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Effective Stiffness of Squat Structural Walls

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Abstract: Reinforced concrete (RC) structural walls are the primary lateral-load carrying elements in many structures designed to resist earthquakes. A review of the technical literature shows considerable uncertainty with regards to the effective stiffness of these structures when subjected to seismic excitations, which many design practices currently deal with by employing a stiffness reduction factor. In an attempt to obtain additional information regarding the stiffness of these structures, an analytical approach, combining the flexure and shear components of deformation, is proposed to evaluate the effective stiffness of the RC walls tested. Based on this proposed analytical approach, a comprehensive parametric study comprising 180 combinations was carried out and a simple equation for assessing effective stiffness of RC squat structural walls then proposed, on the basis of these parametric case studies.

CE Database subject headings: Walls; Stiffness; Reinforced concrete; Load bearing capacity; Earthquake resistant structures.

Author keywords: RC squat structural walls; Stiffness characteristic; Initial stiffness.

Introduction

The past 15 years have seen major developments in seismic design provisions, with a paradigm shift from a force-based approach to a displacement-based one, and an increase in focus on the deformation characteristics of structures. Stiffness properties of reinforced concrete (RC) wall structures can affect the estimation of the fundamental period,

displacements, and distributions of internal force response between walls. The magnitude of the effective stiffness depends on the intensity and distribution of stress on a wall cross section, and the extent of flexural and shear cracking. Flexural cracking causes reduction in the net cross-sectional area and moment of inertia, and hence, a reduction in effective flexural rigidity of the wall section. This leads to an increasing difficulty in making accurate predictions of the effective stiffness of RC members. Thus, stiffness reduction factor is employed in the analysis of RC members under lateral loads. In practice, the values of 0.35 and 0.70, the gross moment of inertia for cracked and uncracked walls, respectively, is widely employed. However, this simplification may not be appropriate in many cases because the recommended moment of inertia for walls is independent of the reinforcement content and axial load level.

In the case of RC walls under lateral loads, Priestley (1998) indicated that the Standards New Zealand (NZS) NZS 3101 (NZS 1995) design code recommendations for estimating the stiffness of RC walls fail to recognize the influence of two important factors: the member strength and the reinforcement grade. For stiffness evaluation of RC walls, the wall strength depends significantly on the axial load and reinforcement content. Thus, the recommendations in the NZS 3101 design code (NZS 1995) for stiffness estimation are oversimplified and may lead to an inaccurate assessment of the wall stiffness. To obtain a more accurate evaluation of wall stiffness as determined from the curvature at first yield and the flexural strength, several proposals on how stiffness values can be more realistically assessed have been presented (Priestley and Kowalsky 1998; Priestley 1998; Adebar and Ibrahim 2002). However, Fenwick and Bull (2000) found that these proposals (Priestley and Kowalsky 1998; Priestley 1998) also proved to be oversimplified. To further improve the effective stiffness predictions, they analytically investigated a range of slender rectangular walls with uniformly spaced reinforcement and proposed an expression for predicting the effective wall stiffness as a function of concrete strength, reinforcement grade, and axial loads. As indicated by the research (Fenwick and Bull 2000), the expression proposed for accurately predicting the wall effective stiffness is only justified for slender rectangular walls for which the response is dominated by flexure, and thus, the proposals cannot be applied to RC walls with low aspect ratios because their response may be controlled by shear deformations. This study

strives to establish more consistency and accuracy in predicting effective stiffness of these squat structural walls with limited transverse reinforcement.

Previous Research in Evaluating Effective Wall Stiffness

The effective moment of inertia, I_e , is defined as the moment of inertia that a uniform, elastically responding wall would have, such that when it is subjected to the lateral force that causes first yield, or a strain of 0.002 in the concrete, it sustains the same deflection. The effective stiffness of walls defined as explained previously is utilized in several previous studies and current design codes. They are briefly reviewed in the subsequent sections.

Research Conducted by Fenwick and Bull

The member effective stiffness can be obtained by the integration of curvatures over the cracked and uncracked sections along the member. The standard beam theory describes the behavior of idealized cracked and uncracked sections and can be used to determine the effective stiffness theoretically. Fenwick and Bull (2000) conducted a parametric study that considered three main parameters: axial loads, yield strength of longitudinal reinforcement, and concrete compressive strength. Fenwick and Bull went on to propose Eq. (1) by relating the effective moment of inertia to the moment of inertia of uncracked concrete section for cantilever walls subjected to flexural deformations:

$$I_e = 0.267 \left(1 + 4.4 \frac{P_u}{f'_c A_g} \right) \left(0.62 + \frac{190}{f_y} \right) (0.76 + 0.005 f'_c) I_g \quad (1)$$

The writers suggested that the expression could be used for slender walls whose behavior is dominated by flexure. Meanwhile, it was assumed that the longitudinal reinforcement was spread uniformly along the walls with rectangular cross sections. On the downside, the expression could not be applied to walls with low aspect ratios because their behavior is normally dominated by deformation associated with shear, nor could it be applied to walls with other types of reinforcement arrangements or other shapes of wall cross section.

Research Conducted by Paulay and Priestley

Paulay and Priestley (1992) proposed the equation shown in Eq. (2) by relating the equivalent moment of inertia of the wall cross section at first yield in the extreme fiber, I_e , to the moment of inertia of the uncracked gross concrete section, I_g , to determine the effective stiffness of RC structural walls that respond in a flexural manner:

$$I_e = \left(\frac{100}{f_y} + \frac{P_u}{f'_c A_g} \right) I_g \quad (2)$$

It is shown in Eq. (2) that two important parameters, yield strength of longitudinal reinforcement, f_y (in MPa), and axial load ratio, $P_u / f'_c A_g$, are considered to reflect the effective stiffness of flexural walls. Moreover, to account for shear deformation contributions to the wall stiffness, Paulay and Priestley presented the following expressions, as shown in Eqs. (3a) and (3b), to evaluate the effective stiffness for structural walls with aspect ratios less than 4.0

$$I_w = \frac{I_e}{1.2 + F} \quad (3a)$$

in which

$$F = \frac{30I_e}{h_w^2 b_w l_w} \quad (3b)$$

In Eqs. (3a) and (3b), the shear deformation contribution to effective wall stiffness is reflected by employing three overall dimensions: wall height, h_w ; wall thickness, b_w ; and wall length, l_w .

NZS 3101

Table 1 lists the stiffness coefficients recommended by the Standards New Zealand (NZS 1995) for the elastic seismic analysis of walls. As shown in Table 1, the stiffness coefficients are dependent on the axial load, N , and the expected inelastic ductility demand, μ . Under the ultimate limit state, three cases designed for ductility $\mu > 1$ ($\mu = 1.25$, 3.0, and 6.0) are all assumed to have passed their yield point. Therefore, the lower bound stiffness values (for example, $0.15\text{--}0.45I_g$, $0.30\text{--}0.80A_g$ for walls under the axial load

ratios, $N / f'_c A_g$ from -0.1 to 0.2) should be used in the analysis. However, under service level state, larger stiffness coefficients can be used for the limited ductile cases; that is, $\mu = 1.25$ or 3 because there would be less yielding compared with walls designed with ductility demands of $\mu = 6$. In the case of $\mu = 1.25$, the walls can be analyzed by using their fully uncracked stiffness under the service level earthquake.

ACI 318-08

The American Concrete Institute's (ACI) ACI 318-08 code [American Concrete Institute (ACI) 2008] recommends the application of the following effective stiffness coefficients for walls in the structure. The effective stiffness, EI_e , is proposed to be $0.70E_c I_g$ and $0.35E_c I_g$ for uncracked and cracked walls, respectively. Alternatively, it can be computed as follows (MacGregor and Hage 1977)

$$I_e = \left(0.8 + 25 \frac{A_{st}}{A_g} \right) \left(1 - \frac{M_u}{P_u h} - 0.5 \frac{P_u}{P_o} \right) I_g \quad (4)$$

In Eq. (4), A_{st} is total area of longitudinal reinforcement, A_g is gross area of concrete section, M_n is nominal flexural strength at section, P_n is nominal axial strength of cross-section, P_o is nominal axial strength at zero eccentricity, and h is overall thickness or height of member.

Stiffness Characteristics

There are two common methods used to define the initial wall stiffness, as shown in Fig. 1. The structure in the first method is cyclic loaded to $\pm 75\%$ of the nominal strength, with the recorded displacement of the structure being extrapolated to the position at the nominal strength level. This defines the yield displacements Δ_{y1} , as shown in Fig. 1. In the second method, the structure is loaded until either the first yield occurs in the longitudinal reinforcement or the maximum compressive strain of concrete reaches 0.002 at the critical section. The displacement at this point, which can be found by integrating the curvatures over the height of the wall, is then extrapolated to the level of the nominal strength and this determines the ductility displacement, Δ_{y2} . Generally, the two methods present similar values. In this study, the second approach is adopted.

Effective Stiffness

Flexural Deformation Determination

The variation of the bending moment at other sections along the wall height can be determined after the bending moment, M_y , is obtained. Moment-curvature analysis is conducted for various sections along the wall height. This analysis derives the different curvatures for the section, that corresponds to the critical section reaching the bending moment. The wall is divided into several segments along its height to determine the flexural deformation at the top of the walls. The total flexural deformation, Δ_{yf} , at the top of the wall is obtained by accumulating the flexural deformation along segments.

Shear Deformation Determination

The concept of modeling cracked RC members as a truss has been around for many years because it provides a more promising way to treat shear. The truss analogy not only presents a clear concept of how a RC wall resists shear, but also properly manages the interaction of flexure, shear, and axial load. In truss analogies, longitudinal reinforcement is represented by the longitudinal chords of a truss whereas transverse steel is represented by transverse tensile ties. The concrete in compression is considered to be the compression chord members. The longitudinal chords and the transverse ties are internally stabilized by the diagonal struts, which model the concrete compressive stress field. For simplicity, the longitudinal chords, transverse tensile ties, and diagonal struts, are assumed to be joined together through rigid nodes.

The contribution of the shear deformation to the total deformation in squat structural walls under large shear forces can be significant because diagonal cracks must be expected at the service limit state. Hence, in the case of squat walls under service conditions, the designer also needs to be able to assess the order of expected shear deformations. Moreover, it is recognized that a greater proportion of the load in squat structural walls is carried by truss action, and hence, the deformation induced by truss mechanism should be emphasized in this study. Based on this, Park and Paulay (1975) proposed a method to calculate the shear stiffness of short or deep rectangular beams of unit length by using the model of the analogous truss. The shear stiffness is defined as the magnitude of the shear force that, when applied to a beam of unit length, will cause unit

shear displacement at one end of the beam relative to the other. The shear deformation of these unit length members under certain shear force can be assessed by using the shear stiffness. Fig. 2 presents the shear distortions according to the truss analogy mechanism for structural walls studied.

As shown in Fig. 2, the horizontal reinforcements and concrete act as tension and strut members, respectively, and the vertical reinforcements form the left and right chords. The total shear distortion includes two components: elongation of the horizontal reinforcements, Δ_S ; and the shortening of the compression strut, Δ_C . As indicated in Fig. 2, the shear distortion, Δ_V , can be defined by:

$$\Delta_V = \Delta_S + \Delta_R = \Delta_S + \Delta_C / \sin \alpha \quad (5)$$

in which α is the inclination of compression strut.

Assuming that the shear force sustained by the wall panel is V_S , the stress of horizontal reinforcement can be expressed as

$$f_S = \frac{V_S \cdot s}{d \cot \alpha \cdot A_h} \quad (6)$$

in which d is the length of wall panel, s is the distance between horizontal reinforcements, and A_h is the area of horizontal reinforcement spaced at a distance, s .

Hence, the elongation of the horizontal reinforcement becomes:

$$\Delta_S = \frac{f_S}{E_S} \cdot d = \frac{V_S \cdot s}{E_S \cot \alpha \cdot A_h} \quad (7)$$

The concrete compression stress is obtained:

$$f_{cd} = \frac{V_S}{b_w L_{CS} \sin \alpha} \quad (8)$$

in which b_w is the depth of wall panel and L_{CS} is the effective depth of the compression strut, as shown in Fig. 3.

Hence, the shortening of the concrete strut is

$$\Delta_C = \frac{f_{cd}}{E_C} \cdot h_w / \cos \alpha = \frac{V_S \cdot h_w}{E_C b_w L_{CS} \sin \alpha \cos \alpha} \quad (9)$$

in which h_w is the height of the wall panel. By making the appropriate substitution for web horizontal steel content, $\rho_h = A_h / sd$, and modular ratio, $n = E_S / E_C$, the shear distortion in the wall panel can be expressed as

$$\theta_V = \frac{\Delta_V}{h_w} = \frac{\Delta_S + \Delta_R}{h_w} = \frac{V_S}{h_w E_S b_w} \left(\frac{1}{\cot \alpha \cdot \rho_h} + \frac{h_w n}{L_{CS} \sin^2 \alpha \cos \alpha} \right) \quad (10)$$

When $\theta_V = 1$ and $L_{CS} = d \cdot \cos \alpha$, the shear stiffness of the wall panel can be defined by the following expression

$$K_V = \frac{\rho_h \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^4 \alpha + n \rho_h} E_S b_w d \quad (11)$$

Eq. (11) indicates that the unit shear stiffness of the wall panel is primarily dependent on the extent of the crack angles. To accurately estimate the theoretical crack angle, Kim and Mander (2007) provided Eq. (12) by considering the energy minimization on the virtual work done by the shear and flexural components:

$$\alpha = \tan^{-1} \sqrt[4]{\frac{\rho_h n + 1.57[(\rho_h \cdot A_v)(\rho_v \cdot A_g)]}{1 + \rho_h n}} \quad (12)$$

There was a good correlation between the predicted crack angle and the published experimental structural wall data (Matthew (2001)). As such, Eq. (12) is adopted in this study to conduct the following validation and parametric studies.

Hence, the shear displacement caused by the yield lateral force, F_y , would be:

$$\Delta_{yv} = \frac{F_y}{K_V} \cdot h_0 \quad (13)$$

Combination of Shear and Flexure Response

After the flexural and shear deformation at the top of wall under yield lateral load are obtained, the effective stiffness of walls can be determined as

$$K_i = \frac{F_y}{\Delta_{yf} + \Delta_{yv}} \quad (14)$$

Hence, the effective moment of inertia for a cantilever wall can be expressed as

$$I_e = \frac{K_i h_w^3}{3E_c} \quad (15)$$

Validation of the Proposed Approach

The proposed approach is validated in the following sections by comparing the theoretical estimations with experimental observations. The experimental results of the eight walls tested by Li and Xiang (2006; Xiang 2008) are considered (see Fig. 4). The theoretical results of the effective stiffness, $K_{i(a)}$, of the eight specimens, together with the experimental results, $K_{i(e)}$, are listed in Table 2. To compare the accuracy of the experimental results with the numerical ones, the error value, γ , measuring the differences between numerical and experimental results, as shown in Eq. (16), is also provided in Table 1:

$$\gamma = \frac{K_{i(a)} - K_{i(e)}}{K_{i(e)}} \quad (16)$$

Table 2 shows that the analytical effective stiffness of the eight RC walls tested correlates with the experimental results because the average error value is observed to be 11.1%. This indicates that the proposed approach can assess effective stiffness of squat RC structural walls with or without axial loads with a satisfactory level of accuracy.

Parametric Study for Effective Stiffness of Squat Structural Walls

A parametric study comprising 180 combinations was carried out to investigate the effect of various parameters on the effective stiffness of squat structural walls with limited transverse reinforcement. The primary parameters investigated were: yield strength of

longitudinal reinforcement in wall boundaries, f_y ; longitudinal reinforcement content in wall boundaries, ρ_b ; the axial load ratio, $N / (f'_c A_g)$; and aspect ratios, h_w / l_w . The investigated range of the parameters is listed in Table 3.

The ranges investigated are typical cases for almost all practical squat structural walls in low- to moderate-earthquake regions. All specimens studied have three common characteristics, namely: (1) aspect ratios less than 2.0; (2) specimens are isolated walls; and (3) solid with symmetrical wall cross section.

In the parametric study, the effect of investigated parameters on the effective stiffness of walls is presented by the dimensionless stiffness ratio, k , defined as follows

$$k = \frac{I_e}{I_g} \quad (17)$$

Figs. 5–7 present the analytical results of the stiffness ratio with the variation of the investigated parameters for longitudinal reinforcement content in wall boundaries of 1.4, 2.8, and 4.2%, respectively.

Influence of Aspect Ratio

As shown in Figs. 5–7, the stiffness ratio generally increases with the addition of aspect ratios for walls with same reinforcement detailing and under the same level of axial loads. With the increase of aspect ratios from 1.125 to 1.625 and 1.925, stiffness ratios for walls without axial loads rise by approximately 70 and 108%, respectively. The rate of increase of stiffness ratios becomes more significant for RC walls under an axial load level of 0.1 because it rises by approximately 85 and 130%, respectively.

This variation of stiffness ratios with the change in aspect ratios can also be observed in Fig. 5 corresponding to three different longitudinal reinforcement ratios in wall boundaries: 1.4, 2.8, and 4.2%, respectively. Because wall aspect ratios rise by approximately 44 and 70%, an increase in stiffness ratios of approximately 80 and 124%, respectively, is obtained for walls subjected to an axial load ratio of 0.05. However, for walls with different longitudinal reinforcement content in wall boundaries, the variation of the stiffness ratios with the change of aspect ratios remain almost the same, which

indicates that the longitudinal reinforcement content in wall boundaries plays a minor role in contributing to the effective stiffness of squat RC walls.

Influence of Axial Load

It is recognized that the effective stiffness of RC walls should increase with added axial loads because of the decrease in depth of flexural cracks. Although the yield curvature does not vary significantly for walls with or without axial loads, the presence of axial load can effectively increase its strength, and thus, lead to larger effective flexural stiffness. As shown in Figs. 5–7, with the added axial loads, the stiffness ratio generally increases. An approximately 6 and 10% stiffness ratio rise is observed for walls with an aspect ratio of 1.125 when the level of axial load ratio increases from 0.0 to 0.05 and 0.1, respectively. Fig. 6 clearly illustrates the increase of stiffness ratio with the added axial loads corresponding to three different levels of longitudinal reinforcement content in wall boundaries: 1.4, 2.8, and 4.2%, respectively. The influence of axial load on the stiffness ratio can be more significant with increase of aspect ratios. It is observed that stiffness ratios for walls with an aspect ratio of 1.625 increase by approximately 13 and 19% with the change of axial load ratio from 0.0 to 0.05 and 0.1, respectively. For walls with an aspect ratio of 1.925, an approximately 16 and 26% increase in stiffness ratios is obtained for walls with the increase of axial load from 0.0 to 0.05 and 0.1, respectively.

Influence of Longitudinal Reinforcement Content in Wall Boundaries

Figs. 5–7 show that with the addition of longitudinal reinforcement content in wall boundaries from 1.4% to 2.8 and 4.2%, stiffness ratios increase slightly by approximately 2 and 5%, respectively, for walls without axial loads. The stiffness ratios increase by approximately 2 and 3%, respectively, for walls under axial loads of 0.05 with the increase of longitudinal reinforcement content from 1.4% to 2.8 and 4.2%. This increase reduces to approximately 1.0 and 3.0%, respectively, for walls under the axial load ratio of 0.1.

The influence of longitudinal reinforcement ratios in wall boundaries on stiffness ratios is also presented in Fig. 6 for three different aspect ratios: 1.125, 1.625, and 1.925. From Fig. 7, stiffness ratios for walls without axial loads are observed to increase slightly for walls corresponding to all aspect ratios. For walls under the axial load ratio of 0.1, the

stiffness ratios for walls with all aspect ratios studied remain approximately the same. This suggests that the influence of longitudinal reinforcement ratios in wall boundaries on stiffness ratios is not significant. For simplicity, the effect of longitudinal reinforcement content in wall boundaries on effective stiffness of RC structural walls with low aspect ratios can be neglected.

Influence of Yield Tensile Strength of Longitudinal Bars in Wall Boundaries

As presented in Figs. 5–7, the stiffness ratios generally decrease with the increase of yield strength of outermost longitudinal bars. With an increase of yield strength from 308–458 MPa, stiffness ratios of walls without axial loads decrease by approximately 6% for walls with an aspect ratio of 1.125. There is an approximately 10 and 14% reduction in stiffness ratios observed for walls with an aspect ratio of 1.625 and 1.925, respectively. The influence of yield strength of longitudinal bars on stiffness ratios becomes insignificant for walls with aspect ratios of 1.125 because a 2.5% and 3% reduction in stiffness ratios is observed for walls with axial load ratios of 0.05 and 0.1, respectively.

Proposed Equation for Moment of Inertia of Structural Walls

Based on the previous parametric study, Eq. (18) considers three previously-mentioned investigation parameters: yield tensile strength of steel bars in wall boundaries, axial loads, and aspect ratios. It is proposed to properly evaluate the effective stiffness of squat structural walls. For simplicity, the influence of longitudinal reinforcement content in wall boundaries on wall effective stiffness is conservatively disregarded:

$$\frac{I_e}{I_g} = 0.19 \left(\frac{100}{f_y} + \frac{N}{f'_c A_g} \right) \left(0.53 + 0.37 \frac{h_w}{L_w} + 0.31 \frac{h_w^2}{L_w^2} \right) \quad (18)$$

Fig. 8 illustrates the comparisons of stiffness ratios proposed by the parametric studies, proposed Eq. (18), and the previous Eq. (3) at different aspect ratios of walls studied. The trend lines of the obtained stiffness ratios are presented in Fig. 9 to indicate the variation of the stiffness ratios with the change of axial loads. As can be seen in Fig. 8, proposed Eq. (18) presents higher value stiffness ratios than the analytical results for walls with an aspect ratio of 1.125.

Comparisons of Analytical Stiffness Ratios with Tested Results

Results from RC squat structural walls tests, as listed in Table 4, are compared with analytical results by using the proposed model, Eq. (18), and other provisions previously reviewed. Experimental effective stiffness values, EI_e , from the tests are back-calculated by dividing the displacements at the yield point by the tested yield strength with an elastic model. All tested walls have aspect ratios not larger than 2 and axial load ratios, which range from 0 to 0.2, which covers almost all conditions likely to be encountered in practice. Yield strengths of outermost longitudinal bars for all specimens ranged from 300–585 MPa. It is believed that the proposed stiffness model is applicable for all values of yield strengths studied. The longitudinal and transverse reinforcement content in the wall web is limited and remains at a low level for all walls selected. It was, however, realized that the proposed equation underestimates the stiffness of the wall compared with the wall data procured by Legas et al. (1990) and Salonikios et al. (2000). This can possibly be attributed to the presence of diagonal reinforcement in the web and heavily reinforced boundary elements that added to their stiffness compared with the walls considered in this study.

Fig. 9 presents a comparison of the experimental and calculated stiffness (EI_e/EI_g) for the proposed model and that presented by other proposals. Fig. 8 illustrates the comparisons of stiffness ratios, EI_e/EI_g , computed by the currently proposed Eq. (18), Eq. (3) proposed by Paulay and Priestley (1992), Eq. (1) proposed by Fenwick and Bull (2000), and the NZS 3101 design code (NZS 1995). Tested-to-calculated values of stiffness ratios in Fig. 8 indicate that Fenwick and Bull's design equation and the NZS 3101 code significantly overestimate the effective stiffness of squat structural walls. Of the three equations, the proposed equation, with a standard deviation of 0.41, appears to be more accurate in effective stiffness evaluation than Eqs. (1) and (3) because a standard deviation of 0.54 and 2.13, respectively, was obtained. This can be explained by the fact that both Fenwick and Bull's design equation and the NZS 3101 code ignored the shear deformation in calculating the effective stiffness of squat walls; whereas, the shear deformation was considered in the proposed model.

Conclusions

An approach assessing the effective stiffness of RC squat walls is proposed in this paper, accounting for deformations attributable to both flexural and shear actions. The analytical stiffness of 29 test squat walls by using the proposed approach is compared with the experimental results. The comparison shows a good agreement between the experimental results and those obtained from the proposed approach. A parametric study comprising 180 case studies also shows that the yield strength of outermost longitudinal reinforcement, applied axial load, and wall aspect ratios can have an influence on the effective stiffness of squat walls. An equation to evaluate the wall effective stiffness that accounts for both flexure and shear deformations, is proposed on the basis of parametric study. The effective squat wall stiffness can be estimated by using Eq. (18):

$$\frac{I_e}{I_g} = 0.19 \left(\frac{100}{f_y} + \frac{N}{f'_c A_g} \right) \left(0.53 + 0.37 \frac{h_w}{L_w} + 0.31 \frac{h_w^2}{L_w^2} \right)$$

Notation

The following symbols are used in this paper:

b_w = depth of wall panel;

d = length of wall panel;

F_y = yield lateral force;

f_y = yielding strength of reinforcement;

h_c = cross-sectional dimension of column core measured center-to-center of confining reinforcement;

h_w = height of cantilever wall;

K_c = analytical cracked member stiffness;

K_e = elastic uncracked stiffness;

K_i = wall effective stiffness;

$K_{i(a)}$ = theoretical effective stiffness;

$K_{i(e)}$ = experimental effective stiffness

K_V = shear stiffness;

k = dimensionless stiffness ratio;

L_{CS} = effective depth of the compression strut;

L_w = length of the cantilever wall;
 M_y = yield moment;
 N = axial loads;
 n = modular ratio;
 s = spacing of transverse reinforcement;
 V_s = shear force resisted by the wall panel;
 α = inclination of compression strut;
 γ = error value;
 Δ_C = shortening of the compression strut;
 Δ_{fl} = flexure deformation;
 Δ_{yf} = flexural displacement caused by the yield lateral force F_y ;
 Δ_{yv} = shear displacement caused by the yield lateral force F_y ;
 Δ_S = elongation of horizontal reinforcements;
 Δ_{sh} = shear deformation;
 Δ_y = yield displacement of the walls;
 Δ_V = shear distortion;
 μ = displacement ductility;
 ρ_b = longitudinal reinforcement content in wall boundaries;
 ρ_h = horizontal steel content;
 ρ_v = longitudinal steel content; and
 ϕ_0 = overall strength moment ratio.

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Type of member		Checks at ultimate limit state	Checks at serviceability limit state		
			$\mu = 1.25$	$\mu = 3.00$	$\mu = 6.00$
Walls	$N/f'_c A_g = 0.2$	$0.45I_g$	I_g	$0.70I_g$	$0.45I_g$
		$0.80A_g$	A_g	$0.90A_g$	$0.80A_g$
	$N/f'_c A_g = 0.0$	$0.25I_g$	I_g	$0.50I_g$	$0.25I_g$
		$0.50A_g$	A_g	$0.75A_g$	$0.50A_g$
	$N/f'_c A_g = -0.1$	$0.151I_g$	I_g	$0.40I_g$	$0.151I_g$
		$0.30A_g$	A_g	$0.65A_g$	$0.30A_g$

Table 1

Specimen	(a) ρ_h	(b) ρ_v	(c) ρ_b	(d) ρ_s	Axial Load ratio	Construction joints	Experiment effective stiffness	Analysis effective stiffness	Error value, γ
	(%)	(%)	(%)	(%)			(kN/mm)	(kN/mm)	(%)
LW1	0.5	0.5	1.4	0.88	0.0	No	58.4	64.4	10.3
LW2	0.5	0.5	1.4	0.88	0.05	No	75.7	67.6	10.7
LW3	0.5	0.5	1.4	0.33	0.05	No	83.3	67.6	18.8
LW4	0.5	0.5	1.4	0.88	0.0	Yes	53.5	64.4	20.4
LW5	0.5	0.5	1.4	0.33	0.05	Yes	83.1	67.6	18.6
MW1	0.5	0.5	1.4	0.88	0.05	No	39.5	40.7	3
MW2	0.5	0.5	1.4	0.33	0.05	No	41.2	40.7	1.2
MW3	0.5	0.5	1.4	0.88	0.05	Yes	38.3	40.7	6.3
Average error value =								11.1	

Note: (a) horizontal web reinforcement ratio; (b) vertical web reinforcement ratio; (c) flexural reinforcement ratio in boundary element; (d) volumetric ratio of transverse reinforcement in boundary element;

Table 2

No.	Name	Description	Range Investigated
1	f_y (MPa)	Yield strength of longitudinal reinforcement in wall boundaries	308, 358, 408, 458
2	ρ_b (%)	Longitudinal reinforcement content in wall boundaries	1.4, 2.8, 4.2
3	$N/(f'_c A_g)$	Axial load ratio	0.00, 0.025, 0.05, 0.075, 0.10
4	h_w/l_w	Aspect ratio	1.125, 1.625, 1.925

Table 3

Units	f_y (MPa)	$\frac{h_c}{L_c}$	$\frac{N}{f_y A_g}$	EI_c/EI_g				
				(1) Tests	(2) Eq. (18)	(3) Ratio = (2)/(1)	(4) Eq. (3)	(5) Ratio = (4)/(1)
Specimens tested by Paulay et al. (1982)								
Wall 1	300	0.5	0.00	0.047	0.050	1.06	0.097	2.06
Wall 3	315	0.5	0.00	0.024	0.048	1.98	0.065	2.71
Specimens tested by Mestyanek (1986)								
Unit 1.0	520	1.0	0.00	0.055	0.044	0.8	0.107	1.95
Unit 1.5	530	1.5	0.00	0.099	0.064	0.64	0.126	1.27
Unit 2.0	500	2	0.00	0.138	0.095	0.69	0.143	1.04
Specimens tested by Lefas et al. (1990)								
SW11	470	1.0	0.00	0.090	0.049	0.54	0.123	1.37
SW12	470	1.0	0.10	0.137	0.072	0.52	0.158	1.15
SW13	470	1.0	0.20	0.124	0.095	0.76	0.185	1.49
SW21	470	2.0	0.00	0.226	0.101	0.45	0.16	0.71
SW22	470	2.0	0.10	0.367	0.149	0.41	0.224	0.61
SW23	470	2.0	0.20	0.368	0.196	0.53	0.283	0.77
Specimens tested by Salonikios et al. (2000)								
LSW1	585	1	0	0.14	0.039	0.28	0.105	0.75
LSW2	585	1	0	0.093	0.039	0.42	0.105	1.13
LSW3	585	1	0.07	0.124	0.055	0.45	0.133	1.07
MSW1	585	1.5	0	0.13	0.058	0.44	0.105	0.81
MSW2	585	1.5	0	0.048	0.058	1.2	0.105	2.19
MSW3	585	1.5	0.07	0.147	0.081	0.55	0.133	0.9
Specimens tested by Oh et al. (2002)								
WR-20	449	2.0	0.10	0.149	0.153	1.03	0.230	1.54
WR-10	449	2.0	0.10	0.178	0.153	0.86	0.230	1.29
WR-0	449	2.0	0.10	0.117	0.153	1.31	0.230	1.97
WB	342	2.0	0.10	0.138	0.187	1.35	0.245	1.78
Specimens tested by Li and Xiang (2006)								
LW1	382	1.125	0.00	0.061	0.082	1.35	0.130	2.13
LW2	382	1.125	0.05	0.082	0.095	1.16	0.142	1.73
LW3	382	1.125	0.05	0.072	0.095	1.32	0.136	1.89
LW4	382	1.125	0.00	0.060	0.082	1.37	0.127	2.12
LW5	382	1.125	0.05	0.073	0.095	1.30	0.140	1.92
MW1	382	1.625	0.05	0.124	0.138	1.12	0.190	1.53
MW2	382	1.625	0.05	0.125	0.138	1.11	0.191	1.53
MW3	382	1.625	0.05	0.113	0.138	1.22	0.186	1.65
Mean=						0.91		1.48
SD=						0.41		0.54

Table 4

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9