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2007

Tan, W. T., Tan, C. K., Lakshminarayanan, S., & Kariwala, V. (2007). Comparison of plant-wide oscillation detection methods. 16th IEEE International Conference on Control Applications (pp. 964-969).

<https://hdl.handle.net/10356/90806>

<https://doi.org/10.1109/CCA.2007.4389358>

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Comparison of Plant-wide Oscillation Detection Methods

Tan Wei Teck, Tan Chee Kiong, Lakshminarayanan Samavedham, and Vinay Kariwala

Abstract—Plant-wide oscillations are common in many industrial processes. During the past few years, various methods have been proposed for the detection of plant-wide oscillations, including methods based on the autocorrelation function, the independent components analysis (ICA) and the spectral envelope. This article is aimed at understanding the advantages and limitations of the available methods using simulated data sets. The ICA based method is also improved by deriving approaches for determining the number of dominant ICs and for differentiating between oscillatory and non-oscillatory ICs.

I. INTRODUCTION

Oscillations are a common form of plant-wide disturbances, which appear frequently in process industries. It is important to detect and diagnose the causes of oscillations in a chemical process because a plant running close to product quality limits or operating constraints is more profitable than a plant that has to back away due to the amplitude of the oscillations [7]. Before a full scale diagnosis exercise is undertaken, it is beneficial to find all signals oscillating at the same frequency as the root cause generally lies within this set.

During the past few years, many methods have been developed for the detection of plant-wide oscillations. Among the available methods, the most promising alternatives include the autocorrelation function (ACF) based method [8], independent components analysis (ICA) [9], [10] and the spectral envelope method [3]. Each of these methods has its advantages and disadvantages, but a thorough quantitative comparison of their performances is lacking in the literature available hitherto. This motivates the present work.

A suitable criterion for comparing the efficiency of a plant-wide oscillation detection method is the accuracy in identifying dominant oscillation frequencies and the variable tags associated with these frequencies. While the computational efficiencies for the various methods differ as well, the difference is only marginal and of less concern, as the analysis is usually carried out off-line. We compare the efficiency of the methods based upon:

- 1) Maximum detectable oscillation frequency;
- 2) Minimum proximity of oscillation frequencies; and
- 3) Maximum tolerable signal to noise ratio

It should be noted that the test to find out the minimum detectable oscillation frequency is extraneous as the results depends on the number of samples available. To prevent

the situation of incorrectly discrediting a method due to the shortage of samples, data sets containing 8000 samples are used for every test. In this paper, we use simulated data sets for all the aforesaid tests as this enables us to ascertain whether the oscillations detected are indeed correct.

As an offshoot, this paper improves the ICA analysis based method [9], [10] by (a) automating the determination of the number of ICs to be detected; (b) redefining a measure for determining the contribution of an IC; and (c) introducing a method for identifying non-oscillatory ICs.

II. DETECTION METHODS

In this section, we provide an overview of the most promising methods available for plant-wide oscillation detection.

A. Autocorrelation Function based Method

The ACF method utilizes the successive zero crossings of the acf of signals to determine the time period of the oscillation. A signal is considered to be oscillating with time period $\bar{T}_p$ if

$$r = \frac{\bar{T}_p}{3\sigma_{T_p}} > 1 \quad (1)$$

where $\bar{T}_p$ and σ_{T_p} are the mean time period and the standard deviation of the zero crossings of the acf. Here, r denotes the regularity of the signal. Presence of multiple oscillations can destroy the regularity of zero crossings and band-pass filters are used to overcome this difficulty. The use of band pass filters also helps in identifying multiple oscillations present in the same signal. To ascertain the plant-wide nature of the oscillations detected in individual signals, a simple heuristic clustering algorithm is used to find all the signals containing an oscillation of same period. The reader is referred to [8] for further details on the acf method. Though useful, the acf method suffers has the following drawbacks [1]:

- The use of band pass filters can give rise to false detections, particularly near the filter boundaries.
- The presence of noise, non-stationary trends and multiple oscillations may destroy the regularity of the zero crossings of the acf. In this case, the automated algorithm may detect none or only one oscillation, despite the spectrum showing multiple distinct peaks.

B. Spectral Envelope Method

Let $x(t)$ be a multivariate vectored-valued time series and X_{pow} denote the power spectral density (psd) matrix. The diagonal matrix X_{var} contains the variance of individual

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signals as its diagonal elements. The spectral envelope of $x(t)$ is given as [5]:

$$\lambda(\omega) = \sup_{\beta \neq 0} \frac{\beta^T X_{\text{pow}}(\omega) \beta}{\beta^T X_{\text{var}}(\omega) \beta} \quad (2)$$

where $\lambda(\omega)$ can be interpreted as the largest proportion of the power that can be obtained at frequency ω for any scaling of the time series $x(t)$ and $\beta(\omega)$ is the particular factor that maximizes $\lambda(\omega)$. It should be noted that the optimum scaling factor $\beta(\omega)$ is not necessarily the same for all ω . When $x(t)$ is normalized to have zero mean and unit variance, $\lambda(\omega)$ is given as the largest eigenvalue of $X_{\text{pow}}(\omega)$ and β is the corresponding eigenvector.

An alternate definition of spectral envelope is obtained by replacing X_{var} by the covariance matrix of $x(t)$ in (2). While both methods usually provide similar results, the use of X_{var} results in a more noisy spectral envelope. In this sense, the use of covariance matrix of $x(t)$ can be advantageous for very noisy data sets.

It is possible to estimate the psd matrix of $x(t)$ as

$$\hat{I}(\omega_k) = x_f(\omega_k) x_f^*(\omega_k); \quad x_f(\omega_k) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} x(t) e^{-2\pi i t \omega_k}$$

where $\omega_k = k/N$, $k = 1, 2, \dots, N$ with N being the data length. $\hat{I}(\omega_k)$, however, does not provide a consistent estimate of the psd matrix. To overcome this difficulty, $\hat{I}(\omega_k)$ is smoothed as

$$\hat{X}_{\text{pow}}(\omega_k) = \sum_{j=-r}^r h_j \hat{I}(\omega_{k+j}) \quad (3)$$

In (3), r denotes the degree of smoothing and is typically selected as 1 or 2. The choice of h is not unique. In this paper, similar to [3], we use $h_j = (r - |j| + 1)/(r + 1)^2$, $j = 0, \pm 1, \pm 2, \dots, \pm r$ with $r = 2$.

The data set is considered to have a dominant oscillation at frequency ω_i if [5]

$$\lambda(\omega_i) > \frac{2}{N} e^{z_\alpha / \nu_N} \sum_{k=1}^{N/2+1} \lambda(\omega_k) \quad (4)$$

where $\nu_N^{-2} = \sum_{j=-r}^r h_j^2$ and z_α is the $(1 - \alpha)$ cutoff of the standard normal distribution. Note that due to smoothing effect, (4) is usually satisfied by a range of frequencies adjacent to the actual oscillation frequency. Define

$$V_\beta(\omega_i) = \nu_N^{-2} \lambda_1(\omega_i) \sum_{\ell=2}^m \frac{\lambda_\ell(\omega_i)}{(\lambda_1(\omega_i) - \lambda_\ell(\omega_i))^2} \beta_\ell(\omega_i) \beta_\ell(\omega_i)^*$$

where $\{\lambda_1(\omega_i), \lambda_2(\omega_i), \dots, \lambda_m(\omega_i)\}$ are the eigenvalues of $X_{\text{pow}}(\omega)$ arranged in decreasing order and $\{\beta_1(\omega_i), \beta_2(\omega_i), \dots, \beta_m(\omega_i)\}$ are the corresponding eigenvectors. Then, the j^{th} variable is considered to contain an oscillation at the frequency ω_i , if $2|\beta_{1,j}(\omega_i)|^2 / \sigma_j > \chi_2^2(\alpha)$. Here, σ_j denotes the j^{th} diagonal term of $V_\beta(\omega_i)$ and $\chi_2^2(\alpha)$ is the cutoff value for a chi-square distribution with 2 degrees of freedom at level α . Based on the recommendation in [5], [3], we use $\alpha = 10^{-3}$ for which $z_\alpha = 3.09$ and $\chi_2^2(\alpha) = 13.82$.

C. Independent Component Analysis

The fundamental idea of ICA is to separate the single-sided power spectra matrix, X , into its underlying informational components which are independent of each other [4], i.e. independent components (ICs). To achieve this, $X \in \mathbb{R}^{m \times N}$ is decomposed into n ICs such that

$$X = A \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = AY$$

where $A \in \mathbb{R}^{m \times n}$ is the mixing matrix, $Y \in \mathbb{R}^{n \times N}$ is the matrix containing the ICs, y_j ($j = 1, \dots, n$) is the j^{th} IC contained in Y and m is the number of process variables. It should be noted that n can at most be equal to m and the ICs detected must be subjected to the scaling algorithm presented by Xia *et al.* in [10].

By decomposing X into its ICs, X can be represented as a linear combination of the ICs. In [10], Xia *et al.* termed each element of A as significance index (SI). The SI provides an idea of the strength of the ICs in each of the process variables. Therefore, the variables containing the spectral features (e.g. oscillations) of an IC are the variables with the highest SI for that particular IC. We point out that in comparison to the ACF and spectral envelope based methods, classification of variables based on SI is vague to some extent and derivation of a rigorous hypothesis test is an issue of future research.

III. IMPROVED ICA BASED METHOD

In this section, we describe the various enhancements made to the ICA based method described in [9], [10].

A. Automatic determination of number of ICs

Computer programs that perform ICA are readily available in the public domain [2]. This code requires the user to have an idea of the number of ICs, n , present in the data. This knowledge is, however, not available except when analyzing synthetic data. It is proposed that n be determined based on the degree of mapping provided by the detected ICs onto the original data. The degree of mapping is measured using Mapping Index (MI) defined as

$$MI = 100 \frac{\|AY\|_{\text{sum}}}{\|X\|_{\text{sum}}} \quad (\%)$$

where $\|\cdot\|_{\text{sum}}$ denotes the sum of all the elements of the matrix. It is recommended that n be selected such that MI is 99% or higher. An iterative algorithm (IterICA) is employed to satisfy this condition, where n is increased sequentially until MI becomes larger than or equal to 99%.

B. Contribution Index

The detected ICs have different degrees of contribution to the overall oscillatory behavior of the plant. To determine

Fig. 1. Oscillatory ICs for test 1

the degree of contribution of each IC in order to rank them, we define Contribution Index (*CI*) as

$$CI = 100 \frac{\|a_j y_j\|_{\text{sum}}}{\|X\|_{\text{sum}}} \quad (\%)$$

where a_j is the j^{th} column of the mixing matrix A . Clearly, an IC with a larger *CI* contributes more towards the oscillatory behavior of the plant. It should be noted that the *CI* proposed here is a modification of the Component-Related Index (*CRI*) suggested by Xia and Howell in [9]. *CI* differs from *CRI* in the sense that the purpose of *CI* is to find the percentage of total energy that can be attributed to individual ICs, while the purpose of *CRI* is to find the significant ICs among the detected ICs.

C. Identifying non-oscillatory ICs

In general, some of the ICs detected by ICA can be non-oscillatory, which are of limited interest for plant-wide oscillation detection studies. To identify the non-oscillatory ICs, similar to the acf method, it is proposed that the regularity of detected ICs be checked. Prior to regularity check, the ICs are scaled as given by (3) with $r = 1$. The regularity is defined in (1). It had been suggested in [8] that an oscillation can be considered as persistent when $r > 1$. This corresponds to rejecting the null hypothesis of a non-persistent oscillation at 95% level of significance. Based on our experience, however, we feel that rejecting the null hypothesis at such high level of significance is too stringent when applied to noisy industrial data sets. In this paper, the null hypothesis is rejected when $r > 0.76$ i.e. an 80% level of significance is considered.

Though useful, the proposed method of identifying non-oscillatory ICs is not foolproof, as it is difficult to decide upon the threshold value of r . It is possible that the regularity check classifies an oscillatory IC as non-oscillatory. This tends to happen when the oscillatory IC has its dominant peak in the low frequency range and the data set does not contain enough samples to generate reliable estimates for $\bar{T}_p$ and σ_{T_p} . In our experience, an IC that has a dominant peak and a high *CI* can be considered as an oscillating IC even though it fails the regularity test.

Fig. 2. Significance indices of oscillatory ICs for test 1

IV. COMPARISON TESTS

For comparing the performance of different methods, the following simulated data set is used:

$$\begin{aligned} x_1(t) &= \gamma_1 \cos(2\pi\omega_1 t) + 2\epsilon(t) + \epsilon(t-1) \\ x_2(t) &= \gamma_2 \cos(2\pi\omega_1(t-5)) + 2\epsilon(t) + \epsilon(t-1) \\ x_3(t) &= \gamma_3 \cos(2\pi\omega_1(t-15)) + 2\epsilon(t) + \epsilon(t-1) \\ x_4(t) &= \gamma_4 \cos(2\pi\omega_1(t-2)) + 2\epsilon(t) + \epsilon(t-1) \\ x_5(t) &= \gamma_5 \cos(2\pi\omega_2 t) + 2\epsilon(t) - \epsilon(t-1) \\ x_6(t) &= \gamma_6 \cos(2\pi\omega_2(t-7)) + 2\epsilon(t) - \epsilon(t-1) \\ x_7(t) &= \gamma_7 \cos(2\pi\omega_2(t-10)) + 2\epsilon(t) - \epsilon(t-1) \\ x_8(t) &= \gamma_8 \cos(2\pi\omega_2(t-20)) + 2\epsilon(t) - \epsilon(t-1) \\ x_9(t) &= 2\epsilon(t) + \epsilon(t-1) \\ x_{10}(t) &= 2\epsilon(t) - \epsilon(t-1) \\ x_{11}(t) &= \gamma_9 \cos(2\pi\omega_1(t-6)) + \gamma_{10} \cos(2\pi\omega_2(t-8)) \\ &\quad + 2\epsilon(t) + \epsilon(t-1) \\ x_{12}(t) &= \gamma_{11} \cos(2\pi\omega_1(t-16)) + \gamma_{12} \cos(2\pi\omega_2(t-4)) \\ &\quad + 2\epsilon(t) - \epsilon(t-1) \end{aligned}$$

where $\epsilon(t)$ is a white noise sequence with unit variance. The performances of the plant-wide oscillation detection methods are compared for different values of the parameters. When ω_1 and ω_2 lie far below 0.5 Hz, are well-separated and signal to noise ratio is high, the efficiencies of the methods described in Section II is found to be nearly identical. In the following discussion, we present some limiting examples, where the performance of one or more methods is superior as compared to the rest. For this purpose, we use

$$\gamma = \kappa [8 \ 6 \ 4 \ 2 \ 9 \ 7 \ 5 \ 3 \ 4 \ 5 \ 8 \ 6] \quad (5)$$

with $\kappa = 0.5$ for Tests 1 and 2 and $\kappa = 0.02$ for Test 3.

A. Maximum Detectable Oscillation Frequency (Test 1)

For this test, ω_1 is chosen as 0.002 Hz and ω_2 is 0.49 Hz. The purpose of this test is to determine whether the three oscillation detection methods are able to detect oscillations with frequency close to the theoretical upper limit of 0.5 Hz. The following results are obtained using different methods.

Fig. 3. Spectral envelope for test 1

Fig. 4. Variable classification test statistic for spectral envelope for test 1

Fig. 5. Oscillatory ICs for test 2

Fig. 6. Significance indices of oscillatory ICs for test 2

Improved ICA method. From Figure 1, it can be seen that both the 0.002 Hz (IC 2) and the 0.49 Hz (IC 1) oscillations are detected by the IterICA algorithm. The *CI* of the IC showing a dominant oscillation at ω_2 (IC 1) is also correctly computed to be higher than the *CI* of the IC showing a dominant oscillation at ω_1 (IC 2). This should indeed be the case because the sum of the weights for ω_2 is 3.5 while the sum of the weights for ω_1 is only 3.2. As shown in Figure 2, all the significance indices of IC 1 and IC 2 are in agreement with the relative weights of their dominant oscillations. Therefore, the identification of the 0.002 Hz and the 0.49 Hz oscillations via ICA can be concluded to be correct.

Spectral envelope method. The spectral envelope method is able to detect the correct oscillation frequencies. The test statistic for variable classification is shown in Figure 4, where the y-axis has been limited to 500 for better visualization. It is noted that the test statistic at both frequencies just exceeds the cutoff value of 13.84 for Tags not containing the oscillation. As these test statistic values are significantly less (by a factor greater than 200) than the corresponding values for other tags, the identification of tags is concluded to be correct.

ACF based method. The ACF method is initialized to search for oscillations in the frequency range 0.001 – 0.5 Hz, but the method finds a single oscillation at frequency 0.5

Hz causing premature termination. When the filter ranges are artificially narrowed to 0.001–0.01 Hz and 0.01–0.5 Hz, the method correctly detects the low-frequency oscillation and also identifies the tags associated with it. The high frequency oscillation is, however, identified at frequency 0.5 Hz instead of 0.49 Hz. Thus, this test highlights the limitation of ACF method regarding premature termination and its inability to handle high frequency oscillations. This is likely due to the fact that the ACF method is better suited for identifying oscillations at low frequencies.

B. Minimum proximity of oscillation frequencies (Test 2)

Here, we investigate whether the methods can discern between oscillations present at frequencies very close to each other. It is known that the level of uncertainties in the power spectrum is highest at low frequencies. With this in mind, we set ω_1 as 0.002 Hz and ω_2 as 0.0025 Hz. The reason for doing so is that if the methods are able to discern such low frequency oscillations, then resolution at higher frequencies should not pose a problem.

Improved ICA method. IterICA is able to recognize that there are two oscillations present in the data. In Figure 5, IC 1 (blue bars in Figure 6) shows a dominant oscillation at 0.0025 Hz and IC 2 (grey bars in Figure 6) shows a dominant oscillation at 0.002 Hz. The *CI* of the IC showing a dominant oscillation at ω_2 (IC 1) is again correctly computed to be

Fig. 7. Spectral envelope for test 2

Fig. 8. Variable classification test statistic for spectral envelope for test 2

higher than the CI of the IC showing a dominant oscillation at ω_1 (IC 2). In Figure 6, all the significance indices of IC 1 and IC 2 are in agreement with the relative weights of their dominant oscillations. Somewhat surprisingly, ICA is able to distinguish between the two oscillations, even when ω_2 is changed to 0.0021 Hz.

Spectral envelope method. It can be observed from Figures 7 and 8 that the spectral envelope method identifies both the frequencies. It detects these frequencies, however, as a broad spectral feature and not distinct oscillations. When ω_2 is changed to 0.0021 Hz, this method detects a single peak only highlighting its limitation in discerning between oscillations with very close frequency content.

ACF based method. The ACF method, as mentioned before, is better suited for detecting oscillations at low frequencies and this point is verified here. It is able to detect the two frequencies correctly, once the filter ranges are narrowed around the oscillation frequencies. The probable sources of the oscillations are also in agreement with the equations used for generating the simulated data. Note that the selection of filter ranges by the user is not necessarily a big limitation, as this can be easily done by looking at the peaks in the power spectrum plots.

Fig. 9. Oscillatory ICs for test 3

Fig. 10. Significance indices of oscillatory ICs for test 3

C. Maximum tolerable signal to noise ratio (Test 3)

Industrial data tends to be corrupted with noise and this motivated us to find out whether the efficiency of the detection methods is compromised if the data have poor signal to noise ratio (SNR). For this purpose, we used ω_1 as 0.02 Hz and ω_2 as 0.3 Hz. The weights of various signals are given by (5).

The SNR is defined as

$$\text{SNR} = \left(\frac{RMS_{\text{signal}}}{RMS_{\text{noise}}} \right)^2$$

where RMS_{signal} and RMS_{noise} are the root mean square of the signal and noise, respectively. Here, the SNR for each of the 12 variables is taken to be approximately one.

Improved ICA method. As can be seen from Figure 9, ICs displaying dominant oscillations at 0.02 Hz (blue bars in Figure 10) and 0.3 Hz (grey bars in Figure 10) are correctly identified. In Figure 10, the variables associated with individual oscillations are also correctly identified except Tags 3, 4 and 8, for which SNR is worst.

Spectral envelope method. When the spectral envelope is computed using X_{var} , the method is unable to identify any oscillations. This is not surprising, as the use of X_{var} results in a noisy spectral envelope, as previously pointed out in Section II.B. Recomputing the spectral envelope by replacing covariance matrix in (2), correctly identifies both the two

Fig. 11. Spectral envelope for test 3 (computed with covariance matrix)

Fig. 12. Variable classification test statistic for spectral envelope for test 3

oscillating frequencies, as shown in Figure 11. The method is also able to find the variables oscillating at individual frequencies, except Tags 3 and 12.

ACF based method. The ACF method is unable to detect any oscillations correctly even after the filter ranges are narrowed around the oscillatory frequencies present in the data. This is due to the fact that the presence of noise disrupts the regularity of the zero crossings of the acf.

V. CONCLUSIONS

Table I provides a comparison of the advantages and disadvantages of the methods studied in this work. With large sample size, high SNR, sufficient spread in the oscillation frequencies and adequate sampling rate, all the methods work equally well and produce accurate results. Even if a large number of samples are available, it is seen that the ACF method may terminate prematurely and may fail to detect any oscillations from very noisy data. As a result, when analyzing industrial data, the improved ICA and spectral envelope methods usually provide more reliable results. The spectral envelope method works very well except in singular cases where the oscillating frequencies are extremely close. The improved ICA technique presented here performs the best as long the non-oscillatory ICs are avoided. At this stage, we conclude that correct detection and diagnosis of oscillations is possible with a consensus between the improved ICA

Analysis method	Advantages	Limitations
ICA	<ul style="list-style-type: none"> • Able to detect oscillations up to the Nyquist frequency • Can handle noisy data • Able to discern signals oscillating at very close frequencies 	<ul style="list-style-type: none"> • Method for identifying variables associated with individual oscillations is somewhat vague • Detection of non-oscillatory ICs is not foolproof
Spectral Envelope	<ul style="list-style-type: none"> • Able to handle noisy data • Able to detect oscillations up to the Nyquist frequency 	<ul style="list-style-type: none"> • May fail to resolve oscillations with very close frequencies
ACF	<ul style="list-style-type: none"> • Well suited for detecting oscillations of low frequency nature 	<ul style="list-style-type: none"> • Prone to false detections due to usage of ideal band pass filters • If SNR is low, it may result in undetected oscillations

TABLE I
PERFORMANCE COMPARISON

method and the spectral envelope method.

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