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Spectral and Energy Efficiency Analysis for SLNR Precoding in Massive MIMO Systems with Imperfect CSI

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Abstract—We derive tractable bound expressions on achievable spectral efficiency (SE) for a multiple-input multiple-output (MIMO) system with a signal-to-leakage-plus-noise ratio precoding scheme (SLNR-PS) under the condition of imperfect channel state information. These bounds are tight and approach exact values when the number of base station (BS) antennas is large. A problem of energy efficiency (EE) maximization is investigated by using a practical power consumption model. The effects of the system parameters and quality of channel estimation including the number of BS antennas, transmit power and training length on the performance metrics are explicitly analyzed. Following that, an alternating optimization algorithm is employed to obtain the optimum EE. It has been shown that the proposed SLNR-PS performs better than matched-filtering and zero-forcing schemes and a deployment of massive MIMO with the optimal transmit power and training length can achieve high EE.

Index Terms—Massive MIMO system, SLNR precoding scheme, spectral efficiency, energy efficiency, imperfect CSI.

I. INTRODUCTION

THE increasing popularity of smart terminals and their multimedia applications such as social networks leads to tremendous growth in demands for capacity and reliability in cellular networks. To meet these demands, multiple-input multiple-output (MIMO) wireless systems have been developed, and they are now part of current standards such as IEEE 802.11 and the fourth generation (4G) long term evolution (LTE) [1]. MIMO systems provide a great improvement in the performance by improving the multiplexing and diversity gains without an additional increase in bandwidth or transmit power. To scale up these gains, massive MIMO systems, which use hundreds of antennas to simultaneously serve tens of users, have been proposed [2]–[4]. The performance of massive MIMO systems with common precoders/detectors including maximal-ratio combining (MRC), maximal-ratio transmission (MRT), zero-forcing (ZF), minimum mean-squared error (MMSE), has been widely studied [4]–[7]. When a relay network employs a massive MIMO technology with MRC and MRT schemes, the scaling law and performance analysis have been considered in [8]. The effect of the numbers of relay antennas and users, the channel state information (CSI) quality, and the transmit powers of the sources and relay on the system sum-rate performance has been analyzed. Moreover, a sequential beamforming strategy, which utilizes a full-duplex

base station (BS) to implement data transmission concurrently with CSI acquisition via in-band closed or open loop training, has been recently investigated [9]. To maximize sum-rate, a joint spatial division multiplexing BF scheme has been proposed by applying an MMSE approach when designing a pre-beamformer and a multi-user precoder sequentially [10].

In downlink (DL) scenarios, one of the main objectives in designing optimal linear multiuser MIMO (MU-MIMO) precoding schemes is to optimize signal-to-interference-plus-noise ratio (SINR). However, this problem is challenging due to its coupled nature [11]. To avoid solving the coupled problem, a zero-forcing precoding scheme (ZF-PS), which can eliminate co-channel interference (CCI) at each user, is usually considered in the literature. Unfortunately, the ZF-PS is sensitive to unmodeled interferences and other sources of distortion. On the other hand, in [12], based on the concept of signal leakage, a signal-to-leakage-plus-noise ratio (SLNR) scheme has been introduced as an optimization metric for the linear precoder design. This metric can transform a coupled optimization problem into a completely decoupled one which is easy to be solved. Different from the ZF solution, the SLNR precoding scheme (SLNR-PS) does not require any restriction on the number of BS antennas. It also takes into account the influence of noise when designing the beamforming vectors for all users. For the conventional MIMO system, a closed-form expression of the leakage-based solution for an MU-MIMO system with single-antenna users has been given in terms of the eigenvector associated to the maximum eigenvalue [12]. It has been extended to study MIMO systems which employ orthogonal space-time coding, such as Alamouti coding, as well as the system with multiple-antenna users [13]. In contrast to [12] and [13], for a generic power allocation, an expression of the leakage-based solution has been shown as a function of user-allocated power and an arbitrary phase shift [14]. For the massive MIMO system, a lower bound on the average SLNR has been derived by employing the Mullen's inequality over a Ricean fading channel [15]. Moreover, a two-stage beamformer design including an ZF-based inner beamforming and a SLNR-based outer beamforming has been presented in [16]. Based on the trace quotient formulation, an iterative algorithm has been introduced to obtain the optimal solution. In addition, the performance bounds for a massive MIMO system with SLNR-PS have been investigated in [17]. Most of the related works only focus on analyzing performance of massive MIMO systems with conventional linear precoders [3], [4] or SLNR-PS when CSI is perfect [15]–[17]. Motivated

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by the above discussions, we study the performance of a massive MIMO system with SLNR-PS in the presence of channel estimation errors in this paper.

With a large number of antennas N , the total power consumption at the BS increases significantly. Therefore, the energy efficiency (EE) problem has attracted a great deal of interest. In [7] and [18], the EE performance of the UL and DL massive MIMO systems employing MRC detector, ZF and matched-filter (MF) precoding schemes has been studied based on simulation results. The authors adopted a power consumption model that only considers the emitted power consumption. For massive MIMO systems, besides the emitted power consumption, the radio frequency circuit power consumption, as well as the power consumption for performing digital signal processing should also be considered [19], [20].

To improve the EE performance, various resource allocation algorithms have been presented in [21]–[24]. This is motivated by the need to optimize energy consumption in a cellular network, which helps to achieve lower operation costs and contributes to reduce CO₂ emission. In particular, an optimal number of antennas N is selected by using sequential search or binary search algorithms in [21]. An iterative algorithm for joint antenna selection and power adaptation based on the exhaustive search has been presented in [22]. In [23], the joint optimization problem of power allocation, N and pilot assignment for EE maximization in the multiuser and multicell massive MIMO network has been investigated. However, the authors did not analytically characterize the effect of changing these parameters on the optimal EE. On the other hand, reliable guidelines for EE maximization of the most common precoders and receivers (such as MRC/MRT, ZF, and MMSE) with respect to N , active users K , and transmit power have been presented in [25] and [26]. Note that these prior works have not considered the necessary overhead signalling for the channel estimation. The contributions of this paper are summarized as follows:

- *Spectral efficiency*: An analytical framework is presented to study the fundamental performance of a massive MIMO system with SLNR-PS. Closed-form expressions of the lower and upper bounds on the achievable rate are derived. An asymptotic limit of the achievable rate is also defined at a high signal-to-noise ratio (SNR) region.
- *Energy efficiency*: By applying a realistic power consumption model, a reliable EE maximization problem of SLNR-PS is formulated in a time-division duplexing (TDD) protocol. This enables us to explicitly investigate the effects of N , the transmit power and length of training sequence on the total EE. The optimal values of each of the three system parameters to maximize the EE are analyzed. This provides an approach to achieve the optimum EE of SLNR-PS by using a standard alternating optimization algorithm.
- *Key insights*: It is shown that (i) the SLNR-PS performs better than MRC and ZF schemes; (ii) an optimum EE of the SLNR-PS is achieved by a massive MIMO setup; (iii) if the quality of channel estimation is low, the SLNR-PS needs to use more antennas to achieve the optimum EE, as compared to that with better channel qualities.

II. SYSTEM MODEL

We consider a downlink MU-MIMO system, where the BS is equipped with N transmit antennas. The users are assumed to be randomly distributed around the BS with a sufficiently high density. Hence, the BS can select K single-antenna users to serve simultaneously from a large set of users within the coverage area and $N > K$ [25]. The channel vector of user k is given by $\mathbf{g}_k = \mathbf{h}_k \sqrt{\beta_k}$, where $\mathbf{h}_k$ is the fast fading vector of user k with $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ and β_k is the large-scale fading coefficient accounting for path loss and shadowing effect. The value of β_k is assumed to be constant over the coherence time interval and it is independent over all N antennas [7]. This assumption is reasonable because the distance between the antennas is much smaller than the distances between the users and the BS and β_k changes very slowly with time. Therefore, we get $\mathbf{G} = \mathbf{H}\mathbf{A}^{1/2}$, where $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_K]$ is a matrix of the fast fading coefficients between the K users and the BS and $\mathbf{A}$ is a $K \times K$ diagonal matrix with $[\mathbf{A}]_{kk} = \beta_k$. We assume that the channel experiences block fading, and the channel reciprocity in a TDD protocol is the same for both forward and reverse links [27]. Based on the TDD protocol, the communication scheme includes two phases. We next describe these two phases briefly.

A. Uplink Training

At the beginning of every coherence interval T , all K users simultaneously transmit orthogonal pilot sequences of length T_t symbols. The pilot sequences adopted by the K users can be shown by a $T_t \times K$ matrix $\sqrt{T_t P_t} \mathbf{\Theta}$, where $T_t \geq K$, P_t is the power level of the training sequence, and $\mathbf{\Theta}^H \mathbf{\Theta} = \mathbf{I}$. The $N \times T_t$ received pilot matrix at the BS is obtained as

$$\mathbf{Y}_{tr} = \sqrt{T_t P_t} \mathbf{G} \mathbf{\Theta}^T + \mathbf{N}, \quad (1)$$

where the noise term $\mathbf{N}$ is an $N \times T_t$ matrix with i.i.d. $\mathcal{CN}(0, \sigma_n^2)$ entries. Based on MMSE estimation, the estimated channel of $\mathbf{G}$ given $\mathbf{Y}_{tr}$ is defined as [7], [27]

$$\hat{\mathbf{G}} = \frac{1}{\sqrt{T_t P_t}} \mathbf{Y}_{tr} \mathbf{\Theta}^H \bar{\mathbf{A}} = \left(\mathbf{G} + \frac{1}{\sqrt{T_t P_t}} \mathbf{N} \mathbf{\Theta}^H \right) \bar{\mathbf{A}}, \quad (2)$$

where $\bar{\mathbf{A}} = \left(\frac{\sigma_n^2}{T_t P_t} \mathbf{A}^{-1} + \mathbf{I}_K \right)^{-1}$. We define $\rho_t = \frac{P_t}{\sigma_n^2}$ as the transmit SNR of the training sequence in the uplink channel. Since $\mathbf{A}$ is a diagonal matrix given by $\mathbf{A} = \text{diag}\{\beta_1, \dots, \beta_K\}$, we have $\bar{\mathbf{A}} = \text{diag}\left\{ \frac{T_t \rho_t \beta_k}{1 + T_t \rho_t \beta_k} \right\}, k \in [1 \dots K]$. We denote $\hat{\mathbf{g}}_k$ as the k^{th} column of $\hat{\mathbf{G}}$. From (2), $\hat{\mathbf{g}}_k$ can be shown to be distributed as $\hat{\mathbf{g}}_k \sim (0, \sigma_{\hat{\mathbf{g}}}^2 \mathbf{I}_N)$, where $\sigma_{\hat{\mathbf{g}}}^2 = \frac{T_t \rho_t \beta_k}{1 + T_t \rho_t \beta_k}$. Due to the orthogonality property of the MMSE estimate, we can decompose the channel vector $\mathbf{g}_k$ as [27]

$$\mathbf{g}_k = \hat{\mathbf{g}}_k + \mathbf{e}_k, \quad (3)$$

where $\mathbf{e}_k$ is the estimation error vector with $\mathbf{e}_k \sim (0, \sigma_e^2 \mathbf{I}_N)$ and $\sigma_e^2 = \frac{\beta_k}{1 + T_t \rho_t \beta_k}$. Note that $\hat{\mathbf{g}}_k$ and $\mathbf{e}_k$ are statistically independent of each other.

B. Downlink Data Transmission

The received signal at user k is given by [14]

$$y_k = \sqrt{p_k} \mathbf{g}_k^H \mathbf{w}_k s_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} \mathbf{g}_k^H \mathbf{w}_i s_i + n_k, \quad (4)$$

where s_k is the data symbol, the beamforming vector $\mathbf{w}_k$ is obtained based on the estimated channel $\hat{\mathbf{g}}_k$ and n_k is the noise term with $n_k \sim \mathcal{CN}(0, \sigma_n^2)$. We denote the transmit power for user k and the total transmit power level of the BS as p_k and P , respectively. The leakage is a measure of how much signal power of a desired user leaks into the other users. Hence, the SLNR at user k is given by [13]

$$\begin{aligned} \varphi_k &= \frac{\mathbb{E}\{\mathbf{w}_k^H \mathbf{g}_k \mathbf{g}_k^H \mathbf{w}_k | \hat{\mathbf{g}}_k\}}{\mathbb{E}\{\mathbf{w}_k^H \bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H \mathbf{w}_k | \hat{\mathbf{G}}_k\} + \frac{\sigma_n^2}{p_k}} \\ &= \frac{\mathbf{w}_k^H (\hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H + \sigma_e^2 \mathbf{I}) \mathbf{w}_k}{\mathbf{w}_k^H \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \left(\frac{\sigma_n^2}{p_k} + (K-1)\sigma_e^2 \right) \mathbf{I} \right) \mathbf{w}_k}, \end{aligned} \quad (5)$$

where the expectation is conditioned on the knowledge of $\hat{\mathbf{g}}_k$, $\bar{\mathbf{G}}_k = [\mathbf{g}_1, \dots, \mathbf{g}_{k-1}, \mathbf{g}_{k+1}, \dots, \mathbf{g}_K]$ and $\hat{\mathbf{G}}_k = [\hat{\mathbf{g}}_1, \dots, \hat{\mathbf{g}}_{k-1}, \hat{\mathbf{g}}_{k+1}, \dots, \hat{\mathbf{g}}_K]$. For the SLNR-PS, the optimal beamforming vector $\mathbf{w}_k$ for maximizing φ_k is given by [13]

$$\mathbf{w}_k \propto \text{max.eigenvector}(\mathbf{B}_k), \quad (6)$$

$$\mathbf{B}_k = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \left(\hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H + \sigma_e^2 \mathbf{I} \right), \quad (7)$$

where $\rho_k = \frac{\sigma_n^2}{p_k} + (K-1)\sigma_e^2$ and $\|\mathbf{w}_k\|^2 = 1$. Let us define ε_k and ε_j as the normalization factors of the precoding vectors for the k^{th} user and the j^{th} user such that $\mathbf{w}_k^H \mathbf{w}_k = 1$ and $\mathbf{w}_j^H \mathbf{w}_j = 1$, respectively. Similar to [14], it is assumed that $\mathbb{E}\{\varepsilon_k^2\} = \mathbb{E}\{\varepsilon_j^2\}$, the power of the interference plus noise can be estimated from the power of the leakage plus noise, i.e.,

$$\sum_{i \neq k}^K p_i \|\mathbf{g}_k^H \mathbf{w}_i\|^2 + \sigma_n^2 \approx \sum_{i \neq k}^K p_i \|\mathbf{g}_i^H \mathbf{w}_k\|^2 + \sigma_n^2. \quad (8)$$

When the leakage power is small as compared with the noise power, this estimation is generally tight. That holds true as $N > K$ because the leakage power converges to zero at low and high SNR regions [14, Eq. (19)]. Moreover, it has been shown in [13] and [14] that the resultant maximum SLNR value φ_k is the largest eigenvalue $\lambda_{\max}$ of $\mathbf{B}_k$. Therefore, under these assumptions, the SINR of the k^{th} user, denoted as γ_k , can be approximated by the corresponding SLNR, i.e., $\gamma_k = \varphi_k = \lambda_{\max}(\mathbf{B}_k)$. This result is used for further analysis.

III. ACHIEVABLE ERGODIC RATE

In the literature, to study the fundamental performance of massive MIMO systems, the lower bound (LB) and upper bound (UB) on the achievable rate of the conventional schemes have been analyzed [3], [4], [6], [7]. As a key contribution of this paper, we derive tractable bounds on the achievable ergodic rate of the SLNR-PS under imperfect CSI when the number of BS antennas N is large.

Theorem 1. For massive MIMO systems with SLNR-PS and imperfect CSI, an achievable rate of the k^{th} user is upper bounded by

$$\hat{R}_k = \log_2 \left(1 + v + \frac{\sigma_g^2}{\rho_k} (N - K + 1) \right), \quad (9)$$

where $v = \frac{K-1}{N-K+1}$.

Proof: Please refer to Appendix A. ■

Proposition 1. When both the number of users K and the number of BS antennas N grow large while their ratio remains bounded, i.e., $N, K \rightarrow \infty$ and $\frac{N}{K} = \varpi > 1$, the UB on the achievable rate of the k^{th} user given in (9) is rewritten as

$$\hat{R}_k = \log_2 \left(1 + \frac{1}{\varpi - 1} + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1) \right). \quad (10)$$

Proof: Let us define $\rho_d = \frac{P}{\sigma_n^2}$ as the transmit SNR in the downlink channel. From (9), we can obtain (10). ■

For massive MIMO with ZF-PS, the achievable rate of the k^{th} user is given by [4]

$$R_k^{zf} = \log_2 \left(1 + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1) \right), \quad (11)$$

We show that the achievable rate of SLNR-PS is lower bounded by that of ZF-PS. In particular, we define $\Delta_k = \hat{R}_k - R_k^{zf}$ as a different achievable rate of these two precoding schemes. We can use it to study the tightness of the derived bound. Based on (10) and (11), it is expressed as

$$\Delta_k = \log_2 \frac{1 + \frac{1}{\varpi - 1} + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1)}{1 + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1)}. \quad (12)$$

As $\varpi > 1$, Δ_k is a decreasing function with respect ϖ due to

$$\begin{aligned} \frac{\partial \Delta_k}{\partial \varpi} &= \frac{-1 - \frac{2\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1)}{\ln 2 \left(1 + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1) \right)} \\ &\times \frac{1}{1 + \frac{\rho_d \sigma_e^2}{1 + \rho_d \sigma_g^2} (\varpi - 1)^3 + (\varpi - 1)^2 + (\varpi - 1)} < 0. \end{aligned} \quad (13)$$

Hence, for a given K , the difference Δ_k decreases gradually as N increases and it approaches zero due to $\lim_{\varpi \rightarrow \infty} \Delta_k = 0$. On the other words, the result in (10) becomes the exact one which is defined by employing ZF-PS as $\varpi \gg 1$. That verifies the tightness of the derived UB for a massive MU-MIMO system with the SLNR-PS.

Proposition 2. In the very high SNR region, an asymptotic analysis of achievable rate for the k^{th} user is given by

$$\lim_{\rho_d \rightarrow \infty} \hat{R}_{\infty, k} = \log_2 \left(v + \frac{T_t \rho_t \beta_k}{v} \right). \quad (14)$$

Proof: By using $\log_2(1+x) \approx \log_2(x)$, from (9), (14) can be obtained, where ρ_t and ρ_d are given in Section II-A and Proposition 1. ■

Theorem 2. A lower bound on the achievable rate of the k^{th} user can be approximated as

$$\tilde{R}_k = \log_2(1 + (\psi_k - 1)\alpha_k), \quad (15)$$

where $\psi_k = \frac{(N-K+1+(K-1)\mu)^2}{N-K+1+(K-1)\zeta}$ and $\alpha_k = \frac{N-K+1+(K-1)\zeta d_k}{N-K+1+(K-1)\mu\rho_k}$ with $d_k = \frac{T_t \rho_t \beta_k^2}{1+T_t \rho_t \beta_k}$. The values of the parameters μ and ζ are defined by solving the following equations

$$\begin{cases} \mu \\ \zeta \left(1 + \sum_{i=1, i \neq k}^K \frac{d_i}{\rho_i \phi_i^2}\right) \end{cases} = \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{1}{\phi_i}, \quad (16)$$

where $\phi_i = N \frac{d_i}{\rho_i} (1 - \frac{K-1}{N} + \frac{K-1}{N} \mu) + 1$, $m_i = \frac{d_i}{\rho_i} \mu (K-1) + 1$, $\rho_i = \frac{\sigma_n^2}{p_i} + (K-1)\sigma_e^2$, and $d_i = \frac{T_t \rho_t \beta_i^2}{1+T_t \rho_t \beta_i}$, respectively.

Proof: Please refer to Appendix B. ■

It is more convenient to obtain (9) than (15) due to its low complexity. Moreover, numerical results in the Section V show that the rate bound in (9) is tight even when N is a finite number. Hence, the closed-form UB expression in (9) is used for subsequent analysis. Similar to [3]–[8], the above bounds are obtained for the k^{th} user. For a general case, an average achievable rate per user can be defined with respect to β_k . We leave the study of this issue as a potential future work.

An advantage of massive MIMO systems is to enable us to reduce the transmit power [7]. To enhance battery life at the user terminals, the potential for power savings was usually studied in the uplink. The solution is obtained by analyzing the power-scaling law that the transmit power level of the users should satisfy while achieving a desirable quality of service (QoS) [7]. However, the BS transmit power is greater than that of the terminals and the solution of the power-scaling law would not help to optimize the EE performance in the downlink. On the other hand, a solution for optimizing the EE can be achieved by determining an appropriate transmit power level to use. It will be investigated in the following section. Therefore, similar to [4], [5], [15], [25], [26], the power-scaling law issue is not considered in this paper.

IV. ENERGY EFFICIENCY

In order to study the EE performance of massive MIMO systems, the power consumption of the BS needs to be captured as well. In particular, the BS consists of multiple transmitters, each of which serves one transmit antenna element [19], [20], [28]. Each transmitter comprises a power amplifier, a radio frequency (RF) module, a baseband unit, a direct current-direct current (DC-DC) power supply, an active cooling system, and an alternating current-direct current (AC-DC) unit (main supply) for connection to an electrical power grid [20]. The baseband unit performs digital signal processing including modulation/demodulation, signal detection (synchronization, channel estimation, equalization), as well as channel decoding/coding. For the RF module, the circuit blocks along the signal path consist of digital-to-analog converter (DAC), filter, mixer, and frequency synthesizer [28]. All the antenna paths share a frequency synthesizer, i.e., a local oscillator. We assume that the BS equipped with a large number of antennas contributes main power consumption in the downlink. Thus, the total power consumption is modelled as [19], [20]

$$P_{tt}(P, N) = \frac{1}{\omega} \left(\frac{P}{\sigma_{am}(1 - \sigma_{feed})} + P_c + P_{bb} \right), \quad (17)$$

where σ_{am} is the efficiency of the power amplifier and $\omega = (1 - \sigma_{DC})(1 - \sigma_{MS})(1 - \sigma_{cool})$. The parameters $\sigma_{feed}, \sigma_{DC}, \sigma_{MS}, \sigma_{cool}$ are the loss factors of antenna feeder, DC-DC power supply, main power supply and active cooling system, respectively. P_{bb} is the power consumption of the baseband unit. P_c is the circuit power consumption, i.e., $P_c = N(p_{dac} + p_{mix} + p_{filt}) + p_{syn}$, where $p_{dac}, p_{mix}, p_{filt}, p_{syn}$ denote the power consumptions from the DAC, the mixer, the two filters, and the frequency synthesizer, respectively.

Without loss of generality, we assume that the system has unit bandwidth to simplify notation [7]. We formulate a reliable problem of EE optimization based on (9) and (17) as

$$\begin{aligned} \max_{P, N, T_t} \quad & \eta_{EE} = \frac{(1 - \frac{T_t}{T})K\hat{R}_k}{P_{tt}(P, N)}, \\ \text{subject to} \quad & P_{\min} \leq P \leq P_{\max}, K \leq N \leq N_{\max}, K \leq T_t \leq T, \end{aligned} \quad (18)$$

where the pre-log factor $(1 - \frac{T_t}{T})$ takes into account the necessary pilot overhead in the TDD protocol, $P_{\min}$ and $P_{\max}$ are the minimum and maximum transmit power levels at the BS, and $N_{\max}$ is the maximum number of BS antennas, respectively. Next, we employ an alternative method to solve problem (18). To do so, we obtain the optimal value of each parameter (P, N or T_t) for maximizing the EE independently.

A. Optimal Transmit Power

We begin by obtaining the optimal value of P for maximizing the EE when N and T_t are given. We define $p_k = P/K$, substitute $\rho_k = K\sigma_n^2/P + (K-1)\sigma_e^2$ into (18), and denote $c_1 = \sigma_g^2(N-K+1)$, $c_2 = (K-1)\sigma_e^2$, $\xi = \frac{1}{\omega\sigma_{am}(1-\sigma_{feed})}$ and $u = (P_c + P_{bb})/\omega$. For given N and T_t , (18) is rewritten as

$$\max_P \quad \eta_{EE} = \frac{(1 - \frac{T_t}{T})K \log_2(1 + v + \frac{c_1 P}{c_2 P + K\sigma_n^2})}{\xi P + u}, \quad (19)$$

subject to $P_{\min} \leq P \leq P_{\max}$. Due to [29, Proposition 4], the objective function in (19) is quasi-concave with P . There exists a global maximizer due to the property of quasi-concavity. By setting $\frac{\partial \eta_{EE}}{\partial P} = 0$, an optimal P is given by

$$P_{opt} = \min \{P^*, P_{\max}\}, \quad (20)$$

$$P^* = \left\{ p | LW(f_1(p)) = \frac{f_1(p)}{f_2(p)}, P_{\min} \leq p \leq P_{\max} \right\}, \quad (21)$$

where $f_1(p) = \frac{K\sigma_n^2 c_1(p+u/\xi)}{(c_2 p + K\sigma_n^2)^2}$, $f_2(p) = 1 + v + \frac{c_1 p}{c_2 p + K\sigma_n^2}$ and $LW(i)$ is the Lambert-W function defined as $i = LW(i)e^{LW(i)}$. Although (20) is not a closed-form expression, this expression can be efficiently computed by numerical methods, instead of using Monte-Carlo simulation methods.

B. Optimal Number of BS Antennas

The total power consumption of a massive MIMO system increases with N , which can significantly affect the EE. Therefore, it is important to determine the optimal number of BS antennas that maximizes the EE. Let us define $c_3 = \xi P + (P_{bb} + p_{syn})/\omega$, $c_4 = (p_{dac} + p_{mix} + p_{filt})/\omega$, $c_5 =$

$\frac{\sigma_d^2 P}{K\sigma_n^2 + (K-1)\sigma_e^2 P}$ and $t = N - K + 1$. For given P and T_t , problem (18) can be rewritten as

$$\underset{1 \leq t \leq t_{\max}}{\text{maximize}} \eta_{EE} = \frac{(1 - \frac{T_t}{T})K \log_2(1 + c_5 t + \frac{K-1}{t})}{c_3 + c_4(t + K - 1)}, \quad (22)$$

where $t_{\max} = N_{\max} - K + 1$. The objective function in (22) is quasi-concave with respect to N [29]. By setting $\frac{\partial \eta_{EE}}{\partial t} = 0$, we have

$$f_3(t) \ln(f_3(t)) = f_4(t), \quad (23)$$

where $f_3(t) = (1 + c_5 t + \frac{K-1}{t})$ and $f_4(t) = (c_3 - \frac{K-1}{t^2})(\frac{c_3}{c_4} + t + K - 1)$, respectively. The solution of (23) is obtained as

$$t^* = \left\{ t | LW(f_4(t)) = \frac{f_4(t)}{f_3(t)}, 1 \leq t \leq t_{\max} \right\}. \quad (24)$$

Therefore, the optimal number of BS antennas for maximizing the EE is defined as

$$N_{opt} = \min \{t^* + K - 1, N_{\max}\}. \quad (25)$$

C. Optimal Channel Training

The training length T_t and total power consumption P_{tt} at the BS are independent of each other. Therefore, for given N and P , the EE optimization problem (18) is equivalent to the SE optimization problem which is formulated as

$$\underset{K \leq T_t \leq T}{\text{max}} \tau_{SE} = \left(1 - \frac{T_t}{T}\right) K \times \log_2 \left(1 + v + \frac{c_6 T_t}{(1 + T_t \rho_t \beta_k) K \sigma_n^2 + c_7}\right), \quad (26)$$

where $c_6 = \rho_t \beta_k^2 P(N - K + 1)$ and $c_7 = (K - 1)P\beta_k$, respectively. It can be shown that the cost function τ_{SE} is strictly concave with respect to T_t in the interval $[K, T]$, where K is the minimum number of training symbols for channel estimation, due to the orthogonality constraint of the pilot sequence. By setting $\frac{\partial \tau_{SE}}{\partial T_t} = 0$, an optimal training length T_t^{opt} is defined as

$$T_t^{opt} = \max \{T_t^*, K\}, \quad (27)$$

$$T_t^* = \left\{ j | LW(f_5(j)) = \frac{f_5(j)}{f_6(j)}, K \leq j \leq T \right\}, \quad (28)$$

where $f_5(j) = \frac{(T-j)(c_6 \sigma_n^2 K + c_6 c_7)}{((1 + \rho_t \beta_k j) K \sigma_n^2 + c_7)^2}$ and $f_6(j) = 1 + v + \frac{c_6 j}{(1 + \rho_t \beta_k j) K \sigma_n^2 + c_7}$, respectively. Moreover, at a high SNR region, we can derive a closed-form expression of the training length T_t^{opt} , which is presented in the following theorem.

Theorem 3. Let $\rho_d \rightarrow \infty$ and assume ρ_t is finite, i.e. $\rho_t \ll \rho_d$, for given K, N and T , the optimum training length T_t^{opt} for maximizing the SE is given by

$$T_t^{opt} = \max \{T_t^*, K\}, \quad (29)$$

$$T_t^* = \frac{v}{\beta_k \rho_t} \left(\frac{c_8}{LW(c_8 e)} - v \right), \quad (30)$$

$$c_8 = v + \frac{T \rho_t \beta_k}{v}, \quad (31)$$

where e is Euler's number.

TABLE I: EE maximization with the AO algorithm

1. Initial: generate feasible sets (N, P, T_t) randomly and an accuracy ϵ
2. Repeat
3. Obtain P_{opt} from (20),
4. Obtain N_{opt} from (25), given P defined in the step 3,
5. Obtain T_t^{opt} from (27), given P and N defined in the steps 3 and 4,
6. Until $ \eta_{EE}(i+1) - \eta_{EE}(i) \leq \epsilon$ (or convergence)
7. Output: $(N_{opt}^*, P_{opt}^*, T_t^{opt*})$

Proof: When $\rho_d \rightarrow \infty$ and ρ_t is finite, from (14), the objective function τ_{SE} in (26) is obtained as

$$\tau_{SE} = \left(1 - \frac{T_t}{T}\right) K \log_2 \left(v + \frac{T_t \rho_t \beta_k}{v}\right). \quad (32)$$

By setting $\frac{\partial \tau_{SE}}{\partial T_t} = 0$, we have

$$\ln \left(v + T_t \rho_t \beta_k / v\right) = \frac{v + T \rho_t \beta_k / v}{v + T_t \rho_t \beta_k / v} - 1. \quad (33)$$

Let us define $\tilde{y}(T_t) = \frac{c_8}{v + T_t \rho_t \beta_k / v}$, where c_8 is defined in (31). From (33), we get

$$c_8 e = \tilde{y}(T_t) e^{\tilde{y}(T_t)}. \quad (34)$$

By applying the solution for the Lambert-W function, after some algebraic manipulations, the optimal T_t^* is obtained. ■

For a specific case, when $\rho_d \rightarrow \infty$, $\rho_t \rightarrow 0$ and $v \rightarrow 0$ (due to assuming $N \gg K$), applying $\ln(1 + x) \approx x + O(x^2)$ to (26), we have

$$\tau_{SE} = \left(1 - \frac{T_t}{T}\right) \frac{K T_t \rho_t \beta_k}{v \ln 2}. \quad (35)$$

It is clear that the optimum training length for maximizing SE in (35) under the SLNR precoding scheme converges to

$$\lim_{\rho_t \rightarrow 0} T_t^{opt} = \frac{T}{2}. \quad (36)$$

This implies that T should be at least greater than $2K$. For ZF-PS, the same limit has been provided in [27].

D. Alternating Optimization Algorithm

In the previous subsections, the optimal values of P, N and T_t for maximizing the EE are obtained separately when the other two parameters are fixed. However, the globally maximal value of the EE can only be achieved by joint optimization. Since N and T_t are integers, by obtaining optimal value of P in (20), we can adopt an exhaustive search (ES) on N and T_t over all reasonable pair (N, T_t) in order to achieve global optimum EE. Herein, we adopt an alternating optimization (AO) algorithm to solve (18) which is summarized in Table I.

Although a general AO algorithm is not analytically guaranteed to obtain the global optimum solution, it is locally convergent [30], [31]. In our EE maximization problem, the algorithm optimizes these parameters (N, P, T_t) sequentially until $|\eta_{EE}(i+1) - \eta_{EE}(i)| \leq \epsilon$. The local convergence of the algorithm is guaranteed for any initial set (N, P, T_t) . This is because the EE metric has a finite upper bound and it is a quasi-concave function with N, P and T_t , respectively, hence the achieved value of EE in the algorithm is non-decreasing in every step. It is expressed as $\eta_{EE}(N_{opt}^{(i+1)}, P_{opt}^{(i+1)}, T_t^{(i), opt}) \leq$

$\eta_{EE}(N_{opt}^{(i+1)}, P_{opt}^{(i+1)}, T_t^{(i+1),opt})$, where the inequality is obtained by applying the solution of the sub-problem in (26). The real-valued solutions (N_{opt} and T_t^{opt}) defined from the AO algorithm can be used as the starting point for achieving the integer-valued solutions. It can be one of the four integer points obtained by respectively taking the floor and ceiling of N_{opt} and T_t^{opt} . As the integers N and T_t are left unchanged in an iteration, the convergence is achieved. The actual convergence speed of the algorithm depends on specific value of the accuracy ϵ [31]. To avoid local convergence, we randomly generate several initial sets and choose the one which results in the overall maximum EE. The output of the chosen initial set via the algorithm is considered as a final solution.

Let us define Δ_p as a small step of transmit power level, which is used to obtain P^* in (21). The complexity of this algorithm to find the optimal value of P_{opt} is on the order of $o(\lfloor \frac{P_{max}-P_{min}}{\Delta_p} \rfloor + 1)$, where the additional comparison is due to the fact that the algorithm needs to compare the two values in (20). Similarly, since N and T_t are integers, the complexities of the algorithm to find the optimal values of N_{opt} in (25) and T_t^{opt} in (27) are on the orders of $o(N_{max} - K + 1)$ and $o(T - K + 1)$, respectively. Hence, the total computational complexity of the AO algorithm is on the order of $M_1 M_2 (o(\lfloor \frac{P_{max}-P_{min}}{\Delta_p} \rfloor + 1) + o(N_{max} - K + 1) + o(T - K + 1))$, where M_1 is the number of iterations and M_2 is the number of randomly generated initial sets to obtain the final solution. As compared with the Brute-force ES algorithm, numerical results presented in the following section show that the AO algorithm performs well and it is almost guaranteed to obtain the global optimum EE after a few iterations with $M_2 = 10$ random sets. This approach is practical because it has a very low complexity and it has been commonly used in the literature [25], [26].

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, simulation results are presented to verify the accuracy of our theoretical analyses, to investigate the system performance with difference parameter settings, and also to evaluate the effectiveness of the proposed algorithm. The active users are assumed to be uniformly distributed in the cell with a radius of $D = 288$ m and a minimum distance of $D_0 = 40$ m. The distribution of the users along the radius of the cell is given by $f_r(r) = \frac{2r}{D^2 - D_0^2}$ [32]. The shadowing coefficient δ_k is a log-normal random variable with standard deviation σ_{sh} , i.e., the quantity $10 \log_{10}(\delta_k)$ is zero-mean Gaussian distributed with a standard deviation of σ_{sh} [3]. The transmitted signals are modulated with the orthogonal frequency-division multiplexing (OFDM) scheme. Based on LTE standards, an OFDM symbol duration is approximated as $71.4 \mu s$. We set the channel coherence time to be $T_c = 1$ ms, hence $T = 196$ symbols; $K = 10$ and $\sigma_{sh} = 8$ dB [3].

Without loss of generality, we fix the noise variance to be 1 to study the SE performance for both cases of perfect CSI and imperfect of CSI [4], [7]. The form of large-scale fading coefficient follows the approaches of [3] and [7]. It is defined as $\beta_k = \delta_k (r_k/r_0)^{-\varrho}$, where r_k is the distance between user k and the BS, ϱ is the path loss exponent, and r_0 is a fixed reference distance. Herein, we choose $\varrho = 3.76$ and $r_0 = 40$

Fig. 1: The SE bound versus SNR for SLNR and ZF precoding schemes with perfect CSI when $N = 30$ and $T_t = 10$.

Fig. 2: The SE bounds versus N for SLNR and ZF schemes with perfect and imperfect CSI when $\rho_d = 5$ dB and $T_t = 10$. When CSI is imperfect, we set $\rho_t = 0$ and 4 dB, respectively.

m. If the BS can estimate CSI perfectly, i.e., $\sigma_e^2 = 0$, the UB on the achievable rate of the k^{th} user in (9) (and also in (57)) can be expressed as

$$\hat{R}_k = \log_2 \left(1 + v + \frac{\rho_d \beta_k}{K} (N - K + 1) \right), \quad (37)$$

whereas the LB on the achievable rate is defined as

$$\bar{R}_k = \log_2 (1 + (\bar{\psi}_k - 1) \bar{\alpha}_k), \quad (38)$$

where $\bar{\psi}_k = \frac{(N-K+1+(K-1)\bar{\mu})^2}{N-K+1+(K-1)\bar{\zeta}}$ and $\bar{\alpha}_k = \frac{N-K+1+(K-1)\bar{\zeta}\beta_k p_k}{N-K+1+(K-1)\bar{\mu}\sigma_n^2}$. The values of the parameters $\bar{\mu}$ and $\bar{\zeta}$ are defined by solving the following equations

$$\begin{cases} \bar{\mu} &= \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{1}{\bar{\phi}_i}, \\ \bar{\zeta} \left(1 + \sum_{i=1, i \neq k}^K \frac{p_i \beta_i}{\bar{\phi}_i^2 \sigma_n^2} \right) &= \frac{1}{K-1} \sum_{i=1, i \neq k}^K \frac{\bar{m}_i}{\bar{\phi}_i^2}, \end{cases} \quad (39)$$

Fig. 3: Bounds and asymptotic limit of the SE for SLNR-PS with imperfect CSI when $N = 60$ and $T_t = 10$.

where $\bar{\phi}_i = N\beta_i \frac{p_i}{\sigma_n^2} (1 - \frac{K-1}{N} + \frac{K-1}{N}\bar{\mu}) + 1$ and $\bar{m}_i = \frac{p_i}{\sigma_n^2} \beta_i \bar{\mu} (K-1) + 1$, respectively. The bounds on the system SE are rewritten as $\tau_{SE} = (1 - \frac{T_t}{T})KR_k$. When CSI is perfect, in Figure 1, the SE bounds of SLNR-PS and the SEs of ZF-based, MF-based and MMSE-based precoding schemes [4], [5] are presented. The curves labelled “Analytical UB (Based on Eq. (37))” and “Analytical LB (Based on Eq. (38))” refer to the corresponding upper bound and lower bound on the SE which are defined by using equations (37) and (38), respectively. In this example, we set $p_k = P/K$, $N = 30$ and $T_t = 10$. We observe that the bounds on the SE for SLNR-PS are closely matched with the corresponding simulation results when SNR is greater than 10 dB. When N is increased, the SE bounds also perform well as shown in Figure 2. Furthermore, the MMSE precoding vector for any user k can be written as $\mathbf{w}_k = (\hat{\mathbf{G}}\hat{\mathbf{G}}^H + \frac{K\sigma_n^2}{P}\mathbf{I})^{-1}\hat{\mathbf{g}}_k$ [33]. This MMSE solution utilizes the same regularization factor equal to $\frac{K\sigma_n^2}{P}$ for all users, whereas the SLNR solution employs a regularization factor of each user equal to the inverse of its individual SNR $\frac{\sigma_n^2}{p_k}$. Therefore, under the assumption of equal power allocation (EPA), i.e., $p_k = P/K$, the SLNR-PS performs as well as the MMSE scheme as shown in Figure 1. A similar observation for the conventional MIMO systems can be found in [14], where the issue on the equivalence between SLNR and MMSE schemes was studied. Moreover, the SLNR scheme performs better than the ZF and MF schemes over the entire range of SNRs. At the low SNR region, MF is better than ZF, and vice versa at the medium and high SNR regions.

Figure 2 shows the SE bounds for SLNR and ZF schemes under the imperfect CSI condition when $\rho_d = 5$ dB. The curves labelled “Analytical UB 1 (Based on Eq. (57))”, “Analytical UB 2 (Based on Eq. (9))”, and “Analytical LB (Based on Eq. (15))” refer to the SE bounds which are defined by equations (57), (9) and (15), respectively. For a given $T_t = 10$, as ρ_t increases, the error variance σ_e^2 is decreased, hence the system performance is improved. This is the reason why as ρ_t increases from 0 dB to 4 dB, the SEs for SLNR and

Fig. 4: SE with different T_t for SLNR-PS with $K = 10$, $N = 200$, and $T = 196$ for the case of imperfect CSI.

ZF precoding schemes are significantly improved. We observe that these SE bounds of the SLNR-PS are generally tight with respect to their simulation results when $N \geq 50$. On a different note, we observe from Figures 1 and 2 that the SE of the SLNR-PS converges to that of the ZF-PS when N is large due to $v = \frac{K-1}{N-K+1} \rightarrow 0$, or when SNR is high, i.e., $\frac{\sigma_n^2}{p_k} \rightarrow 0$. Furthermore, due to $\sigma_e^2 \ll N$, the “Analytical UB 1 (Based on Eq. (57))” is very close to “Analytical UB 2 (Based on Eq. (9))”, which has been mentioned in Appendix A. This fact can be shown more evidently by observing Figure 3 where SNR is increased. Clearly, all bounds are also tight. Moreover, as $\rho_d \rightarrow \infty$, the SE of SLNR-PS with imperfect CSI converges to an asymptotic limit due to the convergence of the achievable rate of each user, which has been shown in (14). This asymptotic limit strongly depends on the quality of the channel estimation. If σ_e^2 is small, the asymptotic limit is high and vice versa. In conclusion, numerical results validate our analysis. The UBs on the SE are accurate even when N is not very large. Due to the low computation complexity, we use the closed-form expression for the UB on the SE based on (9) in the subsequent discussions.

In Figure 4, we investigate the effect of the length of training sequence on the achievable SE for SLNR-PS with imperfect CSI. The optimum training lengths for maximizing the SE have been discussed in Section IV-C. As observed from Figure 4, if channel estimation error is large, i.e., ρ_t is small, by applying the optimum training length, the system SE can be significantly increased. In particular, when $\rho_t = 0$ dB, the system with T_t^{opt} has an 22.5% improvement of the EE at SNR = 25 dB as compared with that of $T_t = K$. When $\rho_t = 10$ dB and SNR is greater than 20 dB, the system SE with T_t^{opt} also outperforms that of $T_t = K$. Therefore, for practical implementation, due to low received signal power levels from the cell-edge users, the BS can employ the optimum training length in order to improve their achievable rates. The second observation is that, when $\rho_t = 10$ dB, at low and medium SNR regions, there is a small SE performance loss based on the closed-form expression of T_t^{opt} in (29) (which is derived

TABLE II: Simulation parameters

Parameter	Value	Parameter	Value
Maximum transmit power $P_{\max}$ [20]	43 dBm	Noise variance σ_n^2 [25]	-96 dBm
DAC power p_{dac} [28]	15.6 mW	Mixer power p_{mix} [28]	30.3 mW
Frequency synthesizer power p_{syn} [28]	50 mW	Feeder loss σ_{feed} [20]	-3 dB
Power amplifier efficiency σ_{am} [20]	0.38	Main supply loss σ_{MS} [20]	9%
Baseband power P_{bb} [20]	29.6 W	Filter power p_{filt} [28]	20 mW
DC-DC loss σ_{DC} [20]	7.5%	Cooling loss σ_{cool} [20]	10%

Fig. 5: EE of the SLNR-PS with imperfect CSI when $\sigma_n^2 = -96$ dBm, $K = 10$, $P_t = 10$ dBm, and $T = 196$. The convergence of the proposed algorithm is shown with circles.

for the case that $\rho_d \rightarrow \infty$ and ρ_t is finite), as compared with that of T_t^{opt} in (27). It is clear that the closed-form expression in (29) achieves good performance, therefore it can be utilized to obtain the optimal training length effectively.

To study the system EE performance, we employ a more practical channel model. Specifically, the noise variance is assumed to be $\sigma_n^2 = -96$ dBm [25]. From Figure 2, it has been shown that the UB on the achievable SE is very tight even when N is not large and SNR is low, i.e., $N = 50$ and $\rho_d = 5$ dB. Hence, for a given $\sigma_n^2 = -96$ dBm, we can select $P_{\min} = -25$ dBm to achieve a reliable EE maximization problem in (18). The large-scale fading coefficient β_k is generated based on the shadowing factor and the distance-dependent path loss factor $L(r_k)$. The shadowing factor δ_k is also a log-normal random variable with $\sigma_{sh} = 8$ dB [3]. For the macro-cell BS and carrier frequency of 2 GHz, the path loss factor $L(r_k)$ is given in dB by $128.1 + 10 \log_{10}(r_k)$, where r_k is measured in km [34]. Moreover, the values of other parameters follow the settings of various works [20], [25], [28]. They are summarized in Table II.

Next, we obtain the optimal values of N , P and T_t that are needed to achieve the maximal value of the EE. Particularly, Figure 5 shows the EE of SLNR-PS as a function of N and P when the system has unit bandwidth, $\sigma_n^2 = -96$ dBm, $K = 10$, $P_t = 10$ dBm and $T = 196$. Each point is obtained by using the optimal value of T_t in (27). An optimal solution obtained by the Brute-force exhaustive search algorithm, named as the global optimum EE, is also provided

for comparisons. As observed from Figure 5, there is a global optimum value of EE, which is equal to 1.74 (bits/Joule), at $N_{opt} = 99$ and $P_{opt} = 24.8$ dBm, and it can be achieved by $T_t^{opt} = 10$. At this global optimum EE, the SE per user is 8.682 (bits/Hz). By applying the AO algorithm, the iterative progression for maximizing the EE is also demonstrated in Figure 5. In this example, the number of feasibly initial sets, which is randomly generated in the AO algorithm, is 10. We set $\epsilon = 10^{-5}$, the AO algorithm converges quickly after 4 iterations to achieve the global optimum EE. The convergence of the algorithm is presented with circles, whereas the globally optimal value of EE is presented with a asterisk. This fact can be easily observed in Figure 6, in which the EE results for SLNR-PS with imperfect CSI (i.e., $P_t = 0$ dBm and $P_t = 10$ dBm) and perfect CSI are also presented. As expected, for a given $P_t = 0$ dBm, the AO algorithm is also able to obtain the global optimum EE numerically, which is 1.217 (bits/Joule) at $N_{opt} = 137$, $P_{opt} = 21.3$ dBm and $T_t^{opt} = 10$. In addition, as P_t increases from 0 dBm to 10 dBm, the global optimum value of EE for SLNR-PS is significantly increased. We observe that the system with $P_t = 0$ dBm (i.e., the quality of channel estimation is low) needs to use more antennas to achieve the global optimum EE, as compared to the system with $P_t = 10$ dBm due to better quality of channel estimation. Moreover, with perfect CSI, the global optimum EE is 2.224 (bits/Joule) at $N_{opt} = 80$, $P_{opt} = 29.5$ dBm and $T_t^{opt} = 10$. In general, we observe that the optimal energy efficient system is a massive MIMO setup because there are many BS antennas at the three global optimum EEs. This is the output by solving the optimization problem, in which the system parameters are not restricted. Therefore, we can conclude that the AO algorithm provides an effective solution to achieve the global optimum EE with a low-computational complexity and it performs well.

Moreover, the EE optimization problem under both transmit power and QoS constraints can be formulated as

$$\max_{p_k} \frac{(1 - \frac{T_t}{T}) \sum_k^K \hat{R}_k(p_k)}{\xi \sum_k^K p_k + u}, \quad (40)$$

$$\text{subject to } (\text{Co}_1) : \hat{R}_k(p_k) \geq R_k^{min}, (\text{Co}_2) : \sum_k^K p_k \leq P.$$

The result in (40) is a fractional programming (FP) problem which is nonconvex. By applying properties of the FP, the objective function is equivalent to $(1 - \frac{T_t}{T}) \sum_k^K \hat{R}_k(p_k) - \eta_{EE}^* (\xi \sum_k^K p_k + u)$, where η_{EE}^* is the EE when p_k is equal to the optimal value p_k^* , i.e., $\eta_{EE}^* = \frac{(1 - \frac{T_t}{T}) \sum_k^K \hat{R}_k(p_k^*)}{\xi \sum_k^K p_k^* + u}$ [24]. Thus, the optimization problem (40) can be rewritten as

$$\max_{p_k} \left(1 - \frac{T_t}{T}\right) \sum_k^K \hat{R}_k(p_k) - \eta_{EE}^* (\xi \sum_k^K p_k + u), \quad (41)$$

Fig. 6: Convergence of the AO algorithm for maximizing total EE when $K = 10$, $\sigma_n^2 = -96$ dBm and $T = 196$.

subject to (Co_1) and (Co_2) . The optimization in (41) has been transformed into a convex optimization problem. Hence, we can obtain the solution to (41) by using the Lagrange multiplier approach associated with the Karush-Kuhn-Tucker conditions. In Figure 7, we investigate the energy-efficient power allocation scheme for the SLNR-PS with perfect CSI when $K = 10$, $N = 80$, $T_t = 10$. In this example, we set $R_k^{min} = 1$ (bit/s/Hz). We observe that the SLNR-PS with optimal power allocation (OPA) provides an improvement in the EE as compared to that of the EPA. When P is greater than $P_{opt} = 29.5$ dBm (i.e., the EE is maximum at P_{opt}), the EE of SLNR-PS with EPA decreases, i.e., named as baseline in Figure 7. This is because the EE is strictly quasi-concave with respect to P . Hence, it increases with $P \in [0, P_{opt}]$ and it decreases with $P \in [P_{opt}, P_{max}]$. However, by using the energy-efficient power allocation scheme, the BS wisely selects the optimal value of P to use in order to achieve an optimal EE, instead of using the maximum available power.

In Figure 8, the SEs of SLNR-PS with the assumptions of EPA and OPA are presented when CSI is perfect. Herein, the OPA is obtained by using the popular Water-Filling method. When N is not large, such as $N = 50$, the OPA algorithm can provide an improvement in the SE as compared to the SLNR-PS with EPA at the low and medium SNR regions. However, when the number of BS antennas is large, the channel hardens in most propagation environments [3]. It means that the effective channel gain becomes a deterministic value [3], [4]. Therefore, the SE improvement by using the OPA is limited. In particular, when $N = 100$ or 150 , the SE of SLNR-PS with OPA is close to that of EPA.

VI. CONCLUSION

In this paper, the SE bounds of the SLNR-PS when N is large have been analyzed for the case of imperfect CSI. The optimal values of N , P and T_t for maximizing the EE have been investigated. The closed-form expression of the optimal training length for maximizing SE (hence to maximize the EE) has also been derived at the high SNR region. An alternating

Fig. 7: EE of SLNR-PS for both cases of EPA and OPA when $K = 10$, $N = 80$, and $\sigma_n^2 = -96$ dBm, and CSI is perfect.

Fig. 8: SE of SLNR-PS for both cases of EPA and OPA when $T_t = 10$, and $\sigma_n^2 = -96$ dBm in the case of perfect CSI.

optimization algorithm has been adopted to achieve the global optimum EE based on the optimal values of N , P and T_t . Numerical results have validated our theoretical analyses. It has been shown that the global optimum EE is achieved by a massive MIMO setup while using optimal values of P and T_t . The number of BS antennas should increase to achieve the optimum EE if the quality of channel estimation is low.

APPENDIX

A. Proof of Theorem 1

The achievable rate of the k^{th} user is defined as

$$R_k = \mathbb{E} \left\{ \log_2(1 + \gamma_k) \right\} = \mathbb{E} \left\{ \log_2(1 + \lambda_{\max}(\mathbf{B}_k)) \right\}. \quad (42)$$

In order to obtain $\lambda_{\max}(\mathbf{B}_k)$, we employ the Weyl's inequality in matrix theory. In particular, if $\mathbf{C}, \mathbf{D} \in \mathbf{M}_n$ are Hermitian matrices and the eigenvalues $\lambda_i(\mathbf{C})$, $\lambda_i(\mathbf{D})$, and $\lambda_i(\mathbf{C} + \mathbf{D})$ are arranged in an increasing order $\lambda_{\min} = \lambda_1 \leq \dots \leq \lambda_n =$

$\lambda_{\max}$, for each $l = 1, \dots, n$, we have [35, Eq. (4.3.2)]

$$\lambda_l(\mathbf{C}) + \lambda_{\min}(\mathbf{D}) \leq \lambda_l(\mathbf{C} + \mathbf{D}) \leq \lambda_l(\mathbf{C}) + \lambda_{\max}(\mathbf{D}). \quad (43)$$

Based on (43) and (7), we have

$$\lambda_{\max}(\mathbf{B}_k) \leq \lambda_{\max}(\mathbf{C}_k) + \lambda_{\max}(\mathbf{D}_k), \quad (44)$$

$$\mathbf{C}_k = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H, \quad (45)$$

$$\mathbf{D}_k = \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \sigma_e^2 \mathbf{I}, \quad (46)$$

where $\lambda_{\max}(\mathbf{C}_k)$ and $\lambda_{\max}(\mathbf{D}_k)$ are the corresponding maximum eigenvalues of $\mathbf{C}_k$ and $\mathbf{D}_k$, respectively. According to [14], $\lambda_{\max}(\mathbf{C}_k)$ is defined by solving the following equations

$$\det \left[\lambda \mathbf{I} - \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right)^{-1} \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \right] = 0, \quad (47)$$

$$\det \left[\left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I} \right) - \left(\frac{\lambda + 1}{\lambda} \right) \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \right] = 0, \quad (48)$$

where $\hat{\mathbf{G}}_k = [\hat{\mathbf{g}}_1 \dots \hat{\mathbf{g}}_K]$. Based on matrix determinant lemma and after some mathematical manipulations, $\lambda_{\max}(\mathbf{C}_k)$ is obtained as [14]

$$\begin{aligned} \lambda_{\max}(\mathbf{C}_k) &= \frac{1}{1 - \hat{\mathbf{g}}_k^H (\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I}) \hat{\mathbf{g}}_k} - 1 \\ &= \frac{1/\rho_k}{[(\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k + \rho_k \mathbf{I})^{-1}]_{k,k}} - 1. \end{aligned} \quad (49)$$

Moreover, based on the approaches in [14] and [33], (49) can also be decomposed as

$$\lambda_{\max}(\mathbf{C}_k) = \frac{1}{\rho_k} \hat{\mathbf{g}}_k^H \Sigma_k \hat{\mathbf{g}}_k + \hat{\mathbf{g}}_k^H \Omega_k \hat{\mathbf{g}}_k, \quad (50)$$

where $\Sigma_k = \mathbf{I} - \hat{\mathbf{G}}_k (\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^H$ and $\Omega_k = \hat{\mathbf{G}}_k (\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k + \rho_k \mathbf{I})^{-1} (\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k)^{-1} \hat{\mathbf{G}}_k^H$. Next, we define the expected value of $\lambda_{\max}(\mathbf{C}_k)$. Let us denote $\theta_{zf,k} = \frac{1}{\rho_k} \hat{\mathbf{g}}_k^H \Sigma_k \hat{\mathbf{g}}_k$ and $\theta_{ad,k} = \hat{\mathbf{g}}_k^H \Omega_k \hat{\mathbf{g}}_k$. It has been shown that $\theta_{ad,k}$ is a nondecreasing function of $1/\rho_k$; $\theta_{zf,k}$ and $\theta_{ad,k}$ are statistically independent [33]. The expected value of $\theta_{zf,k}$ is given by

$$\mathbb{E}\{\theta_{zf,k}\} \stackrel{(a_1)}{=} \mathbb{E}\left\{ \frac{1/\rho_k}{[(\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k)^{-1}]_{k,k}} \right\} \stackrel{(a_2)}{=} \frac{\sigma_g^2}{\rho_k} (N - K + 1), \quad (51)$$

where (a_1) is obtained based on the result presented in [33] and (a_2) is obtained due to $1/[(\mathbf{H}_k^H \mathbf{H}_k)^{-1}]_{kk} \sim \chi_{2(N-K+1)}^2$, where $\mathbf{H}_k = [\mathbf{h}_1, \dots, \mathbf{h}_K]$. The challenge in deriving $\mathbb{E}\{\lambda_{\max}(\mathbf{C}_k)\}$ is to define the $\mathbb{E}\{\theta_{ad,k}\}$. We denote $\mathbf{q}_k = \mathbf{U}_k^H \hat{\mathbf{g}}_k \in \mathbb{C}^{(K-1) \times 1}$, where $\mathbf{U}_k$ is obtained from the singular value decomposition of $\hat{\mathbf{G}}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$. Let us define a matrix $\mathbf{Q}_k \in \mathbb{C}^{N \times (K-1)}$ which has the same distribution of $\hat{\mathbf{G}}_k$, and it is independent of $\mathbf{q}_k$. Following [33, Eq. (32)], at the high SNR region, $\theta_{ad,k}$ has an identical distribution of $\|\mathbf{q}_k\|^2 [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii}$ with $i = 1 \dots K-1$. Using lemma 2.10 of [36], i.e., $\mathbb{E}\{\text{trace}(\Psi^{-1})\} = \frac{m}{n-m}$ and $\mathbb{E}\{\text{trace}(\Psi)\} = mn$, where $\Psi \sim \mathcal{W}_m(n, \mathbf{I}_n)$ is an $m \times m$ complex Wishart matrix with n degrees of freedom ($n > m$),

we have $\mathbb{E}\{\|\mathbf{q}_k\|^2\} = \sigma_g^2(K-1)$ and

$$\begin{aligned} \mathbb{E}\left\{ [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii} \right\} &= \frac{1}{\sigma_g^2} \mathbb{E}\left\{ [(\bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k)^{-1}]_{ii} \right\} \\ &= \frac{\mathbb{E}\left\{ \text{trace}[(\bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k)^{-1}] \right\}}{\sigma_g^2(K-1)} = \frac{1}{\sigma_g^2(N-K+1)}, \end{aligned} \quad (52)$$

where $\bar{\mathbf{H}}_k = [\mathbf{h}_1, \dots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \dots, \mathbf{h}_K]$. Therefore, we get

$$\mathbb{E}\{\theta_{ad,k}\} = \mathbb{E}\left\{ \|\mathbf{q}_k\|^2 [(\mathbf{Q}_k^H \mathbf{Q}_k)^{-1}]_{ii} \right\} = \frac{K-1}{N-K+1}. \quad (53)$$

Moreover, from (43) and (46), we obtain the $\lambda_{\max}$ of $\mathbf{D}_k$ as

$$\begin{aligned} \lambda_{\max}(\mathbf{D}_k) &= \lambda_{\max}\left(\frac{\sigma_e^2 \mathbf{I}}{\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I}} \right) \\ &= \frac{\sigma_e^2}{\lambda_{\min}(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \rho_k \mathbf{I})} \leq \frac{\sigma_e^2}{\lambda_{\min}(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k) + \rho_k}. \end{aligned} \quad (54)$$

When N is very large, the eigenvalues of $\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k$ converge to a fixed deterministic distribution known as the Marchenko-Pastur distribution [4]. We define $z = \frac{K-1}{N}$. According to [37] and [38], as $N \rightarrow \infty$, the deterministic approximation of the smallest eigenvalue of $\hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k$ converges to

$$\lambda_{\min}\left(\frac{1}{N} \hat{\mathbf{G}}_k^H \hat{\mathbf{G}}_k \right) \rightarrow (1 - \sqrt{z})^2 \sigma_g^2. \quad (55)$$

Substituting (55) into (54), as N is very large, we obtain

$$\lambda_{\max}(\mathbf{D}_k) \leq \frac{\sigma_e^2}{(1 - \sqrt{z})^2 \sigma_g^2 N + \rho_k}. \quad (56)$$

Applying the Jensen's inequality, i.e., $\mathbb{E}\{\log_2(1+x)\} \leq \log_2(1 + \mathbb{E}\{x\})$, and using the results in (51), (53) and (56), from (42), the UB on the achievable rate is defined as

$$\begin{aligned} R_k &\leq \hat{R}_k = \log_2 \left(1 + \mathbb{E}\{\lambda_{\max}(\mathbf{B}_k)\} \right) \\ &= \log_2 \left(1 + v + \Upsilon + \frac{\sigma_g^2(N-K+1)}{\rho_k} \right), \end{aligned} \quad (57)$$

where v is defined in (9) and $\Upsilon = \frac{\sigma_e^2}{(1-\sqrt{z})^2 \sigma_g^2 N + \rho_k}$. We observe that $\sigma_e^2 \in [0, 1]$ is much less than N , i.e., $\lim_{N \rightarrow \infty} \Upsilon = 0$. By substituting this result into (57), we can obtain (9).

B. Proof of Theorem 2

According to (43), we have

$$\lambda_{\max}(\mathbf{B}_k) \geq \lambda_{\max}(\mathbf{C}_k) + \lambda_{\min}(\mathbf{D}_k). \quad (58)$$

Hence, the achievable rate of the k^{th} user is lower bounded as

$$R_k \stackrel{(i)}{\geq} \log_2 \left(1 + \left(\mathbb{E}\left\{ \frac{1}{\lambda_{\max}(\mathbf{C}_k)} \right\} \right)^{-1} \right) = \check{R}_k^p, \quad (59)$$

where $\lambda_{\max}(\mathbf{C}_k)$ is given in (49), (i) is obtained by using the Jensen's inequality and the convexity of $\log_2(1 + \frac{1}{x})$. The distribution of $\lambda_{\max}(\mathbf{C}_k)$ can be obtained by using the approach shown in [39]. However, it is analytically intractable. For a tractable analysis, the probability density function of $\lambda_{\max}(\mathbf{C}_k)$ can be approximated by a Gamma function, i.e.,

$$f_{\lambda_{\max}(\mathbf{C}_k)}(x) = \frac{x^{\psi_k-1} e^{-\frac{x}{\alpha_k}}}{\Gamma(\psi_k) \alpha_k^{\psi_k}}, \quad (60)$$

where the shape parameter ψ_k and the scale parameter α_k are defined in (15) [40]. Substituting (60) into (59) and after some algebraic manipulations, (15) can be obtained.

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