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Optimal Quality and Quantity Provisions for Centralized vs. Decentralized Distribution: Market Size Uncertainty Effects*

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Abstract

We provide insights on how market size uncertainty affects the optimal quality (quantity) provision in distribution channels when consumers are heterogeneous in their willingness to pay for product quality. In this context, we denote the difference between the manufacturer's optimal quality (quantity) provision for a centralized channel and its optimal analog for a decentralized channel as the quality (quantity) differential. We find that market size uncertainty creates or increases the quantity differential, but it does not affect the differential's polarity. In contrast, market size uncertainty decreases the quality differential and it does so to the extent that, depending on the level of consumer heterogeneity, it reverses the differential's polarity. Moreover, we find that the higher the inherent uncertainty level, the more pronounced are the effects. We likewise find that the effects of market size uncertainty are amplified if the notion of consumer heterogeneity is replaced with retail-level competition.

Key words: OR in marketing, market uncertainty, newsvendor model, product quality, distribution channel

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1. Introduction

A manufacturer's optimal product quality provision as well as its correspondingly optimal product quantity provision depend on whether it sells its product in a centralized channel structure or in a decentralized channel structure. In this context, product quality refers to one or more product attributes that are such that all consumers agree on the direction of preference, as is the case for example with processing speed and storage capacity in a computer, with driving range per unit of fuel in an automobile, or with thread count in an apparel product. A decentralized channel structure refers to the situation in which a manufacturer exclusively sells its product to an independent retailer that, in turn, sells to end consumers; and a centralized channel refers to the alternative situation in which a manufacturer exclusively sells its product directly to end consumers. Examples of decentralized distribution channels include companies such as North Face and Benetton in the apparel industry and LG in the electronics industry. Examples of centralized distribution channels, in contrast, include such companies as the apparel manufacturers Zara and Gap and the appliance manufacturer Kenmore. Accordingly, scholars in marketing and economics have studied how channel structure affects a manufacturer's product quality provision. One standard result, for example, prescribes an optimal quality provision for a decentralized channel that is lower than its analog for a centralized channel when market size is deterministic and consumers are uniformly heterogeneous in their willingness to pay for product quality (see, for example, Economides 1999).

However, regardless of whether a manufacturer sells its product in a given market through a centralized or a decentralized channel structure, it typically is the case not only that individual consumers within the market differ in their willingness to pay per unit quality, but also that uncertainty surrounds the overall population size of the market. For example, again in the electronics and apparel industries, not only is it typical for consumers to differ vertically in their willingness to pay for product quality, but also is imprecise forecasting a hallmark, especially for new product introductions ranging from Apple's iPod innovations (Gibson 2004) to Microsoft's Surface RT (Reed 2013) to essentially any seasonal introduction in fashion and apparel (Fisher et al. 1994, He et al. 2008, Foster et al. 2004, Desai et al. 2007, Kuksov and Wang 2013, Ofek and Turut 2013). Indeed, due to the long lead times involved, manufacturers in the apparel industry including, for example, Zara (which sells its product directly to end consumers through a centralized channel structure) and Benetton (which sells its product to retailers through a decentralized channel structure) must decide on both their product quality and their production quantity several months before their selling seasons start (Ghemawat and Nueno 2006). Hence, market size uncertainty is another important factor to consider in addition to consumer heterogeneity in determining optimal quality and quantity provisions within a given distribution channel structure.

Thus, in this paper, we study how market size uncertainty impacts the standard effect that channel structure has on a manufacturer's optimal product quality and quantity provisions. In particular, we compare a manufacturer's optimal quality (quantity) provision given that the manufacturer sells its product exclusively through a decentralized channel structure to the manufacturer's optimal quality (quantity) provision given that it instead sells its product exclusively through a centralized channel structure, and we assess how market size uncertainty affects the direction and magnitude of that comparison. For purpose of comparison, we denote the difference between the manufacturer's optimal quality (quantity) provision in centralized distribution and its optimal quality (quantity) provision in decentralized distribution as the *channel differential* with respect to quality (quantity). For shorthand, we refer to this difference as the *quality differential* (*quantity differential*). Toward that purpose, we adopt a well-established game-theoretic model of consumer utility in which consumers are defined to be uniformly heterogeneous in their willingness-to-pay for product quality but we incorporate a random variable to capture market size uncertainty. We then determine the manufacturer's optimal quantity and quality provisions for centralized distribution, followed by its optimal decisions for decentralized distribution, analyze how market uncertainty affects the quantity and quality differentials, and develop insights accordingly. We also explore the scope of our results first by expanding our initial uniform definitions of consumer heterogeneity and market uncertainty to non-uniform dispersions, and second by replacing our definition of consumer heterogeneity with the notion of retail-level competition.

Our stylized analysis makes a technical contribution by incorporating uncertainty effects into the literature realm focused on assessing the impact of distribution channel structure on marketing strategy. In so doing, we establish that, in contrast to the standard quality-decreasing effect of decentralization that otherwise would prevail absent the uncertainty, market size uncertainty fuels an opposing quality-increasing effect that potentially reverses the polarity of the quality differential from positive to negative, depending on whether consumer heterogeneity is moderate or extreme. In particular, we find that if consumer heterogeneity is moderate, then market size uncertainty decreases the quality differential that would exist without uncertainty, but it does not affect the polarity of that differential. However, if consumer heterogeneity is either low or high, then market size uncertainty reverses the polarity of the differential such that the optimal quality provision in the decentralized channel becomes higher than that in the centralized channel. This happens because uncertainty produces the functional equivalent of an increased marginal cost of quality production per unit sold, which drives down the corresponding optimal quality provision in either distribution channel. But this downward pressure is mitigated in a decentralized channel relative to its centralized analog because double marginalization reduces product availability, thus translating

into a comparatively lower marginal cost of quality per unit sold. Accordingly, if consumer heterogeneity is either low or high, then market size uncertainty creates a negative quality differential that otherwise would not exist; but if consumer heterogeneity is intermediate, then market size uncertainty serves only to decrease the positive quality differential that otherwise would exist.

In a similar vein, we find that if consumer heterogeneity is comparatively low, then market size uncertainty creates a quantity differential that otherwise would not exist; and if consumer heterogeneity is comparatively high, then market size uncertainty increases the quantity differential that otherwise would exist. Moreover, we find, the higher is the market size uncertainty, the larger is the quantity differential. This happens because uncertainty creates a need to supplement a target market decision with a product availability decision to mitigate the risk of supply and demand mismatch that is introduced, and double marginalization amplifies that decision in a decentralized channel.

This paper fills a gap situated between two related streams of literature. The first stream, which is concentrated in the marketing and economics literature, is that which studies the polarity of a manufacturer's quality differential. See, for example, Jeuland and Shugan (1983), Villas-Boas (1998), Economides (1999), Xu (2009), Zhao et al. (2009), Shi et al. (2013), Chen et al. (2017) and Zhu and He (2017). Within this stream, Jeuland and Shugan (1983), Villas-Boas (1998), and Economides (1999) study market conditions that lead to a positive quality differential, while Xu (2009) and Shi et al. (2013) characterize market conditions without uncertainty that lead to a negative quality differential. We complement this literature stream by demonstrating how market uncertainty, rather than skewed heterogeneity among consumer willingness to pay, can drive the polarity of the quality differential negative.

The second stream, which is concentrated in the operations management literature, is that which studies the effects of uncertainty on the manufacturer's optimal pricing, quantity, or quality provision. See, for example, Gerchak and Mossman (1992), Federgruen and Heching (1999), Petruzzzi and Dada (1999), Lariviere and Porteus (2001), Bernstein and Federgruen (2004), Wang (2006), Geng (2007), Carlton and Dana (2008), He et al. (2008), Allon and Federgruen (2007, 2008, 2009), Qi et al. (2016), Rong et al. (2015), and Jiang and Tian (2016). Among these, the following are especially relevant to the analytical underpinnings of our model. Carlton and Dana (2008) apply a newsvendor framework to study the related problem of product line design and show that uncertainty can drive down a manufacturer's optimal product quality when consumer preferences are homogeneous. Geng (2007) complements Carlton and Dana (2008) by exploring the effects of uncertainty on a manufacturer's jointly optimal pricing and quality decisions for different aggregate demand functions; and Bernstein and Federgruen (2004) and Allon and Federgruen (2007, 2008, 2009) extend these analyses by considering competition effects. In a related vein, Rong et al. (2015)

study the effects of demand uncertainty on a manufacturer’s optimal product line design in a market consisting of two segments of consumers, each with different willingness to pay for product quality. However, unlike our paper, these studies do not consider any underlying effects of the distribution channel structure on the manufacturer’s optimal quality and quantity provisions.

One notable exception is Jerath et al. (2017), which studies the effects of production cost, price postponement, and distribution channel structure on a manufacturer’s optimal quantity and quality provisions. They show in their context that quality and inventory are substitutes, channel decentralization may lead to higher product quality, and a wholesale price contract cannot coordinate the channel. Our work thus complements theirs: We study the interplay between consumer heterogeneity, market size uncertainty, and ex ante pricing to examine how market uncertainty affects the manufacturer’s quality differential, to explore the generalizability of the effects to broader distributions of either consumer heterogeneity or market uncertainty, and to compare and contrast the effects to similar effects arising from retail-level competition.

This paper also indirectly relates to the vast literature on dual-channel supply chains in which product quality is defined to be either exogenously given (e.g., Chiang et al. 2003, Tsay and Agrawal 2004, Chiang and Monahan 2005, Cattani et al. 2006, Arya et al. 2007, Dumrongsiri et al. 2008, Huang and Swaminathan 2009, Cai 2010, Chiang 2010, and Hua et al. 2010), or endogenously chosen (e.g., Ha et al. 2015). Consider Dumrongsiri et al. (2008), for example. They examine the effects of marginal costs and demand variability on equilibrium prices and the manufacturer’s incentive to introduce a direct channel to supplement its indirect channel in a market in which (service) quality is exogenous. Ha et al. (2015), who study a manufacturer’s optimal quality decision when both centralized and decentralized channels exist simultaneously in a market without demand uncertainty, provide a useful synthesis of this literature. Our work, however, differs because we study a single-channel supply chain that is either centralized or decentralized to compare and contrast the results between the two alternatives.

The remainder of this paper is organized as follows. In Section 2, we develop a stylized model and analyze the optimal product quantity and quality decisions first for a centralized channel and second for a decentralized channel. In Section 3, we first analyze the impact of market size uncertainty on the quantity differential, and then we analyze the corresponding impact on the quality differential. In Section 4, we explore the scope of our results by generalizing our initial definitions of consumer heterogeneity and market uncertainty. Then, in Section 5, we consider as an extension the complementary case in which vertical consumer heterogeneity is replaced with horizontal heterogeneity by introducing retail-level competition. We conclude with Section 6. Proofs are in Appendix A.

2. The Model

Consider a manufacturer that chooses a product quality level q , produces that product at marginal cost $c = q^2$, and then sells it through a distribution channel that is either centralized or decentralized (e.g., Moorthy 1988, Desai 2001, Tyagi 2004, Shi et al. 2013). In the case of a centralized channel, the manufacturer chooses both the product quality q and the retail price p to sell the product directly to consumers. In the case of a decentralized channel, the manufacturer instead chooses the product quality q and the wholesale price w to sell the product to an independent retailer that, in turn, chooses the retail price p to sell the product to consumers.

To capture consumer heterogeneity, we allow consumers to be vertically heterogeneous with respect to their willingness to pay per unit quality (θ). In this section, we define θ to follow a uniform distribution over $[1 - a, 1 + a]$, where $0 < a \leq 1$, so that a parameterizes the level of consumer heterogeneity in the market such that the higher is a , the more heterogeneous are consumers relative to a constant mean willingness to pay.¹ Later, in Section 4, we generalize this definition of heterogeneity to consider the case in which consumer willingness to pay is not necessarily uniformly dispersed around a constant mean. For any given q and p , consumer utility is defined as $u = \theta q - p$. Each consumer with non-negative utility will want to purchase one unit of product. Accordingly, let $s = p/q$ denote the retail price per unit quality so that the proportion of heterogeneous consumers in the market that is targeted to visit the store can be written succinctly as $T(s) := \text{prob}\{\theta \geq s\} = \frac{1+a-\max\{s, 1-a\}}{2a}$. We refer to $T(s)$ as the *target market*.

To capture market uncertainty, we allow the market size to be a random variable, denoted by ξ , where $f(\cdot)$, $F(\cdot)$, and $\bar{F}(\cdot)$ denote the density function, cumulative distribution function, and complement cumulative distribution function, respectively, for ξ . Here, we define $F(\cdot)$ to be a uniform distribution over $[1 - b, 1 + b]$, where $0 < b \leq 1$, to isolate the effects of uncertainty by the parameter b such that the higher is b , the higher is the coefficient of variation associated with the market size.² However, in Section 4, we expand the definition of $F(\cdot)$ to include a more general class of distributions. Given that ξ is the overall size of the market whereas $T(s)$ is the fraction of consumers in that market who visit the store, consumer demand, which is random, can be written as $D = T(s)\xi$.

Because demand is unknown at the time of production, production quantity is a decision variable and hence any given distribution channel is exposed to the operational risk of supply leftovers versus supply shortages. Therefore, in the spirit of Petruzzi and Dada (1999), we define z such that the production quantity can be written as $T(s)z$, where z is a decision variable. Because each consumer

¹ Normalizing the mean willingness to pay equal to 1 in this section is without loss of generality. Derivations and results that follow scale naturally for arbitrary means. Derivation details are available upon request.

² Similarly, normalizing the expected market size to 1 in this section is without loss of generality.

in the target market can purchase a unit of product if and only if a unit is available to purchase, the number of units actually sold is constrained by the minimum of z and ξ : $Sales = T(s) \min\{z, \xi\}$. This means that although $T(s)$ represents the target market, product shortages are averted if and only if $z \geq \xi$. For this reason, we interpret z in our context as a proxy for *product availability*.

For any given distribution channel structure, we assume that the manufacturer as well as the retailer, if applicable, are risk neutral. Accordingly, define $R(z) = E[\min\{z, \xi\}]$ so that expected sales can be written as $E[Sales] = T(s)R(z)$.

In the case of a centralized channel, the manufacturer chooses product quality q , retail price p , and product availability z to maximize its expected profit, which is the difference between its expected sales revenue $pT(s)R(z)$ and its deterministic production cost $q^2T(s)z$. **Given the definition $s = p/q$, the expected profit for the centralized channel therefore can be written succinctly as:**

$$\pi_c = T(s)[sR(z) - qz]q, \quad (1)$$

where the subscript c denotes the case of a centralized channel. Notice from (1) that optimizing π_c jointly over q , p , and z is equivalent to maximizing π_c jointly over q , s , and z instead. Thus, for any given q , the manufacturer's decision problem can be thought of as a standard price-setting newsvendor problem in which s effectively is a decision variable in lieu of p .³ Maximizing (1) jointly over q , s , and z leads to the following lemma, which characterizes the manufacturer's optimal quality and quantity provisions for a centralized channel.

LEMMA 1. (i) *Optimal quality and quantity provision:* $q_c^* = \bar{F}(z_c^*)s_c^*$ and $Q_c^* = z_c^*T(s_c^*)$.
(ii) *Target market:* Let $\hat{a}_c = \frac{1}{5}$, then $T(s_c^*) = 1$ if $a \leq \hat{a}_c$ and $T(s_c^*) = \frac{1+a}{6a} < 1$ if $a > \hat{a}_c$.
(iii) *Product pricing and availability:* If $a \leq \hat{a}_c$, then $s_c^* = 1 - a$; if $a > \hat{a}_c$, then $s_c^* = \frac{2(1+a)}{3}$; and regardless of a , $z_c^* = \frac{b+1+2\sqrt{b^2-b+1}}{3}$.

Thus, the optimal policy prescription for a centralized channel boils down to a market coverage heterogeneity threshold that is *independent* of any market size uncertainty. Specifically, this threshold (namely, $\hat{a}_c = \frac{1}{5}$) dictates that, regardless of the level of uncertainty surrounding the size of the market (b), the manufacturer should price its product such that everyone in the market will visit the store with the desire to purchase the product if and only if the market is characterized by a low enough degree of heterogeneity ($a \leq \hat{a}_c$). In essence, this means that any effects of market size uncertainty in the centralized channel can be separated from the effects of consumer heterogeneity and thus isolated so that the uncertainty can be managed through the product availability decision (z) while heterogeneity is managed through the product pricing decision (s). As a consequence, because the manufacturer's optimal quantity provision moves in lockstep with product

³ The optimal value of p then can be recovered from the optimal s and q from the relation $p = sq$.

availability (z_c^*) whereas in contrast its optimal quality provision moves in lockstep with product scarcity ($\bar{F}(z_c^*)$), market size uncertainty in a centralized channel drives the manufacturer's optimal quantity and quality provisions in opposite directions.

In the case of a decentralized channel, the choice of quantity provision (and ownership of the operational risk associated with it) lies with the retailer. Accordingly, the manufacturer chooses the product quality q and wholesale price w to maximize the profit associated with producing and selling the quantity of units ordered by the retailer to stock for potential sale to consumers, $\pi_m = (w - q^2)T(s)z$, given that the retailer then responds by choosing the retail price p and product availability z to maximize its expected profit. Analogous to (1), the retailer's expected profit can be written as:

$$\pi_r = T(s)[sR(z) - \alpha z]q, \quad (2)$$

where $s = p/q$ as before and $\alpha := w/q$ similarly denotes the manufacturer's wholesale price per unit quality.

Given (2), we define $\hat{z}(\alpha)$ as the unique z that satisfies $\frac{(1+a)\bar{F}(z)}{2-\epsilon(z)} = \alpha$ and we define $\hat{s}(\alpha) = \frac{1+a}{2-\epsilon(\hat{z}(\alpha))}$, where $\epsilon(z) := \frac{dE[\text{Sales}]/E[\text{Sales}]}{dz/z} = z\bar{F}'(z)/\bar{F}(z)$ denotes the z -elasticity of expected sales and is a decreasing function of z (see, for example, Petruzzi et al. 2009). As a result, for any given (w, q) , the retailer's optimal response functions can be characterized uniquely as $s_d = \max\{\hat{s}(\alpha), 1 - a\}$ and $z_d = \bar{F}^{-1}(\frac{\alpha}{s_d})$ where the subscript d denotes the case of a decentralized channel. Hence, the manufacturer's profit in a decentralized channel can be written as

$$\pi_m = T(s_d)z_d(\alpha - q)q \quad (3)$$

where s_d and z_d depend on the manufacturer's choice of α but not q . Thus, maximizing π_m jointly over w and q is equivalent to maximizing it jointly over α and q instead. This leads to the following optimal solution for a decentralized channel.

LEMMA 2. (i) *Optimal quality and quantity provision:* $q_d^* = \frac{1}{2}\bar{F}(z_d^*)s_d^*$ and $Q_d^* = z_d^*T(s_d^*)$.
(ii) *Target market:* Let $\hat{a}_d = \frac{1-\epsilon(\hat{z}_d^*)}{3-\epsilon(\hat{z}_d^*)} < \frac{1}{5}$, then $T(s_d^*) = 1$ if $a \leq \hat{a}_d$ and $T(s_d^*) = \frac{1+a}{2a} \frac{1-\epsilon(\hat{z}_d^*)}{2-\epsilon(\hat{z}_d^*)} < 1$ if $a > \hat{a}_d$, where $\hat{z}_d^*$ is the unique value of z that maximizes $\hat{\pi}_m(z) = \frac{z[1-\epsilon(z)]\bar{F}(z)^2}{[2-\epsilon(z)]^3}$.
(iii) *Product pricing and availability:* If $a \leq \hat{a}_d$, then $s_d^* = 1 - a$ and $z_d^* = \max\{\frac{1+b}{3}, \epsilon^{-1}(\frac{1-3a}{1-a})\}$; if $a > \hat{a}_d$, then $s_d^* = \frac{1+a}{2-\epsilon(\hat{z}_d^*)}$ and $z_d^* = \hat{z}_d^*$.

By comparing Lemma 2 to Lemma 1, notice that although the optimal policy prescription for a decentralized channel boils down to a market coverage heterogeneity threshold (namely, $\hat{a}_d$) in the same spirit as its analog for a centralized channel, in stark contrast to the centralized case, the heterogeneity threshold in the decentralized case depends on the level of uncertainty surrounding

the size of the market (b). Moreover, notice that the heterogeneity threshold $\hat{a}_d$ is increasing in b . Intuitively, the more uncertain is the size of the market, the more important is the market coverage to the manufacturer, hence the less is the homogeneity required among consumers to justify targeting full market coverage. In a related vein, also in stark contrast to the centralized case, the effects of market size uncertainty in a decentralized channel cannot be isolated and managed independently of the effects of consumer heterogeneity. Specifically, in a decentralized channel, if consumer heterogeneity is low enough for full coverage to be optimal (i.e., if $a \leq \hat{a}_d$), then although uncertainty can be managed using nothing other than product availability as the lever, both pricing and availability need to be used in combination to respond effectively to the heterogeneity; and if consumer heterogeneity is high enough for partial coverage to be optimal (i.e., if $a > \hat{a}_d$), then although the heterogeneity can be managed using only product pricing as a lever, it nevertheless would take the combination of pricing and availability to respond effectively to the uncertainty.

All told, this means that market size uncertainty in a decentralized channel not only induces a product availability response that pushes the channel's optimal quality and quantity provisions in opposite directions (which, recall, also is the case in a centralized channel), but also potentially induces a product pricing response that further drives the channel's optimal quality and quantity provisions in opposite directions (which, recall, is not the case in a centralized channel). Intuitively, this suggests that although market size uncertainty undoubtedly affects any quality differential between the centralized and decentralized channels, as well as any quantity differential between the channels, it is not obvious to what extent those effects are in magnitude only or in both magnitude and polarity. We therefore proceed to study the issue in Section 3 by using Lemmas 1 and 2 to define $Q_c^* - Q_d^* = T(s_c^*)z_c^* - T(s_d^*)z_d^*$ and $q_c^* - q_d^* = \bar{F}(z_c^*)s_c^* - \frac{1}{2}\bar{F}(z_d^*)s_d^*$, where the former difference denotes the quantity differential affected by decentralization and the latter difference denotes the corresponding quality differential, and then by analyzing how market size uncertainty (b) affects each of these differences.

3. The Effects of Market Size Uncertainty

In this section, we first analyze the impact of market size uncertainty on the manufacturer's quantity differential. Then, we analyze the corresponding impact on its quality differential. To set the stage for those analyses, recall from Section 2 that the two heterogeneity thresholds $\hat{a}_c = \frac{1}{5}$ and $\hat{a}_d < \hat{a}_c$ imply three distinct regions defined by the optimal target market strategy of the centralized and decentralized channels: The region characterized by lower levels of consumer heterogeneity (namely, $a \leq \hat{a}_d$) corresponds to the region in which targeting full market coverage is optimal regardless of whether the channel is centralized or decentralized, the region characterized by relatively intermediate levels of consumer heterogeneity (namely, $\hat{a}_d < a \leq \hat{a}_c$) corresponds to the

region in which targeting full market coverage is optimal for the centralized channel but targeting partial market coverage is optimal for the decentralized channel, and the region characterized by relatively higher levels of consumer heterogeneity (namely, $a > \hat{a}_c$) corresponds to the region in which targeting partial coverage is optimal for both channels.

PROPOSITION 1. (i) For any given b , $Q_c^* > Q_d^*$. More precisely, $T(s_c^*) \geq T(s_d^*)$ but $z_c^* > z_d^*$.
(ii) Moreover, $Q_c^* - Q_d^*$ increases as b increases.

To frame the subtle impact of uncertainty highlighted by Proposition 1, consider for benchmark comparison if there were no market uncertainty, which is the standard base case in the literature (see, for example, Jeuland and Shugan 1983, Villas-Boas 1998 and Economides 1999). In the context of our model, this corresponds to the asymptotic case in which $b := 0$. From (1) to (3), if $b = 0$, then $Q_c^* > Q_d^*$ would continue to be true for $a > \hat{a}_d$ because $T(s_c^*) > T(s_d^*)$, but $Q_c^* = Q_d^*$ would be true for $a \leq \hat{a}_d$ because $T(s_c^*) = T(s_d^*) = 1$. Thus, qualitatively, Proposition 1 demonstrates that market size uncertainty creates a quantity differential between centralized and decentralized distribution where there otherwise would not be one, namely where markets are characterized by comparatively high levels of consumer *homogeneity* (i.e., where $a \leq \hat{a}_d$).

This is true because uncertainty creates the need to supplement the target market coverage decision with a product availability decision (z) to mitigate the risk of supply and demand mismatch that is introduced. Everything else being equal, this produces a strong quantity differential between the centralized and decentralized channels (namely, $z_c^* > z_d^*$) that would not exist without uncertainty while preserving the weak quantity differential (namely, $T(s_c^*) \geq T(s_d^*)$) that otherwise would exist even without uncertainty. For insight, note that $z_c^* > z_d^*$ would follow directly from double marginalization if price and quality were exogenous (see, for example, Lariviere and Porteus 2001) because double marginalization effectively increases inventory stocking costs and, ceteris paribus, increased stocking costs drive down product availability. Proposition 1(i) therefore extrapolates conventional wisdom by establishing that the decreasing effect of decentralization on a channel's optimal quantity provision remains true even under the case of endogenous price and quality. In essence, market size uncertainty amplifies the double marginalization effect that decentralization has on quantity and, as Proposition 1(ii) highlights and Figure 1 illustrates, the greater is the uncertainty, the larger is the effect.

Although market size uncertainty produces unambiguous effects on both the polarity and magnitude of the manufacturer's quantity differential, Proposition 2 next demonstrates that such is not the case with regard to the manufacturer's quality differential.

PROPOSITION 2. (i) Let $\underline{a} = \frac{2-3(1+b)\hat{q}_c^*}{21(1+b)\hat{q}_c^*-26}$ and $\bar{a} = \frac{3\hat{q}_c^*-2\hat{q}_d^*}{3\hat{q}_c^*+2\hat{q}_d^*}$, where $\hat{q}_c^* = \frac{2(1+b-z_c^*)}{3b}$ and $\hat{q}_d^* = \frac{\bar{F}(\hat{z}_d^*)}{2-\epsilon(\hat{z}_d^*)}$. Then, for any given $b \in (0, 1)$, $q_c^* > q_d^*$ if $\underline{a} < a < \bar{a}$, but $q_c^* < q_d^*$ if either $0 < a < \underline{a}$ or $\bar{a} < a \leq 1$. Also,

Figure 1: The effect of market size uncertainty on the quantity differential

$$0 < \underline{a} < \hat{a}_d < \bar{a} < \hat{a}_c.$$

(ii) Moreover, $\underline{a}$ is increasing in b .

Thus, unlike its effect on the manufacturer's quantity differential, market size uncertainty could result in either a positive or a negative quality differential, depending on whether consumer heterogeneity is moderate or extreme. To probe deeper, consider again for benchmark comparison the standard base case defined by no uncertainty: From (1) to (3), if $b = 0$, then $q_c^* \geq q_d^*$ would be true for all a , which is consistent with the quality-decreasing effect of decentralization highlighted in seminal works such as Jeuland and Shugan (1983), Villas-Boas (1998) and Economides (1999). In particular, $b = 0$ would imply $q_c^* = \frac{1-a}{2} > q_d^* = \frac{1-3a}{2}$ for $a < \hat{a}_d$, $q_c^* = \frac{1-a}{2} > q_d^* = \frac{1+a}{3}$ for $\hat{a}_d \leq a < \hat{a}_c$, and $q_c^* = q_d^* = \frac{1+a}{3}$ for $a \geq \hat{a}_c$. Thus, Proposition 2(i) demonstrates that market size uncertainty qualitatively affects the manufacturer's quality differential by invoking a polarity change in that differential for markets characterized either by comparatively high levels of consumer heterogeneity (i.e., $a > \bar{a}$) or by comparatively high levels of consumer homogeneity (i.e., $a < \underline{a}$). Figure 2 provides an illustration.

The intuition is as follows. By virtue of introducing the operational risk of supply and demand mismatch, market size uncertainty creates a reality in which not all units manufactured in a given distribution channel are necessarily destined to be sold in that channel. Thus, compared to a deterministic market baseline, uncertainty produces the functional equivalent of an increased marginal cost of quality production *per unit sold*. The increased marginal cost of quality, in turn, drives down the corresponding optimal quality provision. Thus, on the one hand, any given distribution channel serving a heterogeneous market of uncertain size needs to provide a lower quality product than it otherwise could provide if the size of that market instead were deterministic. But, on the other hand, the pressure from uncertainty driving quality downward is mitigated in a decentralized

Figure 2: The quality differential for a given level of market size uncertainty

channel relative to its magnitude in a centralized channel because the comparatively lower product availability stemming from double marginalization implies a comparatively lower risk of leftovers, which in turn translates into a comparatively lower marginal cost of quality per unit sold. All told, this means that uncertainty works to push the quality provision in the centralized channel lower than its decentralized analog, *ceteris paribus*.

For markets characterized by higher levels of heterogeneity, the absence of uncertainty would imply the absence of a quality differential because the optimal response to double marginalization in such markets is to adjust the selling price without adjusting the quality provision. Therefore, any uncertainty surrounding the size of such markets necessarily would produce a negative quality differential. For markets characterized by higher levels of *homogeneity*, in contrast, the optimal response to double marginalization requires an adjustment to the quality provision because a selling price adjustment loses its efficacy if a market is fully covered such that, absent uncertainty, such markets imply a positive quality differential inversely related to the level of homogeneity. Therefore, any uncertainty surrounding the size of relatively more homogenous markets serves primarily as a countereffect to the positive quality differential that otherwise would prevail such that any given level of market size uncertainty ultimately yields a positive quality differential of reduced magnitude for comparatively lower levels of homogeneity yet converts a positive quality differential into a negative quality differential for comparatively higher levels of uncertainty. Moreover, as Proposition 2(ii) highlights, the greater is the uncertainty, the larger is the region of homogeneity that converts from a positive quality differential to a negative one so that what remains overall is a relatively narrow band of intermediate levels of heterogeneity for which a positive quality differential endures under uncertainty. Figure 3 provides an illustration.

Nevertheless, Propositions 1 and 2 are derived from a definition of consumer heterogeneity that imposes a uniform dispersion around a fixed mean willingness to pay and from a similar definition

Figure 3: The effect of market size uncertainty on the polarity of the quality differential

of market uncertainty that imposes a uniform dispersion around a fixed mean market size. We therefore explore more fully the scope of this conclusion next, in Section 4, by generalizing the willingness-to-pay and market size distributions such that their respective means are not necessarily constant for a given parameterization.

4. Non-uniform Dispersions

In this section, we extend the analytical results and qualitative insights from Sections 2-3 by expanding the scope of the underlying technical properties that drive them. In particular, we establish the existence and uniqueness of the jointly optimal pricing, stocking, and quality solution for the centralized channel and the corresponding equilibrium solution for the decentralized channel for more general classes of distributions characterizing market size uncertainty, on the one hand, and consumer heterogeneity, on the other hand.

To generalize the notion of market size uncertainty, in this section we expand the definition of $\xi \sim F(\cdot)$ to let F denote any distribution that satisfies the property of increasing generalized failure rate (IGFR). That is, we define F such that $g_\xi(x) := xf(x)/\bar{F}(x)$ is non-decreasing in x . This property is satisfied for a preponderance of theoretical distributions applied in related modeling contexts, including but not limited to, the uniform, power, Weibull, gamma, normal, and lognormal distributions as well as truncated variants of those distributions (Lariviere and Porteus 2001, Lariviere 2006). Similarly, to generalize the notion of consumer heterogeneity, in this section, we expand the definition of θ to let $\theta \sim H(\cdot)$, where H is any distribution that satisfies the following two properties: (i) $g_\theta(x) := xh(x)/[1 - H(x)]$ is non-decreasing in x , and (ii) $r(x) := g_\theta(x)/\eta(x)$, where $\eta(x) := xg'_\theta(x)/g_\theta(x)$, is non-decreasing in x . Given that $g_\theta(x)$ denotes the generalized failure rate of θ , note that $g_\theta(x)$ also represents demand elasticity. Thus, the two properties stipulated here on H are the functional equivalent of stipulating not only that demand elasticity is a non-decreasing function, but also that the ratio of demand elasticity to the elasticity of that demand

elasticity is non-decreasing as well. These properties are satisfied, for example, for the power and Weibull distributions, among others. Thus, this class of distributions is robust in the sense that it provides opportunity to model a wide range of consumer willingness-to-pay dispersions throughout a given market.

Given that, in this section, $\theta \sim H(\cdot)$ replaces $\theta \sim U[1 - a, 1 + a]$ from Sections 2-3 as the willingness-to-pay distribution of the consumer market, $T(s) := 1 - H(s)$ correspondingly now denotes the manufacturer's resulting target market for any given value of s , regardless of whether the channel structure is centralized or decentralized. Thus, as in Section 3, $T(s)\xi$ denotes the overall consumer demand, and $T(s)z$ and $T(s)R(z)$ denote the production quantity and expected sales, respectively. Accordingly, as in (1), the manufacturer's profit in a centralized channel for any given s , z , and q is given by $\pi_c = T(s)[sR(z) - qz]q$; and analogous to (3), the manufacturer's profit in a decentralized channel for any given α and q is given by $\pi_m = T(s(\alpha))z(\alpha)[\alpha - q]q$, where $s(\alpha)$ and $z(\alpha)$ are the unique values of s and z , respectively, that jointly maximize $\pi_r = T(s)[sR(z) - \alpha z]q$. Accordingly, let (s_c^*, z_c^*, q_c^*) denote the jointly optimal pricing, stocking, and quality solution for the centralized channel, and let (s_d^*, z_d^*, q_d^*) denote the corresponding equilibrium solution for the decentralized channel. Then the following proposition provides the key technical building blocks for extending the results and insights from Section 3.

PROPOSITION 3. (i) *In the centralized channel, the jointly optimal solution (s_c^*, z_c^*, q_c^*) is characterized uniquely and explicitly as follows: $s_c^* = g_\theta^{-1}(2)$, $z_c^* = \epsilon^{-1}(0.5)$, and $q_c^* = \bar{F}(z_c^*)s_c^*$.*

(ii) *In the decentralized channel, the equilibrium solution (s_d^*, z_d^*, q_d^*) is characterized uniquely and implicitly as follows: $g_\theta(s_d^*) = \frac{1}{1 - \epsilon(z_d^*)}$, $r(s_d^*) = \frac{[1 - \epsilon(z_d^*)][2g_\xi(z_d^*) - 1]}{\epsilon(z_d^*)[2\epsilon(z_d^*) - 1][g_\xi(z_d^*) + \epsilon(z_d^*) - 1]}$, and $q_d^* = \frac{\bar{F}(z_d^*)s_d^*}{2}$.*

Proposition 3 is an extension that effectively expands the scope of Lemmas 1-2. Perhaps most notably, Proposition 3 highlights the broad applicability of the key structural property from Section 2 indicating that the effects of consumer heterogeneity and market size uncertainty ultimately are separable in the centralized channel, but comingled in the decentralized channel. In a similar vein, Proposition 4 likewise expands the scope of Proposition 1 while partially expanding the scope of Proposition 2.

PROPOSITION 4. (i) *Given $\theta \sim H(\cdot)$ and $\xi \sim F(\cdot)$, $z_c^* > z_d^*$, $T(s_c^*) > T(s_d^*)$, and, thus, $Q_c^* > Q_d^*$.*

(ii) *If θ and ξ follow a uniform and power distribution, respectively, so that $H(x) = \frac{x-1+a}{2a}$ for $0 < a < 1$ and $F(x) = x^b$ for $b > 0$, then $q_c^* \geq q_d^*$ if and only if $a \leq \tilde{a} := \frac{3\hat{q}_c^* - 2\hat{q}_d^*}{3\hat{q}_c^* + 2\hat{q}_d^*}$, where $\hat{q}_c^* = \frac{4b}{3(2b+1)}$, $\hat{q}_d^* = \frac{\bar{F}(\hat{z}_d^*)}{2 - \epsilon(\hat{z}_d^*)}$ and $\hat{z}_d^*$ is the unique value of z that maximizes $\hat{\pi}_m(z) = \frac{z[1 - \epsilon(z)]\bar{F}(z)^2}{[2 - \epsilon(z)]^3}$.*

Thus, the qualitative insights derived from Propositions 1 and 2 do not depend on either market size uncertainty or consumer heterogeneity being characterized by a uniform distribution.

Indeed, Proposition 4(i) analytically extends Proposition 1 without limitation to the generalized heterogeneity-uncertainty model of this section. And, Proposition 4(ii) similarly indicates that the qualitative results of Proposition 2, as depicted by Figure 3, apply more generally although the analytical scope of that generalization is limited by intractability. Thus, we note here that analogs of Proposition 4(ii) and Figure 3 can be reproduced numerically to similar effect for other IGFR specifications of F parametrized by b including, but not limited to the Weibull, Gamma, and lognormal distributions. (See Appendix B for details.)

Accordingly, to close this section, we provide Figure 4 as a representative extension of Proposition 4. In particular, we generate Figure 4 by applying Proposition 3 for the 250,000 parameter combinations defined by the unit square $(a, b) \in [0, 1]$ using an increment of 0.002 both for $0 < a < 1$ and for $0 < b < 1$ to compute the quality differential numerically for the general case in which consumer heterogeneity is characterized by $H(x) = x^{k(a)}$ for $x \in [0, 1]$, where $k(a) = \frac{1}{2a}$ for $0 < a \leq \frac{1}{2}$ and $k(a) = 2(1 - a)$ for $\frac{1}{2} < a < 1$, and in which market size uncertainty is similarly characterized by $F(y) = y^{k(b)}$ for $y \in [0, 1]$, where $k(b) = \frac{1}{2b}$ for $0 < b \leq \frac{1}{2}$ and $k(b) = 2(1 - b)$ for $\frac{1}{2} < b < 1$. Notice that, in this representative case, the parameter b reflects the level of uncertainty surrounding the market size because a higher b indicates a higher coefficient of variation for the market size, whereas the parameter a reflects a homogenization of consumers toward one extreme or the other because a lower a indicates a higher density of consumers toward a higher willingness to pay while a higher a denotes a higher density of consumers toward a lower willingness to pay.

Figure 4: Quality differential for the representative case in which both consumer heterogeneity and market size uncertainty follow power distributions

Figure 4 illustrates that, qualitatively similar to Figure 3, the greater is the uncertainty, the larger is the region of homogeneity that converts from a positive quality differential to a negative one. Thus, we conclude that the effect of market size uncertainty on the manufacturer's quality differential is consistent regardless of whether that homogenization reflects a symmetric shift toward

the center (re: Section 3) or a skewed shift toward an extreme willingness to pay per unit quality (re: Section 4).

5. Retail Competition

In Sections 3-4, we analyzed the effect of market size uncertainty on the difference between the manufacturer's optimal quality (quantity) provision for centralized distribution and its optimal quality (quantity) provision for decentralized distribution. As an extension, we explore in this section the extent to which that effect prevails if retail-level competition replaced differentiated willingness to pay as the defining feature complementing a market of uncertain size. Accordingly, let $\theta = 1$ identically for all consumers, but assume that consumers are uniformly located on a Hotelling line ($x \in [0, 1]$) characterized by a retail store located at either end (at $x = 0$ and at $x = 1$) (Hotelling 1929). The consumer located at x must incur traveling cost tx to visit the store located at $x = 0$ or $t(1 - x)$ to visit the store located at $x = 1$, where t is the traveling cost per unit distance. Thus, the parameter t represents the level of horizontal consumer heterogeneity (in contrast to the vertical consumer heterogeneity in previous sections) or the intensity of competition such that the lower is t the more homogeneous are consumers and the more intense is competition. As in Section 2, we assume that the uncertain market size ξ follows a uniform distribution over $[1 - b, 1 + b]$, however, the technical and qualitative results of this section continue to hold if market size uncertainty is instead characterized by a Weibull, Gamma, or lognormal distribution, among others (see Appendix B). In this context, we define a centralized channel to mean that the manufacturer owns both retail stores situated at either end of the Hotelling line, and we define a decentralized channel to mean that each retail store is independently owned and operated.

In the centralized channel, the manufacturer chooses its product quality q as well as determines the price p and product availability z for its stores. Then, given q , p , and z , the consumer located at x will choose which store to visit, if either, by comparing its expected utility of purchase $R(z)(q - p)$ to its travel costs tx and $t(1 - x)$. Note that $R(z) = E(\text{Sales})/E(\text{Demand})$ represents the probability that the product is available when a consumer visits a store (e.g., Bernstein and Federgruen 2004, Çil and Larivière 2013). Accordingly, the manufacturer's resulting target market is $T = 1$ if $R(z)(q - p) \geq t/2$ and $T = 2R(z)(q - p)/t$ if $R(z)(q - p) < t/2$. Correspondingly, the manufacturer's expected profit function, analogous to (1), for any given z , q and p is given by $\pi_c^c = T[pR(z) - q^2z]$, where the superscript c denotes the case of competition. Maximizing π_c^c jointly over z , q and p implies a unique optimal quality q_c^{c*} and a unique optimal quantity Q_c^{c*} (see Lemma 5 in Appendix A for derivation details).

In the decentralized channel, the manufacturer first chooses its product quality q and wholesale price w , and then the two retailers compete for consumers on price p and product availability z

in a simultaneous game. Accordingly, the resulting target markets for retailer 1 and retailer 2 are $T_1 = \frac{1}{2} + \frac{R(z_1)(q-p_1)-R(z_2)(q-p_2)}{2t}$ and $T_2 = \frac{1}{2} - \frac{R(z_1)(q-p_1)-R(z_2)(q-p_2)}{2t}$ if $R(z_1)(q-p_1) + R(z_2)(q-p_2) \geq t$ and $T_1 = \frac{R(z_1)(q-p_1)}{t}$ and $T_2 = \frac{R(z_2)(q-p_2)}{t}$ otherwise, respectively. Hence, the manufacturer's total market coverage is $T_d^c = T_1 + T_2$ and its aggregate demand is $T_1 z_1 + T_2 z_2$. This implies that the manufacturer's profit and the retailers' expected profits are given by $\pi_m^c = [T_1 z_1 + T_2 z_2](\alpha - q)q$, $\pi_{r1} = T_1[p_1 R(z_1) - w z_1]$ and $\pi_{r2} = T_2[p_2 R(z_2) - w z_2]$, respectively. Using backward induction to obtain the two competing retailers' best response functions $(z_1(w, q), p_1(w, q), z_2(w, q), p_2(w, q))$ and then maximizing the manufacturer's profit jointly over w and q leads to a unique optimal quality q_d^{c*} and a unique optimal quantity Q_d^{c*} (see Lemma 5 in Appendix A for derivation details). Comparing Q_c^{c*} to Q_d^{c*} and q_c^{c*} to q_d^{c*} hence yields the following.

PROPOSITION 5. (i) For any given $b \in (0, 1)$, $Q_c^{c*} > Q_d^{c*}$. More precisely, $T_c^{c*} \geq T_d^{c*}$ but $z_c^{c*} > z_d^{c*}$. (ii) For any given $b \in (0, 1)$, $q_c^{c*} < q_d^{c*}$. Moreover, $\frac{d}{dt}(q_d^{c*} - q_c^{c*}) \leq 0$.

Note that for the benchmark case in which there is no uncertainty, the manufacturer's optimal quality provision in the centralized channel should be no lower than its analog in the decentralized channel (Shi et al. 2013). Proposition 5, in contrast, prescribes the opposite by indicating that market size uncertainty invokes a polarity change in the quality differential. Thus, retail-level competition accentuates the effects of market size uncertainty from Sections 3-4 in the sense that, if competing retailers replaced heterogeneous willingness to pay, then market size uncertainty not only would produce an unambiguously positive quantity differential, but also would it likewise produce an unambiguously negative quality differential. Indeed, according to Proposition 5(ii), the more intense is the competition between the retailers, the larger is that differential. Figure 5 helps illustrate this. Therefore, all told, we find that increased retail competition is qualitatively similar to increased consumer homogenization toward either a constant mean (re: Section 3) or an extreme willingness to pay (re: Section 4).

6. Conclusion

This paper provides insights into how market size uncertainty affects the effect of channel distribution structure on a manufacturer's optimal quality and quantity provisions for a market defined by consumers who are heterogeneous in their willingness to pay for product quality. In comparing the difference between the optimal quality (quantity) provision for centralized versus decentralized distribution, we find, first, that market size uncertainty creates or amplifies a positive quantity differential, and the higher is the inherent uncertainty level, generally the greater is the differential. And we find, second, that market size uncertainty decreases the quality differential such that it reverses its polarity for all but a relatively narrow band of consumer heterogeneity, thereby

Figure 5: The effect of market size uncertainty on quality differential with retail competition

demonstrating that market size uncertainty alone, rather than skewed heterogeneity among consumer willingness to pay, can drive a manufacturer's optimal quality provision for a decentralized channel to be higher than its optimal quality provision for a centralized channel, *ceteris paribus*. This is a particularly important managerial implication because it is in direct contrast to the well-known quality-decreasing effect of decentralization that consumer homogenization would drive absent any market size uncertainty. Moreover, we find that the effect of market size uncertainty on both the quantity differential and the quality differential continues to hold either when the uniform definitions of consumer heterogeneity and market size uncertainty are expanded to non-uniform dispersions or when consumer heterogeneity is replaced by retail-level competition.

From a modeling standpoint, our analysis and results derive analytically because we are able to reduce the manufacturer's expected profit to a function of a single variable regardless of the given distribution channel structure. This in turn facilitates broad-based comparisons of the manufacturer's optimal quality and quantity provisions across the different channel structures through explicit or implicit functional analysis. Accordingly, one natural extension would be to test the robustness of the results and insights of this paper by developing methodology to address the core research questions of this paper for the complementary cases in which consumers in a given market of uncertain size are defined either to be horizontally heterogeneous or to be both vertically and horizontally heterogeneous. Other potentially viable research directions would be to consider the combined effects of vertical and horizontal heterogeneity under competition or a product line with multiple products to assess cannibalization effects.

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Appendix A: Proofs and Derivations

Proof of Lemma 1 For the centralized channel, the manufacturer’s optimization problem is $\max \pi_c(q, z, s)$ s.t. $s \geq 1 - a$, where $\pi_c(q, z, s)$ is from (1) and can be written explicitly as $\pi_c = \frac{1+a-s}{2a} [sR(z) - qz]q$. Notice that $\pi_c(q, z, s)$ is concave in q . Accordingly, let $q_c^*(z, s)$ denote the unique value of q that satisfies $\frac{\partial \pi_c(q, z, s)}{\partial q} = 0$. Then, $q_c^*(z, s) = \frac{sR(z)}{2z}$ is the manufacturer’s optimal q for any given z and s . Substituting $q_c^*(z, s)$ for q in $\pi_c(q, z, s)$ thus reduces $\pi_c(q, z, s)$ to a function of two variables. Accordingly, let $\pi_c(z, s) = \pi_c(q_c^*(z, s), z, s) = \frac{(1+a-s)s^2}{8a} \frac{R(z)^2}{z}$. Then, the manufacturer’s optimization problem reduces to solving two separable subproblems, where one subproblem is $\max \frac{R(z)^2}{z}$ and the other subproblem is $\max (1+a-s)s^2$ s.t. $s \geq 1 - a$. Note that $\frac{d}{dz} \frac{R(z)^2}{z} = \frac{R(z)^2 [2\epsilon(z) - 1]}{z^2}$, which is positive if and only if $\epsilon(z) > \frac{1}{2}$. Because $\epsilon(z)$ is monotone decreasing in z , this

implies that $\frac{R(z)^2}{z}$ is quasiconcave and, hence, is maximized at $z_c^* = \epsilon^1(\frac{1}{2}) = \frac{b+1+2\sqrt{b^2-b+1}}{3}$. Similarly, note that $\frac{d[(1+a-s)s^2]}{ds} = s[2(1+a) - 3s]$, which is positive if and only if $s < \frac{2(1+a)}{3}$. This implies that $(1+a-s)s^2$ is quasiconcave and, hence, is maximized at $\frac{2(1+a)}{3}$. Given the constraint on s , this in turn implies that the manufacturer's optimal s is $s_c^* = \max\{\frac{2(1+a)}{3}, 1-a\}$. Thus, $a \leq \frac{1}{5} \Rightarrow s_c^* = 1-a$ and $a > \frac{1}{5} \Rightarrow s_c^* = \frac{2(1+a)}{3}$. Substituting z_c^* and s_c^* , respectively, for z and s in $q_c^*(z, s)$ then provides the manufacturer's optimal q : $q_c^* = \frac{s_c^* R(z_c^*)}{2z_c^*} = s_c^* \bar{F}(z_c^*)$. $\square$

Proof of Lemma 2 For the decentralized channel, the retailer's optimization problem is, from (2), $\max \pi_r(s, z) = \frac{1+a-s}{2a} [sR(z) - \alpha z]q$ s.t. $s \geq 1-a$, which is a price-setting newsvendor problem. Hence, similar to Petruzzini et al. (2009), the retailer's optimal response $\hat{z}(\alpha)$ is given by $\frac{(1+a)\bar{F}(z)}{2-\epsilon(z)} = \alpha$, and its optimal response $\hat{s}(\alpha)$ is $\hat{s}(\alpha) = \frac{1+a}{2-\epsilon(\hat{z}(\alpha))}$. As $s \geq 1-a$, then the retailer's best response functions are: $s_d = \max\{\hat{s}(\alpha), 1-a\}$ and $z_d = \bar{F}^{-1}(\frac{\alpha}{s_d})$.

We first consider $a = 1$, then the best response functions become $2\bar{F}(z)/[2-\epsilon(z)] = \alpha$ and $s = 2/[2-\epsilon(z)]$. Thus π_m in (3) becomes $\pi_m = \frac{(2-s)}{2} z(\alpha - q)q$. At optimality, $q = \alpha/2$. Substitute the best response function into π_m , with some algebraic calculation, π_m can be rewritten as a function of z , $\pi_m = \frac{z[1-\epsilon(z)]\bar{F}(z)^2}{[2-\epsilon(z)]^3}$. Hence, $\ln \pi_m = \ln z + \ln[1-\epsilon(z)] + 2\ln \bar{F}(z) - 3\ln[2-\epsilon(z)]$ and $z \frac{\pi'_m}{\pi_m} = 1 - \frac{2zf(z)}{F(z)} - \frac{[2\epsilon(z)-1]z\epsilon'(z)}{[1-\epsilon(z)][2-\epsilon(z)]}$. Denote $g(z) = \frac{zf(z)}{F(z)}$, at $\pi'_m = 0$, as $\epsilon'(z) < 0$ and $0 < \epsilon(z) < 1$, then $\frac{2g(z)-1}{2\epsilon(z)-1} = -\frac{z\epsilon'(z)}{[1-\epsilon(z)][2-\epsilon(z)]} > 0$. Therefore, either (A) $2g(z) > 1$ and $2\epsilon(z) > 1$, or (B) $2g(z) < 1$ and $2\epsilon(z) < 1$. Note that $\epsilon'(z) = \epsilon(z)[1-g(z)-\epsilon(z)]/z < 0$, hence $1 < g(z) + \epsilon(z)$, which is in conflict with (B). So at $\pi'_m = 0$, we have

$$2g(z) > 1 \quad \text{and} \quad 2\epsilon(z) > 1. \quad (4)$$

Moreover, as $z\epsilon'(z) = \epsilon(z)[1-\epsilon(z)-g(z)]$, at $\pi'_m = 0$, $\frac{2g(z)-1}{\epsilon(z)+g(z)-1} = \frac{\epsilon(z)[2\epsilon(z)-1]}{[1-\epsilon(z)][2-\epsilon(z)]}$. As $\epsilon'(z) < 0$, $\frac{\epsilon(z)[2\epsilon(z)-1]}{[1-\epsilon(z)][2-\epsilon(z)]}$ is decreasing in z . Let $X_1(z) = \frac{2g(z)-1}{\epsilon(z)+g(z)-1}$, to establish unimodality, it suffices to show $X'_1(z) > 0$. As $g'(z) > 0$, based on (4), $X'_1(z) = \frac{g'(z)[2\epsilon(z)-1]-\epsilon'(z)[2g(z)-1]}{[\epsilon(z)+g(z)-1]^2} > 0$. Thus, at $a = 1$, the optimal solution for the decentralized channel is $(z_d^*, s_d^*, q_d^*) = (\hat{z}_d^*, \frac{2}{2-\epsilon(\hat{z}_d^*)}, \frac{1}{2}\bar{F}(\hat{z}_d^*)s_d^*)$.

Next we consider $a < 1$, the manufacturer's problem is still given by (3), which can be further rewritten as $\pi_m = \frac{1+a-s_d}{2a} z_d[\alpha - q]q$ if $s_d > 1-a$ and $\pi_m = z_d[\alpha - q]q$ otherwise.

If $s_d > 1-a$, as $\hat{s}(\alpha) = \frac{1+a}{2-\epsilon(\hat{z}(\alpha))}$, the manufacturer's problem becomes $\max \pi_m = \frac{1+a}{2a} [1 - \frac{1}{2-\epsilon(\hat{z}(\alpha))}] \hat{z}(\alpha) [\alpha - q]q$. Similar to the case of $a = 1$, at optimality, $q = \alpha/2$, substitute the best response function into π_m , π_m can be rewritten as a function of z , namely, $\pi_m = \frac{(1+a)^3}{8a} \frac{z[1-\epsilon(z)]\bar{F}(z)^2}{[2-\epsilon(z)]^3}$. The optimal product availability is $\hat{z}_d^*$, which is the same as that for the case of $a = 1$. Thus, the optimal solution for the decentralized channel is $(z_d^*, s_d^*, q_d^*) = (\hat{z}_d^*, \frac{1+a}{2-\epsilon(\hat{z}_d^*)}, \frac{1}{2}\bar{F}(\hat{z}_d^*)s_d^*)$ if $\frac{1+a}{2-\epsilon(\hat{z}_d^*)} > 1-a$ (i.e., $a > \frac{1-\epsilon(\hat{z}_d^*)}{3-\epsilon(\hat{z}_d^*)} = \hat{a}_d$). If $s_d \leq 1-a$, as the retailer will never set a price per quality less than $1-a$, $s_d = 1-a$, the manufacturer's problem then becomes $\max \pi_m =$

$\bar{F}^{-1}(\frac{\alpha}{1-a})[\alpha - q]q = (1 + b - \frac{2b\alpha}{1-a})[\alpha - q]q = z[(1 - a)\bar{F}(z) - q]q$ s.t. $\hat{s}(\alpha) \leq 1 - a$. The optimal solution is $(\alpha^{f1}, q^{f1}) = ((1 - a)\bar{F}(g^{-1}(0.5)), (1 - a)\bar{F}(g^{-1}(0.5))/2)$ if $\hat{s}(\alpha^{f1}) \leq 1 - a$, where $g^{-1}(0.5) \equiv 1 - b$ if $g(1 - b) > 0.5$. Otherwise, the constraint is bounded such that $\hat{s}(\alpha) = 1 - a$, namely, $\frac{1+a}{2-\epsilon(\bar{F}^{-1}(\frac{\alpha}{1-a}))} = 1 - a$, and the optimal solution is α^{f2} . As $\hat{s}(\alpha)$ is increasing in α and $\hat{s}(\alpha^{f2}) = 1 - a$, so $\hat{s}(\alpha^{f1}) \leq 1 - a \Leftrightarrow \alpha^{f1} \leq \alpha^{f2}$. Thus, when $a \leq \hat{a}_d$, the optimal solution is $\alpha^f = \min\{\alpha^{f1}, \alpha^{f2}\} = (1 - a) \min\{\bar{F}(g^{-1}(0.5)), \bar{F}(\epsilon^{-1}(\frac{1-3a}{1-a}))\}$. To summarize, the optimal solution for the decentralized channel is: $(z_d^*, s_d^*, q_d^*, T_d^*) = (\hat{z}_d^*, \frac{1+a}{2-\epsilon(\hat{z}_d^*)}, \frac{1}{2}\bar{F}(z_d^*)s_d^*, \frac{1+a}{2a} \frac{1-\epsilon(\hat{z}_d^*)}{2-\epsilon(\hat{z}_d^*)})$ if $a > \hat{a}_d = \frac{1-\epsilon(\hat{z}_d^*)}{3-\epsilon(\hat{z}_d^*)}$ and $(z_d^*, s_d^*, q_d^*, T_d^*) = (\hat{z}, 1 - a, \frac{1}{2}\bar{F}(z_d^*)s_d^*, 1)$ otherwise, where $\hat{z} = \bar{F}^{-1}(\min\{\bar{F}(g^{-1}(0.5)), \bar{F}(\epsilon^{-1}(\frac{1-3a}{1-a}))\}) = \max\{\frac{b+1}{3}, \epsilon^{-1}(\frac{1-3a}{1-a})\}$. $\square$

Proof of Proposition 1 (i) First, we show $s_d^* \geq s_c^*$ and $z_d^* < z_c^*$. For the centralized channel, $\epsilon(z_c^*) = \frac{1}{2}$, $s_c^* = 1 - a$ if $a \leq \frac{1}{5}$ and $s_c^* = \frac{2}{3}(1 + a)$ otherwise. For the decentralized channel, $\hat{a}_d = \frac{1-\epsilon(\hat{z}_d^*)}{3-\epsilon(\hat{z}_d^*)}$. From (4) in the proof of Lemma 2, $\epsilon(\hat{z}_d^*) > \frac{1}{2}$, hence $\hat{a}_d < \frac{1}{5} = \hat{a}_c$. When $a > \hat{a}_c$, $s_d^* = \frac{1+a}{2-\epsilon(\hat{z}_d^*)} > \frac{2}{3}(1 + a) = s_c^*$; $\epsilon(z_d^*) > \frac{1}{2} = \epsilon(z_c^*)$, hence $z_d^* < z_c^*$. When $\hat{a}_d < a \leq \hat{a}_c$, $s_d^* > 1 - a = s_c^*$; $\epsilon(z_d^*) > \frac{1}{2} = \epsilon(z_c^*)$, hence $z_d^* < z_c^*$. When $a \leq \hat{a}_d$, $s_d^* = 1 - a = s_c^*$ and $z_d^* = \max\{g^{-1}(0.5), \epsilon^{-1}(\frac{1-3a}{1-a})\}$. If $z_d^* = \epsilon^{-1}(\frac{1-3a}{1-a})$, then $\epsilon(z_d^*) = \frac{1-3a}{1-a} > \frac{1}{2} = \epsilon(z_c^*)$, hence $z_d^* < z_c^*$. If $z_d^* = g^{-1}(0.5)$, as $\epsilon'(z) = \epsilon(z)[1 - \epsilon(z) - g(z)]/z < 0$ and $\epsilon(z_c^*) = \frac{1}{2}$, then $g(z_c^*) > 0.5$. As $g(z)$ is increasing in z , hence $z_d^* < z_c^*$. In sum, $s_d^* \geq s_c^*$ and $z_d^* < z_c^*$. Last, as $T(s)$ is decreasing in s , $T(s_d^*) \leq T(s_c^*)$ and hence $Q_d^* < Q_c^*$.

(ii) Based on Lemmas 1 and 2,

$$Q_c^* - Q_d^* = \begin{cases} z_c^* - z_d^*, & \text{for } a \leq \hat{a}_d; \\ z_c^* - \frac{(1+a)(1-s_d^*/2)}{2a} \hat{z}_d^*, & \text{for } \hat{a}_d < a \leq \hat{a}_c; \\ \frac{1+a}{6a} z_c^* - \frac{1+a}{2a} (1 - \frac{s_d^*}{2}) \hat{z}_d^*, & \text{for } a > \hat{a}_c. \end{cases}$$

First we consider $a \leq \hat{a}_d$. Comparing $\epsilon^{-1}(\frac{1-3a}{1-a})$ and $g^{-1}(0.5)$, z_d^* can be explicitly rewritten as

$$z_d^* = \begin{cases} \frac{1+b}{3}, & \text{for } b \geq \frac{1}{2} \text{ and } 0 \leq a < \hat{a}_0; \\ \epsilon^{-1}(\frac{1-3a}{1-a}), & \text{otherwise,} \end{cases}$$

where $\hat{a}_0 = \frac{1-\epsilon(\frac{b+1}{3})}{3-\epsilon(\frac{b+1}{3})}$. Recall that $z_c^* = \frac{b+1+2\sqrt{b^2-b+1}}{3}$, hence when $b \geq \frac{1}{2}$ and $0 \leq a < \hat{a}_0$, $Q_c^* - Q_d^* = \frac{2\sqrt{b^2-b+1}}{3}$ which is easily verified that it is increasing in b .

Now it suffices to show $Q_c^* - Q_d^*$ is increasing in b for $\hat{a}_0 \leq a < \hat{a}_d$. Note that $Q_c^* - Q_d^* = z_c^* - z_d^* = z_c^*(1 - \frac{z_d^*}{z_c^*})$ and z_c^* is increasing in b , hence it suffices to show $\frac{z_d^*}{z_c^*}$ is decreasing in b for $\hat{a}_0 \leq a < \hat{a}_d$. To facilitate the proof, let $K(b) = \frac{\epsilon^{-1}(\frac{1-3a}{1-a})}{b+1+2\sqrt{b^2-b+1}}$, it suffices to show $K'(b) < 0$. With some algebraic calculation, $\epsilon^{-1}(\tau) = \frac{(1+b)(1-\tau) + \sqrt{(1+b)^2(1-\tau)^2 + (2-\tau)\tau(1-b)^2}}{2-\tau}$. Let $\tau = \frac{1-3a}{1-a}$, hence

$$\begin{aligned} K(b) &= \frac{(1+b)(1-\tau) + \sqrt{(1+b)^2(1-\tau)^2 + (2-\tau)\tau(1-b)^2}}{(2-\tau)(b+1+2\sqrt{b^2-b+1})} \\ &= \frac{(1-\tau)[1 + \sqrt{1 + \frac{(2-\tau)\tau(1-b)^2}{(1-\tau)^2(1+b)^2}}]}{(2-\tau)[1 + 2\sqrt{3(\frac{1}{1+b} - \frac{1}{2})^2 + \frac{1}{4}}]} \end{aligned}$$

$$= \frac{(1-\tau)K_1(b)}{(2-\tau)K_2(b)}$$

where $K_1(b) = 1 + \sqrt{1 + \frac{(2-\tau)\tau(1-b)^2}{(1-\tau)^2(1+b)^2}}$ and $K_2(b) = 1 + 2\sqrt{3(\frac{1}{1+b} - \frac{1}{2})^2 + \frac{1}{4}}$. To show $K'(b) < 0$, as $1-\tau > 0$, it is equivalent to show $K'_1(b)K_2(b) - K_1(b)K'_2(b) < 0$, namely, $\frac{K'_1(b)}{K_1(b)} < \frac{K'_2(b)}{K_2(b)}$. Note that,

$$\begin{aligned} \frac{K'_1(b)}{K_1(b)} &= \frac{\frac{1}{2}[1 + \frac{(2-\tau)\tau(1-b)^2}{(1-\tau)^2(1+b)^2}]^{-1/2} \frac{(2-\tau)\tau}{(1-\tau)^2} \left[\frac{(1-b)^2}{(1+b)^2} \right]'}{1 + \sqrt{1 + \frac{(2-\tau)\tau(1-b)^2}{(1-\tau)^2(1+b)^2}}} \\ &= \frac{x}{2\sqrt{1 + x\frac{(1-b)^2}{(1+b)^2}} [1 + \sqrt{1 + x\frac{(1-b)^2}{(1+b)^2}}]} \left[\frac{(1-b)^2}{(1+b)^2} \right]' \\ &= \frac{1}{2\sqrt{\frac{1}{x} + \frac{(1-b)^2}{(1+b)^2}} [\frac{1}{\sqrt{x}} + \sqrt{\frac{1}{x} + \frac{(1-b)^2}{(1+b)^2}}]} \left[\frac{(1-b)^2}{(1+b)^2} \right]' \end{aligned}$$

where $x = \frac{(2-\tau)\tau}{(1-\tau)^2}$. As $\frac{1}{\sqrt{\frac{1}{x} + \frac{(1-b)^2}{(1+b)^2}} [\frac{1}{\sqrt{x}} + \sqrt{\frac{1}{x} + \frac{(1-b)^2}{(1+b)^2}}]}$ is increasing in x and $[\frac{(1-b)^2}{(1+b)^2}]' < 0$, to show $\frac{K'_1(b)}{K_1(b)} < \frac{K'_2(b)}{K_2(b)}$, it suffices to show $\frac{K'_1(b)}{K_1(b)} < \frac{K'_2(b)}{K_2(b)}$ for the minimum x . As $x = \frac{(2-\tau)\tau}{(1-\tau)^2}$ is increasing in τ for $0 < \tau < 1$, it suffices to show $K'(b) < 0$ for the minimum τ . As $\tau = \frac{1-3a}{1-a} \geq 0$, when $\tau = 0$, $K(b) = \frac{b+1}{b+1+2\sqrt{b^2-b+1}}$, which is easily verified that $K'(b) < 0$.

For $\hat{a}_d < a \leq \hat{a}_c$, $Q_c^* - Q_d^* = \frac{1+a}{2a} [\frac{2a}{1+a} z_c^* - (1 - \frac{\hat{s}_d^*}{2}) \hat{z}_d^*]$. As z_c^* is increasing in b and $\frac{2a}{1+a}$ is increasing in a , it suffices to show $Q_c^* - Q_d^*$ is increasing in b at $a = \hat{a}_d$, which is a special case of $a \leq \hat{a}_d$. For $a > \hat{a}_c$, as $Q_c^* - Q_d^* = \frac{1+a}{6a} [z_c^* - 3(1 - \frac{\hat{s}_d^*}{2}) \hat{z}_d^*]$, to show $Q_c^* - Q_d^*$ is increasing in b , it suffices to show $Q_c^* - Q_d^*$ is increasing in b for $a = \hat{a}_c$, which is a special case of $\hat{a}_d < a \leq \hat{a}_c$. $\square$

Lemma 3 and Its Proof

LEMMA 3. For any b , $\hat{q}_d^* > \hat{q}_c^*$, where $\hat{q}_c^*$ and $\hat{q}_d^*$ are the optimal quality decisions for the centralized and decentralized channel when $a = 1$, respectively.

Proof. From the proof of Lemma 2, $z \frac{\pi'_m}{\pi_m} = 1 - 2g(z) - \frac{[2\epsilon(z)-1]z\epsilon'(z)}{[1-\epsilon(z)][2-\epsilon(z)]}$. As $\epsilon'(z) = \epsilon(z)[1-g(z)-\epsilon(z)]/z$, $z \frac{\pi'_m}{\pi_m} = 1 - 2g(z) - \frac{[2\epsilon(z)-1]\epsilon(z)[1-g(z)-\epsilon(z)]}{[1-\epsilon(z)][2-\epsilon(z)]} = \frac{L(z)}{[1-\epsilon(z)][2-\epsilon(z)]}$, where $L(z) = 2(1+\epsilon(z))(1-\epsilon(z))^2 - (4-5\epsilon(z))g(z)$. Thus, at optimality, $4-5\epsilon(\hat{z}_d^*) > 0$, i.e., $\epsilon(\hat{z}_d^*) < 4/5$. Recall that $\epsilon(\hat{z}_d^*) > 1/2$. Thus, $1/2 < \epsilon(\hat{z}_d^*) < 4/5$. For market size uncertainty $\xi \sim U[1-b, 1+b]$, let $A = 1-b$, $B = 1+b$ and $y = \bar{F}(z)$. Then, $y = \frac{1+b-z}{2b} = \frac{B-z}{2b}$, $R(z) = \int_0^z \bar{F}(x)dx = \frac{1}{2b}[Bz - \frac{1}{2}z^2] - \frac{A^2}{4b} = 1-by^2$. As $z = B-2by$, $\epsilon(z) = \frac{z\bar{F}(z)}{R(z)} = \frac{(B-2by)y}{1-by^2}$, or $b(2-\epsilon(z))y^2 - By + \epsilon(z) = 0$. Thus, $y = \frac{B \pm \sqrt{B^2-4b\epsilon(z)(2-\epsilon(z))}}{2b(2-\epsilon(z))}$, it is easily verified that $\frac{B + \sqrt{B^2-4b\epsilon(z)(2-\epsilon(z))}}{2b(2-\epsilon(z))} > 1$, hence $y = \frac{B - \sqrt{B^2-4b\epsilon(z)(2-\epsilon(z))}}{2b(2-\epsilon(z))} = \frac{B - \sqrt{A^2+4b(1-\epsilon(z))^2}}{2b(2-\epsilon(z))}$.

Next, we introduce “ v -space”. Let $v = \frac{2-\epsilon(z)}{3}$, then $2/5 < v < 1/2$. Thus, $\epsilon(z) = 2-3v$, and $y = \frac{B - \sqrt{A^2+4b(3v-1)^2}}{6bv} = \frac{B(1-r)}{6bv}$, where $r^2 := \frac{A^2}{B^2} + \frac{4b}{B^2}(3v-1)^2$. Then $g(v) = \frac{zf(z)}{y} = \frac{B-2by}{2by} = \frac{B}{2by} - 1 = \frac{B}{2b\frac{B(1-r)}{6bv}} - 1 = \frac{3v}{1-r} - 1$, and $L(v) = 6(1-v)(3v-1)^2 - 3(5v-2)g(v) = 6(1-v)(3v-1)^2 - 3(5v-2)[\frac{3v}{1-r} - 1]$. As $L(v) = L(z(v))$, $L'(v) = L'(z)z'(v)$, namely, the sign of $L'(v)$ is the same as the

sign of $L(z)$. From the proof of Lemma 2, $L(z) = 0$ has a unique solution, so $L(v) = 0$ also has a unique solution, which is denoted as v_d . Moreover, $L(v) < 0$ if and only if $v > v_d$. For the centralized channel, $\epsilon(\hat{z}_c^*) = 1/2$, then $v_c = 1/2$ and $y_c = \frac{B - \sqrt{A^2 + b}}{3b} = \frac{B - \sqrt{B^2 - 3b}}{3b} = \frac{BK}{3b}$, where $K = 1 - \sqrt{1 - \frac{3b}{B^2}}$. Thus, $\hat{q}_c^* = \frac{4BK}{9b} = \frac{4B}{9b}(1 - t)$, where $t^2 := 1 - \frac{3b}{B^2}$. For the decentralized channel, $q_d(v) = \frac{y}{2 - \epsilon(z)} = \frac{y}{3v}$. Note that $\frac{\bar{F}(z)}{2 - \epsilon(z)}$ is strictly decreasing in z and v is increasing in z , hence $q_d(v)$ is strictly decreasing in z . Let $\hat{v}$ be the unique solution that satisfies $q_d(v) = \hat{q}_c^*$. Thus, $\hat{q}_d^* > \hat{q}_c^* \Leftrightarrow \hat{v} > v_d \Leftrightarrow L(\hat{v}) < 0$.

First we show $L(v) < 0$ for $1 - 1/\sqrt{3} < v < 1/2$. Note that $1 - r^2 = 1 - \frac{A^2}{B^2} - \frac{4b}{B^2}(3v - 1)^2 = \frac{4b}{B^2}[1 - (3v - 1)^2] \leq 1 - (3v - 1)^2$, hence $r \geq 3v - 1$. Thus, $L(v) = 6(1 - v)(3v - 1)^2 - 3(5v - 2)[\frac{3v}{1 - r} - 1] \leq 6(1 - v)(3v - 1)^2 - 3(5v - 2)[\frac{3v}{2 - 3v} - 1] = \frac{18v(3v - 1)}{2 - 3v}(3v^2 - 6v + 2) < 0$ for $1 - 1/\sqrt{3} < v < 1/2$. Thus, $v_d \leq 1 - 1/\sqrt{3}$. To show $\hat{v} > v_d$, there are two cases, either $\hat{v} > 1 - 1/\sqrt{3}$ or $\hat{v} \leq 1 - 1/\sqrt{3}$. If $\hat{v} > 1 - 1/\sqrt{3}$, then $\hat{v} > v_d$. Thus, it suffices to show that $\hat{v} > v_d$ if $\hat{v} \leq 1 - 1/\sqrt{3}$, namely, $L(\hat{v}) < 0$ if $\hat{v} \leq 1 - 1/\sqrt{3}$. Note that $\hat{v}$ satisfies $\frac{y}{3v} = \frac{4B}{9b}(1 - t)$, or $\frac{B(1 - r)}{18bv^2} = \frac{4B}{9b}(1 - t)$. Thus, $1 - r = 8v^2(1 - t)$, and $1 + r = 2 - (1 - r) = 2 - 8v^2(1 - t)$. Hence $1 - r^2 = (1 - r)(1 + r) = 8v^2(1 - t)[2 - 8v^2(1 - t)] = 1 - \frac{A^2}{B^2} - \frac{4b}{B^2}(3v - 1)^2 = \frac{4b}{B^2}(2 - 3v)3v = 4v(2 - 3v)(1 - t^2)$, namely, $8v^2[2 - 8v^2(1 - t)] = 4v(2 - 3v)(1 + t) = 4v(2 - 3v)[2 - (1 - t)]$. That is, $1 - t = \frac{2(5v - 2)}{16v^3 + 3v - 2}$ and thus $1 - r = \frac{16v^2(5v - 2)}{16v^3 + 3v - 2}$. Hence $g(v) = \frac{3v}{1 - r} - 1 = \frac{3(16v^3 + 3v - 2)}{16v(5v - 2)} - 1$. Hence it suffices to show that $L(v) = 6(1 - v)(3v - 1)^2 - 3(5v - 2)g(v) = 6(1 - v)(3v - 1)^2 - 3(5v - 2)[\frac{3(16v^3 + 3v - 2)}{16v(5v - 2)} - 1] < 0$ for $2/5 < v \leq 1 - 1/\sqrt{3}$. After some algebraic calculation, $L(v) = \frac{9(-96v^4 + 144v^3 - 48v^2 - 3v + 2)}{16v}$, hence it suffices to show $\hat{L}(v) = -96v^4 + 144v^3 - 48v^2 - 3v + 2 < 0$ for $v \in (2/5, 1 - 1/\sqrt{3}]$. As $\hat{L}''(v) = -96(12v^2 - 9v + 1) > 0$ for $v \in [2/5, 1 - 1/\sqrt{3}]$, namely, $\hat{L}(v)$ is convex in v for $v \in [2/5, 1 - 1/\sqrt{3}]$. Therefore it only needs to show $\hat{L}(v) < 0$ for the two end-points $v = 2/5$ and $v = 1 - 1/\sqrt{3}$, which are easily verified. $\square$

Proof of Proposition 2 (i) First we consider $a \leq \hat{a}_d$. From the proof of Proposition 1, when $a \leq \hat{a}_d$, $z_d^* = \frac{1+b}{3}$ for $0 \leq a < \hat{a}_0$ and $b \geq \frac{1}{2}$, otherwise $z_d^* = \epsilon^{-1}(\frac{1-3a}{1-a})$, where $\hat{a}_0 = \frac{1 - \epsilon(\frac{b+1}{3})}{3 - \epsilon(\frac{b+1}{3})}$. For $0 \leq a < \hat{a}_0$ and $b \geq \frac{1}{2}$, based on Lemmas 1 and 2, $q_d^* > q_c^* \Leftrightarrow \frac{(1-a)(1+b)}{6b} > (1-a)\frac{b+1 - \sqrt{b^2 - b + 1}}{3b} \Leftrightarrow 3(b-1)^2 > 0 \Leftrightarrow b < 1$. Otherwise (i.e., $\hat{a}_0 \leq a \leq \hat{a}_d$), $q_c^* - q_d^* = (1-a)\bar{F}(z_c^*) - \frac{1}{2}(1-a)\bar{F}(z_d^*) = (1-a)D(a)$, where $D(a) = \bar{F}(z_c^*) - \frac{1}{2}\bar{F}(\epsilon^{-1}(\frac{1-3a}{1-a}))$. From the proof of Proposition 1, $\epsilon^{-1}(\tau) = \frac{(1+b)(1-\tau) + \sqrt{(1+b)^2(1-\tau)^2 + (2-\tau)\tau(1-b)^2}}{2-\tau}$. With some algebraic calculation, $\bar{F}(\epsilon^{-1}(\frac{1-3a}{1-a})) = \frac{(1-a)(1+b)}{2b(1+a)}[1 - \sqrt{1 - \frac{4b(1+a)(1-3a)}{(b+1)^2(1-a)^2}}]$. Let $D(a) = 0$, i.e., $\frac{3}{4}\hat{q}_c^* = \frac{(1-a)(1+b)}{4b(1+a)}[1 - \sqrt{1 - \frac{4b(1+a)(1-3a)}{(b+1)^2(1-a)^2}}]$, after some algebraic calculation, $\underline{a} = \frac{4[3(1+b)\hat{q}_c^*]}{3[3b(\hat{q}_c^*)^2 + 2(1+b)\hat{q}_c^* - 4]} - 1$. Note that $\hat{q}_c^* = \frac{4}{3}\bar{F}(z_c^*) = \frac{4}{9b}(b+1 - \sqrt{b^2 - b + 1})$, hence $(\hat{q}_c^*)^2 = \frac{8}{27b}[3(b+1)\hat{q}_c^* - 2]$. Substitute it into $\underline{a}$, hence $\underline{a} = \frac{2-3(1+b)\hat{q}_c^*}{21(1+b)\hat{q}_c^* - 26}$. Note that $(1+b)\hat{q}_c^* = \frac{4}{3} \frac{1}{1 + \sqrt{3(\frac{1}{b+1} - \frac{1}{2})^2 + \frac{1}{4}}}$ which is increasing in b and $\frac{2}{3} \leq (1+b)\hat{q}_c^* \leq \frac{8}{9}$, hence $\underline{a}$ is increasing in b and $0 < \underline{a} < 1/11$ for $0 < b < 1$. From the proof of Lemma 3, $\epsilon(\hat{z}_d^*) < 4/5$, $\hat{a}_d = \frac{1 - \epsilon(\hat{z}_d^*)}{3 - \epsilon(\hat{z}_d^*)} > 1/11$, hence $\underline{a} < \hat{a}_d$. Recall that $q_c^* - q_d^* = (1-a)D(a)$, where $D(a)$ is increasing in a . Thus, if $a < \underline{a}$, $q_c^* < q_d^*$, if $\underline{a} < a \leq \hat{a}_d$, $q_c^* > q_d^*$.

Next we consider $a > \hat{a}_d$. When $\hat{a}_d < a \leq \hat{a}_c$, $q_c^* - q_d^* = (1-a)\bar{F}(z_c^*) - \frac{1}{2}\bar{F}(\hat{z}_d^*)\frac{1+a}{2-\epsilon(\hat{z}_d^*)} = (1-a)\frac{3}{4}\hat{q}_c^* - \frac{1+a}{2}\hat{q}_d^*$. When $a = \hat{a}_c = 1/5$, $q_c^* - q_d^* = \frac{3}{5}(\hat{q}_c^* - \hat{q}_d^*) < 0$, where the inequality is from Lemma 3. Let $q_c^* - q_d^* = 0$, i.e., $(1-a)\frac{3}{4}\hat{q}_c^* = \frac{1+a}{2}\hat{q}_d^*$, or $\bar{a} = \frac{3\hat{q}_c^* - 2\hat{q}_d^*}{3\hat{q}_c^* + 2\hat{q}_d^*}$. As $q_c^* - q_d^* = (1-a)\frac{3}{4}\hat{q}_c^* - \frac{1+a}{2}\hat{q}_d^*$ is decreasing in a , if $a < \bar{a}$, $q_c^* > q_d^*$, if $\bar{a} < a \leq \hat{a}_c$, $q_c^* < q_d^*$. When $a > \hat{a}_c$, $q_c^* - q_d^* = \frac{2(1+a)}{3}\bar{F}(z_c^*) - \frac{1}{2}\bar{F}(\hat{z}_d^*)\frac{1+a}{2-\epsilon(\hat{z}_d^*)} = \frac{1+a}{2}(\hat{q}_c^* - \hat{q}_d^*) < 0$.

(ii) Clearly, the proof is a part of the proof for Part (i). $\square$

Proof of Proposition 3 (i) For the centralized channel, the manufacturer's expected profit function is $\pi_c = T(s)[sR(z) - qz]q$. Thus, analogous to the proof of Lemma 1, the optimal q for a given z and s can be rewritten as $q_c^*(z, s) = \frac{sR(z)}{2z}$. Accordingly, the manufacturer's problem can be reduced to maximizing $\pi_c(z, s) = \frac{T(s)s^2}{4} \frac{R(z)^2}{z}$, where both $T(s)s^2$ and $\frac{R(z)^2}{z}$ are quasiconcave. Hence, $z_c^* = \arg \max_z \frac{R(z)^2}{z} = \epsilon^{-1}(0.5)$, and $s_c^* = \arg \max_s T(s)s^2 = g_\theta^{-1}(2)$. Accordingly, $q_c^* = q_c^*(z_c^*, s_c^*) = \bar{F}(z_c^*)s_c^*$.

(ii) For the decentralized channel, the retailer's expected profit is $\pi_r = T(s)[sR(z) - \alpha z]q$. Thus, analogous to the proof of Lemma 2, the retailer's optimal response functions for a given α and q are uniquely characterized as follows:

$$s\bar{F}(z) = \alpha, \quad (5)$$

and

$$g_\theta(s)[1 - \epsilon(z)] = 1. \quad (6)$$

From (6), $\ln g_\theta(s) + \ln(1 - \epsilon(z)) = 0$. Taking the derivative of both sides implies $\frac{g'_\theta(s)}{g_\theta(s)}s'(z) = \frac{\epsilon'(z)}{1 - \epsilon(z)}$. By definition, $\eta(s) = sg'_\theta(s)/g_\theta(s)$; hence

$$\eta(s) \frac{s'(z)}{s} = \frac{\epsilon'(z)}{1 - \epsilon(z)}. \quad (7)$$

As $g_\theta(s)$ is increasing in s , from (6), $s(z)$ is uniquely determined. Substituting $s(z)$ into (5) yields $\ln \alpha(z) = \ln \bar{F}(z) + \ln s(z)$. Taking the derivative of both sides implies $\frac{\alpha'(z)}{\alpha(z)} = \frac{s'(z)}{s(z)} - \frac{f(z)}{\bar{F}(z)}$. From (7), $s'(z) = \frac{s(z)\epsilon'(z)}{\eta(s)[1 - \epsilon(z)]}$. Hence

$$\frac{z\alpha'(z)}{\alpha(z)} = \frac{z\epsilon'(z)}{\eta(s)[1 - \epsilon(z)]} - g_\xi(z). \quad (8)$$

Given the retailer's best response, the manufacturer's profit is $\pi_m = T(s(z))z[\alpha(z) - q]q$. Thus, for a given z , the optimal quality provision is $q_d = \alpha(z)/2$. Hence the manufacturer's profit becomes $\pi_m = \frac{1}{4}T(s(z))z\alpha(z)^2$. Analogous to the proof of Lemma 2, $z \frac{d \ln \pi_m}{dz} = 1 + \frac{2z\alpha'(z)}{\alpha(z)} - \frac{zh(s)s}{T(s)} \frac{s'(z)}{s}$. Based on (7) and (8), $z \frac{d \ln \pi_m}{dz} = 1 + 2[\frac{z\epsilon'(z)}{\eta(s)[1 - \epsilon(z)]} - g_\xi(z)] - \frac{g_\theta(s)}{\eta(s)} \frac{z\epsilon'(z)}{1 - \epsilon(z)}$. From (6), $z \frac{\pi'_m}{\pi_m} = 1 - 2g_\xi(z) + \frac{-z\epsilon'(z)}{\eta(s)[1 - \epsilon(z)]} \frac{2\epsilon(z) - 1}{1 - \epsilon(z)}$. Thus, similar to (4), at $\pi'_m = 0$, here we have

$$2g_\xi(z) > 1 \quad \text{and} \quad 2\epsilon(z) > 1. \quad (9)$$

Moreover, since $z\epsilon'(z) = \epsilon(z)[1 - \epsilon(z) - g_\xi(z)]$,

$$\begin{aligned} z \frac{\pi'_m}{\pi_m} &= \frac{\epsilon(z)[\epsilon(z) + g_\xi(z) - 1][2\epsilon(z) - 1]}{[1 - \epsilon(z)]^2} \left[\frac{1}{\eta(s)} - \left(\frac{2g_\xi(z) - 1}{\epsilon(z) + g_\xi(z) - 1} \right) \left(\frac{1 - \epsilon(z)}{\epsilon(z)} \right) \left(\frac{1 - \epsilon(z)}{2\epsilon(z) - 1} \right) \right] \\ &= \frac{\epsilon(z)[\epsilon(z) + g_\xi(z) - 1][2\epsilon(z) - 1]}{1 - \epsilon(z)} \left[\frac{g_\theta(s)}{\eta(s)} - \left(\frac{2g_\xi(z) - 1}{\epsilon(z) + g_\xi(z) - 1} \right) \left(\frac{1 - \epsilon(z)}{\epsilon(z)[2\epsilon(z) - 1]} \right) \right] \end{aligned} \quad (10)$$

where the second equality follows from (6). At $\pi'_m = 0$, from (9), $\frac{\epsilon(z)[\epsilon(z) + g_\xi(z) - 1][2\epsilon(z) - 1]}{1 - \epsilon(z)} > 0$. From (7), we know $s'(z) < 0$. Hence $r(s(z)) = \frac{g_\theta(s(z))}{\eta(s(z))}$ is non-increasing in z . Therefore, analogous to the proof of Lemma 2, one can show $\frac{2g_\xi(z) - 1}{\epsilon(z) + g_\xi(z) - 1} \frac{1 - \epsilon(z)}{\epsilon(z)[2\epsilon(z) - 1]}$ is increasing in z . Hence the solution is unique. $\square$

Lemma 4 and Its Proof

LEMMA 4. If $\theta \sim U[0, 2]$ and $\xi \sim F(z) = z^b$, then $q_d^* > q_c^*$.

Proof. $F(z) = z^b$ implies $R(z) = \frac{z(b+1-z^b)}{b+1} = \frac{z[b+\bar{F}(z)]}{b+1}$. Hence, $\epsilon(z) = z\bar{F}(z)/R(z) = (b+1)\bar{F}(z)/[\bar{F}(z) + b]$, or $\bar{F}(z) = b\epsilon(z)/[b+1-\epsilon(z)]$. Similarly, $\theta \sim U[0, 2]$ implies $g_\theta(s) = \frac{s}{2-s}$ and $\eta(s) = \frac{2}{2-s}$. For the centralized channel, from Proposition 3(i), $s_c^* = g_\theta^{-1}(2) = \frac{4}{3}$, so $q_c^* = \frac{4}{3}\bar{F}(z_c^*) = \frac{4}{3} \frac{b\epsilon(z_c^*)}{[b+1-\epsilon(z_c^*)]}$; as $\epsilon(z_c^*) = \frac{1}{2}$, $q_c^* = \frac{4b}{3(2b+1)}$. For the decentralized channel, as $g_\theta(s) = \frac{s}{2-s}$, from (6), $s(z) = \frac{2}{2-\epsilon(z)}$. Moreover, $\eta(s) = \frac{2}{2-s}$, substitute $g_\theta(s)$ and $\eta(s)$ into (10), at $\pi'_m = 0$,

$$\frac{1}{2 - \epsilon(z)} = \left(\frac{2g_\xi(z) - 1}{\epsilon(z) + g_\xi(z) - 1} \right) \left(\frac{1 - \epsilon(z)}{\epsilon(z)[2\epsilon(z) - 1]} \right).$$

Note that $g_\xi(z) = zf(z)/\bar{F}(z) = b[1 - \bar{F}(z)]/\bar{F}(z) = (b+1)[1 - \epsilon(z)]/\epsilon(z)$ where the last equality follows from $\bar{F}(z) = b\epsilon(z)/[b+1-\epsilon(z)]$, then

$$2(1 - \epsilon^2)(1 - \epsilon) = (4 - 5\epsilon)(b+1)(1 - \epsilon)/\epsilon \quad (11)$$

where $\epsilon = \epsilon(z_d^*)$. Namely, $b+1 = 2\epsilon(1 - \epsilon^2)/(4 - 5\epsilon)$ where $\epsilon = \epsilon(z_d^*)$. From Proposition 3(ii), $q_d^* = \frac{\bar{F}(z_d^*)}{2 - \epsilon(z_d^*)} = \frac{b\epsilon}{(b+1-\epsilon)(2-\epsilon)}$ at $\epsilon = \epsilon(z_d^*)$. Now, consider:

$$\frac{q_d^* - q_c^*}{b} = \frac{\epsilon}{(b+1-\epsilon)(2-\epsilon)} - \frac{4}{3(2b+1)} = \frac{3\epsilon(2b+1) - 4(b+1-\epsilon)(2-\epsilon)}{3(b+1-\epsilon)(2-\epsilon)(2b+1)}.$$

Since $b+1 = 2\epsilon(1 - \epsilon^2)/(4 - 5\epsilon)$, to show $q_d^* > q_c^*$, it suffices to show

$$3\epsilon \left[2 \frac{2\epsilon(1 - \epsilon^2)}{4 - 5\epsilon} - 1 \right] - 4 \left[\frac{2\epsilon(1 - \epsilon^2)}{4 - 5\epsilon} - \epsilon \right] (2 - \epsilon) > 0.$$

As $b+1 = 2\epsilon(1 - \epsilon^2)/(4 - 5\epsilon) > 0$, $\epsilon(z_c^*) < 0.8$. From (9), $0.5 < \epsilon(z_c^*)$. Hence, $0.5 < \epsilon(z_c^*) < 0.8$. Thus, we only need to show the function

$$k(\epsilon) = 3[4\epsilon(1 - \epsilon^2) - (4 - 5\epsilon)] - 4[2(1 - \epsilon^2) - (4 - 5\epsilon)](2 - \epsilon) > 0,$$

for $0.5 < \epsilon < 0.8$. After some algebra, $k(\epsilon) = -20\epsilon^3 + 36\epsilon^2 - 21\epsilon + 4$ and hence $k'(\epsilon) = -60(\epsilon - 0.6)^2 + 0.6$. Thus $k(\epsilon)$ is increasing in $(0.5, 0.7]$ and decreasing in $[0.7, 0.8)$, so $k(\epsilon) > \min\{k(0.5), k(0.8)\} = 0$. $\square$

Proof of Proposition 4 (i) From Proposition 3(i), $\epsilon(z_c^*) = 1/2$ and thus, $s_c^* = g_\theta^{-1}(2) = g_\theta^{-1}(\frac{1}{1-\epsilon(z_c^*)})$. Moreover, from (9), $\epsilon(z_d^*) > 1/2$, and from (6), $s_d^* = g_\theta^{-1}(\frac{1}{1-\epsilon(z_d^*)})$. As $\epsilon(z)$ is decreasing in z , $z_d^* < z_c^*$ and $s_d^* > s_c^*$. Thus, $T(s_c^*) > T(s_d^*)$ and $Q_c^* > Q_d^*$.

(ii) Let $\hat{a}_d = \frac{1-\epsilon(\hat{z}_d^*)}{3-\epsilon(\hat{z}_d^*)}$ and $\underline{a} = \frac{1}{4b+7}$, where $\hat{z}_d^*$ is the optimal product availability when $\theta \sim U[0, 2]$. First we show $\hat{a}_d > \underline{a}$. Note that $\hat{a}_d > \frac{1}{4b+7} \Leftrightarrow \frac{1-\epsilon(\hat{z}_d^*)}{3-\epsilon(\hat{z}_d^*)} > \frac{1}{4b+7} \Leftrightarrow \frac{2(2b+3)}{4b+7} > \frac{2}{3-\epsilon(\hat{z}_d^*)} \Leftrightarrow \epsilon(\hat{z}_d^*) < \frac{2b+2}{2b+3}$. By definition, similar to Lemma 2, one can show $\hat{z}_d^*$ is the unique value of z that maximizes $\hat{\pi}_m(z) = \frac{z[1-\epsilon(z)]\bar{F}(z)^2}{[2-\epsilon(z)]^3}$. From (11), $\epsilon(\hat{z}_d^*)$ uniquely solves $L(\epsilon) = 2(1-\epsilon)^2(1+\epsilon) - (4-5\epsilon)(b+1)(1-\epsilon)/\epsilon$. Moreover, $L(\epsilon) > 0$ for $\epsilon > \epsilon(\hat{z}_d^*)$ and $L(\epsilon) \leq 0$ otherwise. To show $\epsilon(\hat{z}_d^*) < \frac{2b+2}{2b+3}$, it suffices to show $L(\frac{2b+2}{2b+3}) > 0$. With some algebra, $L(\frac{2b+2}{2b+3}) = \frac{1}{2b+3}[\frac{2(4b+5)}{(2b+3)^2} - (1-b)]$. Note that $[\frac{2(4b+5)}{(2b+3)^2} - (1-b)]' = 1 - \frac{8(2b+2)}{(2b+3)^3} > 0$ for $b > 0$ and $[\frac{2(4b+5)}{(2b+3)^2} - (1-b)]|_{b=0} > 0$, hence $L(\frac{2b+2}{2b+3}) > 0$.

When $0 < a \leq \underline{a}$, it is easily verified that $g_\xi^{-1}(\frac{1}{2}) \geq \epsilon^{-1}(\frac{1-3a}{1-a})$. Based on Proposition 3, $q_c^* - q_d^* = (1-a)\frac{b}{2b+1} - (1-a)\frac{1}{2}(1 - \frac{1}{2b+1}) = 0$, i.e., $q_c^* = q_d^*$. When $\underline{a} < a < \hat{a}_d$, $g_\xi^{-1}(\frac{1}{2}) < \epsilon^{-1}(\frac{1-3a}{1-a})$, hence $q_c^* - q_d^* = (1-a)\frac{b}{2b+1} - (1-a)\frac{1}{2}\bar{F}(z_d^*) > (1-a)[\frac{b}{2b+1} - \frac{1}{2}\bar{F}(g_\xi^{-1}(\frac{1}{2}))] = 0$, i.e., $q_c^* > q_d^*$. When $a > \hat{a}_d := \frac{1}{5}$, $q_c^* - q_d^* = \frac{2(1+a)}{3}\bar{F}(z_c^*) - \frac{1}{2}\frac{1+a}{2-\epsilon(z_d^*)}\bar{F}(z_d^*) = \frac{1+a}{2}[\hat{q}_c^* - \hat{q}_d^*] < 0$, where the inequality is from Lemma 4, and $\hat{q}_c^*$ and $\hat{q}_d^*$ are the optimal quality provision for $a = 1$, respectively. When $\hat{a}_d \leq a \leq \hat{a}_c$, $q_c^* - q_d^* = (1-a)\bar{F}(z_c^*) - (1-a)\frac{1+a}{2-\epsilon(z_d^*)}\bar{F}(z_d^*) = (1-a)\frac{3}{4}\hat{q}_c^* - \frac{1+a}{2}\hat{q}_d^*$, which is decreasing in a . Note that $q_c^* - q_d^* > 0$ for $a = \hat{a}_d$ and $q_c^* - q_d^* < 0$ for $a = \hat{a}_c$. Let $(1-a)\frac{3}{4}\hat{q}_c^* = \frac{1+a}{2}\hat{q}_d^*$, i.e., $\tilde{a} = \frac{3\hat{q}_c^* - 2\hat{q}_d^*}{3\hat{q}_c^* + 2\hat{q}_d^*}$, then $q_c^* > q_d^*$ if $\hat{a}_d < a \leq \tilde{a}$, but $q_c^* < q_d^*$ if $a > \tilde{a}$. In sum, $q_c^* \geq q_d^*$ if $a \leq \tilde{a}$, but $q_c^* < q_d^*$ if $a > \tilde{a}$. $\square$

Lemma 5 and Its Proof

LEMMA 5. (i) For the centralized channel, $(q_c^{c*}, Q_c^{c*}) = (\bar{F}(z_c^{c*}), z_c^{c*}T_c^{c*})$ and $T_c^{c*} = \min(\frac{R(z_c^{c*})^2}{4z_c^{c*}t}, 1)$, where $z_c^{c*} = \epsilon^{-1}(\frac{1}{2})$.

(ii) For the decentralized channel, $(q_d^{c*}, Q_d^{c*}) = (\frac{2\bar{F}(z_d^{c*})}{3}, z_d^{c*}T_d^{c*})$ and $(T_d^{c*}, z_d^{c*}) = (\frac{2\bar{F}(z_1^*)[R(z_1^*) - z_1^*\bar{F}(z_1^*)]}{3t}, z_1^*)$ if $t > \frac{2\bar{F}(z_1^*)[R(z_1^*) - z_1^*\bar{F}(z_1^*)]}{3}$, $(q_d^{c*}, Q_d^{c*}) = (\frac{\bar{F}(z_d^{c*})}{2}, z_d^{c*}T_d^{c*})$ and $(T_d^{c*}, z_d^{c*}) = (1, z_2^*)$ if $t < \frac{\bar{F}(z_2^*)[R(z_2^*) - z_2^*\bar{F}(z_2^*)]}{2}$, and $(q_d^{c*}, Q_d^{c*}) = (\frac{t}{R(z_d^{c*}) - z_d^{c*}\bar{F}(z_d^{c*})}, z_d^{c*}T_d^{c*})$ and $(T_d^{c*}, z_d^{c*}) = (1, z_3^*)$ otherwise, where z_1^* , z_2^* and z_3^* are the unique solutions to $\pi_m^1 = \frac{4[R(z) - z\bar{F}(z)]}{27t}\bar{F}(z)^3z$, $\pi_m^2 = \frac{\bar{F}(z)^2z}{4}$ and $\pi_m^3 = \frac{z[R(z)\bar{F}(z) - \bar{F}(z)^2z - t]}{[R(z) - z\bar{F}(z)]^2}t$, respectively.

Proof. (i) In the centralized channel, when the consumer located at $x = \frac{1}{2}$ visits a retailer, i.e., when $R(z)(q-p) \geq \frac{t}{2}$, $T = 1$; otherwise $T = \frac{2R(z)(q-p)}{t}$. Thus, the manufacturer's expected profit is $\pi_c^c = pR(z) - q^2z$ if $R(z)(q-p) \geq \frac{t}{2}$ and $\pi_c^c = \frac{2R(z)(q-p)}{t}[pR(z) - q^2z]$ otherwise. As $R(z)(q-p) > \frac{t}{2}$ cannot be optimal, the firm's profit function becomes $\pi_c^c = pR(z) - q^2z$ for $R(z)(q-p) = \frac{t}{2}$ and $\pi_c^c = 2\frac{R(z)(q-p)}{t}[pR(z) - q^2z]$ for $R(z)[q-p] < \frac{t}{2}$. If $R(z)(q-p) = \frac{t}{2}$, $\pi_c^c = qR(z) - q^2z - \frac{t}{2}$, the

optimal solution is $(p_{c1}^{c*}, q_{c1}^{c*}, z_{c1}^{c*}) = (\bar{F}(z_c^{c*}) - \frac{t}{2R(z_c^{c*})}, \bar{F}(z_c^{c*}), z_c^{c*})$, where $z_c^{c*} = \epsilon^{-1}(\frac{1}{2})$. If $R(z)[q-p] < \frac{t}{2}$, $\pi_c^c = \frac{2R(z)(q-p)}{t}[pR(z) - q^2z]$, the optimal solution is $(p_{c2}^{c*}, q_{c2}^{c*}, z_{c2}^{c*}) = (\frac{3R(z_c^{c*})}{8z_c^{c*}}, \frac{R(z_c^{c*})}{2z_c^{c*}}, z_c^{c*})$. Hence $(p_c^{c*}, q_c^{c*}, z_c^{c*}) = (p_{c2}^{c*}, q_{c2}^{c*}, z_{c2}^{c*})$ if $\frac{R(z_c^{c*})^2}{8z_c^{c*}} < \frac{t}{2}$ and $(p_c^{c*}, q_c^{c*}, z_c^{c*}) = (p_{c1}^{c*}, q_{c1}^{c*}, z_{c1}^{c*})$ otherwise. Namely, the optimal quality $q_c^{c*} = \bar{F}(z_c^{c*})$ and the optimal quantity $Q_c^{c*} = z_c^{c*}T_c^{c*}$, where the optimal target market $T_c^{c*} = \min(\frac{R(z_c^{c*})^2}{4z_c^{c*}t}, 1)$ and $z_c^{c*} = \epsilon^{-1}(\frac{1}{2})$.

(ii) In the decentralized channel, given p_1, p_2, z_1, z_2 , and q , depending on whether the marginal consumer $x = \frac{1}{2} + \frac{R(z_1)(q-p_1) - R(z_2)(q-p_2)}{2t}$ visits a retailer, the retailers' target markets are given by

$$(T_1, T_2) = \begin{cases} (\frac{R(z_1)(q-p_1)}{2}, \frac{R(z_2)(q-p_2)}{2}), & \text{for } R(z_1)(q-p_1) + R(z_2)(q-p_2) \leq t; \\ (\frac{1}{2} + \frac{R(z_1)(q-p_1) - R(z_2)(q-p_2)}{2t}, \frac{1}{2} - \frac{R(z_1)(q-p_1) - R(z_2)(q-p_2)}{2t}), & \text{otherwise.} \end{cases}$$

The two retailers simultaneously maximize their expected profits $\pi_{r1} = T_1[p_1R(z_1) - wz_1]$ and $\pi_{r2} = T_2[p_2R(z_2) - wz_2]$ to obtain their best response functions $(z_1(w, q), p_1(w, q), z_2(w, q), p_2(w, q))$ as

$$\begin{cases} (z(\alpha), \frac{t+\alpha qz(\alpha)}{R(z(\alpha))}, z(\alpha), \frac{t+\alpha qz(\alpha)}{R(z(\alpha))}), & \text{if } \frac{2}{3}q[R(z(\alpha)) - \alpha z(\alpha)] \geq t; \\ (z(\alpha), q - \frac{t}{2R(z(\alpha))}, z(\alpha), q - \frac{t}{2R(z(\alpha))}), & \text{if } \frac{2}{3}q[R(z(\alpha)) - \alpha z(\alpha)] < t \leq q[R(z(\alpha)) - \alpha z(\alpha)]; \\ (z(\alpha), \frac{q}{2}[1 + \frac{\alpha z(\alpha)}{R(z(\alpha))}], z(\alpha), \frac{q}{2}[1 + \frac{\alpha z(\alpha)}{R(z(\alpha))}]), & \text{if } q[R(z(\alpha)) - \alpha z(\alpha)] < t. \end{cases}$$

where $z(\alpha) = \bar{F}^{-1}(\alpha)$ and $\alpha = w/q$. The manufacturer's aggregate demand is $T_1z_1(w, q) + T_2z_2(w, q)$. Thus, its expected profit is $\pi_m^c = z(\alpha)(\alpha - q)q$ if $q[R(z(\alpha)) - \alpha z(\alpha)] > t$ and $\pi_m^c = \frac{q[R(z(\alpha)) - \alpha z(\alpha)]}{t}z(\alpha)(\alpha - q)q$ otherwise.

We first solve $\max \pi_m^{c1} = \frac{q[R(z(\alpha)) - \alpha z(\alpha)]}{t}z(\alpha)(\alpha - q)q$. The optimal quality $q = \frac{2\bar{F}(z)}{3}$, hence $\pi_m^{c1} = \frac{4[R(z) - z\bar{F}(z)]}{27t}\bar{F}(z)^3z$. Thus, $\frac{d\pi_m^{c1}}{dz} = C(z)\{[1 - 3g(z)][1 - \epsilon(z)] + \epsilon(z)g(z)\}$, where $C(z) = \frac{4}{27t}R(z)\bar{F}(z)^3$. Thus, at optimality, $[1 - 3g(z)][1 - \epsilon(z)] + \epsilon(z)g(z) = 0$, i.e., $3 - \frac{1}{g(z)} = \frac{1}{1/\epsilon(z) - 1}$, where the left-hand side is increasing and the right-hand side is decreasing. Hence the solution is unique and we denote it as z_1^* . Hence, for the sub-problem $\max \pi_m^c = \frac{q[R(z(\alpha)) - \alpha z(\alpha)]}{t}z(\alpha)(\alpha - q)q$ s.t. $q[R(z(\alpha)) - \alpha z(\alpha)] \leq t$, the optimal solution is z_1^* if $t_1 := \frac{2\bar{F}(z_1^*)[R(z_1^*) - z_1\bar{F}(z_1^*)]}{3} \leq t$; otherwise, the optimality is achieved at $q[R(z(\alpha)) - \alpha z(\alpha)] = t$ and we denote the solution as z_3^* . Accordingly, when $t_1 \leq t$, the optimal quality $q_d^{c*} = \frac{2\bar{F}(z_1^*)}{3}$ and the optimal quantity $Q_d^{c*} = z_1^*T_d^{c*}$, where the aggregate target market is $T_d^{c*} = \frac{2\bar{F}(z_1^*)[R(z_1^*) - z_1\bar{F}(z_1^*)]}{3t}$.

We then solve $\max \pi_m^{c2} = z(\alpha)(\alpha - q)q$. The optimal quality satisfies $q = \frac{\bar{F}(z)}{2}$, hence $\pi_m^{c2} = \frac{\bar{F}(z)^2z}{4}$. The unique solution is $z_2^* = g^{-1}(\frac{1}{2})$. Hence, for the sub-problem $\max \pi_m^c = z(\alpha)(\alpha - q)q$ s.t. $q[R(z(\alpha)) - \alpha z(\alpha)] > t$, the optimal solution is z_2^* if $t_2 := \frac{\bar{F}(z_2^*)[R(z_2^*) - z_2\bar{F}(z_2^*)]}{2} \geq t$; otherwise, the optimality is achieved at $q[R(z(\alpha)) - \alpha z(\alpha)] = t$ where the optimal solution is z_3^* . Accordingly, when $t_2 \geq t$, the optimal quality $q_d^{c*} = \frac{\bar{F}(z_2^*)}{2}$ and the optimal quantity $Q_d^{c*} = z_2^*T_d^{c*}$, where the aggregate target market $T_d^{c*} = 1$.

Next, we show z_3^* is unique for $t_2 < t < t_1$. Let $z = z(\alpha)$, then the manufacturer's profit becomes $\pi_m^{c3} = \frac{z[R(z)\bar{F}(z) - z\bar{F}(z)^2 - t]}{[R(z) - z\bar{F}(z)]^2}$. With some math calculation, the FOC of π_m^{c3} satisfies $A(z) := \frac{[R(z)\bar{F}(z) - z\bar{F}(z)^2][\epsilon(z) + g(z) - 1]}{[\epsilon(z) + 2\epsilon(z)g(z) - 1]} = t$, note that

$$\begin{aligned} A'(z)[\epsilon(z) + 2\epsilon(z)g(z) - 1]^2 &= \{[2\bar{F}(z)f(z)z - R(z)f(z)][\epsilon(z) + g(z) - 1] \\ &\quad + [R(z)\bar{F}(z) - z\bar{F}(z)^2][\epsilon'(z) + g'(z)]\}[\epsilon(z) + 2\epsilon(z)g(z) - 1] \\ &\quad - [R(z)\bar{F}(z) - z\bar{F}(z)^2][\epsilon(z) + g(z) - 1][\epsilon'(z)(2g(z) + 1) + 2\epsilon(z)g'(z)] \\ &= R(z)\bar{F}(z)[1 - \epsilon(z)]\{\epsilon'(z)g(z)[1 - 2g(z)] + g'(z)[2\epsilon(z) - 1][1 - \epsilon(z)] \\ &\quad + R(z)f(z)[2\epsilon(z) - 1][\epsilon(z) + g(z) - 1][\epsilon(z) + 2\epsilon(z)g(z) - 1]\}. \end{aligned} \quad (12)$$

To show the uniqueness, it suffices to show that $A'(z) > 0$ when evaluated at z_3^* . Recall that $\epsilon'(z) = \epsilon(z)[1 - g(z) - \epsilon(z)]/z < 0$. Thus, $1 < g(z) + \epsilon(z)$. Moreover, $A(z_3^*) = t$ by definition. Thus, $A(z_3^*) > 0$, which implies that $\epsilon(z) + 2\epsilon(z)g(z) - 1 > 0$. Thus, from (12), to complete the proof, it suffices to show, that $g(z_3^*) \geq \frac{1}{2}$ and, that $\epsilon(z_3^*) \geq \frac{1}{2}$. We show $g(z_3^*) \geq \frac{1}{2}$ by contradiction. To that end, for any $t_2 \leq t \leq t_1$, denote $Z_3(t)$ as the set of solutions that for $A(z) = t$ and suppose that $\exists t' \in (t_2, t_1)$ such that $g(z') < \frac{1}{2}$ for some $z' \in Z_3(t')$. Because $A(z) = t$ is continuous, there exists a continuous function $z_3(t) \in Z_3(t)$ such that $z_3(t') = z'$. Moreover, it is easy to see that $z_3(t_2) = z_2$ and $z_3(t_1) = z_1$. Accordingly, let $t_0 = \inf_t \{t : g(z_3(t)) < \frac{1}{2} \text{ and } t_2 < t < t_1\}$ so that $t_0 \in [t_2, t_1]$. Then, on the one hand, from the continuity of $g(z)$ and $z_3(t)$, $g(z_3(t_0)) = \frac{1}{2}$; hence $\epsilon(z_3(t_0)) > \frac{1}{2}$. Substituting $z_3(t_0)$ into (12), this implies $A'(z_3(t_0)) > 0$. Accordingly, because $A'(\cdot)$ and $z_3(t)$ are continuous, there exists $\delta > 0$ such that for any $t_0 < t < t_0 + \delta$, $A'(z_3(t)) > 0$; hence $z_3(t) > z_3(t_0)$ and $g(z_3(t)) > g(z_3(t_0)) = \frac{1}{2}$. On the other hand, from the definition of t_0 , there exists $\delta' > 0$ such that $g(z_3(t)) < \frac{1}{2}$ for any $t_0 < t < t_0 + \delta'$, which is a contradiction. Thus, $g(z_3^*) \geq \frac{1}{2}$.

Similarly, to show $\epsilon(z_3^*) \geq \frac{1}{2}$, suppose $\exists t' \in (t_2, t_1)$ such that $\epsilon(z') < \frac{1}{2}$ for some $z' \in Z_3(t')$. Because $A(z) = t$ is continuous, there exists a continuous function $z_3(t) \in Z_3(t)$ such that $z_3(t') = z'$. Moreover, it is easy to see that $z_3(t_2) = z_2$ and $z_3(t_1) = z_1$. Accordingly, let $t_0 = \sup_t \{t : \epsilon(z_3(t)) < \frac{1}{2} \text{ and } t_2 < t < t_1\}$, so $t_0 \in (t_2, t_1)$.

Then, on the one hand, from the continuity of $g(z)$ and $z_3(t)$, $\epsilon(z_3(t_0)) = \frac{1}{2}$; hence $g(z_3(t_0)) > \frac{1}{2}$. Substituting $z_3(t_0)$ into (12) thus implies $A'(z_3(t_0)) > 0$. Accordingly, because $A'(\cdot)$ and $z_3(t)$ are continuous, there exists $\delta > 0$ such that for any $t_0 - \delta < t < t_0$, $A'(z_3(t)) > 0$; hence $z_3(t) < z_3(t_0)$ and $\epsilon(z_3(t)) > \epsilon(z_3(t_0)) = \frac{1}{2}$. On the other hand, from the definition of t_0 , there exists $\delta' > 0$ such that $\epsilon(z_3(t)) < \frac{1}{2}$ for any $t_0 < t < t_0 + \delta'$, which is a contradiction. Thus, $\epsilon(z_3^*) \geq \frac{1}{2}$. Hence z_3^* is unique for any $t_2 < t < t_1$. Accordingly, the optimal quality satisfies $q[R(z_3^*) - z_3^*\bar{F}(z_3^*)] = t$, hence $q_d^{c*} = \frac{t}{R(z_3^*) - z_3^*\bar{F}(z_3^*)}$ and the optimal quantity $Q_d^{c*} = z_3^*T_d^{c*}$, where the aggregate target market $T_d^{c*} = 1$. $\square$

Proof of Proposition 5 (i) When $t \geq t_1$, we show $z_1^* < z_c^{c*}$ by contradiction. If $z_1^* \geq z_c^{c*}$, then $\epsilon(z_1^*) \leq 1/2$ and $\epsilon(z_1^*) > 1/2$. On the one hand, $3 - \frac{1}{g(z_1^*)} > 1$. On the other hand, $\frac{1}{1/\epsilon(z_1^*) - 1} \leq 1$, which contradicts the optimality condition for z_1^* . Thus, $z_1^* < z_c^{c*}$ and $\epsilon(z_1^*) \geq 1/2$. From Lemma 5, $k(z) = \bar{F}(z)R(z)[1 - \epsilon(z)]$ is increasing in z for $z < z_c^{c*}$, so $t_1 = \frac{2\bar{F}(z_1^*)[R(z_1^*) - z_1^* \bar{F}(z_1^*)]}{3} < \frac{2\bar{F}(z_c^{c*})R(z_c^{c*})[1 - \epsilon(z_c^{c*})]}{3} = \frac{\bar{F}(z_c^{c*})R(z_c^{c*})}{3} < t_c = \frac{R(z_c^{c*})^2}{4z_c^{c*}}$. Hence $T_d^{c*} < T_c^{c*}$ and $Q_d^{c*} < Q_c^{c*}$. When $t \leq t_2$, similar to the proof in Proposition 1, $z_2^* = g^{-1}(1/2) < z_c^{c*} = \epsilon^{-1}(1/2)$. Moreover, $T_d^{c*} = T_c^{c*} = 1$ and hence $Q_d^{c*} \leq Q_c^{c*}$. When $t_2 < t < t_1$, from the proof for Lemma 5(ii) we know that $A'(z_3^*) > 0$, hence z_3^* is increasing in t . Namely, $z_2^* < z_3^* < z_1^*$, hence $z_3^* < z_c^{c*}$. In addition, $T_d^{c*} = T_c^{c*} = 1$ and hence $Q_d^{c*} \leq Q_c^{c*}$.

(ii) First, we show $\frac{d(q_d^{c*} - q_c^{c*})}{dt} \leq 0$. From Lemma 5(i), q_c^{c*} is a constant for any given t , so we only need to show $\frac{dq_d^{c*}}{dt} \leq 0$. Since q_d^{c*} is continuous at $t = t_1$ and $t = t_2$, based on Lemma 5(ii), it suffices to show q_d^{c*} is decreasing for $t_1 < t < t_2$. For $t_1 < t < t_2$, as $A(z_3^*) = t$, $q_d^{c*} = \frac{t}{R(z_3^*) - z_3^* \bar{F}(z_3^*)} = \frac{A(z_3^*)}{R(z_3^*) - z_3^* \bar{F}(z_3^*)} = \frac{\bar{F}(z_3^*)[\epsilon(z_3^*) + g(z_3^*) - 1]}{\epsilon(z_3^*) + 2\epsilon(z_3^*)g(z_3^*) - 1} =: B(z_3^*)$. From (i), z_3^* is increasing in t , hence it suffices to show $B'(z) \leq 0$ at $z = z_3^*$. Note that

$$\begin{aligned} B'(z)[\epsilon(z) + 2\epsilon(z)g(z) - 1]^2 &= \{-f(z)[\epsilon(z) + g(z) - 1] + \bar{F}(z)[\epsilon'(z) + g'(z)]\}[\epsilon(z) + 2\epsilon(z)g(z) - 1] \\ &\quad - \bar{F}(z)[\epsilon(z) + g(z) - 1][\epsilon'(z)(2g(z) + 1) + 2\epsilon(z)g'(z)] \\ &= -f(z)[\epsilon(z) + g(z) - 1][\epsilon(z) + 2\epsilon(z)g(z) - 1] \\ &\quad + \bar{F}(z)\{\epsilon'(z)g(z)[1 - 2g(z)] + g'(z)[2\epsilon(z) - 1][1 - \epsilon(z)]\}. \end{aligned}$$

As $\epsilon'(z) = \epsilon(z)[1 - g(z) - \epsilon(z)]/z < 0$, $B'(z) \leq 0 \Leftrightarrow g'(z)[2\epsilon(z) - 1][1 - \epsilon(z)] \leq [\epsilon(z) + g(z) - 1]g(z)[2\epsilon(z) - 1]/z$. Moreover, $\epsilon(z_3^*) > \frac{1}{2}$, so we only need to show $\frac{zg'(z)[1 - \epsilon(z)]}{g(z)} \leq [\epsilon(z) + g(z) - 1]$. If $\xi \sim U[1 - b, 1 + b]$, then $\frac{zg'(z)}{g(z)} = g(z) + 1$, so $[g(z) + 1][1 - \epsilon(z)] \leq [\epsilon(z) + g(z) - 1] \Leftrightarrow [g(z) + 1][1 - \epsilon(z)] \leq [\epsilon(z) + g(z) - 1] \Leftrightarrow 2 \leq \epsilon(z)[2 + g(z)] \Leftrightarrow \frac{1}{\epsilon(z)} \leq 1 + \frac{g(z)}{2}$. Moreover, we have $g(z) = \frac{z}{1+b-z}$ and $\epsilon(z) = \frac{z(1+b-z)}{(1+b)z - z^2/2 - (1-b)^2/2}$, hence $\frac{1}{\epsilon(z)} \leq 1 + \frac{g(z)}{2} \Leftrightarrow (1-b)^2 \geq 0$.

Second, we show $q_c^{c*} < q_d^{c*}$ for $b \in (0, 1)$. As q_d^{c*} is decreasing in t and q_c^{c*} is independent of t , it suffices to show $q_d^{c*} > q_c^{c*}$ for $t > t_1$, i.e., $\frac{2\bar{F}(z_1^*)}{3} > \bar{F}(z_c^{c*}) \Leftrightarrow \frac{2(1+b-z_1^*)}{3} > 1 + b - z_c^{c*}$. As in Lemma 1, $z_c^{c*} = \frac{b+1+2\sqrt{b^2-b+1}}{3}$, so it suffices to show $z_1^* < \sqrt{b^2-b+1} := z_0$. As z_1^* is the unique solution for $\frac{d\pi_m^{c1}}{dz} = 0$, so it suffices to show $\frac{d\pi_m^{c1}}{dz}|_{z=z_0} < 0$. Namely, $[1 - 3g(z_0)][1 - \epsilon(z_0)] + \epsilon(z_0)g(z_0) < 0$, or $3 - \frac{1}{g(z_0)} > \frac{1}{1/\epsilon(z_0) - 1}$. With some calculation, it suffices to show $3(b-1)^2 > 0$. $\square$

Appendix B: Numerical Analogs of Propositions 4(ii) and 5(ii)

In this appendix, we expand the scope of key results by numerically illustrating the qualitative impact of using a more general distribution to characterize ξ in Propositions 4(ii) and 5(ii). Toward that end, we reproduce the analogs of Figures 3 and 5 for each of three cases, namely (1) $\xi \sim$

Weibull distribution, (2) $\xi \sim \text{Gamma}$ distribution, and (3) $\xi \sim \text{lognormal}$ distribution. Consistent with Sections 4 and 5, we parametrize these distributions using $b \in (0, 1)$ for ξ . Accordingly, for both the Weibull distribution (defined by $F(x) = 1 - \exp(-(\frac{x}{\lambda})^k)$) and the Gamma distribution (defined by $\xi \sim \Gamma(k, \lambda)$), we set the scale parameter $\lambda = 1$ and we specify the shape parameter to be $k(b)$, where $k(b) = \frac{1}{2b}$ if $0 < b \leq \frac{1}{2}$ and $k(b) = 2(1 - b)$ if $\frac{1}{2} < b < 1$. Similarly, for the lognormal distribution (defined by $\ln \xi \sim N(\mu, \sigma)$), we set the scale parameter $\mu = 0$ and we specify the shape parameter to be $\sigma(b)$, where $\sigma(b) = \sqrt{2b}$ if $0 < b \leq \frac{1}{2}$ and $\sigma(b) = \frac{1}{\sqrt{2(1-b)}}$ if $\frac{1}{2} < b < 1$.

Given these specifications, we generate Figure 6 below by applying Proposition 3 for the 2500 parameter combinations defined by the unit square $a \times b$ using an increment of 0.02 both for $0 < a < 1$ and for $0 < b < 1$. Like Figures 3 and 4, Figure 6 illustrates that higher uncertainty correlates to a larger region of homogeneity for which a positive quality differential is converted to a negative one, thus expanding the scope of Proposition 4(ii) qualitatively.

Figure 6: Robustness test of Proposition 4(ii)

Similarly, we generate Figure 7 by applying Lemma 5 (in Appendix A) for the 2500 parameter combinations defined by the rectangle $t \times b$ using an increment of 0.004 for $0 < t < 0.2$ and an increment of 0.02 for $0 < b < 1$. As Figure 7 illustrates, increased competition intensity translates into a (weakly) more positive quality differential, thus expanding the scope of Proposition 5(ii) qualitatively.

Figure 7: Robustness test of Proposition 5(ii)