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2018

Tan, E. Z., & Liew, T. C. H. (2018). Unidirectional flow of lossless exciton-polariton signals. *Journal of Optics*, 20(2), 025503-. doi:10.1088/2040-8986/aaa2c3

<https://hdl.handle.net/10356/90148>

<https://doi.org/10.1088/2040-8986/aaa2c3>

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Unidirectional Flow of Lossless Exciton-Polariton Signals

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(Dated: November 29, 2017)

We consider the propagation of intensity signals in the discrete nonlinear driven-dissipative Schrödinger equation, well-known for the description of a variety of systems from coupled arrays of Kerr nonlinear cavities to exciton-polariton arrays. By periodic switching of a driving laser field, we find that the propagation can be engineered to be unidirectional, while the signals fully withstand dissipation and are resilient against disorder. We anticipate that such a mechanism would be relevant for use in photonic circuits.

I. INTRODUCTION

Optical integrated circuits are already widely used in fibre-optic communications and have proved to be advantageous in long-distance telecommunications, achieving higher transmission rates with lower loss [1]. The advantages of optical circuits persist down to the chip-level, which has motivated research to improve the feasibility of optical circuits in short-distance interconnects [2, 3] to create optical logic circuits that can potentially achieve much higher information densities while consuming much less energy. Nonlinear optics holds much potential in this regard [4, 5], and this has prompted extensive research into nonlinear optical cavities [6] and, in particular, into coupled nonlinear optical cavities [7, 8]. Indeed, coupled nonlinear optical cavities have been demonstrated to have applications in optical generation of microwaves [9, 10], signal regeneration [11] and optical switches [12, 13]. Despite decades of research, new examples of nonlinear effects in photonic lattice systems continue to be discovered, including, for example, the connection to the classical Ising model [14] and two-particle topological states [15], which appeared recently.

Among the requirements for an optical circuit architecture it was pointed out long ago that it is necessary to control feedback [16] and the isolation of input and output signals at all stages remains a major challenge when considering large scale devices. Some of the most rapidly growing fields in modern photonics have had as one of their key results the engineering of *unidirectional* optical transport. Namely, topological photonics [17] has made use of analogies with the quantum Hall effect to demonstrate one-way transport in states at the edge of lattice systems, while chiral quantum optics exploits the transverse spin-momentum locking in tightly confined photonic systems to generate a directionally dependent light-matter interaction [18]. Topological photonics has focused primarily on linear optical systems, while recent works have introduced concepts of nonlinear topological photonics [19–21], which would in principle allow for simple logical operations in these systems.

Typical optical systems are also dissipative and dispersive, so even if unidirectional transport mechanisms can be engineered, separate care is needed to preserve the in-

tensity and form of optical signals propagating through a large network. Solitons have been predicted and studied in waveguide arrays [22, 23] and lattices of dissipative Kerr nonlinear cavities [24, 25] and represent natural candidates for signal propagation, given their shape-preserving property and the possibility of soliton-based logic gates [26]. However, their interplay with disorder is largely unstudied.

In the present work, we introduce and study a simple mechanism of intensity signal propagation in driven-dissipative cavity arrays. We use a generic model based on a discrete nonlinear Schrödinger equation, suitable for the description of waveguide arrays [22, 23, 27], coupled nonlinear cavities [24], photonic crystal arrays [28], arrays of metallic nanoparticles [29], liquid crystal arrays [30], or arrays of exciton-polaritons [31, 32]. In the latter system, very low switching energies due to strong optical nonlinearity were reported recently [33] and several alternatives exist for constructing polariton-based switches [34, 35]. The achievement of cascading devices [34] in the face of dissipation and disorder remains a current research topic in this field. Here, by considering a periodic driving field applied to the coupled cavity system, we show theoretically the existence of a unidirectional flow of a photonic signal that is lossless and resilient to disorder.

II. PRINCIPLES OF OPERATION

A. Single Cavity

To begin, it is instructive to recall the well-known physics of a single driven-dissipative cavity, with wavefunction ψ described by the non-linear Schrödinger equation [36]:

$$i\hbar \frac{\partial \psi}{\partial t} = E_0 \psi + \alpha |\psi|^2 \psi - i\Gamma \psi + F e^{-iE_p t/\hbar}. \quad (1)$$

The first term on the right-hand side contains the bare normal mode energy of the cavity, E_0 . The second term is the (here repulsive or self-defocusing) Kerr term, with nonlinearity parameter α . The third term represents dissipative effects, with Γ the dissipation rate. The last term

is the driving term, where F is the amplitude of a driving field with energy E_p .

It is convenient to rescale the variables, $\psi \rightarrow \psi' e^{-iE_p t/\hbar}/\sqrt{\alpha}$, $t \rightarrow \hbar t/\Gamma$, and $F \rightarrow F/\sqrt{\alpha}$, such that the non-linear Schrödinger equation becomes:

$$\frac{\partial \psi}{\partial t} = -i\Delta\psi - i|\psi|^2\psi - \psi - iF \quad (2)$$

where we defined $\Delta = (E_0 - E_p)/\Gamma$.

Setting $\frac{\partial \psi}{\partial t} = 0$ in Eq. 2, we obtain an equation for the stationary state solutions:

$$n^3 + 2\Delta n^2 + (1 + \Delta^2)n - |F|^2 = 0 \quad (3)$$

where $n = |\psi|^2$. For $\Delta < -2\sqrt{3}$, the steady-state curve $n(|F|^2)$ takes an S-shaped curve [37], as illustrated in Fig. 1a. The multiple solutions appearing for some values of $|F|^2$ is characteristic of an optical bistability in the system [38] (the middle branch in Fig. 1a can be shown to be unstable [37]). By choosing an appropriate value of F in the bistable region, the single cavity becomes a natural encoder of binary information. The cavity can be prepared in either of the two allowed states of the system via the application of pulses [39]. These are readily treated theoretically by adding an additional term to the right-hand side of Eq 2 of the form $F_1 e^{-(t-t_0)^2/\Delta t}$, where F_1 is the pulse amplitude, t_0 the pulse arrival time, and Δt is the pulse duration. We verified that the pulse causes transitions between bistable states as shown in Fig. 1b.

B. System of Coupled Cavities

We can then consider a driven-dissipative chain of M cavities, described by the following equation:

$$\frac{\partial \psi_m}{\partial t} = -i \left(\Delta + |\psi_m|^2 - i \right) \psi_m - iF_m + iJ(\psi_{m-1} + \psi_{m+1}) \quad (4)$$

where the term $-J(\psi_{m-1} + \psi_{m+1})$ represents the coupling of a cavity with its two adjacent cavities. The coupling between two cavities allows the state of one cavity to be transmitted to the other [40], that is, the state of a cavity (either the lower or upper bistable state) influences and is influenced by the state of adjacent cavities. We will work in the regime $J \ll |\Delta|$, where locally each cavity follows a hysteresis curve similar to that illustrated in Fig. 1a, but the presence of J may allow the state of one cavity to switch the state of its neighbours. For example, in the case of two coupled cavities in which one cavity is initially in the upper bistable state, it is possible that the coupling causes the cavity initially in the lower bistable state to transit to the upper bistable state. This is shown in Fig. 1d, in which the blue curve n_1 corresponds to the first cavity initially in the upper bistable state and the red curve n_2 corresponds to the second cavity initially in the lower bistable state. The corresponding driving

FIG. 1: Scheme of Operation: a) Steady-state curve for $\Delta = -5$ b) Time evolution of n for a single cavity subjected to laser pulse at $t_0 = 0.5$ c) Instantaneous switching for $M = 4$, $T = 5$, $\delta = 0.25$ d) Time evolution of 2 coupled cavities with smooth switching

laser fields F_1 and F_2 are shown as well. As seen, the coupling between both cavities causes the second cavity to transition to the upper bistable state and this state of the second cavity is maintained even after switching off the driving laser field for the first cavity.

The coupling term J is bidirectional, that is, propagation from the m -th cavity to the $(m+1)$ -th cavity is as likely as propagation from the m -th cavity to the $(m+1)$ -th cavity. In order to engineer a unidirectional flow of the signal, we can consider switching on and off the driving fields of the cavities in sequence. We will compare two forms of the driving field. The first, illustrated in Fig. 1c, shows the driving profile F corresponding to instantaneous switching in which $F_m(t)$ is given by:

$$F_m(t) = \begin{cases} F_0, & \text{if } (m-1)T \leq t \leq (m+\delta)T \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where δ is a parameter. We will also consider a smooth switching with $F_m(t)$ given by

$$F_m(t)/F_0 =$$

$$\begin{cases} \sin^2\left(\frac{2\pi t}{T}\right), & \text{if } t \in \left[\left(m - \frac{3}{2}\right)T, \left(m - \frac{5}{4}\right)T\right) \\ 1, & \text{if } t \in \left[\left(m - \frac{5}{4}\right)T, \left(m - \frac{3}{4} + \delta\right)T\right) \\ \sin^2\left(\frac{2\pi t}{T} - 2\pi\delta\right), & \text{if } t \in \left[\left(m - \frac{3}{4} + \delta\right)T, \left(m - \frac{1}{2} + \delta\right)T\right) \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

This gives a similar dependence of the driving field to that illustrated in Fig. 1c, but with smooth rather than abrupt changes. As we will show, these time-dependent driving fields will prevent backward propagation of the switching signal.

Practically, such time varying fields can be achieved with a sequence of periodically repeating laser pulses that excite the system of cavities at different times. Pulse lasers at present are capable of picosecond scale duration with GHz scale repetition rate [41]. The time difference between consecutive pulses can be introduced by splitting a single pulse into four channels and introducing a pathlength difference before the pulse reaches the cavity.

III. SIMULATION RESULTS

A. Demonstration of Unidirectional Propagation

It is shown in Fig. 2a and 2b that unidirectional propagation of an intensity signal has been achieved with a chain of cavities. We consider two cases, where the first and last cavity in the chain are initialized in the high intensity state of the local hysteresis curve, corresponding to Fig. 2a and 2b. In the former case, propagation of the intensity signal to the last cavity is archived, while in the latter case backward propagation does not occur.

We can also see from Fig. 2a that the propagation is highly stable and the dissipation in the system is fully balanced by the driving fields such that the signal arrives at the end of the chain with no loss of fidelity. While similar effects were demonstrated previously in exciton-polariton systems, there was no suppression of backward propagation [42]. Despite the presence of fluctuations when the driving field of a particular cavity is turned on or off, we observe a fairly consistent propagation across all cavities. While we have only simulated a chain of finite length, we expect the propagation to be stable for larger numbers of cavities as well, and we verified this separately for a chain of 100 cavities.

A realistic driving field may not be instantaneous, and, for this reason we have considered also application of smooth changes in the driving field amplitude. The results are shown in Figs. 2c and 2d, and show a similar behaviour to the instantaneous switching case. The fluctuations in the system are noticeably smaller for smooth switching, as expected since there are more gradual changes in the driving laser field amplitude.

FIG. 2: Demonstration of Unidirectional Propagation in a system of 10 coupled cavities with $\Delta = -5$, $J = 0.5$ and $F_0 = 4$ with switching period of $T = 5$: a) Forward propagation with instantaneous switching; b) Failure in back propagation with instantaneous switching; c) Forward propagation with smooth switching; d) Failure in back propagation with smooth switching.

B. Dependence of T_{min} on parameters F_0 , J and Δ

For the switching to be practically achievable it should ideally be possible for a range of parameters. As a figure of merit, we define T_{min} as the fastest period for which the unidirectional switching is successful for any given choice of parameters F_0 , J and Δ . To calculate T_{min} , we gradually reduce the period of switching of the driving laser field T , as specified in Eq. 5 and 6, determining the success of unidirectional switching based on the fulfilment of the following two conditions:

1. If the first cavity is initially in the upper bistable state, all the cavities must transit to the upper bistable state within the period when the corresponding driving laser field is turned on.
2. If any particular cavity and all its preceding cavities are in the lower bistable state, the cavity must remain in the lower bistable state at all times.

For some choices of parameters, these conditions can not be met and no T_{min} can be defined. In particular $|\Delta|$ must be sufficiently large for bistability to even exist in the first place and F_0 must be chosen to be close to the

higher turning point of the bistable curve but still within the bistable region. Regarding the coupling constant J , if J is too high, the positive feedback between the cavities will lead to the amplification of fluctuations caused by the switching of the driving laser field, leading to massive chaotic fluctuations that dominate the signal, and thus prevent successful signal transmission. At the same time, if J is too weak it might be insufficient to trigger the transition.

1. Dependence of T_{min} on parameters F_0 and J

In Fig. 3a, the plot of the minimum switching period is shown on the $|F_0|^2$ - J plane for the case of instantaneous switching for $\Delta = -5$. The range of T considered in searching for T_{min} is $T \in [0, 15]$, since large T_{min} corresponds to slow transmission and therefore large T_{min} can be deemed as failure for practical purposes. As can be seen, there is a clear boundary for the set of parameters for successful unidirectional propagation as expected. We see that the minimum switching period is the shortest near the upper boundaries for $|F_0|^2$ and J . This is to be expected since nearer to the upper boundary for either parameter, there is a greater tendency to undergo a transition to the upper bistable state, since a higher F_0 means the system is closer to the upper turning point on the bistable curve, while a higher J corresponding to a stronger coupling means that an adjacent cavity that is in the upper bistable state exerts a stronger influence its neighbour. This allows for the transmission of the state with a shorter period of coupling between adjacent cavities.

FIG. 3: T_{min} in the $|F_0|^2$ - J plane: a) Instantaneous switching; b) Smooth switching.

The analogous plot for the case of smooth switching of the periodic driving laser field is shown in Fig. 3b. As can be seen, there is also a clearly defined boundary for the parameters for which successful unidirectional propagation can be achieved. It is also apparent that the shorter minimum switching periods occur near the upper boundaries for F_0 and J . Comparing the plots obtained from the instantaneous switching and smooth switching cases, we see that the region of allowed parameters is smaller for the case of smooth switching, arising from

the lower boundary for the smooth switching case being higher than in the instantaneous switching case. This is due to the more gradual change in the amplitude of the driving laser field that leads to a correspondingly more gradual change in the cavity, and as such, a stronger coupling or stronger driving laser field is necessary to trigger the transition.

2. Dependence of T_{min} on parameters Δ and J

FIG. 4: T_{min} on the Δ - J plane: a) Instantaneous switching b) Smooth switching

In Fig. 4a and 4b, the plot of T_{min} is shown on the Δ - J plane for the cases of instantaneous and smooth switching respectively. The driving laser field amplitude F_0 at which the minimum switching period is determined is dependent on the Δ value: The F_0 value is chosen such that the corresponding $|F_0|^2$ is positioned at some fixed relative point between the lower and higher turning points on the bistable curve as shown in Fig. 1a. The turning points are denoted n_1 and n_2 respectively, and can be obtained by differentiating Eq. 3 and setting $\frac{dF}{dn} = 0$. We have found that fixing $|F_0|^2$ as

$$|F_0|^2 = \frac{1}{4} (n_1 + 3n_2) \quad (7)$$

gives an adequately large parameter boundary for both cases of switching. For $\Delta = -5$, this is denoted by the blue line in Fig. 1a. Even so, it should be noted that fixing $|F_0|^2$ at other points, such as the midway between the turning points, still gives an observable range of parameters for which the propagation scheme is successful. While the range of successful parameters when $|F_0|^2$ was positioned midway between the turning points was larger for the case of instantaneous switching, it was small for smooth switching with large areas of intermittent success. This is probably due to the more gradual change in the driving laser field with smooth switching leading to correspondingly less drastic changes in the cavity; consequently, the system requires a larger driving laser field to trigger the transition. We observed a larger parameter boundary by positioning F_0 at a higher point as expected.

We note also that the range of T considered in searching for T_{min} is $T \in [0, 15]$, hence it is possible that the parameter boundary could be larger with a larger range of T considered. From Figure 4a, we see that the instantaneous switching scheme is successful over continuous range of Δ and J , and that there is also a continuous range of short T_{min} , that is $T_{min} < 5$. Furthermore, we observe that the minimum J for successful propagation increases with Δ , and this is probably since larger Δ corresponds to larger separation of the turning points leading to a stronger coupling required to push the cavity past the higher turning point and trigger the transition. It is also likely that a larger T_{min} corresponding to a longer coupling time can achieve the same result, since we can see that there is an increase in T_{min} with decrease in J from the center of the valid parameter set towards the parameter boundary for a fixed Δ , and also since simulation with larger range of considered T indeed yielded larger parameter boundaries.

Another feature of the plot is that there seems to be an upper bound on J at $J = 1$ across which the minimum switching periods sharply increase. As the strength of the coupling increases towards $J = 1$, the stronger coupling predictably leads to shorter switching times. However, there is a blue-green region to the right of this dark blue region corresponding to an abrupt increase in switching times. There is also a distinct yellow region to the bottom right.

These distinct regions can be attributed to different reasons for the failure of the unidirectional propagation. Considering only the periodic switching of the driving field, we see from Fig. 2b and 2d that there are two fluctuations when the driving field is on, the first and larger one due to the switching on, and the second and smaller one due to the switching off of the previous cavity's driving field. In the dark blue region for example, the failure at shorter times occur due to the first fluctuation in itself being sufficient to trigger a transition to the upper bistable state, and hence violating condition 2. However, in the blue-green region, the failure occurs due to the second fluctuation being sufficient in itself to trigger the transition, and since this fluctuation is induced by change in the previous cavity, it will only be large enough at higher J values when there is stronger coupling. Since the second fluctuation occurs some time after the first, when the previous cavity is switched off, a longer period is required for the fluctuation to decay, which explains the abrupt increase in switching time.

In Fig. 4b, it is immediately apparent that there is a much larger range of parameters over which the propagation scheme is successful for smooth switching, with the propagation scheme being continuously successful over the range $J \in [0.5, 1]$ at all $\Delta \geq -5$, with short switching periods $T_{min} < 4$. For $J \geq 1$, the plot shows that the propagation is intermittently successful for $\Delta > -8$ while there is a clearer parameter boundary for $\Delta \leq -8$. The intermittent success and larger T_{min} at higher values of J can be attributed to amplification of transient oscillations

arising from overly strong coupling. However, the upper bound on J is higher for smooth switching, probably because with the more gradual change in the driving laser field for smooth switching, the transient oscillations that can cause unwanted switching are suppressed. Lastly, it is apparent that there are few instances of large T_{min} , and this is probably because T_{min} corresponds to the rate of change of the driving laser field amplitude during the switching on of the field, and hence large T_{min} corresponds to a more gradual change that may be unable to trigger the transition.

C. Demonstration of Unidirectional Propagation with Disorder

We see from Section IIIB1 and IIIB2, particularly the latter, that smooth switching is the most feasible method to realise unidirectional propagation with disorder since the propagation scheme is successful over the largest range of parameters. As such, we shall be investigating primarily smooth switching in this section, although we have found that propagation with disorder can be realised with instantaneous switching as well.

FIG. 5: Demonstration of Unidirectional Propagation of with Smooth Switching in a System with Disorder: a) Forward propagation of states with smooth switching; b) Failure in back propagation with smooth switching.

Fig. 5 and 5b show the forward propagation of intensity signals and failure to backward propagate respectively in a system with smooth switching of the periodic driving laser field with disorder. The disorder is introduced in the form of randomised Δ , coupling constants J and driving field amplitudes F_0 across the 10 cavities within certain ranges. Fig. 5 and 5b have been generated with the same set of randomised J , Δ and F_0 . The range of randomised constants are $J \in [0.6, 0.9]$, $\Delta \in [-6.8, 7.2]$ and $F \in [6.2, 6.7]$ are determined based on testing the success of the propagation scheme with 10 different sets of randomised constants within the range, and we have found that the range is successful for at least 100 different sets, meaning that the propagation is likely to be resistant to disorder of this magnitude. The ranges

of J and Δ are centered on values approximately in the centre of the valid parameter range in Fig. 4b, and the center of the range of F_0 is $F_0 \approx 6.45$ derived from Eq. 10 (see later) where n_1 and n_2 are the turning points of the bistable curve with $\Delta = -7$, i.e. the center of the Δ range.

We see from Fig. 5 that for forward propagation, the peaks corresponding to the transitions of the cavities are of different heights, and this consequently leads to larger fluctuations relative to those in Figure 2c. The fluctuations are also more rapid, seen from the fact that there are noticeably more peaks for each excitation in Fig. 2c than in Fig. 5. Similarly, the low amplitude fluctuations in the cavities when the corresponding driving laser fields are switched off are noticeably larger. The same can be seen for Fig. 5b showing the failure of backward propagation, indicating that the introduction of disorder has a predictably adverse effect on the stability of the propagation. Nevertheless, there remains a successful propagation of a binary intensity coded signal that is always clearly corresponding to a high or low intensity state.

D. Dependence of the minimum switching period on size of range of randomised parameter values

We investigate here the dependence of the minimum switching period on the size of the range of randomised parameter values. Dependence is investigated for three pairs of parameters, namely J and Δ , F and Δ , as well as $d\tau$ and $d\phi$. The minimum switching periods are calculated in the same way as described in Section III C for 100 different sets of randomised parameter values at each point in the phase diagram, and the minimum switching period is obtained by taking the average of the minimum switching periods for the 100 different sets. The propagation is determined to be a failure if the propagation fails for any of the 100 sets of randomised parameters, that is, the propagation is successful and a minimum switching period is given if and only if the propagation is successful for all 100 sets of randomised parameters.

1. Dependence of the minimum switching period on size of range of J and Δ

The minimum switching period is shown in Fig. 6a on the $d\Delta$ - dJ plane. The minimum switching periods calculated at each $d\Delta$ and dJ correspond to a particular range of the randomised J and Δ values, given at each dJ and $d\Delta$ by

$$J \in \left[0.75 - \frac{dJ}{2}, 0.75 + \frac{dJ}{2} \right] \quad (8)$$

$$\Delta \in \left[-7 - \frac{d\Delta}{2}, -7 + \frac{d\Delta}{2} \right] \quad (9)$$

The F_0 value at which the minimum switching periods are evaluated is $F_0 \approx 6.45$.

FIG. 6: a) T_{min} on the $d\Delta$ - dJ plane for smooth switching with J range centred at $J = 0.75$ and Δ range centred at $\Delta = -7$. b) T_{min} on the $d\Delta$ - dF plane for smooth switching with F range centred at $F = 6.45$ and Δ range centred at $\Delta = -7$.

From Fig. 6a, we see that there is a boundary within which the propagation is successful, although the propagation is only intermittently stable beyond this boundary. This boundary is approximately given by $dJ \leq 0.35$ and $d\Delta \leq 0.7$. This corresponds to a range of J given by $J \in [0.55, 0.95]$, which lies within the range of valid parameters from Fig. 4b. Although Fig. 4b shows that the range of Δ is indefinitely large, nor is there any reason in theory to believe that an upper bound exists on the range of valid Δ , the range of Δ in the system with disorder arises from the fact that the driving laser field amplitude F_0 is constant and does not vary with the randomised Δ values, and as such, Δ values that are too high will mean that F_0 is correspondingly too low to trigger a transition to the upper bistable branch, while Δ values that are too low means that F_0 is too high and will trigger a transition without any contribution from the coupling to adjacent cavities. Even so, we see that there is a boundary within which the system of 10 coupled cavities is resilient to disorder.

Along the dJ axis, we see that beyond $dJ = 0.4$, the success of the propagation scheme becomes intermittently successful, even though T_{min} within the range $dJ < 0.4$ is small, that is, there is no gradual increase in the T_{min} . This is consistent with Fig. 4b, which showed a clear boundary on the range of valid J beyond which the system is intermittently successful. Within the boundary we observe from Figure 4b that $T_{min} < 3$, which is also true of the system with disorder.

Along the $d\Delta$ axis, it is apparent from Fig. 6a that there is a gradual increase in T_{min} . We attribute this to the fact that longer switching periods allow more time for the oscillations arising from the transitions to decay. This reduction in the size of the fluctuations is needed to compensate for the larger fluctuations caused by the coupling of cavities with larger variation in Δ . We can see, however, that there too exists a certain boundary beyond which the propagation fails without further decrease in

T_{min} . This is expected since we see from Fig. 4b that there is an upper boundary for Δ for a given J beyond which the propagation fails.

2. Dependence of the minimum switching period on size of range of F_0 and Δ

The minimum switching period is shown in Fig. 6b on the $d\Delta$ - dF_0 plane. The minimum switching periods at each dF_0 and $d\Delta$ are determined with the same method as detailed in Section III D and are evaluated at $J = 0.75$. The range of the randomised Δ values is given by Eq. 9. The limits of the range of the randomised F_0 values are given as follows:

$$0.45n_1 + 0.55n_2 \leq |F_0|^2 \leq 0.05n_1 + 0.95n_2 \quad (10)$$

From Fig. 6b, we see that there is a well-defined region within which the propagation is successful. The boundary of this region is noticeably a linear relation between $d\Delta$ and dF_0 that is given approximately by $d\Delta = -0.75dF_0 + 0.9$. We observe that for constant Δ corresponding to $d\Delta = 0$ there is successful propagation for a large range of dF_0 , and complete failure when dF_0 exceeds a certain value. This is in good agreement with results obtained in III B 1 since there is also a clearly defined upper boundary for most J values as shown in Fig. 3b. Failure at large values of dF_0 likely occurs because overly high values of F_0 together with the transient oscillations is in itself able to cause a transition to the upper bistable state without adjacent cavities being initially excited, which is a failure by condition 2 in III B. It is furthermore apparent that the minimum switching period increases gradually as the boundary is approached, and this is also reflected in Fig. 3b.

As $d\Delta$ increases, the maximum value of dF_0 for which unidirectional propagation can be achieved decreases linearly. The decrease in range of dF_0 is due to the change in Δ leading to a change in the turning points n_1 and n_2 while the center of the F_0 range remains unchanged leading to a decrease in the allowed range of randomised F_0 values.

From Fig. 6b, it is also apparent that T_{min} increases as dF_0 or $d\Delta$ increases, up to the boundary. This can be explained similarly to the case of section III D 1, where larger disorder in parameter ranges caused higher amplitude transient oscillations which require more time to decay as explained in III D 1.

3. Dependence of the minimum switching period on size of range of phase ϕ and the switching time variation τ

We have so far assumed that the phase of the driving laser field are perfectly in phase. Another assumption is that the driving laser field is perfectly synchronised, that

is the switching on and off of the driving laser fields occurs with perfect precision. These two assumptions will limit the practical applicability of the mechanism since both assumptions are in practice difficult to implement. As such, we show here that the success of the unidirectional propagation is not dependent on both assumptions being true.

The minimum switching period is shown in Fig. 7 on the $d\phi$ - $d\tau$ plane. The minimum switching periods at each dF_0 and $d\Delta$ are determined with the same method as detailed in Section III D and are evaluated at $J = 0.75$ and $\Delta = -7$. Here, ϕ is the phase of the driving laser field, and is implemented by replacing the F_m term in Eq 4 by $F_m e^{i\phi}$. τ refers to the displacement in switching times of the driving laser field and is implemented by replacing $F_m(t)$ by $F_m(t + \tau T)$. The minimum switching periods calculated at each $d\phi$ and $d\tau$ correspond to a particular range randomised ϕ and τ values, where $d\phi$ and $d\tau$ gives the size of the ϕ and τ range respectively, both centered at zero.

FIG. 7: T_{min} on the $d\phi$ - $d\tau$ plane for smooth switching with $J = 0.75$ and $\Delta = -7$ and ϕ and τ range centered at zero

From Fig. 7, we see a well defined region within which the propagation is successful. The boundary is given approximately by $d\tau \leq 0.25$ and $d\phi \leq 3\pi/8$. As such, we have shown that unidirectional propagation can be achieved without the driving laser field being perfectly in phase, or being perfectly synchronised in time. Furthermore, within this region, there is an increase in T_{min} as $d\phi$ increases. It is therefore also possible to achieve rapid switching times by limiting the phase variation of the driving laser field.

IV. CONCLUSION

In this paper, we have demonstrated a method of unidirectional signal propagation in coupled driven-dissipative nonlinear arrays and demonstrated that it is fully resilient to dissipation and a finite level of disorder. The

binary signal is here encoded in intensity, making use of the local bistability arising from a Kerr type nonlinearity. The cavities are coupled by a usual linear coupling, which allows the state of one cavity to influence and switch the state of its neighbours. To engineer unidirectional signal propagation, we consider illumination by a time dependent driving field. This supports the forward propagation of the pulse in an analogous fashion to a wave pushing forward a surfer, while at the same time providing the correct gain to compensate dissipation. We characterized the range of parameters in which the unidirectional signal propagation operates and because such a range exists we recover robust propagation within a disordered environment.

Our calculations are based on a discrete nonlinear Schrödinger equation, which offers a well-known description of a variety of physical systems, including photonic crystal cavity and exciton-polariton arrays. We believe the engineering of unidirectional signal propagation in

these systems is a key requirement for photonic information processing schemes. An interesting extension can be to investigate the polarization degree of freedom. In principle different polarizations could be made to propagate either in the same or opposite directions, allowing spin currents. However, the presence of spin-orbit coupling and anisotropic nonlinearity can be expected to affect different specific physical realizations.

While we considered a near resonant coherent injection, it can be noted that nonlinear cavities based on exciton-polaritons have also demonstrated forms of bistability with non-resonant pumps [43] and electrical injection [44]. In principle, it might also be possible to obtain unidirectional switching in these systems, using a similar scheme. By negating the need for a coherent laser source, these could allow the creation of hybrid electro-optic and more compact devices.

This research was supported by the CNYang Research Program and Ministry of Education (grant 2015-T2-1-055).

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