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Vibration measurement of a micro-structure by digital holographic microscopy

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Abstract

A method for the vibration measurement of a micro-electro-mechanical system (MEMS) structure based on digital holographic microscopy (DHM) is presented. A sequence of digital holograms of a vibrating micro-beam excited by varying voltage is captured by a high-speed CCD camera. The phase changes of the object at different instants are obtained by digital reconstruction. The displacement, velocity and acceleration are evaluated by both spatial and temporal Fourier and windowed Fourier analyses. The results show that digital holographic microscopy with Fourier analyses is an effective method for evaluating the dynamic characteristics of a MEMS component.

Keywords: vibration measurement, MEMS characterization, digital holography, temporal analysis

1. Introduction

A micro-electro-mechanical system (MEMS) is an integration of mechanical elements, sensors, actuators and electronics on a common silicon substrate through micro-fabrication technology [1]. Many MEMS devices such as pressure-sensing, inkjet print heads, airbag accelerometers, micro-gyroscope and micro-actuators/motors are emerging. Application areas of MEMS devices include aerospace, appliances, automotive, biotechnology, chemical systems and communications. With the widespread use of MEMS devices, the evaluation of their performance behavior and reliability are urgently needed. In many cases, MEMS devices are actuated by the use of electrostatic force. Hence, it is important to study the static and dynamic behavior of MEMS devices that are subjected to electrostatic voltages. Numerical simulations are often employed; however, a complete and accurate modeling of MEMS is not always possible since it is difficult to take into account all the real boundary conditions. To verify the accuracy of numerical analysis, it is necessary to employ techniques for the characterization of the real mechanical behavior of MEMS devices under both static and dynamic conditions. Optical methods enjoy the

virtues of being whole-field, noninvasive and high-resolution (in both space and time), and are widely used in industries for the measurement of shape, displacement and mechanical properties of microelements [2–4]. However, the optical assessment of a micro-object would encounter more challenges compared to a normal size specimen. The constraints of a microscope, such as short working distance, limited depth of field and difficulty in alignment are some of the considerations in the experimental work. Due to high magnification of the microscopic system, the signal-to-noise ratio may also be reduced by environmental factors, such as vibration.

In recent years, dynamic behavior of micro elements, such as vibration and transient deformation are of more interest. Time-averaged microscopic interferometry [5] has been used to obtain vibration amplitude and mode shape of structure for vibration measurements. Microscopic interferometry with stroboscopic illumination [6–8] has been also reported to measure the vibration mode shapes of MEMS devices. In this method, a periodical movement is repeated many times with different time delayed stroboscopic pulses. In many cases, the transient displacement or surface profiling of vibrating micro-objects provides useful information on the dynamic response and deformation of the objects concerned. Due to the rapid development of high-speed digital recording devices, it is now possible to record interferograms with a high imaging rate. Retrieving precise spatial phase maps from these transient interferograms enable the measurement of instantaneous 3D profiles, deformations as well as dynamic responses. Different algorithms [9–11] are applied to analyze this interferogram sequence. Among them, the temporal phase analysis technique has the advantage of eliminating speckle noise and is more popular in high-speed interferometry [12]. However, it does have a disadvantage: if only the intensity variations of pixels are analyzed, it is not able to extract the phase values from a part of an object that is not moving with the rest or from objects that deform in different directions at different parts. This limits the technique to the measurement of deformation in one known direction. Adding a temporal carrier [13, 14] to the image acquisition process is a method to overcome these problems. However, it limits the measurement range of phase variation due to the constraints of the Nyquist sampling theorem and the acquisition speed of the camera [15].

In the last decade, a novel computer-aided optical technology, digital holography (DH) [16] has been applied in static and dynamic measurement. The holograms are recorded directly by a camera and reconstructed digitally by computer simulation. Both the amplitude and phase of an object can be obtained by digital reconstruction from a single recorded hologram [17]. This advantage makes it more suitable for dynamic measurement. In addition, no phase ambiguity problem exists in further processing of the reconstructed phase. In recent years, high-speed digital holographic interferometry [18] was used for vibration measurement. With temporal phase unwrapping of digital hologram sequences [19], it is possible to obtain displacement, including the direction of a moving object as a function of time. Previous research has shown that the kinematic parameters of a vibrating object can be obtained by further processing of the reconstructed phase maps [20, 21].

Digital holography has also been applied with microscope to obtain the quantitative amplitude and phase-contrast image of a micro-object [22–25]. Detection of refractive-index changes and variations in the shape of biological objects, such as undyed cellular samples, has also been reported [26–28]. In this paper, high-speed image-plane digital holographic microscopy is applied to capture a sequence of digital holograms on a vibrating micro-beam. The

complex amplitude of the object wave is reconstructed using a 2D digital Fourier transform method [29, 30]. The displacement, velocity and acceleration of the vibrating object are obtained using a windowed Fourier and Fourier analysis.

2. Theoretical analysis

2.1. High-speed digital holographic microscopy

Figure 1 shows a schematic layout of digital holographic microscopy. Unlike the classical image-plane digital holography, the setup requires the use of a microscope objective. A laser beam is divided into object and reference waves by a beam splitter. The reference wave is directed at a high speed CCD sensor via a single-mode optical fiber. An image-plane hologram is recorded on the CCD sensor as a result of the interference between a magnified object wave produced by a microscope objective and the reference wave. Let $O(x, y)$ and $R(x, y)$ denote the object and reference wave respectively, the intensity recorded on the CCD sensor is given by

$$I_H(x, y) = |O_H(x, y)|^2 + |R_H(x, y)|^2 + R_H(x, y) O_H^*(x, y) + R_H^*(x, y) O_H(x, y), \quad (1)$$

where subscript H indicates the hologram plane and $*$ denotes the complex conjugate amplitude. The last two terms of equation (1) contain information about the amplitude and phase of the object wave. Figure 2(a) shows a typical digital hologram of a vibrating micro-beam captured by the high-speed CCD camera indicated in figure 1. By performing a Fourier transform on the intensity function, the spectrum can be separated as a result of the angular offset of the reference wave. The complex object wave $O_H(x, y)$ can be obtained after filtering out one of the last two terms in equation (1) and inverse Fourier transformation. The Fourier spectrum and filtering window are shown in figure 2(b). From the complex object wave $O_H(x, y)$, the phase values are calculated by

$$\phi = \arctan \frac{\text{Im}[O_H(x, y)]}{\text{Re}[O_H(x, y)]}, \quad (2)$$

where Im and Re denote the imaginary and real parts, respectively. In this application, only the phase difference of the object wave between two exposures is of interest as it is directly related to the out-of-plane deformation z of the object. The relationship between the phase change $\Delta\phi = \phi_{t_2} - \phi_{t_1}$ and out-of-plane displacement z is given by

$$\Delta\phi = \frac{2\pi z}{\lambda} S, \quad (3)$$

where $S = k_i - k_o$ is the sensitivity vector given by the geometry of the setup, and k_i and k_o are the unit vectors of illumination and observation, respectively. The phase difference between two digital holograms recorded at t_1 and t_n can be calculated by

$$\Delta\phi = \arctan \frac{\text{Im}[O(x, y; t_n)O^*(x, y; t_1)]}{\text{Re}[O(x, y; t_n)O^*(x, y; t_1)]}, \quad (4)$$

where t_n and t_1 denote two different times and $\Delta\phi$ is wrapped within $(-\pi, \pi)$. Figure 2(c) shows a typical instantaneous phase difference on a vibrating micro-beam.

When a sequence of digital holograms is recorded, we are able to obtain a series of phase difference $\Delta\phi(x, y; t_n)$ if the first hologram is considered as a reference, $n = 1, 2 \dots m$, where m is the total number of frames captured. After temporal phase unwrapping, a continuous phase variation $\varphi(x, y; t)$ is obtained.

2.2. Windowed Fourier and Fourier analysis

A one-dimensional windowed Fourier transform (WFT) of a signal $f(t)$ and its inverse windowed Fourier transform (IWFT) can be written as [31]

$$Sf(u, \xi) = \int_{-\infty}^{+\infty} f(t)g(t - u) \exp(-j t \xi) dt, \quad (5)$$

and

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} Sf(u, \xi) g(t - u) \exp(j t \xi) dt d\xi, \quad (6)$$

where $Sf(u, \xi)$ denotes the WFT spectrum; the window function $g(t)$ is usually chosen as a Gaussian function,

$$g(t) = \exp(-t^2/2\sigma^2), \quad (7)$$

where the parameter σ controls the extension of $g(t)$. For a point $P(x, y)$ on the object, we calculate a series of phase values $\Delta\phi(x, y; t_n) = \arg [U(x, y, t_n) \cdot U^*(x, y, t_1)]$, $n = 1, 2 \dots m$, where m is the total number of frames captured. The unwrapped phase φ_P is converted to an exponential function $C_P = \exp(j\varphi_P)$. The windowed Fourier transform of this complex function is

$$SC_P(u, \xi) = \frac{\sqrt{s}}{2} A_{xy}(u) \exp\{j[\varphi(u) - \xi u]\} \{\hat{g}(s[\xi - \varphi'(u)]) + \varepsilon(u, \xi)\}, \quad (8)$$

where s is a scaling factor. For a fixed value of s , $g_s(t) = s^{-1/2}g(t/s)$ has a support size of s . $A_{xy}(u)$ is the modulus of C_P (in our case, $A_{xy}(u) = 1$) and $\varepsilon(u, \xi)$ is a corrective term which can be neglected if $A(u)$ and $\varphi'(u)$ have small relative variations over the support of window g . $\hat{g}(\omega)$ denotes the Fourier transform of $g(t)$. The trajectory of maximum $|SC_P(u, \xi)|^2$ on the u - ξ plane is called a windowed Fourier ‘ridge’. The instantaneous frequency of the signal is obtained by extracting the ridge on the time–frequency domain:

$$\xi(u) = \varphi'(u). \quad (9)$$

In this case, $\varphi'(u)$ is proportional to the velocity of the deformation. The acceleration can also be evaluated by a similar process to the first derivative of phase values. Equation (9) is based on the assumption that the phase in the local area determined by the extension of Gaussian function g is linear. However, the real phase of a vibrating object is rarely linear, and hence there exists a linear phase approximation error. The error is small when the window size is reduced. However, a large window size is more effective to reduce noise. In our case, the linear phase approximation error is mainly concerned as the noise is not so obvious in temporal signals.

Fourier transform with low-pass filtering and numerical differentiation is an alternative method for extracting velocity and acceleration. The first derivative of the phase variation $\varphi'(x, y; t) = d\varphi/dt$ is calculated by numerical differentiation, and then it is converted to an exponential signal $\exp(j * \varphi')$, where $j = \sqrt{-1}$. The velocity is obtained after a low-pass Fourier filter is applied and the acceleration can be evaluated after a similar process. A proper filtering window is required to encompass the signal spectrum. However, if the signal spectrum is broad, more noise will be included in the filtering window.

These two methods were used for extracting the first and second derivatives of a simulated phase variation with random noise [20]. In the first derivative evaluation, the result from the windowed Fourier ridge method may be better than that from Fourier analysis because of the broad spectrum. In the second derivative evaluation, both methods perform well as the spectrum is narrow.

3. Experimental work

Micro-beams are often used as on/off switches in micro-devices. Typically, the beam is placed over a substrate with a small gap between them. When a voltage is applied on the beam, electrostatic forces are exerted on the beam causing it to bend. The specimen investigated in this study is a micro-beam fixed at both ends, and the beam is excited by an electrostatic actuation force, as shown in figure 3. The length, width and thickness of the micro-beam are 550, 30 and 2.8 μm , respectively. When a fluctuating voltage is applied between the top and bottom electrodes by two probes linked to a function generator, out-of-plane vibration of the micro-beam is produced.

Figure 1 shows the experimental setup. A He-Ne laser (75 mW, $\lambda = 632.8 \text{ nm}$) is split into an object and a reference wave by a beam splitter (B1). The object is illuminated and observed at right angles. In this arrangement, a 1 rad phase change is equivalent to a 50.36 nm displacement in the z direction. A microscopic objective (Mitutoyo Plan APO 20 $\times$, NA = 0.42) is used in the object wave path to magnify the specimen. The reference wave is delivered by a single-mode optical fiber located close to an aperture. To reduce external vibrations, the setup is arranged on a vibration-isolation table (Newport RS-4000). As the recording period is very short, measurement error introduced by the environment is limited. A sinusoidal voltage with a frequency of 11 Hz is applied between two electrodes. A high-speed camera (KODAK Motion Corder Analyzer, SR-Ultra) with a pixel size of $7.4 \times 7.4 \mu\text{m}^2$ at a sampling rate of 500 frames s^{-1}

is used. Two hundred image-plane holograms are recorded at a time interval of 0.4 s. In many cases, the shutter speed is an important parameter in the measurement. If the speed is too low, the contrast of intensity variation will decrease. However, in this study as the maximum phase change between two adjacent frames is less than $\pm\pi$ the imaging rate is a more important criterion. The shutter speed in this case is set as 1/500 s to accommodate the requirements of high-intensity illumination.

4. Results and discussion

Figure 4(a) shows an instantaneous wrapped phase map with speckle noise. A smoothed wrapped phase after 2D WFT filtering [32] is shown in figure 4(b). The window sizes along x and y directions for 2D filtering are $\sigma_x = 20$ and $\sigma_y = 6$. A noise spectrum with low coefficients is distributed widely. Hence the noise can be suppressed by discarding spectrum coefficients with low amplitude. It is observed that the 2D WFT filter performs well. Comparisons with Fourier and orthogonal wavelet filtering have shown that it is usually able to reduce the noise more effectively [32, 33]. Similarly each instantaneous wrapped phase map is filtered by 2D WFT. The displacement of the micro-beam at any instant is obtained after phase unwrapping. Figure 4(d) shows the displacement distributions on cross section A–A (indicated in figure 2(c)) at different instants.

For each point $P(x, y)$ of the micro-beam, we obtain the continuous phase variation $\varphi(x, y; t)$ after 1D unwrapping along the time axis. Figure 5(a) shows the displacement of point B (indicated in figure 2(c)) during deformation. The resonance frequency of point B is evaluated to be around 22 Hz, which is twice that of the driving frequency. This is because the attractive electrostatic force is proportional to the square of the applied voltage [34]. The vibration amplitudes are not uniform due to a bias voltage superimposed on the driving voltage.

The phase difference is converted to an exponential signal $\exp[j\varphi(x, y; t)]$, and the windowed Fourier ridge method is used to evaluate the instantaneous frequency. Figure 5(b) shows the spectrogram of WFT, the velocity of a point is evaluated from the ridge. $\sigma_t = 2$ is selected as the window size to limit the error due to the linear phase approximation.

The maximum velocity of B is around $45 \mu\text{m s}^{-1}$. The velocity extracted by a numerical differentiation and band-pass Fourier filter is shown in figure 5(c). Figure 6(a) shows the acceleration of point B, the dashed line indicates the ridge. The window size is selected as $\sigma_t = 2$. The maximum acceleration of point B is around 5 mm s^{-2} . Figure 6(b) shows the results obtained by a numerical differentiation and a band-pass Fourier filter. In this case, the temporal signal is not noisy since the phase values have been filtered in the spatial domain and the spectrum of the signal is narrow. It can be observed that similar results can be obtained with a proper filtering window.

The measuring range of the velocity and acceleration is limited due to the constraint of the Nyquist sampling theorem and the imaging rate of the camera. The maximum phase change between two adjacent frames is $\pm\pi$, which is equivalent to a velocity of $\pm 79.1 \mu\text{m s}^{-1}$ using the experimental setup and the imaging rate mentioned above. It is worth noting that

as the interferometric signal is processed along the time axis the maximum frequency obtained is also constrained by the maximum velocity. When the amplitude of vibration is less than a π phase change, theoretically a frequency of up to 250 Hz can be obtained with an imaging rate of 500 frames s^{-1} . On the other hand, the measurement range of frequency decreases when the vibration amplitude increases. Within certain vibration amplitude, the frequency range can be expanded by increasing the imaging rate or by multi-wavelength digital holographic microscopy [35, 36]. In the latter method, the displacement measured in a π phase change would be dramatically increased.

The spectrum of the temporal signal evaluated using continuous phase difference contains values from negative to positive. When Fourier transform with band-pass filtering is used to evaluate the phase derivatives, the processing speed is much faster than the windowed Fourier ridge algorithm. However, it is not able to remove the noise whose frequency is within the filtering window. The accuracy increases with decreasing width of the spectrum. The windowed Fourier ridge method however shows its capability and better performance in noise reduction when the spectrum is broad. However, the resolution is reduced due to the time–frequency uncertainty principle.

5. Uncertainty analysis

Uncertainty in the displacement measurement of a microstructure can be addressed in accordance with ISO *Guide to the Expression of Uncertainty in Measurement* [37]. The displacement measurements are influenced by wavelength stability, phase difference evaluation and misalignment of the optical setup. The maximum error associated with the wavelength λ is ± 2 nm, the half-interval limit is 2 nm and the standard uncertainty using a rectangular distribution is given by $u_{(\lambda)} = 2/\sqrt{3}$ nm. The error resulting from phase difference evaluation is mainly associated with a vertical resolution of $\lambda/100$ and can be as high as $\pi/25$ rad. The corresponding standard uncertainty assuming a rectangular distribution can be obtained by $u_{(\Delta\phi)} = 0.1256/\sqrt{3}$ rad. The misalignment error between the illumination and observation direction in the setup is estimated to be less than 2° . The resulting error for the modulus of sensitivity vector of the setup is evaluated from $s = 2(1 - \cos 2^\circ)$. The standard uncertainty is given by $u_{(s)} = 2(1 - \cos 2^\circ)/\sqrt{3}$.

The sources of uncertainty and their contributions are shown in table 1, the uncertainty contribution (U_i) is the multiplication of sensitivity coefficient (c_i) and standard uncertainty (u_i). In accordance with the ISO guide [37], the combined standard uncertainty attributed to the out-of-plane displacement z based on equation (3) is given by

$$u_c(z) = \left[\left(\frac{\partial z}{\partial \lambda} \right)^2 u_{(\lambda)}^2 + \left(\frac{\partial z}{\partial \Delta\phi} \right)^2 u_{(\Delta\phi)}^2 + \left(\frac{\partial z}{\partial s} \right)^2 u_{(s)}^2 \right]^{1/2}, \quad (10)$$

where the partial derivatives $\left(\frac{\partial z}{\partial \lambda}, \frac{\partial z}{\partial \Delta\phi}, \frac{\partial z}{\partial s} \right)$ are the sensitivity coefficients. The effective degree of freedom (ν_{eff}) for $u_c(z)$ and expanded uncertainty $U(z)$ can be calculated from the Welch-

Scatterthwaite formula:

$$v_{\text{eff}} = \frac{u_c^4(z)}{\sum_{i=1}^3 [(c_i \times u_i)^4 / v_i]}. \quad (11)$$

In our case, $v_{\text{eff}} \rightarrow \infty$. From the t -distribution table, $k_p = 1.96$ for $v_{\text{eff}} > 100$ and a confidence level of approximately 95%, the expanded uncertainty is given by

$$U(z) = k_p u_c(z) = 1.96 \times 3.88 \approx 7.60 \text{ nm}. \quad (12)$$

Hence, the uncertainty for a maximum displacement of 347 nm is ± 7.60 nm.

6. Concluding remarks

A high-speed digital holographic microscopy has been presented for low-frequency vibration measurement on a micro-beam. The method employed 2D windowed Fourier and temporal Fourier analyses for image processing. Experimental results show the possibility of evaluating the whole-field instantaneous displacement, velocity and acceleration of a micro vibrating beam. This is a distinct advantage over a conventional laser Doppler vibrometer which requires point by point scanning to carry out similar measurements. In addition, the proposed method is able to utilize spatial filtering to remove the noise. It can also be applied to measurement of non-periodic deformation. A disadvantage of the present method is the relatively small frequency range measured. For high-frequency measurement, the pointwise laser Doppler vibrometry [38] is still a dominant method. This disadvantage however will become inconspicuous with the rapid development in the capacity of high-speed cameras and computers.

References

- [1] Leondes C T 2006 *MEMS/NEMS Handbook Technique and Applications* (Berlin: Springer)
- [2] Osten W, Jupter W, Seebacher S and Baumbach T 1999 The qualification of optical measurement techniques for the investigation of material parameters of microcomponents *Proc. SPIE* **3825** 152–63
- [3] Salbut L and Kujawinska M 2001 Optical measurement station for complex testing of microelement *Opt. Lasers Eng.* **36** 225–40
- [4] Wang S H, Quan C, Tay C J, Reading I and Fang Z P 2003 Deformation measurement of MEMS components using optical interferometry *Meas. Sci. Technol.* **14** 909–15
- [5] Petitgrand S R, Yahiaoui R, Bosseboeuf A and Danaie K 2001 Quantitative time-averaged microscopic interferometry for micromechanical device vibration mode characterization *Proc. SPIE* **4400** 51–60
- [6] Petitgrand S, Yahiaoui R, Danaie K, Bosseboeuf A and Gilles J P 2001 3D measurement of micromechanical devices vibration mode shapes with a stroboscopic interferometric microscope *Opt. Lasers Eng.* **36** 77–101
- [7] Petitgrand S and Bosseboeuf A 2003 Simultaneous mapping of phase and amplitude of MEMS vibration by microscopic interferometry with stroboscopic illumination *Proc. SPIE* **5145** 33–44
- [8] Montfort F, Emery Y, Marquet F, Cuche E, Aspert N, Solanas E, Mehdaoui A, Ionescu A and Depeursinge C 2007 Process engineering and failure analysis of MEMS and MOEMS by Digital Holography Microscopy (DHM) *Proc. SPIE* **6463** 64630G
- [9] Takeda M, Ina H and Kobayashi S 1982 Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry *J. Opt. Soc. Am.* **72** 156–60
- [10] Huntley J M, Kaufmann G H and Kerr D 1999 Phase-shifted dynamic speckle pattern interferometry at 1 kHz *Appl. Opt.* **38** 6556–63
- [11] Huntley J M and Saldner H 1993 Temporal phase-unwrapping algorithm for automated interferogram analysis *Appl. Opt.* **32** 3047–52
- [12] Joenathan C, Franze B, Haible P and Tiziani H J 1998 Speckle interferometry with temporal phase evaluation for measuring large-object deformation *Appl. Opt.* **37** 2608–14
- [13] Kaufmann G H 2003 Phase measurement in temporal speckle pattern using the Fourier transform method with and without temporal carrier *Opt. Commun.* **217** 141–9
- [14] Joenathan C, Franze B, Haible P and Tiziani H J 1998 Novel temporal Fourier transform

speckle pattern shearing interferometer *Opt. Eng.* **37** 1790–5

- [15] Fu Y, Tay C J, Quan C and Miao H 2005 Wavelet analysis of speckle patterns with a temporal carrier *Appl. Opt.* **44** 959–65
- [16] Goodman J W and Lawrence R W 1967 Digital image formation from electronically detected holograms *Appl. Phys. Lett.* **11** 77–9
- [17] Schnars U and Juptner W 1994 Direct recording of holograms by a CCD target and numerical reconstruction *Appl. Opt.* **33** 179–81
- [18] Pedrini G, Osten W and Gusev M E 2006 High-speed digital holographic interferometry for vibration measurement *Appl. Opt.* **45** 3456–62
- [19] Pedrini G, Alexeenko I, Osten W and Tiziani H J 2003 Temporal phase unwrapping of digital hologram sequences *Appl. Opt.* **42** 5846–54
- [20] Fu Y, Pedrini G and Osten W 2007 Vibration measurement by temporal Fourier analysis of a digital hologram sequence *Appl. Opt.* **46** 5719–27
- [21] Fu Y, Groves R M, Pedrini G and Osten W 2007 Kinematic and deformation parameter measurement by spatiotemporal analysis of an interferogram sequence *Appl. Opt.* **46** 8645–55
- [22] Xu L, Peng X, Asundi A K and Miao J 2001 Studies of digital microscopic holography with applications to microstructure testing *Appl. Opt.* **40** 5046–52
- [23] Cuche E, Marquet P and Depeursinge C 1999 Simultaneous amplitude-contrast and quantitative phase-contrast microscopy by numerical reconstruction of Fresnel off-axis holograms *Appl. Opt.* **38** 6994–7001
- [24] Kuhn J, Charriere F, Colomb T, Cuche E, Montfort F, Emery Y, Marquet P and Depeursinge C 2008 Axial sub-nanometer accuracy in digital holographic microscopy *Meas. Sci. Technol.* **19** 074007
- [25] Lobera J and Coupland J M 2008 Contrast enhancing techniques in digital holographic microscopy *Meas. Sci. Technol.* **19** 025501
- [26] Marquet P, Rappaz B, Magistretti P J, Cuche E, Emery Y, Colomb T and Depeursinge C 2005 Digital holographic microscopy: a noninvasive contrast imaging technique allowing quantitative visualization of living cells with subwavelength axial accuracy *Opt. Lett.* **30** 468–70
- [27] Charriere F, Marian A, Montfort F, Kuehn J, Colomb T, Cuche E, Marquet P and Depeursinge C 2006 Cell refractive index tomography by digital holographic microscopy *Opt. Lett.* **31** 178–80

- [28] Kemper B and Bally G V 2008 Digital holographic microscopy for live cell applications and technical inspection *Appl. Opt.* **47** A52–61
- [29] Salder H O, Molin N E and Stetson K A 1996 Fourier-transform evaluation of phase data in spatially phase-biased TV holograms *Appl. Opt.* **35** 332–6
- [30] Cuche E, Marquet P and Depeursinge C 2000 Spatial filtering for zero-order and twin-image elimination in digital off-axis holography *Appl. Opt.* **39** 4070–5
- [31] Mallet S 1998 *A Wavelet Tour of Signal Processing* (New York: Academic)
- [32] Qian K and Soon S H 2005 Two-dimensional windowed Fourier frames for noise reduction in fringe pattern analysis *Opt. Eng.* **44** 075601
- [33] Qian K 2004 Windowed Fourier transform for fringe pattern analysis *Appl. Opt.* **43** 2695–702
- [34] Ijntema D J and Tilmans H A C 1992 Static and dynamic aspects of an air-gap capacitor *Sensors Actuators A* **35** 121–8
- [35] Kühn J, Colomb T, Montfort F, Charrière F, Emery Y, Cuche E, Marquet P and Depeursinge C 2007 Real-time dual-wavelength digital holographic microscopy with a single hologram acquisition *Opt. Exp.* **15** 7231–42
- [36] Fu Y, Pedrini G, Hennelly B M, Groves R M and Osten W 2009 Dual-wavelength image-length digital holography for dynamic measurement *Opt. Lasers Eng.* **47** 552–7
- [37] International Organization for Standardization 1995 *Guide to the Expression of Uncertainty in Measurement* (Geneva:ISO)
- [38] Castellini P, Martarelli M and Tomasini EP 2006 Laser Doppler vibrometry: development of advanced solutions answering to technology's needs *Mech. Syst. Signal Process.* **20** 1265–85

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Fig. 6 (a) Acceleration of point B by a windowed Fourier ridge; (b) acceleration of point B by numerical differentiation and low-pass filtering.

	Sources of uncertainty	Standard uncertainty (u_i)	Sensitivity coefficient (c_i)	Probability distribution	Uncertainty contribution (U_i)	Degree of freedom (ν_i)
1	Wavelength stability, $u_{(\lambda)}$	1.15 nm	0.58 rad	Rectangular	0.667 nm	∞
2	Phase difference counting error, $u_{(\Delta\phi)}$	0.0725 rad	52.73 nm	Rectangular	3.82 nm	∞
3	Misalignment error, $u_{(\delta)}$	0.0007	-184.56 nm	Rectangular	-0.129 nm	∞
-	Combined uncertainty, $u_c(z)$	-	-	t -distribution	3.88 nm	∞
-	Expanded uncertainty, $U(z)$	-	-	t -distribution	7.60 nm	∞

Table 1

Fig. 1

Fig. 2

(a)

(b)

Fig. 3

Fig. 4

(a)

(b)

(c)

Fig. 5

(a)

(b)

Fig. 6