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A quantitative strain energy indicator for predicting the failure of laboratory-scale rock samples: application to shale rock

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ABSTRACT

Strain energy is one of the key factors that control the brittle failure process of rock mass, such as the onset of accelerating displacement. The phenomenon of accelerating displacement is a necessary and important precursor to predict the brittle failure of rock, and it is attributed to the unstable propagation of micro-cracks when the load reaches or exceeds the crack damage stress threshold. Thus, for the design of underground rock engineering and to ensure proper safety practices, it is important to propose a suitable strain energy criterion that uses the strain energy at the crack damage stress threshold (W_{cd}) to predict the strain energy at final failure (W_{ucs}). For this purpose, a quantitative strain energy indicator (W_{ucs}/W_{cd}) was proposed based on two-dimensional renormalization group theory for the prediction of the brittle failure of laboratory-scale rock samples subjected to uniaxial pressure. The indicator is a function of the homogeneity index, m , of the Weibull distribution. The reliability of the strain energy indicator was verified by a series of uniaxial compression tests on 12 shale samples that had different homogeneity indices, m . The heterogeneity of the sample was dominated by different bedding dip angles. The experimental results showed that the quantitative indicator could be used to predict the brittle failure of samples of any type of rock at the laboratory scale using their m values. In addition, the effect of the bedding dip angle on the homogeneity index, m , and the strain energy indicator were discussed based on the experimental results to provide a better understanding of the relationships among heterogeneity, the strain energy indicator, and the other physical and mechanical properties of the shale samples that were tested. Also, the discussion included the effects of the homogeneity of the sample and the strain energy indicator on the physical and mechanical properties of the samples. As a result, some conclusions were obtained that might be applicable to laboratory-scale shale samples.

Keywords: Rock; Strain energy; Renormalization group theory; Weibull distribution; Homogeneity index

1 Introduction

Brittle failure in a stressed rock mass, such as flaking, spalling, and bursting, is a concern in all underground excavations in mining and civil tunneling projects. Strain energy, one of the key factors that induces and controls the brittle failure of rock mass (Wang and Park 2001), is the result of the redistribution of stresses near underground excavations. This redistribution increases the strain energy in zones of increased compression. If the resulting imbalance of the energy of the system is severe enough, crack initiation and the progressive degradation of the strength of the rock mass can occur through the brittle failure process (Eberhardt et al., 1998). This is a complicated process and comprehensive understanding of the processes involved is required in order to propose suitable criteria to predict the final failure.

The initiation and propagation of stress-induced micro-crack damage is known as the micro-failure mechanism in all brittle failures of rock mass (Cai et al. 2004; Ma et al 2009; Fan et al. 2012; Wu et al. 2017). Extensive theoretical, experimental, and numerical studies have been conducted at the laboratory scale on crack initiation and the propagation of brittle rock mass (Brace 1964; Hoek and Bieniawski 1965; Bieniawski 1967; Lajtai and Lajtai 1974; Martin 1993; Hatzor and Palchik 1997; Eberhardt et al. 1997, 1998, 1999; Vásárhelyi and Bobet 2000; Diederichs et al. 2004; Zhu et al. 2007; Park and Bobet 2010; Nicksiar and Martin 2012). In these studies, it was well recognized that the stress-strain curves of brittle rock mass subjected to compression tests can be characterized by five critical stages. Fig. 1 shows that the five stages, i.e., (1) crack closure, (2) linear elastic deformation, (3) crack initiation and stable crack growth, (4) critical energy release and unstable crack growth, and (5) failure and post-peak behavior, comprise a continuous process from the macro-response to the micro-crack initiation and

propagation. Three stress thresholds were used to quantify the different crack levels. The first is the crack closure stress, σ_{cc} , which occurs during the initial stages of loading when existing cracks are orientated at an angle to the applied load and close. As a result, a linear elastic deformation occurs after the majority of the existing cracks have closed. The second crack level is the crack initiation stress, σ_{ci} , which represents the stress level at which micro-fracturing begins. At this stage, crack growth can be stopped by controlling the applied load. The third crack level is the crack damage stress, σ_{cd} , which defines the stress level at which crack propagation cannot be stopped even when the applied load is held constant. Therefore, it is worthwhile to propose a suitable strain energy criterion by using the strain energy at these stress thresholds to predict the strain energy at final failure for underground rock engineering design and safety practice.

Strain energy criteria related to σ_{cc} , σ_{ci} , and σ_{cd} have not yet to be reported. Conversely, in previous studies, some other criteria, i.e., the so-called crack initiation and crack damage thresholds, have been proposed in term of stress or strain based on laboratory test results and the results of analyzing the monitoring data. Brace et al. (1966), Stacey and Page (1986), Wagner (1987), Castro et al. (1996), Grimstad and Bhasin (1997), and Diederichs (2000) showed that stress-induced fracturing around tunnels was initiated (crack initiation stress, σ_{ci}) at approximately $0.3-0.5 \sigma_c$ and that the crack initiation stress function for rock strength in terms of maximum principal stress could be approximated by a simple linear function, i.e., $\sigma_{ci} = (0.3-0.5)\sigma_{ucs} + (1-1.5)\sigma_3$, where σ_{ucs} is the uniaxial compressive strength of the intact rock mass, and σ_3 is the confining stress. Cai et al. (2004) proposed generalized crack initiation and crack damage thresholds of brittle rock masses near underground excavations as $\sigma_{ci} - \sigma_3 = A\sigma_{ucs}$ and $\sigma_{cd} - \sigma_3 = B\sigma_{ucs}$, respectively, for jointed rock masses, where A and B are material constants of the intact or jointed

rock masses. Xue et al. (2014a) investigated the relationship between the σ_{cd} and σ_{ucs} of different types of rock samples at the laboratory scale. The results showed that the mean values and standard deviations of σ_{cd}/σ_{ucs} for igneous, metamorphic, and sedimentary rocks were 0.78 ± 0.09 , 0.85 ± 0.09 , and 0.78 ± 0.11 , respectively. Also, Xue et al. (2015) found that normalized ratio values between the axial strain at the crack damage stress threshold and the axial strain at the uniaxial compressive strength threshold for different types of rock typically are about 1.34 ± 0.24 . It is well known that strain energy is a parameter that is linked to both strain and stress, so there should be relationships of strain energies between σ_{ucs} and σ_{cd} , σ_{ci} , and σ_{cc} . Thus, proposing a strain energy criterion associated with σ_{ucs} and either σ_{cd} , σ_{ci} , or σ_{cc} may be feasible.

The proposed strain energy criterion should be calculated easily. It demands that we be able to identify the stress thresholds directly. However, σ_{cd} , σ_{ci} , and σ_{cc} cannot be identified directly from stress-strain curves, but σ_{cd} can be identified directly from a typical creep rupture curve. Fig. 1 shows that a typical creep rupture curve usually can be divided into three stages, i.e., primary creep (strain hardening), secondary creep (steady), and tertiary creep (accelerating) (Saito 1979; Crosta and Agliardi 2003). Reches and Lockner (1994) suggested that the accelerating creep in the creep rupture curve was attributable to the unstable propagation of the crack, which means that the tertiary creep begins (point C_2) when the load reaches or exceeds the stress threshold at point C (σ_{cd}). Therefore, σ_{cd} in the stress-strain curve is equivalent to point C_2 in the typical creep rupture curve, which is the onset of the accelerating deformation (Qin et al. 2010; Heap et al. 2011; Xue et al. 2014b). Fig. 1 shows the connection between the typical creep curve and the stress-strain curve of brittle rock mass subjected to compression tests. Point C_2 can be identified directly and easily from the creep rupture curve due to the increasing slope of the tangent line at

point C_2 . Therefore, σ_{cd} can be identified directly from the creep rupture curve through the onset of the accelerating deformation. In addition, Saito (1979) indicated that accelerating displacement was a necessary precursor to all brittle failure of rock mass at the laboratory scale and in actual project and natural hazard scales. Several models have been proposed to predict the failure of rock mass using the observed displacement data associated with the onset of the accelerating deformation (point C_2) (Fukuzono 1985; Voight 1989; Cornelius and Scott 1993; Petley et al. 2005; Federico et al. 2012). Since brittle failures usually occur near the excavation surfaces where the rock masses are subject to uniaxial pressure or low confining pressures, it is important to propose a strain energy indicator, W_{ucs}/W_{cd} . This can be done by using the strain energy at crack damage stress, W_{cd} , to predict the strain energy at final failure, W_{ucs} , for rock masses under uniaxial pressure.

The aim of this paper was to present the procedure for the derivation of an equation for the strain energy indicator. The procedure was based on two-dimensional renormalization group theory and a critical equation proposed by Pan et al. (2017). Subsequently, in order to verify the reliability of the proposed strain energy indicator, 12 shale samples with four different bedding dip angles were prepared to perform uniaxial compression tests. It is well known that heterogeneity is one of the most important factors that affect the physical and mechanical properties of rock masses. To better understand the relationships among heterogeneity, the strain energy indicator, and other physical and mechanical properties of shale samples, corresponding discussions were conducted based on the test results irrespective of the limited number of samples. Some of the conclusions were applicable to shale samples at the laboratory scale.

Fig. 1 Stress-strain diagram showing the elements of crack development and relationship with typical creep rupture curve (modified version of Martin 1993)

2 Theoretical study of strain energy indicator

The failure process of the cylindrical sample was simplified as a two-dimensional problem on a fracture plane under uniaxial compression. The extensively used two-dimensional renormalization group theory was used to model this failure problem and derive the equation of the strain energy indicator. The procedure used to derive the equation was as follows, i.e., 1) the two-dimensional renormalization group model with four cells of each order block was established in Section 2.1, and critical equations proposed in a previous study (Pan et al. 2017) were used to derive the equation of the strain energy indicator; 2) based on the critical equations and a constitutive formula of the stress-strain curves from the uniaxial compression test, the strain energy indicator, W_{ucs}/W_{cd} , which is a function of the homogeneity index, m , of the Weibull distribution, was derived in Section 2.2.

2.1 Two-dimensional renormalization group model and failure probability

The cylindrical core is the most extensively used type of sample when performing uniaxial compression tests. During the failure process, cracks develop and coalesce along a two-dimensional macroscopic fracture plane, as shown in Fig. 2a. The potential plane controls the stability of the cylindrical sample. The failure process of the cylindrical sample is simplified as a two-dimensional problem involving uniaxial compression. Therefore, two-dimensional renormalization group theory, which has been used successfully to study the fractures of rocks, is used to describe this two-dimensional problem (Allegre et al. 1982; Madden 1983; Smalley et al. 1985; Qin et al. 2010; Qin and Pan 2011; Xue et al. 2014b).

The macroscopic fracture plane with unit-thickness can be renormalized into many different blocks of different orders, each of which contains four cells.

Fig. 2b shows an example of blocks of three orders is illustrated in Fig. 2b, e.g., a second-order block contains four second-order cells (marked with the number 2 in Fig. 2b). Similarly, each first-order block has four first-order cells, which are marked with the number 1. Each cell in different order blocks is assigned a non-uniform failure strength, σ_f , which is a Weibull distribution that represents the inherent heterogeneity in the strength of the rock (Hudson and Fairhurst 1969; Smalley et al. 1985) as shown in Eq. (1):

$$f(\sigma_f) = \frac{m}{\sigma_0} \left(\frac{\sigma_f}{\sigma_0} \right)^{m-1} e^{-\left(\frac{\sigma_f}{\sigma_0} \right)^m} \quad (1)$$

where σ_0 is the mean failure strength of all of the cells for the rock sample, and m is the homogeneity index of the sample. An infinitely high m value corresponds to a homogeneous structure with uniform strength, whereas a heterogeneous structure with a broad distribution of local strength is associated with a relatively low m value (Tang et al. 2000; Wong et al. 2006). Fig. 3 shows a case of Weibull strength distribution for a different homogeneity index, m .

When an external load is applied to a rock sample, if part of the cells in a block fail, the stresses from the failed cells will transfer equally to the adjacent un-failed cells. The block fails only when all four cells that make up the block fail. The statistics of block failure also follows the same process. An iterative equation that describes the probability of failure, $P_1^{(n+1)}$, of an $(n+1)$ -order block from n -order block, $P_1^{(n)}$, was proposed by Pan et al. (2017), i.e.,:

$$\begin{aligned} P_1^{(n+1)} = & (P_1^{(n)})^4 + 4(P_1^{(n)})^3 \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{4^m} \right] + 6(P_1^{(n)})^2 \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{2^m} \right]^2 \\ & + 12(P_1^{(n)})^2 \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{2^m} \right] \left[(1 - P_1^{(n)})^{2^m} - (1 - P_1^{(n)})^{4^m} \right] + 4P_1^{(n)} \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} \right]^3 \\ & + 12P_1^{(n)} \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} \right]^2 \left[(1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} - (1 - P_1^{(n)})^{4^m} \right] \\ & + 12P_1^{(n)} \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} \right] \left[(1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} - (1 - P_1^{(n)})^{2^m} \right]^2 \\ & + 24P_1^{(n)} \left[1 - P_1^{(n)} - (1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} \right] \left[(1 - P_1^{(n)})^{\left(\frac{4}{3}\right)^m} - (1 - P_1^{(n)})^{2^m} \right] \left[(1 - P_1^{(n)})^{2^m} - (1 - P_1^{(n)})^{4^m} \right] \end{aligned} \quad (2)$$

Fig. 4 shows the dependences of $P_1^{(n+1)}$ on $P_1^{(n)}$ for different m values. The points 0.0 and 1.0 are stable, fixed points of the system, which correspond to the completely stable and completely unstable states of the fracture plane, respectively (Wilson 1979). The straight line that corresponds to $P_1^{(n+1)} = P_1^{(n)}$ crosses the iterative relationship at the critical failure point, P^* , which is an unstable fixed point that separates the regions of stable and unstable behavior. Fig. 4 shows that it is important to emphasize that the critical failure point, P^* , is equivalent to the point of the crack damage stress threshold, σ_{cd} , which controls the stability of the brittle rock mass.

Fig. 4 shows that the value of the critical failure point P^* becomes smaller as the homogeneity index, m , becomes larger. The relationship between m and P^* can be determined by regression of the interior trust region (Coleman and Li Y 1996) as shown in Eq. (3) (Pan et al. 2017).

$$P^* = a - b \frac{1}{c^m - d} \quad (3)$$

where $a = 1.004$, $b = 0.768$, $c = 1.603$, and $d = 1.132$.

Fig. 2. Illustration of the two-dimensional Renormalization Group (RG) theory model of a macroscopic fracture plane in a rock sample: (a) A shale rock sample with a macroscopic fracture plane; (b) Two-dimensional RG model with four cells per block of the macroscopic fracture plane. The numbers 1, 2, and 3 represent the cells of the first-order block, the second-order block, and the third-order block, respectively

Fig. 3. Distribution of Weibull failure strength for different homogeneity indexes, m

Fig. 4 Dependences of the failure probability for the $n+1$ cell, $P_1^{(n+1)}$, on the failure probability for the n cell, $P_1^{(n)}$ with different values of m . The critical failure point, P^* , gives the bifurcation point for the fracture plane. If $P_1 < P^*$, the solution iterates to 0 and no failure occurs. If $P_1 > P^*$, the solution iterates to 1, and the fracture plane has failed (modified version of Pan et al., 2017).

2.2 Strain energy indicator (W_{ucs}/W_{cd})

The failure probability, P_1 , of a cell can be expressed as follows (Qin et al., 2006):

$$P_1 = 1 - e^{-\left(\frac{\varepsilon}{\varepsilon_0}\right)^m} \quad (4)$$

where ε_0 is the mean uniaxial strain of the cells for the rock sample, and ε is the uniaxial strain of a cell.

Furthermore, the stress-strain curves obtained from the uniaxial compression tests can be described by a constitutive formula (Chen et al. 2006):

$$\sigma = E\varepsilon e^{-\left(\frac{\varepsilon}{\varepsilon_0}\right)^m} \quad (5)$$

where σ is the uniaxial stress, and E is the Young's modulus of the rock sample.

When the first order derivative of Eq. (5) is set to zero, the stress point is the uniaxial compressive strength point. Therefore, the relationship of the strain at the uniaxial compressive strength, ε_{ucs} , and mean strain, ε_0 , can be expressed as:

$$\frac{\varepsilon_{ucs}}{\varepsilon_0} = \left(\frac{1}{m}\right)^{\frac{1}{m}} \quad (6)$$

Substituting Eq. (3) into Eq. (4), we obtain the relationship between the strain relating to the crack damage stress threshold, ε_{cd} , and the reference strain, ε_0 :

$$\frac{\varepsilon_{cd}}{\varepsilon_0} = \left[-\ln \left(1 - a + b^{\frac{1}{c^m - d}} \right) \right]^{\frac{1}{m}} \quad (7)$$

When subjected to stress, the energy of the rock sample is stored as strain energy (Wiebols and Cook 1968), and the accumulation of strain energy is the key factor that induces brittle failure (Eberhardt et al. 1998; Wang and Park 2001). The strain energy, W , of a rock sample subjected to uniaxial stress can be expressed as:

$$W = \int_0^\varepsilon \sigma d\varepsilon \cdot V \quad (8)$$

where V is the volume of the rock sample. Eq. (8) can be converted into:

$$W = \int_0^\varepsilon E \varepsilon e^{-\left(\frac{\varepsilon}{\varepsilon_0}\right)^m} d\varepsilon \cdot V \quad (9)$$

Therefore, the strain energy at the uniaxial compressive strength, W_{ucs} , can be obtained as:

$$W_{ucs} = \int_0^{\varepsilon_{ucs}} E \varepsilon e^{-\left(\frac{\varepsilon}{\varepsilon_0}\right)^m} d\varepsilon \cdot V \quad (10)$$

In addition, the strain energy at the crack damage stress threshold, W_{cd} , also can be obtained:

$$W_{cd} = \int_0^{\varepsilon_{cd}} E \varepsilon e^{-\left(\frac{\varepsilon}{\varepsilon_0}\right)^m} d\varepsilon \cdot V \quad (11)$$

If $\varepsilon = k\varepsilon_0$, Eqs. (10) and (11) are converted into Eqs. (12) and (13), respectively.

$$W_{ucs} = E \varepsilon_0^2 \int_0^{\frac{\varepsilon_{ucs}}{\varepsilon_0}} k e^{-k^m} dk \cdot V \quad (12)$$

$$W_{cd} = E \varepsilon_0^2 \int_0^{\frac{\varepsilon_{cd}}{\varepsilon_0}} k e^{-k^m} dk \cdot V \quad (13)$$

Substituting Eq. (6) into Eq. (12), we obtain:

$$W_{ucs} = E \varepsilon_0^2 \int_0^{\left(\frac{1}{m}\right)^{\frac{1}{m}}} k e^{-k^m} dk \cdot V \quad (14)$$

Similarly, substituting Eq. (7) into Eq. (13), we obtain:

$$W_{cd} = E \varepsilon_0^2 \int_0^{\left[-\ln\left(1-a+b\frac{1}{c^{m-d}}\right)\right]^{\frac{1}{m}}} k e^{-k^m} dk \cdot V \quad (15)$$

The ratio of Eq. (14) to Eq. (15) gives:

$$\frac{W_{ucs}}{W_{cd}} = \frac{\int_0^{\left[\frac{1}{m}\right]^{\frac{1}{m}}} k e^{-k^m} dk}{\int_0^{\left[-\ln\left(1-a+b\frac{1}{c^{m-d}}\right)\right]^{\frac{1}{m}}} k e^{-k^m} dk} \quad (16)$$

which indicates that the strain energy indicator, W_{ucs}/W_{cd} (hereafter referred to as ‘theoretical indicator’) is a function only of m . In this paper, the value of W_{ucs}/W_{cd} was calculated using a computer program that we developed in MATLAB. The results of the ratio, with m increasing from 1 to 14.5, are listed in Table 1 and shown in Fig. 5, and they indicate that the ratio gradually

decreases as m increases.

In order to verify the reliability of the theoretical indicator, W_{ucs}/W_{cd} , shale samples with different homogeneity index values (i.e., m values) were prepared for use in performing uniaxial compression tests, as discussed in the following Section. Bedding dip angle is the dominate effect that affect the heterogeneity of the sample.

Fig. 5 Gradually decreasing relationship between W_{ucs}/W_{cd} and the homogeneity index, m , and the results of the tests of the shale samples

Table 1. Theoretical relationship between W_{ucs}/W_{cd} and the homogeneity index, m

3 Experimental Verification

3.1 Sample preparation and test procedure

The shale samples that were tested were cored from an outcrop shale block (Fig. 6a). The block was collected from the Lower Silurian Longmaxi Formation of Sichuan basin in Chongqing, China. It was composed of quartz, feldspar, pyrite, clay, calcite, mica, and it had low porosity and low permeability (Chen et al 2014; Liang et al 2015; Wang et al. 2016). Bedding dip angle is the dominate effect that affect the heterogeneity of the sample. Fig. 6b shows the four bedding dip angles that were considered, i.e., 0° , 30° , 60° , and 90° , and they were made with the end surfaces of the sample. Duplicate samples of each type of bedding dip angle were prepared. In order to reduce the damage to the shale samples during the drilling process, first, large shale samples (75 mm in diameter and 120 mm in length) were cored from the block. Then, a waterjet cutting machine was used to cut the large samples into the standard size samples recommended by the International Society for Rock Mechanics Standards which was referred to as ‘ISRM’ after here (Bieniawski and Bernede 1979). The diameter of each sample was 50 mm, and the ratio of each

sample's length to its diameter was two. The end surfaces were flat with an accuracy of 0.02 mm. The density of the samples was 2.6 g/cm³. Before the experiments were performed, the samples were dried by heating at 60 °C for 24 h to reduce the effect of the water content on the physical and mechanical parameters. This procedure might generate shrinkage and micro-cracking, and it was not considered in this paper.

Two layers of folded sticky bands with silicon grease lubrication between each layer were placed on the top and bottom of the sample to eliminate the effect of friction. Each sample was tested using an MTS machine with a constant axial strain loading rate (1.0×10^{-5} /s). An eight-channel acoustic emission (AE) system manufactured by Physical Acoustic Corporation (PAC) was used for the acquisition of AE data. Six NANO-30 AE sensors were attached to the sides of the sample to monitor the AE activity. Fig. 6b shows the configuration of the sensors. The operation range of the sensors was 125 - 750 kHz, with a resonant frequency of 300 kHz related to V/ μ bar. Superglue was used to attach the sensor tightly to the sample. This glue provided good and constant contact between the sensors and the samples throughout the tests. The data were acquired in the hit-based mode. The threshold and PDT/HDT/HLT were selected as 39 dB and 600/600/300, respectively. PAC 2/4/6 preamplifiers with selectable 20/40/60 dB gains were used for each active channel. The AE signals were amplified for 40 dB gain by the preamplifiers; then they were passed through 100 kHz/400 kHz lower and upper analog filters, respectively, and digitalized and recorded by the 3MSPS sampling rate. Before the test, the Auto Sensor Test (AST) and pencil lead were conducted to confirm the sensor coupling and the selected data acquisition parameters (threshold, PDT/HDT/HLT). When the test began, the MTS loading system and the AE data acquisition were triggered simultaneously. Therefore, the loading data and the AE data

can be compared based on the same time line.

Fig. 6 (a) Plot of shale samples cored from the block of shale with different drilling angles; (b) Standard shale samples with four types of bedding dip angles made with sample end surfaces and the AE sensors' configuration

3.2 Techniques for determining strain at crack damage stress

1. AE technique

A critical issue for successfully verifying the theoretical indicator, W_{ucs}/W_{cd} , is how to determine ε_{ucs} and ε_{cd} . It is well known that ε_{ucs} can be obtained directly from the stress–strain curve. Conversely, ε_{cd} is difficult to identify directly from the stress–strain curve.

Correlating the parametric changes of the detected AE signals with a growing crack is considered to be a very good tool for investigating the cracking sequences of stressed rocks (Eberhardt et al. 1998; Nicksiar and Martin 2012; Moradian et al. 2016). The parametric features of AE, such as the number of AE hits and the AE energy, were extracted, and they provided useful information about the failure behavior of materials. The number of AE hits was proportional to the number of growing cracks, and the energy of the AE signals was proportional to the magnitude of the cracking events in materials (Lockner 1993). We can use the parametric changes of the AE hits and the AE energy to estimate the evolution of the cracking process, especially the fourth failure stage of the critical energy release and unstable crack growth. Fig. 7a shows that the AE hit rate at the critical point of σ_{cd} begins to increase dramatically. This phenomenon is induced by the coalescence of the micro-cracks, which always accompanies the occurrence of a high AE energy rate (Fig. 7b). Macro-cracks can be detected as evidence of the transition after the critical point from a micro-cracking-dominated process to a process dominated by the propagation of a few large cracks (Moradian et al. 2016). As the load increases, additional cracks increase

significantly and contribute to the degradation of the inherent strength of the rock. As a result, the mean AE hits rate, the AE energy rate of the fourth failure stage of the critical energy release, and unstable crack growth are much greater than they are in the other stages.

Therefore, σ_{cd} can be identified as the point at which a dramatic acceleration of the AE hits rate is initiated, accompanied by the occurrence of a high AE energy rate. Therefore, ε_{cd} can be obtained from the stress-strain curve.

Fig. 7. (a) AE hits rate (per second); (b) AE energy rate (per second) versus the applied uniaxial stress normalized by the peak strength for the granite specimen AE-GR-2a-30-30, where (1) is crack closure, (2) is linear elastic deformation, (3) is micro-crack growth (stable crack growth), and (4) is macro-crack growth (unstable crack growth) (modified version of Moradian et al., 2016)

2. Volumetric strain reversal point, ε_{vr}

Volumetric strain, ε_v , is a pervasive volumetric property of the rock mass (Cook 1970). For a cylindrical rock sample subjected to axial loading and under small strain, the volumetric strain is given as (Martin and Chandler 1994):

$$\varepsilon_v = \varepsilon_{axial} + 2\varepsilon_{lateral} \quad (24)$$

Fig. 1 shows an example of axial, lateral strain and volumetric strain versus axial stress curves. The value of the axial strain at which the reversal of the volumetric strain occurs (volumetric strain reversal point, ε_{vr}) marks the beginning of Region IV and represents the onset of unstable crack growth (Bieniawski 1967; Martin and Chandler 1994). Therefore, ε_{cd} is equal to the volumetric strain reversal point, ε_{vr} .

3.3 Using $\varepsilon_{ucs}/\varepsilon_{cd}$ to determine the homogeneity index, m

In order to verify the reliability of the proposed relationship between W_{ucs}/W_{cd} and m values of shale samples that were tested in Eq. (16) (hereafter referred to as the ‘theoretical indicator’), the

W_{ucs}/W_{cd} calculated based on stain and stress directly (subsequently referred to as the ‘experimental indicator’) after σ_{cd} and ε_{cd} have been determined using the AE technique were used to compare with theoretical result. The comparing process requires that the value of m must be obtained for each sample.

Comparing Eqs. (6) and (7), we can derive the quantitative strain ratio relationship proposed in a previous study (Pan et al. 2017) between the strain at uniaxial compressive strength, ε_{ucs} , and the strain at the crack damage stress threshold, ε_{cd} :

$$\frac{\varepsilon_{ucs}}{\varepsilon_{cd}} = \left[-\frac{1}{m \ln \left(1 - a + b^{\frac{1}{c^m - d}} \right)} \right]^{\frac{1}{m}} \quad (17)$$

which indicates that $\varepsilon_{ucs}/\varepsilon_{cd}$ decreases gradually as m increases. Thus, the value of m can be obtained once the value of $\varepsilon_{ucs}/\varepsilon_{cd}$ is known. The value of ε_{ucs} can be obtained directly from the stress–strain curve. The value of ε_{cd} can be estimated using the AE technique.

3.4 Test results

Table 2 presents the data obtained for the uniaxial compressive strength, σ_{ucs} ; the strain at uniaxial compressive strength, ε_{ucs} ; Young's modulus, E ; the strain at the crack damage stress threshold, ε_{cd} , determined by the AE technique; the strain ratio, $\varepsilon_{ucs}/\varepsilon_{cd}$, determined by AE technique; the homogeneity index, m ; the theoretical indicator, W_{ucs}/W_{cd} ; the experimental indicator, W_{ucs}/W_{cd} ; and the strain ratio, $\varepsilon_{ucs}/\varepsilon_{cd}$, determined by the volumetric strain reversal point, ε_{vr} .

The values of σ_{ucs} and ε_{ucs} were obtained directly from the stress–strain curve. Young's modulus was measured at a stress level equal to 50% of the uniaxial compressive strength. The values of ε_{cd} were obtained using the AE technique based on the results obtained for AE using various

samples, such as hit rate, cumulative hits, energy rate, and cumulative energy. As examples, Figs. 8-11 show four typical AE results of four types of samples with dip angles increasing from 0° to 90° to obtain ε_{cd} . The figures show that σ_{cd} could be identified by the point at which the AE hits rate begins to accelerate dramatically accompanied by the occurrence of a high AE energy rate. The values of ε_{cd} were determined at 78%, 75%, 69%, and 71% of ε_{ucs} of samples 1, 4, 7, and 10, respectively. The ε_{cd} values of other samples were obtained by the same procedure. Thus, $\varepsilon_{ucs}/\varepsilon_{cd}$ was determined by calculating the AE technique of each sample using the values of ε_{ucs} and ε_{cd} that were obtained. In order to verify the validity of using the AE technique to determine $\varepsilon_{ucs}/\varepsilon_{cd}$, the values of $\varepsilon_{ucs}/\varepsilon_{cd}$ determined by volumetric strain reversal point, ε_{vr} , also were calculated, and they are shown in Table 2. The results of the comparison presented in Fig. 12 showed that the values and trends of $\varepsilon_{ucs}/\varepsilon_{cd}$ obtained from these two methods were consistent. Subsequently, the m values were calculated based on the $\varepsilon_{ucs}/\varepsilon_{cd}$ that was obtained by AE technique and Eq. (17). The value of the theoretical indicator (theoretical result), W_{ucs}/W_{cd} , of each sample can be obtained directly by comparing the m values that were determined with the quantitative theoretical values of m for different W_{ucs}/W_{cd} values that are listed in Table 1. The value of the experimental indicator (experimental result), W_{ucs}/W_{cd} , of each sample was the result of the integration of the stress-strain product.

Fig. 13 shows that the value of the theoretical indicator, W_{ucs}/W_{cd} , of each sample was essentially consistent with the values of the experimental indicator W_{ucs}/W_{cd} , which suggests that the strain energy indicator (Eq. (17)) with a reasonable m value can be used to predict the failure of brittle rock samples at the laboratory scale.

Fig. 8. (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive

strength for sample 1

Fig. 9. (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 4

Fig. 10. (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 7

Fig. 11. (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 10

Fig. 12. Values of $\varepsilon_{ucs}/\varepsilon_{cd}$ for the tested shale samples that were determined by the AE technique and the volumetric strain reversal point, ε_{vr}

Fig. 13. Theoretical results and experimental results for the values of the W_{ucs}/W_{cd} ratios of the shale samples that were tested

Table 2 Shale samples and uniaxial compression test results

4 Discussion

In order to better understand the relationships among heterogeneity, strain energy indicator, and other physical and mechanical properties of the shale samples that were tested, the effects of the bedding dip angle on the homogeneity index, m , and W_{ucs}/W_{cd} were discussed based on the test results; we also discussed the effects of the homogeneity of the samples and W_{ucs}/W_{cd} on the physical and mechanical properties.

4.1 Effect of the bedding dip angle on the homogeneity index, m

The comparison between the 12 shale specimens that were tested (Fig. 14a) indicated that the mean m values decreased as the dip angles increased from 0° to 60° , and the mean m values increase as the dip angles increased from 60° to 90° . As defined in Section 2.1, the smaller the m value, the more heterogeneous the sample is. Thus, the sample with the 60° dip angle was the most heterogeneous sample. The mean m value of samples 1, 2, and 3, which had 0° dip angles,

was 11.3; the mean m value of samples 4, 5, and 6, which had 30° dip angles, was 8.5; the mean m value of samples 7, 8, and 9, which had 60° dip angles, was 3.6; the mean m value of samples 10, 11, and 12, which had 90° dip angles, was 8.4. The larger the m value, the more homogeneous the sample is. Therefore, the sample with a 0° dip angle was more homogeneous than the sample with a 30° dip angle; the sample with a 30° dip angle was more homogeneous than the sample with a 60° dip angle; and the sample with a 90° dip angle was more homogeneous than the sample with a 60° dip angle.

4.2 Effect of the homogeneity of a sample on the physical and mechanical properties

(1) Fig. 14b indicates that the mean σ_{ucs} values of the samples that were tested with the same dip angle increased, while their mean m values were in ascending order, indicating that the more homogeneous samples had the larger mean σ_{ucs} values. The m values of samples 1, 2, and 3 with 0° dip angles were 11.5, 10.2, and 12.3, respectively. Their mean m value was 11.3. The corresponding values of σ_{ucs} were 187.9, 180.1, and 196.7 MPa, respectively. Thus, the mean σ_{ucs} value was 188.2 MPa. Following the same process, the mean m value and the mean σ_{ucs} value of samples 4, 5, and 6 with 30° dip angles were 8.5 and 165.3 MPa, respectively. The mean m value and the mean σ_{ucs} value of samples 7, 8, and 9 with 60° dip angles were 3.6 and 136.6 MPa, respectively. The mean m value and the mean σ_{ucs} value of samples 10, 11, and 12, whose dip angle was 90°, are 8.4 and 164.3 MPa, respectively. Therefore, the mean σ_{ucs} values of the samples with 60° dip angles, the samples with 90° dip angles, the samples with 30° dip angles, and the samples with 0° dip angles increased (136.6, 164.3, 165.3, and 188.3 MPa, respectively), while their mean m values were in ascending order (3.6, 8.4, 8.5, and 11.3, respectively).

(2) Following the same comparison process (Fig. 14c), the mean ε_{ucs} values of the samples

with 60° dip angles, the samples with 90° dip angles, the samples with 30° dip angles, and the samples with 0° dip angles also increased (0.487, 0.566, 0.567, and 0.657%, respectively), while their mean m values were in ascending order. This indicated that the more homogeneous the sample is, the larger the mean ε_{ucs} value will be.

(3) Fig. 14d shows that the larger the m value, the smaller the value of $\varepsilon_{ucs}/\varepsilon_{cd}$ is. The $\varepsilon_{ucs}/\varepsilon_{cd}$ values of samples 8, 9, 7, 10, 6, 5, 12, 4, 2, 11, 1, and 3 were 1.552, 1.490, 1.458, 1.408, 1.371, 1.355, 1.354, 1.333, 1.310, 1.304, 1.275, and 1.258, respectively. The corresponding m values were in ascending order (2.7, 3.7, 4.5, 6.3, 7.8, 8.4, 8.5, 9.3, 10.2, 10.4, 11.5, and 12.3, respectively). However, the mean $\varepsilon_{ucs}/\varepsilon_{cd}$ values of the samples with 60° dip angles, the samples with 90° dip angles, the samples with 30° dip angles, and the samples with 0° dip angles were 1.500, 1.355, 1.353, and 1.281, respectively. Their mean m values also were in ascending order, i.e., 3.6, 8.4, 8.5, and 11.3, respectively. These findings indicate that the more homogeneous the sample, the smaller the value of $\varepsilon_{ucs}/\varepsilon_{cd}$ will be.

(4) Table 2 indicates that the Young's modulus has a wide range of values for samples with the same bedding dip angle; for example, the values of Young's modulus for samples 1, 2, and 3 with 0° bedding dip angles were 41.3, 26.3, and 29.1 GPa, respectively. It is well known that a heterogeneous micro-structure is one of the most important factors that affect the microscopic cracking and the macroscopic non-linear stress-strain behaviors of rock masses. Although the tested shale samples were cored from the same shale block, the micro-structures of the samples with the same bedding dip angle may be different to some extent. Therefore, the microscopic cracking and the macroscopic stress-strain behaviors would be different, such as strains at different stress levels. However, Young's modulus was defined by ISRM as the ratio of the stress

change to the strain produced by the stress change at the stress level equal to 50% of the uniaxial compressive strength. The result measured by two mechanical property parameters would be amplified. Therefore, the value range of Young's modulus was large among the different samples.

4.3 Effect of the bedding dip angle on W_{ucs}/W_{cd}

The comparison shown in Fig. 15a indicates that the mean W_{ucs}/W_{cd} values increased when the dip angles increased from 0° to 60° , and they decreased when the dip angles increased from 60° to 90° . For example, the mean W_{ucs}/W_{cd} value of samples 1, 2, and 3, which had 0° dip angles, was 1.620; the mean W_{ucs}/W_{cd} value of samples 4, 5, and 6, which had 30° dip angles, was 1.794; the mean W_{ucs}/W_{cd} value of samples 7, 8, and 9, which had 60° dip angles, was 2.088, and the mean W_{ucs}/W_{cd} value of samples 10, 11, and 12, which had 90° dip angles, was 1.796.

4.4 Effect of W_{ucs}/W_{cd} on the physical and mechanical properties

(1) By comparison, the results shown in Fig. 15b indicate that the larger the mean W_{ucs}/W_{cd} value, the smaller the mean σ_{ucs} value becomes. The W_{ucs}/W_{cd} values of samples 1, 2, and 3 with 0° dip angles were 1.608, 1.690, and 1.563, respectively. Their mean W_{ucs}/W_{cd} value was 1.620. The corresponding σ_{ucs} values were 187.9, 180.1, and 196.7 MPa, respectively. Thus, the mean σ_{ucs} value was 188.2 MPa. Following the same process, the mean W_{ucs}/W_{cd} value and the mean σ_{ucs} value of samples 4, 5, and 6 with 30° dip angles were 1.794 and 165.3 MPa, respectively. The mean W_{ucs}/W_{cd} value and the mean σ_{ucs} value of samples 7, 8, and 9 with 60° dip angles were 2.088 and 136.6 MPa, respectively. The mean W_{ucs}/W_{cd} value and the mean σ_{ucs} value of samples 10, 11 and 12 with 90° dip angles were 1.796 and 164.3 MPa, respectively. Therefore, the mean σ_{ucs} values of the samples with 0° dip angles, the samples with 30° dip angles, the samples with 90° dip angles, and the samples with 60° dip angles decreased, i.e., 188.2, 165.3, 164.3 and 136.6 MPa,

respectively, while their mean $W_{\text{ucs}}/W_{\text{cd}}$ values were in ascending order, i.e., 1.620, 1.794, 1.796, and 2.088, respectively.

(2) Following the same comparison process (Fig. 15c), the mean ε_{ucs} values of the samples with 0° dip angles, the samples with 30° dip angles, the samples with 90° dip angles, and the samples with 60° dip angles also decreased (0.657, 0.567, 0.566, and 0.487%, respectively) while their mean $W_{\text{ucs}}/W_{\text{cd}}$ values were in ascending order, indicating that the larger the mean $W_{\text{ucs}}/W_{\text{cd}}$ value is, the smaller the mean ε_{ucs} value becomes.

(3) As shown in Fig. 15d, the larger the $W_{\text{ucs}}/W_{\text{cd}}$ value is, the larger the $\varepsilon_{\text{ucs}}/\varepsilon_{\text{cd}}$ value becomes. The $\varepsilon_{\text{ucs}}/\varepsilon_{\text{cd}}$ values of samples 3, 1, 11, 2, 4, 12, 5, 6, 10, 7, 9, and 8 were 1.258, 1.275, 1.305, 1.310, 1.333, 1.354, 1.356, 1.372, 1.408, 1.458, 1.490, and 1.552, respectively. Their corresponding $W_{\text{ucs}}/W_{\text{cd}}$ values were in ascending order (1.563, 1.608, 1.677, 1.690, 1.746, 1.794, 1.801, 1.835, 1.916, 2.014, 2.072, and 2.177, respectively). However, the mean $\varepsilon_{\text{ucs}}/\varepsilon_{\text{cd}}$ values of the samples with 0° dip angles, the samples with 30° dip angles, the samples with 90° dip angles, and the samples with 60° dip angles were 1.281, 1.354, 1.356, and 1.500, respectively. Their mean $W_{\text{ucs}}/W_{\text{cd}}$ values also were in ascending order (1.620, 1.794, 1.796, and 2.088, respectively).

Fig. 14. (a) m values of tested samples with different dip angles; (b) σ_{ucs} of tested samples with m values in ascending order; (c) ε_{ucs} of tested samples with m values in ascending order; (d) $\varepsilon_{\text{ucs}}/\varepsilon_{\text{cd}}$ of tested samples with m values in ascending order

Fig. 15. (a) $W_{\text{ucs}}/W_{\text{cd}}$ values of tested samples with different dip angles; (b) σ_{ucs} of tested samples with $W_{\text{ucs}}/W_{\text{cd}}$ values in ascending order; (c) ε_{ucs} of tested samples with $W_{\text{ucs}}/W_{\text{cd}}$ values in ascending order; (d) $\varepsilon_{\text{ucs}}/\varepsilon_{\text{cd}}$ of tested samples with $W_{\text{ucs}}/W_{\text{cd}}$ values in ascending order

5 Conclusions

Based on the two-dimensional renormalization group theory, an equation between the critical failure point P^* and the homogeneity index m was proposed by Pan et al. (2017), as well as a

constitutive formula of stress-strain curves in uniaxial compression tests was proposed by Chen et al. (2006), a quantitative strain energy indicator (W_{ucs}/W_{cd}) was proposed for predicting the brittle failure of laboratory-scale rock samples subjected to uniaxial pressure in this paper. The W_{ucs}/W_{cd} was a function of the homogeneity index, m , of the Weibull distribution. It established the relationship between the strain energy at the uniaxial compressive strength (W_{ucs}) and the strain at the crack damage stress (W_{cd}). W_{ucs}/W_{cd} decreased gradually as m increased from 1 to 14.5. The reliability of the strain energy indicator was verified by a series of uniaxial compression tests on 12 shale samples with different homogeneity, m , values induced by different bedding dip angles. The experimental results showed that the values of the proposed strain energy indicator (W_{ucs}/W_{cd}) of the tested samples were almost consistent with the experimental results. This suggested that the proposed strain energy indicator could be used to predict the brittle failure of samples of any type of rocks at the laboratory scale based on their m values.

Based on the discussion of the results, some conclusions related to the relationships among heterogeneity, strain energy indicator, and other physical and mechanical properties of the tested shale samples were apparent, and they are presented below:

- (1) The mean m values decreased as the dip angles increased from 0° to 60° , and they increases as the dip angles increased from 60° to 90° . Conversely, the mean W_{ucs}/W_{cd} values increased as the dip angles increased from 0° to 60° , and they decreased as the dip angles increased from 60° to 90° .
- (2) The more homogeneous the sample was, the larger the mean σ_{ucs} and the mean ε_{ucs} values were, but the smaller the mean $\varepsilon_{ucs}/\varepsilon_{cd}$ value was.
- (3) The larger the mean W_{ucs}/W_{cd} value was, the smaller the mean σ_{ucs} and mean ε_{ucs} values

were, but the larger the mean $\varepsilon_{ucs}/\varepsilon_{cd}$ value was.

- (4) The above conclusions were reached based on a limited number of samples, and they are applicable only to shale samples at the laboratory scale.

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Fig. 1 Stress-strain diagram showing the elements of crack development and relationship with typical creep rupture curve (modified version of Martin 1993)

Fig. 2. Illustration of the two-dimensional Renormalization Group (RG) theory model of a macroscopic fracture plane in a rock sample: (a) A shale rock sample with a macroscopic fracture plane; (b) Two-dimensional RG model with four cells per block of the macroscopic fracture plane. The numbers 1, 2, and 3 represent the cells of the first-order block, the second-order block, and the third-order block, respectively

Fig. 3. Distribution of Weibull failure strength for different homogeneity indexes, m

Fig. 4 Dependences of the failure probability for the $n+1$ cell, $P_1^{(n+1)}$, on the failure probability for the n cell, $P_1^{(n)}$, with different values of m . The critical failure point, P^* , gives the bifurcation point for the fracture plane. If $P_1 < P^*$, the solution iterates to 0 and no failure occurs. If $P_1 > P^*$, the solution iterates to 1 and the fracture plane has failed (modified version of Pan et al. 2017)

Fig. 5 Gradually decreasing relationship between W_{ucs}/W_{cd} and the homogeneity index, m , and the results of the tests of the shale samples

Fig. 6 (a) Plots of shale samples cored from the block of shale with different drilling angles; (b) Standard shale samples with four types of bedding dip angles made with sample end surfaces and the AE sensors' configuration

Fig. 7 (a) AE hits rate (per second); (b) AE energy rate (per second) versus the applied uniaxial stress normalized by the peak strength for the granite specimen AE-GR-2a-30-30, where (1) is crack closure, (2) is linear elastic deformation, (3) is micro-crack growth (stable crack growth), (4) is macro-crack growth (unstable crack growth) (modified version of Moradian et al. 2016)

(a)

(b)

Fig. 8 (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 1

(a)

(b)

Fig. 9 (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 4

(a)

(b)

Fig. 10 (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 7

(a)

(b)

Fig. 11 (a) AE hit rate (per second) and cumulative hits; (b) AE energy rate (per second) and cumulative energy versus the axial strain normalized by the strain at the uniaxial compressive strength for sample 10

Fig. 12 Values of $\epsilon_{ucs}/\epsilon_{cd}$ for the tested shale samples that were determined by the AE technique and the volumetric strain reversal point ϵ_{vr}

Fig. 13 Theoretical results and experimental results for the values of the W_{ucs} / W_{cd} ratios of the shale samples that were tested

Fig. 14 (a) m values of tested samples with different dip angles; (b) σ_{ucs} of tested samples with mean m values in ascending order; (c) ϵ_{ucs} of tested samples with mean m values in ascending order; (d) $\epsilon_{ucs}/\epsilon_{cd}$ of tested samples with mean m values in ascending order

(a)

(b)

(c)

(d)

Fig. 15 (a) W_{ucs}/W_{cd} values of tested samples with different dip angles; (b) σ_{ucs} of tested samples with W_{ucs}/W_{cd} values in ascending order; (c) ϵ_{ucs} of tested samples with W_{ucs}/W_{cd} values in ascending order; (d) $\epsilon_{ucs}/\epsilon_{cd}$ of tested samples with W_{ucs}/W_{cd} values in ascending order

Table 1 Theoretical relationship between $W_{\text{ucs}}/W_{\text{cd}}$ and the homogeneity index, m

m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$
1.1	2.410	2.0	2.291	2.9	2.151	3.8	2.064	4.7	2.002
1.2	2.419	2.1	2.272	3.0	2.139	3.9	2.055	4.8	1.995
1.3	2.416	2.2	2.255	3.1	2.129	4.0	2.048	4.9	1.989
1.4	2.404	2.3	2.237	3.2	2.118	4.1	2.041	5.0	1.984
1.5	2.389	2.4	2.221	3.3	2.107	4.2	2.034	5.1	1.978
1.6	2.370	2.5	2.205	3.4	2.098	4.3	2.027	5.2	1.973
1.7	2.350	2.6	2.191	3.5	2.089	4.4	2.021	5.3	1.967
1.8	2.330	2.7	2.177	3.6	2.080	4.5	2.014	5.4	1.962
1.9	2.310	2.8	2.164	3.7	2.072	4.6	2.007	5.5	1.957
m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$
5.6	1.952	6.5	1.905	7.4	1.857	8.3	1.806	9.2	1.752
5.7	1.946	6.6	1.900	7.5	1.852	8.4	1.801	9.3	1.746
5.8	1.941	6.7	1.895	7.6	1.847	8.5	1.794	9.4	1.740
5.9	1.936	6.8	1.889	7.7	1.841	8.6	1.789	9.5	1.734
6.0	1.931	6.9	1.884	7.8	1.835	8.7	1.783	9.6	1.728
6.1	1.926	7.0	1.879	7.9	1.830	8.8	1.776	9.7	1.721
6.2	1.920	7.1	1.874	8.0	1.824	8.9	1.771	9.8	1.715
6.3	1.916	7.2	1.868	8.1	1.818	9.0	1.765	9.9	1.709
6.4	1.910	7.3	1.863	8.2	1.813	9.1	1.758	10.0	1.703
m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$	m	$W_{\text{ucs}}/W_{\text{cd}}$
10.1	1.696	11.0	1.640	11.9	1.586	12.8	1.535	13.7	1.489
10.2	1.690	11.1	1.634	12.0	1.580	12.9	1.530	13.8	1.484
10.3	1.684	11.2	1.628	12.1	1.574	13.0	1.524	13.9	1.479
10.4	1.677	11.3	1.622	12.2	1.568	13.1	1.519	14.0	1.475
10.5	1.671	11.4	1.615	12.3	1.563	13.2	1.514	14.1	1.470
10.6	1.665	11.5	1.608	12.4	1.557	13.3	1.509	14.2	1.465
10.7	1.658	11.6	1.604	12.5	1.551	13.4	1.504	14.3	1.461
10.8	1.653	11.7	1.598	12.6	1.546	13.5	1.499	14.4	1.456
10.9	1.646	11.8	1.592	12.7	1.541	13.6	1.494	14.5	1.452

Table 2 Shale samples and uniaxial compression test results

Sample No.	Bedding dip angle °	σ_{ucs} MPa	ϵ_{ucs} %	E GPa	ϵ_{cd}	$\epsilon_{ucs}/\epsilon_{cd}$	m	W_{ucs}/W_{cd}	W_{ucs}/W_{cd}	$\epsilon_{ucs}/\epsilon_{cd}$
					AE technique			Theoretical results	Experimental results	ϵ_{vr}
1	0	187.9	0.593	41.3	0.465	1.275	11.5	1.608	1.741	1.308
2	0	180.1	0.676	26.3	0.516	1.310	10.2	1.690	1.862	1.299
3	0	196.7	0.702	29.1	0.558	1.258	12.3	1.563	1.656	1.272
4	30	176.2	0.528	36.6	0.396	1.333	9.3	1.746	1.844	1.386
5	30	158.2	0.594	34.5	0.438	1.356	8.4	1.801	1.876	1.328
6	30	161.6	0.579	29.3	0.422	1.372	7.8	1.835	2.013	1.363
7	60	128.1	0.468	29.6	0.321	1.458	4.5	2.014	2.239	1.399
8	60	136.3	0.481	27.7	0.310	1.552	2.7	2.177	2.569	1.462
9	60	145.4	0.511	38.5	0.343	1.490	3.7	2.072	2.367	1.427
10	90	164.2	0.576	32.2	0.409	1.408	6.3	1.916	2.133	1.398
11	90	173.4	0.608	25.2	0.466	1.305	10.4	1.677	1.842	1.366
12	90	155.3	0.513	37.1	0.379	1.354	8.5	1.794	1.953	1.331