appendix B for the proof.

In this subsection, we also provide essential proofs for the Nash equilibrium' existence of the proposed game algorithm

Theorem 3. For given utility function $f_i = \pi_i$ and strategy function $s_i = p_i$, a pure Nash equilibrium for the proposed pricing game algorithm exists.

Please refer to Appendix C for the proof.

IV. NUMERICAL RESULTS

In this section, numerical results are provided to testify the effects of our proposed pricing method, the iterative convergence as well as the existence of Nash equilibrium. In this dynamic access networks, we suppose the spectrum leased is controlled by the licensed users and we ignore the internet interference caused by adjacent cells. The secondary users who want to access the spectrum must participant in the spectrum trading and pay for the cost to the primary systems. Since our proposed pricing method is an iterative algorithm, we thus set the initial spectrum pricing for different kinds qualities of channels to be $p_i^{(0)} = 0.01$. Other relevant parameters are set as $\kappa = 10, \alpha = \phi = 8, \varepsilon = 0.5, \mu = 0.2, 0.1 \leq C_i \leq 3(bps), 1 \leq \theta_i \leq 10, \Delta I = 0.001mW, \tau = 0.2$. Channel qualities are initialized to be $C_1 = 0.5, C_2 = 1.2, C_3 = 2, C_4 = 2.8$. Besides, the preference region for the secondary buyer are set as $\theta_1 = 1, \theta_2 = 4, \theta_3 = 7, \theta_4 = 10$. We also set $\kappa > \varepsilon$ to put the emphasis on network capacity for the secondary users, since this is the only intention for them to involve in the spectrum deal. Furthermore, we set C_i in unit bandwidth where $C_h > C_l$ if $h > l$ holds. In addition, we give a small ΔI , because the interference power level is relatively low. As the curves of normal distribution cover the whole region by 99.8%, we fix $\sigma = (\theta_i - \theta_j)$ where $i = j + 1$. Then, we will testify how the different parameters affect the channel pricing and primary profits of several kinds of leased spectrum.

In Fig. 8, the pricing of different kinds of channel types is given. In this section, different channel types correspond to various spectrum qualities leased by primary system I, II, III and IV as shown in the section of system model. In Fig. 8, α represents the user distribution in which there are $N_1 = 25, N_2 = 15, N_3 = 10$. Furthermore, β denotes $N_1 = 11, N_2 = 23, N_3 = 16$ and γ denotes $N_1 = 14, N_2 = 29, N_3 = 7$. As shown from the figure, the channel pricing is increasing with upgrading user number. Otherwise, the channel pricing has to decrease in order to attract more potential

Fig. 8. Channel pricing with different kinds of user distributions

Fig. 9. Pricing of channel I with different quality settings

secondary users to improve system profits. Besides, we can also obtain from the figure that the pricing of high-quality channel is relatively higher than that of low-quality channel which is necessary to compensate increasing marginal costs of primary systems. The pricing of channel II is not only affected by N_1 , but also N_2 , since the secondary users with selection preference located in both regions are all the potential customers for channel II. Same principle applies to channel III.

In Fig. 9, the pricing of low-quality channel I with different marginal cost coefficient μ are given. In this case, the distribution of user number is $N_1 = 25, N_2 = 15, N_3 = 10$. We can achieve from Fig. 9 that the price of high-quality channel increases obviously with growing marginal cost coefficient μ in which the other parameters involved in the proposal are fixed. As shown in Fig. 9, the pricing algorithms converge in a relative fast speed in which the proposed algorithm can achieve a steady result after about ten iterations. For the secondary

Fig. 10. Pricing of channel II with different quality settings

Fig. 12. Pricing of channel IV with different quality settings

Fig. 11. Pricing of channel III with different quality settings

Fig. 13. Pricing of channel I with different secondary preference types

users in this spectrum trading, uploading marginal costs mean the preference degree of the primary systems to preserve the idle channels for future transmission. From another prospect, the primary systems need to increase channel pricing so as to defend high cost.

In Fig. 10, we give the pricing of channel II with different channel quality parameter C_1 . In general, the channel with high quality will reap high pricing. In this case, we investigate the effects of pricing of channel II with the quality change of adjacent channels. In fact, in adjacent selection region, two kinds of channels have to confront market competition with the basic intention of attracting more customer. Thus, as shown in Fig. 10, when the spectrum quality of channel I increases, the pricing of channel II decreases in fixed C_2 . Furthermore, the low-quality channel's variation trend tells that it is predictable for the primary system that the improvement of channel quality will increase the channel marginal cost and

attract more potential consumers. In this case, any increase of channel's quality is expected to stimulate the consumption. As shown in Fig. 11, the pricing of channel III is increasing when its channel quality becomes better. In this situation, the primary system also needs to afford more marginal cost when the spectrum quality is improved, thus it is reasonable for the primary system to increase the price so as to compensate higher potential cost. Furthermore, the pricing performances of channel IV with different quality settings are also shown in Fig. 12.

We compare three kinds of situation where the secondary user's preference comply with the linear distribution, the uniform distribution and normal distribution, respectively. As shown in Fig. 13, the optimal pricing for channel I in linear distribution exceeds that in uniform distribution generally. When secondary user's selection preference follows the linear distribution which implies more demands for high-quality

Fig. 14. Primary system's profits

channels, we can obtain the buyers would like to pay higher price for the idle spectrum. It can be clearly concluded from the figure that secondary user's channel selection is apparently impacted by its own demand preference which also leads to a diversity of dynamic spectrum pricing for the primary systems as a result.

We can conclude from Fig. 9 to Fig. 13 that the pricing curves for channel I to channel IV converge no more than 20 iterations. In fact, by choosing proper parameters, the proposed pricing algorithm can be guaranteed to converge at a fixed point.

Based on the pricing algorithm given in (16) to (19), the equations can be rewritten in the following matrix form

$$P^{k+1} = U^{-1}WP^k + U^{-1}b, \quad k = 0, 1, 2, \dots \quad (21)$$

According to Reference [30], if $\rho(U^{-1}W) < 1$ holds, the iterative algorithm p^k will be convergent. Based on the proposed pricing algorithm, we can easily achieve the Jacobi matrix and the corresponding maximal eigenvalue. Hence, properly setting the algorithm's parameter, we can ensure $\rho(U^{-1}W) < 1$. In addition, the oscillating point appears when $\rho(U^{-1}W) = 1$.

Fig. 14 shows the changes of the system profits with different channel pricing. Other parameters are settled as $\kappa = 10, \alpha = \phi = 8, \varepsilon = 0.5, C_1 = 0.5, C_2 = 1.2, C_3 = 2, C_4 = 2.8, \theta_1 = 1, \theta_2 = 4, \theta_3 = 7, \theta_4 = 10, N_1 = 25, N_2 = 15, N_3 = 10$ and $\triangle I = 0.001$. As shown in Fig. 14, the curves which represent the profits of four kinds of channels behave as convex function with maximum values. The peaks of the curves mean the maximum system profits. We can obtain from the optimal value when the primary system's spectrum pricing reaches the Nash equilibrium point, which is similar to real market that over price will discourage customer's demand and lead to the decrease of system's income in the end. Additionally, Fig. 14 shows that the system profit of low-quality channels is not less than that of high-quality channels in same channel quantity. Since the secondary user's

spectrum preference is normally distributed, the system profits are affected by many factors.

We further provide the comparative analyses between our proposal and the other two kinds of algorithms focusing on the spectrum trading with heterogeneous users. In [24], a Stackelberg-game-based algorithm (SGA) is designed to address the differentiated pricing problem. The authors model the spectrum procurement and trading process as a five-stage Stackelberg game. Due to the heterogeneity of the secondary users, price differentiation is introduced to improve the profit of the cognitive virtual network operator. The system profit of SGA method can be expressed as

$$Pr^* = \lambda^* \alpha B_s + \sum_{i=1}^I \theta_i G_i \exp\{-2 - \frac{\lambda^*}{\theta_i}\} \quad (22)$$

$$s.t. \quad D > \alpha B_s \geq A$$

where $A \triangleq \sum_{i=1}^I G_i \exp\{-2 - \frac{C_i}{\theta_i}\}$, $D \triangleq \sum_{i=1}^I G_i e^{-2}$. C_s and C_l denote the sensing cost and leasing cost per bandwidth, respectively. B_s is the sensing bandwidth, and α is the sensing realization factor. $G_i(\tilde{G}_i)$ denote the aggregate wireless characteristics of the secondary users in $S_i(\tilde{S}_i)$ where S_i is the index set of the set of secondary users with willingness-to-pay θ_i and $\tilde{S}_i$ is the admitted secondary users in S_i . Fig. 15 shows the expected profit of SGA method with changing sensing cost and bandwidth. Since in the system model of SGA method, the cognitive virtual operator provides surplus spectrum for the secondary users to choose which enables the possibility of diverse spectrum selection for the secondary customers. Therefore, in this case, the system profit will decrease when sensing cost is increasing or more spectrum band is available in the market.

Besides, Fig. 16 gives the performance of another differentiated pricing algorithm indexed in [25], which is an iterative pricing algorithm (IPA). Two kinds of channels with diverse channel qualities are formulated in IPA method, and the corresponding system profits and channel prices for the channels are as follows

$$Pr^H = \frac{N(p_1 - \alpha s_1)(\bar{\theta} - \frac{p_1 - p_2}{s_1 - s_2})}{\bar{\theta} - \underline{\theta}}, \quad (23)$$

$$Pr^L = \frac{N(p_2 - \alpha s_2)(\frac{p_1 - p_2}{s_1 - s_2} - \underline{\theta})}{\bar{\theta} - \underline{\theta}}, \quad (24)$$

$$p_i^{k+1} = p_i^k + \beta \frac{\partial Pr^j}{\partial p_i}, \quad (25)$$

where p_i denotes the channel pricing, s_i denotes the channel quality, N denotes the total number of the leased channels. $\bar{\theta}, \underline{\theta}$ are the preference thresholds. α, β are the monetary coefficients. In Fig. 16, the channel prices and system profits for two kinds of channels in IPA method are given. Due to the obvious diversity of the utility function and parameter setting between our solution and the methods proposed in [24] and [25], we do not perform the comparison tests by putting the curves into one figure. The system model and utility

Fig. 15. Expected profit of SGA method

Fig. 16. Channel pricing and system profit of IPA method

function of IPA method are more simple than ours. Since the secondary user's preference is assumed to be complied with ideal uniform distribution, the performances of system profits are quite different to those of our solution.

Both the SGA and IPA methods are introducing secondary user's preference parameter to system models. Alike to our proposal, both of them design the pricing algorithm from the perspective of primary systems by predicting the secondary customer's spectrum preference. In this section, we do not provide comparison curves combining the performances of spectrum pricing and system benefits for three kinds of proposals including our method in same figure, since we found the comparative effectiveness is not ideal due to obviously different utility functions and parameter settings adopted by these methods. In SGA method, homogenous spectrum and sensing cost are considered in the utility function which are unlike to our system model. In IPA method, the user preference is supposed to comply with uniform distribution for various kinds of spectrum which is too idealistic in real scenarios. Unlike to SGA method focusing on homogenous and surplus spectrum with sensing cost, or IPA method with simplified and uniform secondary demands, our goal is to solve the problem of differentiated spectrum pricing in more complex case of heterogeneous spectrum and stochastic user demands. By considering a primary virtual operator serving as the processing fusion, our proposal can be centrally carried out and avoids the sensing cost meanwhile. In addition, the spectrum pricing and system profits of the three methods are not in same order of magnitude which makes part of simulated curves vague. Therefore, we provide the numerical performances of our proposal, SGA and IPA method separately, to make them clearer.

V. CONCLUSION

In this paper, we have proposed a pricing-based method for spectrum allocation in dynamic spectrum access networks by taking into account the heterogeneous demands of secondary users and channel diversity of primary systems. We diversified the users' characteristics and then priced the spectrum more specifically. We have introduced the concept of spectrum pool for the primary systems to facilitate and unify the channel selling. The channels with different qualities need to be priced variously. On the other hand, the primary systems should also predict and judge the actions of secondary buyers on what kind of purchasing habits they have. Therefore, we proposed a channel pricing algorithm based on the spectrum selection preference of the secondary users. To be more practical, we assumed the channel selections of the secondary users are random, thus the users' preferences comply with normal distribution in a given area. We have further formulated the channel pricing by using Hotelling game model which is proper to describe the pricing problem caused by product diversity. Furthermore, to ensure the rationality of the proposed method, we gave the relevant proof for utility function and Nash Equilibrium.

APPENDIX

A. Proof of Theorem 1

When the secondary user's selection preference complies with normal distribution, the demand function for spectrum I

can be expressed as

$$\begin{aligned}
D_1 &= N_1 \int_{\theta_1}^{\theta_b} g(\theta) d\theta \\
&= N_1 \int_{\theta_1}^{\theta_b} \frac{1}{\sqrt{2\pi}\sigma} \exp[-\frac{1}{2}(\frac{\theta - \mu}{\sigma})^2] d\theta \\
&= N_1 \int_{\frac{\theta_b - \mu}{\sigma}}^{\frac{\theta_1 - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{t}{2})^2 dt \\
&= \frac{N_1}{\sqrt{2\pi}} [\int_{\frac{\theta_b - \mu}{\sigma}}^{-\infty} \exp(-\frac{t}{2})^2 dt - \int_{\frac{\theta_1 - \mu}{\sigma}}^{-\infty} \exp(-\frac{t}{2})^2 dt].
\end{aligned} \tag{26}$$

For the front part of the integral sum in (26), in condition of $\theta_b \geq (1 + \sigma)\mu$, we have

$$\begin{aligned}
D'_1 &= \int_{\frac{\theta_b - \mu}{\sigma}}^{-\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{t}{2})^2 dt \\
&= \frac{1}{\sqrt{2\pi}} [\int_{-\infty}^0 \exp(-\frac{t}{2})^2 dt - \int_0^{\frac{\theta_b - \mu}{\sigma}} \exp(-\frac{t}{2})^2 dt] \\
&= \frac{1}{2\sqrt{2\pi}} (1 + \int_{\mu - \frac{\theta_b}{\sigma}}^{\frac{\theta_b - \mu}{\sigma}} \exp(-\frac{t}{2})^2 dt).
\end{aligned} \tag{27}$$

We use approximate circle solution in two dimensions to solve the problem of integration, thus we can obtain the following equation

$$\begin{aligned}
f(t) &= \int_{\frac{\mu - \theta_b}{\sigma}}^{\frac{\theta_b - \mu}{\sigma}} \exp(-\frac{t}{2})^2 dt \\
\Rightarrow [f(t)]^2 &= \int_{\frac{\mu - \theta_b}{\sigma}}^{\frac{\theta_b - \mu}{\sigma}} \int_{\frac{\mu - \theta_b}{\sigma}}^{\frac{\theta_b - \mu}{\sigma}} \exp[-\frac{(x^2 + y^2)}{2}] dx dy
\end{aligned} \tag{28}$$

We further use a circular area with radius R to approach integral region, where $\pi R^2 = \frac{4(\theta_b - \mu)^2}{\sigma^2}$, then we obtain

$$\begin{aligned}
[f(t)]^2 &= \int_0^{2\pi} \int_0^R \exp[-\frac{\rho^2}{2}] \rho d\rho d\varphi = 2\pi(1 - e^{-\frac{R^2}{2}}) \\
\Rightarrow f(t) &= \sqrt{2\pi(1 - e^{-\frac{R^2}{2}})}.
\end{aligned} \tag{29}$$

Thus, there is

$$F(\theta) = \int_{-\infty}^{\frac{\theta_b - \mu}{\sigma}} \frac{1}{2\pi} \exp(-\frac{t}{2})^2 dt = \sqrt{1 - \exp[-\frac{2(\theta_b - \mu)^2}{\pi\sigma^2}]} \tag{30}$$

Substituting the result in (30) to (26), due to $\theta_1 \leq \mu$, so we have

$$D_1 = \frac{N_1}{2} \left\{ \sqrt{1 - \exp[-\frac{2(\theta_b - \mu)^2}{\pi\sigma^2}]} + \sqrt{1 - \exp[-\frac{2(\theta_1 - \mu)^2}{\pi\sigma^2}]} \right\} \tag{31}$$

Furthermore, when $\theta_b < (1 + \sigma)\mu$, we have

$$D_1 = \frac{N_1}{2} \left\{ \sqrt{1 - \exp[-\frac{2(\theta_1 - \mu)^2}{\pi\sigma^2}]} + \sqrt{1 - \exp[-\frac{2(\theta_b - \mu)^2}{\pi\sigma^2}]} \right\} \tag{32}$$

Specifically, if $\theta_b = \mu$, there is

$$D_1 = \frac{N_1}{2} \left\{ \sqrt{1 - \exp[-\frac{2(\theta_1 - \theta_b)^2}{\pi\sigma^2}]} \right\} \tag{33}$$

At the balancing point θ_b , secondary users would receive identical benefits from the two kinds of spectrum adjacent. On the other hand, at the central point μ , secondary users' preferences concentrate on the point. When $\theta_b \Rightarrow \mu$, it means the spectrum pricing of adjacent primary systems is balanced which means none of the primary systems try to attract secondary customers by excessively decreasing pricing. Due to space limitation, we mainly discuss the situation when $\theta_b \approx \mu$, thus the profit of spectrum I can be expressed as

$$\begin{aligned}
\pi_1 &= (p_1 - M_1)D_1 \\
&= \frac{N_1(p_1 - \mu C_1)}{2} \left\{ \sqrt{1 - \exp[-\frac{2(\theta_1 - \theta_b)^2}{\pi\sigma^2}]} \right\}.
\end{aligned} \tag{34}$$

Thus, the theorem can be proved.

B. Proof of Theorem 2

When $\sup_{i \in N} E|\zeta_i|^\tau < \infty$ can be met, then we have the following equation in condition of $\chi > \chi_\varepsilon$

$$P(|\zeta_i| \geq \chi) \leq E|\zeta_i|/\chi \leq \sup_{i \in N} E|\zeta_i|^\tau / \chi < \delta_\varepsilon, \tag{35}$$

where ζ_i denotes the spectrum preference of secondary user i . Therefore, $E|\zeta_i|I_{\{|\zeta_i| \geq \chi\}} \leq \varepsilon$ holds for any $t \in N$, then we have

$$\sup_{i \in t} E|\zeta_i|^\tau I_{\{|\zeta_i| \geq \chi\}} \leq \varepsilon, \quad \tau \geq 0, \tag{36}$$

wherein for $\omega \in \Omega$

$$I_A(\omega) = \begin{cases} 1, & \omega \in H \\ 0, & \omega \in H^C := \omega \setminus H \end{cases} \tag{37}$$

Besides, for any $H \in \mathfrak{R}$ and $\chi > 0$, we have

$$\begin{aligned}
&\sup_{i \in N} E|\zeta_i|^\tau I_H \\
&= \sup_{i \in N} (E|\zeta_i|^\tau I_{H \cap \{|\zeta_i| < \chi\}} + E|\zeta_i|^\tau I_{H \cap \{|\zeta_i| \geq \chi\}}) \\
&= \chi P(H) + \sup_{i \in N} E|\zeta_i|^\tau I_{H \cap \{|\zeta_i| \geq \chi\}}
\end{aligned} \tag{38}$$

If $\{\zeta_i, i \in N\}$ is consistent integrable, $H = \Omega$ can be set which makes χ_0 larger to obtain $\sup_{i \in N} E|\zeta_i|^\tau I_{H \cap \{|\zeta_i| \geq \chi_0\}} < 1$, then we further have $\sup_{i \in N} E|\zeta_i|^\tau \leq \chi_0 + 1 < \infty$. Furthermore, for any $\varepsilon > 0$, letting χ_ε large enough can guarantee $\sup_{i \in N} E|\zeta_i|^\tau I_{H \cap \{|\zeta_i| \geq \chi_\varepsilon\}} < \varepsilon/2$. Let $\delta_\varepsilon = \varepsilon/(2\chi_\varepsilon)$, for any $A \in \mathfrak{R}$, only if $P(H) < \delta_\varepsilon$, we can achieve

$$\sup_{i \in t} E|\zeta_i|^\tau I_H \leq \chi_\varepsilon P(H) + \varepsilon/2 < \varepsilon, \tag{39}$$

Thus, the sufficient conditions can be satisfied to ensure the consistent integrability of the proposed utility function.

C. Proof of Theorem 3

Based on the theorem of Debreu's equilibrium existence [28], for the functions s_i and f_i given above, a pure strategy Nash equilibrium exists once the following sufficient conditions can be satisfied: (1) In limited Euclidean space, strategy function s_i is a nonempty and compact subset. (2) For a

strategy combination S , f_i is continuous and concave. First of all, we define $S : \Sigma \rightarrow \Sigma$ to be a Cartesian direct product for s_i and Σ_i is a simplex in dimension $|s_i|$. s_i can be a compact subset in limited Euclidean space only en the condition that s_i are symmetrically bounded and continuous functions is met. In order to prove s_i are symmetrically bounded, thus $\forall \varepsilon_1$, it requires certifying that $\exists \delta = \delta(\varepsilon_1)$ can make $\forall \psi \in F$ workable which can be

$$|\psi(x_1) - \psi(x_2)| < \varepsilon_1 \quad (\text{when } \rho(x_1, x_2) < \delta), \quad (40)$$

where $\rho(x, y) \triangleq \max_{a \leq t \leq b} |x(t) - y(t)|$ and $\psi(x_i) \Rightarrow s_i$. As the subset $\varepsilon_1/3$ is finite $\bar{N}(\varepsilon_1/3) = \{\psi_1, \psi_2, \dots, \psi_n\}$, then based on the characteristic of continuity, $\exists \delta = \delta(\varepsilon/3)$. Therefore, in condition of $\rho(x_1, x_2)$, we have

$$|\psi_i(x_1) - \psi_i(x_2)| < \varepsilon_1/3 \quad (i = 1, 2, \dots, n), \quad (41)$$

Because of $\forall \psi \in F, \psi_i \in N(\varepsilon/3)$ enabling $d(\psi, \psi_i) < \varepsilon/3$, we can obtain

$$\begin{aligned} & |\psi(x) - \psi(x')| \\ & \leq |\psi(x) - \psi_i(x)| + |\psi_i(x) - \psi_i(x')| + |\psi_i(x) - \psi(x')| \\ & \leq 2d(\psi, \psi_i) + |\psi_i(x) - \psi_i(x')| < \varepsilon_1 \quad (\text{when } \rho(x, x') < \delta), \end{aligned} \quad (42)$$

where $d(u, v) = \max |u(x) - v(x)|$. Besides, it is apparent that s_i can be continuous and differentiable, and we can concluded s_i is a nonempty and compact subset in limited Euclidean space.

Furthermore, since the requirement for concave function is that $f(tx + (1-t)y) \geq tf(x) + (1-t)f(y)$, and f_i is concave if, we needs the following equation for the high-quality channels

$$\pi_i'' < 0, \quad (43)$$

Obviously, proper parameter settings can satisfy the condition. Hence, the existence of a pure Nash equilibrium for our proposed algorithm has been proved in this case which is also unique [29].

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