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An Algebraic MIDO-MISO Code Construction

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Abstract—Multiple-input double-output (MIDO) codes are important in future wireless communications, where the portable end-user device is physically small and uses only two receive antennas. In this paper, we address the design of 4×2 MIDO codes. Starting from a 4×4 space-time block code matrix built from a cyclic division algebra, two ways of puncturing the code are presented, resulting in either a well-shaped MIDO code, or a MIDO code with some orthonormal columns, yielding fast maximum-likelihood (ML) decodability. The well-shaped MIDO code outperforms the fast decodable one through simulations, an indication that the shaping property stays an important code design criterion. We then provide a slightly modified version of the well-shaped MIDO code which both preserves the shaping and makes the code fast ML decodable. Finally, we show that a multiple-input single output (MISO) code is actually embedded in the MIDO code, allowing the transmitter to choose between sending a MIDO or MISO code, without having to change the encoder. All the proposed codes have the non-vanishing determinant (NVD) property.

I. INTRODUCTION

One of the most interesting wireless applications currently is the design of 4×2 multiple input-double output (MIDO) codes. Such asymmetric systems can be used in the communication between, for instance, a TV station and a portable digital-TV device. Codes with the so-called non-vanishing determinant (NVD) property and excellent performance for this purpose have been designed in e.g. [4], [9], [5], but all the codes require high maximum-likelihood (ML) complexity decoding, namely full-dimensional sphere decoding.

The decoding complexity induced by sphere decoding has been a bottleneck for many efficient multiple input-multiple output (MIMO) space-time codes. In order to reduce the ML decoding complexity, an approach similar to the construction of the Silver code [6], [3] was adopted in [2]. Namely, the Silver code is formed by adding together two copies of the Alamouti code: one is the original Alamouti code, the other copy consists of elements that come from a unitary transformation of the original information symbols. Before addition, the unitarily transformed copy is yet to be multiplied with a diagonal twist matrix. The combination of two Alamouti codes brings fast decodability. Furthermore, the Silver code lattice has an orthogonal basis and the NVD property — putting it into the family of Perfect codes [10]. The MIDO code (referred to as BHV-code later on) proposed in [2], for its part, is constructed in a similar manner by combining a quasi-

orthogonal code [13] with a twisted unitary transformation of another quasi-orthogonal code. This resulted in a MIDO code that does have lower decoding complexity, but unfortunately does not have full rank. This is of course acceptable, because good performance is still achieved at low-to-moderate SNRs and with one dimension less in the sphere decoder. The most recent results on fast decodable codes have appeared in [12], where new constructions with optimized performance have been presented.

In this paper, the goal is to construct MIDO codes that have comparable performance with the BHV-code at low-to-moderate SNRs, admit fast ML decoding, and in addition have the NVD property, which in particular implies full rank. The latter property is expected to improve the performance at high SNRs. As a by-product, we show that one of the constructed codes also has a MISO code embedded in it, giving the operator the freedom to choose between two transmission schemes without having to change the encoder. E.g. when high correlation is to be expected, one may want to choose a MISO code over a MIDO code, whereas with no or low correlation, a MIDO code is to be preferred. Note that throughout the paper we consider 4 Tx-antennas and 2 Rx-antennas, and transmission over a coherent Rayleigh fading channel with perfect channel state information at the receiver (CSIR). By MISO code we refer to a code that only carries one symbol per channel use, while a MIDO code carries two symbols per channel use. When high correlation is present when receiving with two antennas, the situation resembles the one where we only have one receiver: the correlation destroys the variety of the two different versions of the signal received, and the amount of information that can be fully supported in such a scenario is only one symbol per channel use.

The paper is organized around the different code constructions: a 4×4 space-time code is presented in Section II, that illustrates the process of constructing space-time codes from cyclic division algebras. The same code is then punctured in Section III, yielding two MIDO codes, one well-shaped and one fast decodable. The well-shaped code is furthermore modified in Section IV to be made fast-decodable. Finally, we describe in Section V how a further puncturing of the aforementioned code reveals a MISO code.

II. BACKGROUND AND FIRST CONSTRUCTION

Since the work of Sethuraman et al. [11], a popular technique for constructing space-time block codes is to use cyclic division algebra. The procedure goes as follows (this is a rough rather intuitive explanation, details can be found in [11]). First information symbols, say QAM symbols, are encoded as linear combinations in a number field L of degree n whose Galois group is cyclic. For example, take $\zeta_5 = e^{2i\pi/5}$ a primitive 5th root of unity, and consider the number field $L = \mathbb{Q}(i, \zeta_5)$, given by

$$\mathbb{Q}(i, \zeta_5) = \{x = a + b\zeta_5 + c\zeta_5^2 + d\zeta_5^3, a, b, c, d \in \mathbb{Q}(i)\}.$$

It is of degree 4 (that is of dimension 4 as a vector space over $\mathbb{Q}(i)$). Since QAM symbols can be seen as elements in $\mathbb{Z}[i] \subset \mathbb{Q}(i)$, we have that one element x in $\mathbb{Q}(i, \zeta_5)$ encodes 4 QAM symbols, namely a, b, c, d . The Galois group of $\mathbb{Q}(i, \zeta_5)/\mathbb{Q}(i)$ describes maps that permute ζ_5 and its conjugates ζ_5^j , $j = 2, 3, 4$ while fixing $\mathbb{Q}(i)$. If $\sigma(\zeta_5) = \zeta_5^2$, we have that

$$\sigma^2(\zeta_5) = \zeta_5^4, \sigma^3(\zeta_5) = \zeta_5^3, \sigma^4(\zeta_5) = \zeta_5$$

yielding a cyclic Galois group. We now build a non-commutative algebra $\mathcal{A}$ based on L . As a vector space, $\mathcal{A}$ can be seen as a sum of n copies of the chosen number field L of degree n . In our example, this gives

$$\mathcal{A} = \mathbb{Q}(i, \zeta_5) \oplus e\mathbb{Q}(i, \zeta_5) \oplus e^2\mathbb{Q}(i, \zeta_5) \oplus e^3\mathbb{Q}(i, \zeta_5)$$

where $1, e, e^2, e^3$ forms a basis and e^4 must be an element of the base field $\mathbb{Q}(i)$, say $e^4 = i$. A space-time block code can be obtained by considering the matrix of left multiplication in this given basis. If $x_0 + ex_1 + e^2x_2 + e^3x_3 \in \mathcal{A}$, $x_0, x_1, x_2, x_3 \in \mathbb{Q}(i, \zeta_5)$, then its corresponding multiplication matrix is

$$X = \begin{pmatrix} x_0 & i\sigma(x_3) & i\sigma^2(x_2) & i\sigma^3(x_1) \\ x_1 & \sigma(x_0) & i\sigma^2(x_3) & i\sigma^3(x_2) \\ x_2 & \sigma(x_1) & \sigma^2(x_0) & i\sigma^3(x_3) \\ x_3 & \sigma(x_2) & \sigma^2(x_1) & \sigma^3(x_0) \end{pmatrix} \quad (1)$$

where the factor i comes from $e^4 = i$, and σ^j , $j = 1, 2, 3, 4$ are the elements of the Galois group, appearing due to the non-commutative multiplication defined on $\mathcal{A}$, by $xe = e\sigma(x)$ for $x \in L$.

Let $\mathcal{C}$ be the codebook formed by codewords X of the above form. For it to be fully diverse, it is enough to have

$$\det(X' - X'') \neq 0$$

for $X' \neq X''$ in $\mathcal{C}$, or equivalently by linearity

$$\det(X) \neq 0$$

for $X \neq 0$ in $\mathcal{C}$. This can be obtained by asking for $\mathcal{A}$ to be a division algebra, property that depends on the choice of the value of e^n (or $e^4 = i$ in our example). If there exists no element $x \in \mathbb{Q}(i, \zeta_5)$ such that its norm is i, i^2 or i^3 , i.e., $N_{\mathbb{Q}(i, \zeta_5)/\mathbb{Q}(i)}(x) = i, -1$ or $-i$, then $\mathcal{A}$ will be a division algebra [11].

Let us check that $\mathcal{A}$ is indeed a division algebra. Note for this purpose that $\mathbb{Q}(\zeta_5 + \zeta_5^{-1}) = \mathbb{Q}(\sqrt{5})$ is a subfield of $\mathbb{Q}(\zeta_5)$.

Suppose now that there exists an element $x \in L$ such that $N_{\mathbb{Q}(i, \zeta_5)/\mathbb{Q}(i)}(x) = i$, then, by transitivity of the norm

$$N_{\mathbb{Q}(i, \zeta_5)/\mathbb{Q}(i)}(x) = N_{\mathbb{Q}(i, \sqrt{5})/\mathbb{Q}(i)} N_{\mathbb{Q}(i, \zeta_5)/\mathbb{Q}(i, \sqrt{5})}(x) = i,$$

which implies the existence of an element $y = N_{\mathbb{Q}(i, \zeta_5)/\mathbb{Q}(i, \sqrt{5})}(x)$ such that

$$N_{\mathbb{Q}(i, \sqrt{5})/\mathbb{Q}(i)}(y) = i,$$

a contradiction [1]. The case of a norm of $-i$ can be dealt similarly, since using the same argument of transitivity of the norm, it is enough to show that there cannot be an element with norm $-i$ over $\mathbb{Q}(i, \sqrt{5})/\mathbb{Q}(i)$. If there were an element x with $N_{\mathbb{Q}(i, \sqrt{5})/\mathbb{Q}(i)}(x) = -i$, then ix would have norm

$$i^2 N_{\mathbb{Q}(i, \sqrt{5})/\mathbb{Q}(i)}(x) = i,$$

a contradiction. The case of a norm of -1 is tougher though. However, there are several ways to deal with it. We refer the reader to [14, Section 8], where the proof used for the algebra D_4 can be used here verbatim.

We have thus constructed in our example a fully-diverse 4×4 space-time block code. It furthermore has the so-called non-vanishing determinant property [10], since when the information symbols are restricted to algebraic integers in L , then its minimum determinant belongs to $\mathbb{Z}[i]$, and thus $\min_{X \neq 0} |\det(X)| = 1$, which, prior to normalization, gives a lower bound which does not depend on the size of the QAM constellation used. There is one condition missing for this code to be called a perfect code [10], namely the so-called shaping property: the encoding matrix should be unitary, which is not the case here.

III. MIDO CODES THROUGH PUNCTURING

The goal of this paper is the design of 4×2 MIDO codes. To get such codes, we start from the matrix (1) and perform some puncturing in two different ways: one which will give a MIDO code with the shaping property, another one which will have some orthonormal columns.

A. A well-shaped MIDO code

Let us first repeat a remark made above, namely that $\mathbb{Q}(\zeta_5 + \zeta_5^{-1}) = \mathbb{Q}(\sqrt{5})$ is a subfield of $\mathbb{Q}(\zeta_5)$. As a first puncturing, we restrict ourselves to elements in $\mathbb{Q}(\sqrt{5})$ instead of $\mathbb{Q}(\zeta_5)$. Note that since $\sigma^2(\zeta_5) = \zeta_5^4$, we further have

$$\sigma^2(\zeta_5 + \zeta_5^{-1}) = \zeta_5^4 + \zeta_5 + 5^{-4} = \zeta_5^{-1} + \zeta_5$$

and thus $\mathbb{Q}(\sqrt{5})$ is fixed by σ^2 . This yields a codebook $\mathcal{C}_1$ made of codewords of the form

$$X = \begin{pmatrix} x_0 & i\sigma(x_3) & ix_2 & i\sigma(x_1) \\ x_1 & \sigma(x_0) & ix_3 & i\sigma(x_2) \\ x_2 & \sigma(x_1) & x_0 & i\sigma(x_3) \\ x_3 & \sigma(x_2) & x_1 & \sigma(x_0) \end{pmatrix}. \quad (2)$$

It is now enough to notice that we are working on the same field extension as the Golden code [1], meaning that we can

use the same shaping technique. Denote:

$$\begin{aligned}\theta &= \frac{1 + \sqrt{5}}{2}, \\ \sigma(\theta) &= \frac{1 - \sqrt{5}}{2} = 1 - \theta, \\ \alpha &= 1 + i - i\theta, \\ \sigma(\alpha) &= 1 + i - i\sigma(\theta).\end{aligned}$$

Every entry x_j in the above matrix is now taken of the form

$$x_j = \frac{\alpha(a_j + b_j\theta)}{\sqrt{5}}, \quad j = 1, 2, 3, 4,$$

where a_j, b_j are QAM symbols. We thus indeed get a MIDO code carrying 8 complex QAM symbols, with unitary encoding matrix yielding the shaping property. Note that the $\sqrt{5}$ as the denominator is a normalization factor.

B. A fast-decodable MIDO code

We now propose another puncturing, which focuses this time on having orthonormal columns, in order to have a provable fast (ML) decodability. Since $\mathbb{Q}(\zeta_5, i) = \mathbb{Q}(\zeta_{20})$, where $\zeta_{20} = e^{2i\pi/20}$ is a primitive 20th root of unity, we can alternatively take as basis for $\mathbb{Q}(i, \zeta_5)$ the set $1, \zeta := \zeta_{20}, \zeta^2, \zeta^3$. An element x is then written as

$$x = a + b\zeta + c\zeta^2 + d\zeta^3, \quad a, b, c, d \in \mathbb{Q}(i).$$

We now propose the following puncturing. Take x_0 and x_1 of the form

$$a + ib\zeta + c\zeta^2 + id\zeta^3, \quad a, b, c, d \in \mathbb{Q}$$

so that $\sigma^2(x_0) = \bar{x}_0$, $\sigma^2(x_1) = \bar{x}_1$. For x_2 and x_3 , take instead

$$a(1+i) + b(1-i)\zeta + c(1+i)\zeta^2 + d(1-i)\zeta^3, \quad a, b, c, d \in \mathbb{Q}$$

to get this time $\sigma^2(x_2) = -\bar{x}_2$, $\sigma^2(x_3) = -\bar{x}_3$. This puncturing results in a codebook $\mathcal{C}_2$ where codewords are given by

$$X = \begin{pmatrix} x_0 & i\sigma(x_3) & -\bar{x}_2 & \overline{i\sigma(x_1)} \\ x_1 & \sigma(x_0) & -\bar{x}_3 & -\overline{\sigma(x_2)} \\ x_2 & \sigma(x_1) & \bar{x}_0 & -\overline{\sigma(x_3)} \\ x_3 & \sigma(x_2) & \bar{x}_1 & \overline{\sigma(x_0)} \end{pmatrix}. \quad (3)$$

An easy computation shows that the 1st and 3rd row, respectively the 2nd and 4th row, are orthonormal. Alternatively, by permuting the 2nd and 3rd rows and columns respectively, we get

$$X = \begin{pmatrix} x_0 & -\bar{x}_2 & i\sigma(x_3) & \overline{i\sigma(x_1)} \\ x_2 & \bar{x}_0 & \sigma(x_1) & -\overline{\sigma(x_3)} \\ x_1 & -\bar{x}_3 & \sigma(x_0) & -\overline{\sigma(x_2)} \\ x_3 & \bar{x}_1 & \sigma(x_2) & \overline{\sigma(x_0)} \end{pmatrix}$$

which clearly exhibit the Alamouti block structure of the code. This MIDO code is then fast-decodable. By fast-decodable, we mean that the complexity of a sphere decoding is reduced. The complexity of a sphere decoder (see e.g. [2]) is defined as the number of points visited in a search tree, or equally,

the number of metrics computed during the search for the closest point. This number cannot exceed M^κ , where M is the constellation size and κ is the number of complex information symbols (e.g. QAM symbols) in the encoding matrix. Here κ is referred as the dimension of a sphere decoder. If we can drop the exponent, i.e., the dimensionality of a sphere decoder, we say that the code is fast-decodable.

C. Simulations and comments

The performances of the MIDO codes $\mathcal{C}_1$ and $\mathcal{C}_2$ are shown in Fig. 1. We see that the fast-decodable code $\mathcal{C}_2$ loses 0.5 – 0.7 dB to $\mathcal{C}_1$. Though both codes are coming from the same division algebra, this suggests that the loss of shaping for $\mathcal{C}_2$ worsens its performance.

IV. A WELL-SHAPED FAST-DECODABLE MIDO CODE

The simulations shown in Fig. 1 suggest as already mentioned above that it is favorable for the code performance to maintain the shaping of the code. We thus get back to the first well-shaped MIDO code $\mathcal{C}_1$ we proposed in (2), and propose a small variation to help the fast decodability.

A. Code construction

Take again a codeword

$$\begin{pmatrix} x_0 & i\sigma(x_3) & ix_2 & i\sigma(x_1) \\ x_1 & \sigma(x_0) & ix_3 & i\sigma(x_2) \\ x_2 & \sigma(x_1) & x_0 & i\sigma(x_3) \\ x_3 & \sigma(x_2) & x_1 & \sigma(x_0) \end{pmatrix}$$

and multiply both the 3rd and 4th column by ζ_8^{-1} , where $\zeta_8 = e^{2i\pi/8}$ is a primitive 8th root of unity. Then multiply the 3rd and 4th row this time by ζ_8 . Note that this of course bring the matrix entries out of the algebra we started with, but without changing the determinant! We further note that we can use $e^4 = -i$ instead of $e^4 = i$, since $-i$ is not a norm, as already shown in Section II. We obtain for the codebook $\mathcal{C}_3$

$$\begin{pmatrix} x_0 & -i\sigma(x_3) & -\zeta_8 x_2 & -\zeta_8 \sigma(x_1) \\ x_1 & \sigma(x_0) & -\zeta_8 x_3 & -\zeta_8 \sigma(x_2) \\ \zeta_8 x_2 & \zeta_8 \sigma(x_1) & x_0 & -i\sigma(x_3) \\ \zeta_8 x_3 & \zeta_8 \sigma(x_2) & x_1 & \sigma(x_0) \end{pmatrix}. \quad (4)$$

Let us denote by $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$ and $\mathbf{c}_4$ the 4 columns of the above matrix. It can be easily seen that the above manipulations result in having columns 1 and 3, and 2 and 4 respectively, orthogonal (that is $\mathbf{c}_1^T \mathbf{c}_3 = 0$, $\mathbf{c}_2^T \mathbf{c}_4 = 0$) rather than orthonormal (with respect to the complex inner product), without changing the shaping. We now discuss how this helps fast decodability.

B. Fast decodability

The analysis follows the one proposed in [2] to evaluate the sphere decoder complexity. We first vectorize the codeword X as given in (4) to get a column vector $\text{vec}(X)$, which we write as

$$\text{vec}(X) = G\mathbf{s}$$

where $\mathbf{s}$ contains the QAM information symbols

$$\mathbf{s} = (a_0, b_0, a_1, b_1, a_2, b_2, a_3, b_3)^T$$

and G is the corresponding encoding matrix. Let

$$Y = HX + V$$

be the channel equation, where H is the 2×4 channel matrix and V the noise at the receiver. We again vectorize this equation to get

$$\text{vec}(Y) = (I_4 \otimes H)Gs + \text{vec}(V),$$

where I_4 denotes the 4-dimensional identity matrix, and $\otimes$ is the tensor (Kronecker) product of matrices. To perform a sphere decoding, we start by computing the QR decomposition of the matrix

$$F = (I_4 \otimes H)G$$

that is $F = QR$ where Q is a unitary matrix and R is an upper triangular matrix. The number of zero entries appearing in the upper part of R is a measure of the fast decodability of the code via sphere decoding. For example, it is shown in [2] that when such a computation is done for the Silver code, two zeros appear in the upper part of R . For this code, we observe that 4 zeros appear in the upper part of R .

C. Comparison with coding gain bounds

In order to measure how far from the optimum the (asymptotic) coding gain of the codes is, we adopt the language used in [8]. The proposed MIDO codes can be seen as 16-dimensional lattices in a 32-dimensional real inner product space $M_{4 \times 4}(\mathbb{C})$, where the inner product is defined as

$$\langle A, B \rangle = \Re(\text{tr}(AB^\dagger)).$$

To compare the coding gain of two different MIDO codes, we first scale the code lattices so that the used energy for given n codewords is approximately the same. This scaling is done by multiplying the code lattice so that the volume of the code lattice's fundamental parallelotope is one. We then have the following.

Proposition 1: [8] Let us suppose that L is an orthogonally shaped MIDO lattice-code in $M_{4 \times 4}(\mathbb{C})$. If we further assume that the volume of the fundamental parallelotope of L is scaled to 1, we have that

$$\min_{X \in L, X \neq 0} |\det(X)| = 1 \leq \frac{1}{16}.$$

A straightforward calculation gives that the volume of the fundamental parallelotope of the code $\mathcal{C}_3$ is $5^4 \cdot 2^8$. At the same time, the minimum determinant of the code is already 1. If we now scale the code $\mathcal{C}_3$ with $(1/(5^4 \cdot 2^8))^{1/16}$, the resulting code lattice $\mathcal{C}_3^* = ((1/(5^4 \cdot 2^8))^{1/16}) \cdot \mathcal{C}_3$ has fundamental parallelotope of volume 1. We now see that the minimum determinant of the lattice $\mathcal{C}_3^*$ is

$$((1/(5^4 \cdot 2^8))^{1/16})^4 = 1/20.$$

Comparing this to Proposition 1, we conclude that the normalized minimum determinant of the code $\mathcal{C}_3$ is very close to the optimum minimum determinant of codes having orthogonal shaping.

Fig. 1. Comparison of the new proposed codes $\mathcal{C}_1$ (Code 1, shaped), $\mathcal{C}_2$ (Code 2, FD), $\mathcal{C}_3$ (Code 3, FD, shaped) versus the code proposed by Biglieri-Hong-Viterbo (BHV), and a maximal order code.

D. Simulation results

We compare the performance of the code $\mathcal{C}_3$ with respect to the previously proposed MIDO codes $\mathcal{C}_1, \mathcal{C}_2$, as well as the BHV-code in Fig. 1.

The code $\mathcal{C}_3$ with shaping and fast-decodability is equally good as the first (shaped) code $\mathcal{C}_1$. The code $\mathcal{C}_3$ only loses about 0.2 dB to the BHV-code, until the full diversity starts to show at 15dB — from which point our code beats the BHV-code with a gap that increases with SNR.

The code proposed by Srinath and Rajan (SR-code) [12] is equally good as the BHV-code at low-moderate SNRs, and then due to its full diversity gets better than BHV at higher SNRs. This means that our code slightly loses to the SR-code, but with such a small difference that it is insignificant in practice. On the other hand, the code proposed here has a nice algebraic structure and provable NVD.

The code $I_{A_{MAX}}$ [4] is the best known MIDO code, but requires full sphere decoding complexity. In addition, it does not have shaping (it is constructed using a maximal order) and thus sphere encoding [7] is required to achieve the performance shown in the picture. Hence it has totally different nature and purpose as the codes proposed in this paper, and is simply added for the sake of completeness in comparison.

V. A MISO CODE EMBEDDING

We consider a final puncturing of the code (4) which reveals an embedded MISO code. Recall that coefficients x_j , $j = 0, 1, 2, 3$ in the codeword (2) are of the form

$$x_j = \frac{\alpha(a_j + b_j\theta)}{\sqrt{5}}, \quad j = 1, 2, 3, 4,$$

where a_j, b_j are QAM symbols. Let us set the coefficients b_j to zero, $j = 0, 1, 2, 3$, so that only the 4 QAM symbols a_j are

left. We thus get a codeword of the form

$$\begin{pmatrix} x_0 & -ix_3 & -\zeta_8 x_2 & -\zeta_8 x_1 \\ x_1 & x_0 & -\zeta_8 x_3 & -\zeta_8 x_2 \\ \zeta_8 x_2 & \zeta_8 x_1 & x_0 & -ix_3 \\ \zeta_8 x_3 & \zeta_8 x_2 & x_1 & x_0 \end{pmatrix}$$

since σ fixes $\mathbb{Q}(i)$. This code inherits fast decodability from the MIDO code (4) by noting that similarly, the first and 3rd columns, 2nd and 4th columns respectively, are orthogonal.

VI. CONCLUSION

The construction of 4×2 multiple-input double-output codes are motivated by the introduction in future wireless communications of systems where the portable end-user device is physically small and uses two receive antennas. In this paper, we proposed such MIDO codes which enjoy the properties of good shaping, non-vanishing determinant (in particular full rank), and reduced decoding complexity. This set of properties result in performances that compare to the best known fast-decodable MIDO codes. One of the codes further has a MISO code embedded within, enabling an easy change-of-scheme e.g., in the presence of correlation without having to change the encoder. The codes proposed in this paper are the first with both provable NVD and excellent performance. Finally, all the codes have an algebraic structure making them easy to describe and aiding to proof of non-vanishing determinant.

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