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2006

Quan, C., Yu, F., & Miao, H. (2006). Wavelet analysis of digital shearing speckle patterns with a temporal carrier. *Optics Communications*, 260(1), 97-104.

<https://hdl.handle.net/10356/91295>

<https://doi.org/10.1016/j.optcom.2005.10.023>

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[DOI:<http://dx.doi.org/10.1016/j.optcom.2005.10.023>].

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Wavelet analysis of digital shearing speckle patterns with a temporal carrier

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Abstract

Shearography is an optical technique allows direct measurement of deflection derivatives. This paper presents a novel temporal phase analysis technique based on wavelet transform when shearography is applied to measure a continuously deforming object. A series of shearing speckle patterns is captured by a high-speed camera during the deformation. To avoid the phase ambiguity problem, a temporal carrier is generated by a piezoelectrical transducer (PZT) stage in one beam of the modified Michelson interferometer. The intensity variation of each pixel on recorded images is then analyzed along time axis by a robust mathematical tool --- complex Morlet wavelet transform. After the temporal carrier is removed, the absolute phase change representing the first-order derivative of the continuous deformation is obtained without the need of temporal or spatial phase unwrapping process. The results obtained by wavelet transform are compared with those from temporal Fourier transform.

Keywords: Digital shearing speckle interferometry; Temporal carrier; Wavelet transform; Phase retrieval.

1. Introduction

Optical methods such as holography and shearography [1] have already gained many industrial applications for non-destructive evaluation [2-5]. Among them, shearography [6] is a well-known whole-field technique for the measurement of spatial derivative along one axis of surface displacement, thus eliminating the need of numerical differentiation of displacement data to yield strains. Compared with holography, shearography does not require a reference light beam. This feature leads to simple optical setups and alleviation of the coherence length requirement of laser and environmental stability demand [7]. With the rapid development of computer and image processing technologies, digital shearography received wide acceptance in the last two decades. Direct and automated measurement of phase changes can be realized without relying on fringe pattern reconstruction and human interpretation using digital shearography. Generally, phase-shifting technique is the most popular phase-retrieving techniques applied with digital shearography, but it requires several, normally three to five images with prescribed phase steps at one status; subsequently, it is not easily accomplished [8] when a continuously deforming object is measured. Spatial phase unwrapping is also compulsory to remove the 2π phase discontinuities [9]. However, the existing of noise effect and low modulated pixels in speckle patterns may produce breaks in wrapped phase map and generate large errors when two-dimensional unwrapping process is performed.

Temporal phase analysis and temporal phase unwrapping technique [9-12] have been reported to overcome these problems and successfully applied on the measurement of a continuously deforming object in recent years. In this technique, a series of fringe or speckle patterns is recorded throughout the entire deformation history of the object. Each pixel is then analyzed as a function of time. Joenathan et al. [13] used this technique to obtain the transient derivative of a continuously deformation which is over 500 μm . This is one of the advantages

of temporal phase analysis as measurement of a displacement or its derivatives in this range is very difficult to be accomplished with conventional ESPI or digital shearography. In addition, they introduced a temporal carrier by tilting the object with a constant angular velocity. The frequency of temporal carrier is high enough so that even the highest negative frequency is pushed to positive. This avoids the phase ambiguity problem involved in almost all the temporal phase analysis techniques.

There are several temporal analysis algorithms [14]. Among them, Fourier transform [11] is a method that receives the widest acceptance in temporal analysis of speckle patterns. The intensity fluctuation of each pixel is first transformed, and one side of the frequency spectrum is filtered with a bandpass filter. The filtered spectrum is inverse-transformed to obtain the wrapped phase. The phase values are then unwrapped along the time axis at each pixel independently of other pixels in the image. The Fourier analysis gains high accuracy when the spectrum of temporal frequency is narrow. However, in most cases, the spectrum is wide due to the non-linear phase change along the time axis. Various spectrums at different pixels increase the difficulties of automatic filtering process as the width and position of the bandpass filter must be different at each pixel, otherwise it has to be broadened, and introduces further errors in phase extraction.

The shortcoming of Fourier analysis mentioned above can be overcome by the use of wavelet transform. Wavelet transform [15] is a robust mathematical tool for signal analysis. It has been applied in optical interferometry recently to denoise speckle patterns [16,17]. Continuous wavelet transform (CWT) was also used for phase extraction on different types of fringe patterns with spatial carrier [18, 19]. These applications use one-dimensional CWT along one spatial axis. The phase gradient can be obtained by extracting the ‘ridge’ of the wavelet coefficients, followed by an integration process to retrieve the phase. Wavelet transform has also been applied to temporal phase analysis of fringe or speckle interferometry.

The concept was first introduced by Colonna de Lega in 1996 and some preliminary results were presented [20]. Cherbuliez and Jacquot [21] extended the study on applying different processing algorithms to develop a useful software tool for phase retrieval from a series of speckle patterns. Previous researches [22-24] also show the advantages of wavelet transform over the Fourier transform in temporal phase analysis.

In this study, continuous wavelet transform is applied to extract the phase change from a series of shearing speckle patterns. Similar as Fourier transform, temporal wavelet transform still has phase ambiguity problem. This limits the technique to phase extraction in one direction which is already known. However, in digital shearography, the phase change represents the deflection derivative, which is most likely in opposite directions at different pixels even the deflection itself is in one direction. Adding a carrier frequency to the image acquisition process is the only method so far to avoid this problem. In this application, the temporal carrier is generated by shifting one mirror constantly in Michelson type shearing device using a piezoelectrical transducer (PZT) stage, which is easier to realize comparing to rotating the object. The phase variation due to temporal carrier is also measured experimentally. After remove the effect of temporal carrier, the absolute displacement derivative of a continuously deforming plate is obtained. The results are also compared with those from temporal Fourier transform.

2. Theoretical analysis

Fig. 1 shows the schematic layout of digital shearography with temporal carrier. Instead of a glass wedge [6] or a birefringent prism [2,7], a modified Michelson shearing interferometer [25,26] is selected as the shearing device so that the temporal carrier can be generated by shifting mirror M2 constantly with a PZT stage. A series of speckle patterns are captured by a high-speed CCD camera with a telecentric lens during deformation. When the

shearing is in the x -direction and with near normal illumination and viewing conditions, the intensity variation of each pixel can be expressed as

$$\begin{aligned}
 I_{xy}(t) &= I_{0xy}(t) + A_{xy}(t) \cos[\phi_{xy}(t)] = I_{0xy}(t) + A_{xy}(t) \cos[\phi_C(t) + \phi_{xy}(t)] \\
 &= I_{0xy}(t) \left\{ 1 + V \cos \left[\phi_{0xy} + 2\pi f_C t + \frac{4\pi \frac{\partial w_{xy}(t)}{\partial x} \delta x}{\lambda} \right] \right\}, \tag{1}
 \end{aligned}$$

where $I_{0xy}(t)$ is the intensity bias of the speckle pattern, V is the visibility, ϕ_{0xy} is the initial random phase, f_C is the temporal carrier frequency, $\phi_C(t) = 2\pi f_C t$ is the phase change due to temporal carrier, δx is the amount of image shearing in the x -direction, and $w_{xy}(t)$ is the out-of-plane deformation of the object. At each pixel the temporal intensity variation is analyzed by continuous wavelet transform.

A wavelet is a real or complex function $\psi(t)$ with a zero average and localized both in time and in frequency. The continuous wavelet transform decomposes a signal $s(t)$ over dilated and translated wavelets $\psi_{a,b}(t)$

$$W_S(a, b) = \int_{-\infty}^{+\infty} s(t) \psi_{a,b}^*(t) dt, \tag{2}$$

where

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right), \quad a, b \in R, \quad a > 0, \tag{3}$$

where a is the scaling factor related to the frequency, b is the time shift, and $*$ denotes the complex conjugate. The factor $1/\sqrt{a}$ in Eq. (3) is used to keep the energy of $\psi_{a,b}(t)$ constant during scaling and translation.

In this investigation, an analytic wavelet is selected for analysis of phase related properties of real functions (e.g. determination of instantaneous frequency). The most commonly used mother wavelet for such applications is the complex Morlet wavelet because it gives the smallest Heisenberg box [27]. The Morlet wavelet is defined by

$$\psi(t) = g(t) \exp(i\omega_0 t) = \exp\left(-\frac{t^2}{2}\right) \exp(i\omega_0 t). \quad (4)$$

Thus, ω_0 is the “mother” frequency, a parameter that has to be properly chosen. In order to remove negative frequencies and avoid DC signals, the Fourier transform at $\omega = 0, \hat{\psi}(0)$, should be numerically negligible. This condition is achievable when $\omega_0 > 5$. Here, the $\omega_0 = 2\pi$ was chosen to satisfy the admissibility condition [15] so that the wavelet function is able to remove the negative frequencies as well as the DC term of the signals. The computation of Eq. (2) requires an extension of the signal at its left- and right-hand edges to avoid the large error generated at the boundary of the signal. Symmetrical or zero padding extension algorithms are commonly used. A linear predictive extrapolation method [28] is selected in this study. The advantage of this extrapolation method is that phase and frequency of intensity variations are maintained. After CWT has been carried out on the extended data, the wavelet coefficients are truncated appropriately.

CWT expands a one-dimensional temporal intensity variation of certain pixels into a two-dimensional plane of scaling a (which is related to the frequency) and position b (which is the time axis). Substituting Eqs. (1) and (4) into Eq. (2), the wavelet transform of temporal intensity variation can be expressed as [27]

$$W_{xy}(a, b) = \frac{\sqrt{a}}{2} A_{xy}(b) \left(\hat{g} \{ a [\zeta - \phi'_{xy}(b)] \} + \varepsilon(b, \zeta) \right) \{ \exp[i\phi_{xy}(b)] \}, \quad (5)$$

where $\zeta = \omega_0/a$, and ε is a corrective term which normally can be negligible. $\hat{g}(\omega)$ is the Fourier spectrum of $g(t)$. The trajectory of maximum $|W_{xy}(a,b)|^2$ on the a - b plane is called a ‘ridge’. Since $|\hat{g}(\omega)|$ is maximum at $\omega = 0$, and if ε is negligible, $|W_{xy}(a,b)|^2$ reaches maximum when

$$\varphi'_{xy}(b) = \zeta_{rb} = \frac{\omega_0}{a_{rb}}, \quad (6)$$

where $\varphi'_{xy}(b)$ is defined as instantaneous frequency of signal, and a_{rb} denotes the value of a at instant b on the ridge. The wavelet transform on the ridge is then given by

$$W_{xy}(a_{rb}, b) \approx \frac{\sqrt{a_{rb}}}{2} A_{xy}(b) \hat{g}(0) \exp[i\varphi_{xy}(b)]. \quad (7)$$

The phase value $\varphi_{xy}(b)$, which represents the temporal deflection derivative in this case, can be retrieved by two methods. One method is to calculate the arctangent of the ratio of the imaginary and real parts of the wavelet transform on the ridge.

$$\varphi_{xy}(b) = \tan^{-1} \left(\frac{\text{Im}[W_{xy}(a_{rb}, b)]}{\text{Re}[W_{xy}(a_{rb}, b)]} \right), \quad (8)$$

where Re and Im denote the real and imaginary part of the wavelet transform. However, $\varphi_{xy}(b)$ obtained from Eq. (8) is within $[0, 2\pi)$ and one-dimensional phase unwrapping cannot be avoided. As only the phase change $\Delta\varphi_{xy}(b)$ is concerned in this study, it can be calculated by integration of the instantaneous frequency in Eq. (6), and phase unwrapping procedure is not needed in temporal and spatial domain. The phase change between two instants T_1 and T_2 at point $P(x,y)$ can be expressed by

$$\Delta\varphi = \varphi_{xy}(T_2) - \varphi_{xy}(T_1) = 2\pi f_C(T_2 - T_1) + \frac{4\pi\delta x}{\lambda} \cdot \left[\frac{\partial w_{xy}(T_2)}{\partial x} - \frac{\partial w_{xy}(T_1)}{\partial x} \right], \quad (9)$$

where $w_{xy}(T)$ is the out-of-plane displacement of point P at instant T . The phase change due to temporal carrier can also be measured experimentally and removed from Eq. (9).

3. Experimental illustration

Fig. 1 shows the experimental set up. The test specimen in this study is a square plate with a circular blind hole, clamped at the edges by rows of screws. The width and thickness of the plate are 80 and 5 mm, respectively. The plate is loaded by a uniformly distributed pressure applied with compressed air, and continuously deformed by increments of pressure in the chamber. The plate is with near normal illumination using a He-Ne laser (30 mW, $\lambda = 632.8$ nm). The image shearing $\delta x = 3\text{mm}$ is generated by rotating mirror M1 with a small angle. To generate a temporal carrier, mirror M2 is mounted on a computer-controlled PZT stage. During the deformation of the plate, it is applied with a linear rigid body motion at certain velocity. To retrieve the phase change of the temporal carrier, a stationary reference block with a diffuse surface is mounted besides the object and its image is captured along with the plate. During deformation of the object, a series of speckle patterns is captured by a high-speed CCD camera (KODAK Motion Corder Analyzer, SR-Ultra) with a recording rate of 60 frame/s (fps). Fig. 2 shows the typical shearography fringes of the test specimen with the reference block at an instant $T = 1.5\text{s}$, which is obtained by subtraction of two speckle patterns.

4. Results and discussion

Five hundred and forty images were recorded within 9 s, and 300 speckle patterns were selected to process pixel-by-pixel along the time axis. The interested area on the plate

(shown in Fig. 2) contains 330×330 pixels with the actual dimension of $56.3\text{mm} \times 56.3\text{mm}$. For each pixel, 300 sampling points along the time axis were obtained. Fig. 3(a) shows the intensity variation of point R (indicated in Fig. 2) on the reference block. The modulus of the Morlet wavelet transform of intensity variation of points R is shown in Fig. 3(b). The dashed line shows the ridge of the wavelet transform where the maximum modulus are found. Although the a_{rb} on the ridge is fairly constant, some variation of a_{rb} due to noise on certain pixel are still observed. To eliminate the noise effect, an average value of a_{rb} was calculated in an area of 20×200 pixels (shown in Fig.2) on reference block. Fig. 3(c) shows the averaged scaling a_{rb} on the ridge. As the reference block is not moving, the ridge value represents the effect of temporal carrier, which was applied on mirror M2 by a PZT stage. Little variation of averaged a_{rb} is observed, which implies the temporal carrier is constant along time axis. Integration of $2\pi/a_{rb}$ was carried out along the time axis to generate a continuous phase change $\Delta\phi_c(t)$ due to temporal carrier.

Fig. 4 shows the intensity variations on points A and B (indicated in Fig. 2). It can be observed that the frequency of intensity variation of point R is higher than that of point B, but lower than that of point A, which implies that the phase changes at these two points are of opposite sign. Similar to point R, integration was carried out on each pixel to generate a temporal phase change $\Delta\phi_{xy}(t) = \Delta\phi_c(t) + \Delta\phi_{xy}(t)$. Fig. 5(a) shows the temporal phase change obtained on points A and B and the averaged phase change on reference block. Subtracting the phase change $\Delta\phi_c(t)$ due to temporal carrier, the absolute phase change $\Delta\phi_{xy}(t)$ can be obtained on each pixel. Fig. 5(b) shows the absolute phase change on points A and B. Combining the phase change of each point at certain instant, a high-quality spatial distribution of phase change is obtained. Fig. 6(a) shows the 3-D plot of phase variation

obtained by temporal wavelet analysis at $T = 3s$. Fig. 7(a) shows the phase variation on cross-section C-C (shown in Fig.2).

For comparison, temporal Fourier analysis was also applied on the same speckle patterns. On the reference block, a narrow bandpass filter was applied as the spectrum of the signal was concentrated. On the plate, a relatively wider filter was applied as it should include all frequencies at different pixels. A one-dimensional phase unwrapping was then applied along the time axis, as all phase values retrieved by inverse Fourier transform fall within a $[0, 2\pi)$ range. Fig. 6(b) shows the 3-D plot of phase variation obtained by temporal Fourier analysis at $T = 3s$. Fig. 7(b) shows the phase variation on cross-section C-C obtained by Fourier analysis. Similar as temporal wavelet analysis, a 3×3 median filter was also applied on phase maps. However, large errors are still found at some pixels, especially in the area where the phase changes are of high positive values. In these areas, the phase change measured is the sum of temporal carrier and object. The frequency of signal is high so that it overlaps with some noise frequencies. Fourier transform shows the lack of flexibility when the signal frequency is extracted from noise. From Fig. 7, it can be observed that CWT generates a much smoother spatial phase distribution at different instants compared to a Fourier transform.

From the above comparison between the results of wavelet and Fourier analysis, it can be observed that wavelet analysis shows better results in temporal phase measurement. As wavelet analysis extracts the instantaneous frequency with the highest energy (which is the frequency of the signal), it performs an adaptive band-pass filtering of the measured signal, thus, limits the influence of various noise sources and increases the resolution of measurement significantly. In contrast, Fourier transform uses a broader filter which is less efficient in eliminating noise effect.

Continuous wavelet transform maps a one-dimensional intensity variation signal to a two-dimensional plane of time and frequency, and then extracts the instantaneous frequencies with the highest energy. Obviously it is a time-consuming process and requires high computing speed and memory. In this investigation, the computation time is about 10 times larger than that of temporal Fourier transform. However, this disadvantage becomes inconspicuous due to the rapid improvement in capacity of computers. Similar to temporal Fourier analysis, temporal wavelet transform is also limited by Nyquist sampling theorem. It is impossible to analyze signals with a frequency higher than half of the acquisition rate. Because of it, selecting a suitable temporal carrier frequency to the image acquisition process, as was mentioned above, is not so easy. The frequency of temporal carrier should be high enough so that even the highest negative frequency of the object is pushed to positive. However, it cannot be too high due to the limitation of Nyquist sampling theorem, because the phase changes at some areas are the sum of temporal carrier and object as mentioned above. Sometimes a compromise is not easily reached if the capturing rate of the camera is not high enough.

5. Concluding remarks

We have proposed a novel method to retrieve the transient displacement derivatives of a continuously deforming object using combination of temporal wavelet analysis and temporal carrier technique. The introducing of temporal carrier ensures that even with shearography configuration, the phase change of each point on the object is in one direction, so that temporal phase analysis methods can be applied. A complex Morlet wavelet is selected as the wavelet basis. Comparing to Fourier transform, wavelet has advantages on extracting the instantaneous frequencies, so that high quality phase map of the object can be obtained without any phase unwrapping processes. A comparison between temporal wavelet

transform and Fourier transform shows that wavelet analysis can limit the influence of various noise sources, and significantly improve the result in temporal phase measurement.

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Fig. 1

Fig. 2

(a)

(b)

(c)

Fig. 3

(a)

(b)

Fig. 4

(a)

(b)

Fig. 5

(a)

(b)

Fig. 6

(a)

(b)

Fig. 7