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Fracture analysis of load-carrying cruciform fillet welded joints with multiple cracks

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Abstract

Load-carrying cruciform fillet welded joints configuration is frequently encountered in many welded structures. Different from non-load-carrying joints, the crack in load-carrying joints can propagate from the weld root or from the weld toe. Hence, the crack propagation location is critical for a given weld size, transverse and main plates thickness. In this study, a finite element (FE) mesh generator developed previously is extended to generate 3-D models of load-carrying joints containing cracks at the weld root and the weld toe. Then, extensive parametric study is carried out to calculate the stress intensity factors (SIFs). Corresponding SIF equations are proposed according to multiple regression analysis. Based on these equations obtained for the weld root and weld toe cracks, the critical crack and weld sizes at which the crack propagates from the weld root to the weld toe are determined.

Keywords: Crack propagation; Load-carrying cruciform joints; Stress intensity factor; Weld root crack; Weld toe crack

1. Introduction

Cruciform fillet welded joint is a common connection encountered in many welded structures. They are classified as non-load-carrying and load-carrying. For the non-load-carrying joints, the crack always propagates from the weld toe [1,2]. However, the crack may propagate from the weld toe [3] or the weld root [4-9] for the load-carrying joints. This is mainly due to the presence of non-penetrating region between the transverse and main plates. For the past decades, considerable research works have been performed to evaluate the fatigue and fracture strength of

Nomenclature

a	weld toe crack depth
c	weld toe crack half length
E	Young's modulus
f_w	finite width correction factor
h	weld leg length
K_{root}	stress intensity factor for weld root crack
$K_{\text{toe}}^{\text{deep}}, K_{\text{toe}}^{\text{ends}}$	stress intensity factors for the weld toe crack front deepest point and weld toe crack front ends, respectively
M	bulging correction factor
M_a	stress intensity magnification factor for a plain plate containing a surface crack
$M_{k,\text{toe}}^{\text{deep}}, M_{k,\text{toe}}^{\text{ends}}$	stress intensity magnification factors for the weld toe crack front deepest point and weld toe crack front ends, respectively
T	main and transverse plate thickness
Y_{root}	dimensionless stress intensity factor for weld root crack
a/c	weld toe crack aspect ratio
a/T	weld toe crack depth ratio
h/T	weld leg length ratio
L/W	weld root crack depth ratio
ν	Poisson's ratio
σ_a	applied axial stress

Abbreviations

BEM	boundary element method
BS	British Standard
DBEM	dual boundary element method
FE	finite element
SGBEM	symmetric Galerkin boundary element method

this type of cracked configuration [10-16]. A 2-D stress intensity factor (SIF) formula was presented by Frank and Fisher [11] by considering the non-penetrating region at the weld root as an initial crack. By combining Paris' law [17] and SIF formula developed by Frank and Fisher [11], Guha [12] proposed new fracture mechanics equations to predict the fatigue life of load-carrying cruciform welded joints. Xu et al. [15] studied the behaviour of load-carrying joints using 2-D boundary element method (BEM). In the paper [15], symmetric Galerkin boundary element method (SGBEM) was employed to analyse the multiple cracks problem in the welded joints, and the results obtained were compared with those given by dual boundary element method (DBEM). The critical weld size for the crack to propagate from the weld root was determined by equating the SIF values at the weld root with that at the weld toe. However, the analysis is limited to the fatigue and fracture assessment of 2-D configuration cases. Besides, the crack depth at the weld toe is assumed to be constant equal to 0.15 mm, limiting the application range of the obtained results.

3-D linear elastic finite element (FE) analysis on the load-carrying cruciform fillet welded joints with a weld toe crack and two weld root cracks is carried out in the study. Due to large numbers of complicated FE models are required to be created, a FE mesh generator developed previously [18,19] is extended to serve for the load-carrying joints with multiple crack so as to simplify the cumbersome and time-consuming task. A Python script communicating with ABAQUS [20] is written to create, process and post-process these FE models. Then, the SIF values of the weld root and weld toe cracks are determined from the FE analysis. It is noted that increasing weld toe crack depth has a noticeable influence on the SIFs at the weld root and vice versa. Hence, a new set of SIF equations for the two types of cracks are proposed based on multiple regression analysis. Finally, by comparing the SIFs obtained at the weld root and the weld toe, it enables the authors to predict the critical crack and weld sizes whereby the crack will propagate from the

weld root of the load-carrying filled welded joint.

2. Details of numerical analyses

2.1. Dimensions of weld and crack parameters

The load-carrying cruciform fillet welded joints containing multiple cracks are investigated in the study (Fig. 1). The thickness (T) and leg length (h) of the main and transverse plates are equal, and the leg length ratio (h/T) is introduced to represent the relative dimension. The width (B) of the main plate selected for the analyses are as follows: $B = 10c$ but $\geq 10T$, which minimizes finite geometry effect [21]. The cruciform joint contains a total of three cracks, viz., one weld toe crack and two weld root cracks. The weld toe crack is modelled as a semi-elliptical shape and it is perpendicular to the main plate face as indicated in Fig. 1. 9 different crack depth ratios (a/T) are considered, ranging from a very shallow crack depth ratio of 0.05 to a deep crack depth ratio of 0.7. The crack aspect ratio (a/c) varies from 0.1 to 1.0, which takes into account most crack sizes encountered in practice. The non-penetrating regions at the weld root are assumed to act as two initial cracks [5,15], and the incomplete penetration ratio (L/W) used in the previous work [6,12] is renamed as the weld root crack depth ratio in the study to characterize the crack profile with $W = T + 2h$. Therefore, four basic parameters, namely, weld root crack depth ratio (L/W), weld leg length ratio (h/T), weld toe crack depth ratio (a/T) and weld toe crack aspect ratio (a/c) are used in the parametric study and reported in Tables 1 and 2.

2.2. Finite element modelling

In this study, hundreds of different geometries of the welded joints are analysed, and a FE mesh generator developed earlier [18,22,23] enables the authors to generate all these 3-D models rapidly. The analyses are performed using ABAQUS [20], and the element type used is a 20-node brick element with reduced integration (C3D20R). The boundary conditions for the model are presented in Fig. 2. Based on symmetry, only half of the plate model is considered and a

symmetric boundary condition about Y-axis is applied on the plate face of $Y = 0$. One end of the main plate parallel to crack face is restrained and a uniform axial stress is applied at the other end of the plate to provide the loading condition. Fig. 3 shows a typical FE model of load-carrying cruciform fillet welded joints for $h/T = 0.7$, $L/W = 0.3$, $a/T = 0.4$ and $a/c = 0.4$. A fine mesh along the crack front is required to ensure convergence, especially at the weld toe crack front ends [21,24]. The mesh design of the crack section is depicted in Fig. 4 by using the same crack and weld sizes as that in Fig. 3. The material employed in the analyses is SM490B [2] with Young's modulus (E) and Poisson's ratio (ν) being 210 kN/mm² and 0.3, respectively. The SIFs of the weld root and weld toe cracks are computed from the J -integral [25] implemented in ABAQUS [20]. For the weld root crack case, since the crack front uniformly penetrates through the main plate along the width direction, a plane strain condition is employed at the crack front except the region near the plate boundary. The SIF is calculated along the crack front using the following expression:

$$K_{\text{root}} = \sqrt{K_I^2 + K_{II}^2} = \sqrt{\frac{JE}{(1 - \nu^2)}} \quad (1)$$

where K_I and K_{II} are the Mode-I and Mode-II SIF, respectively. Due to the SIF values changing with specific crack sizes and loading conditions, the dimensionless SIF (Y_{root}) is introduced to characterize the fatigue and fracture strength of cracked welded joints, and it is given as

$$Y_{\text{root}} = \frac{K_{\text{root}}}{\sigma_a \sqrt{\pi L/2}} \quad (2)$$

where σ_a is the applied axial stress. For the weld toe crack case, the plane strain condition is used along the crack front except at the crack front ends [18,19,24], and the SIF (K_{toe}) is defined as [5]

$$K_{\text{toe}} = \sqrt{K_I^2 + K_{II}^2} = \sqrt{\frac{JE}{(1 - \nu^2)}} \quad (3)$$

At the crack front ends, the plane stress condition is assumed and $E/(1 - \nu^2)$ in Eq. (3) is

replaced by E to determine the corresponding SIF values [18,19,24]. The SIF for the weld toe crack is frequently described in terms of a stress intensity magnification factor ($M_{k,toe}$), which is expressed as the product of the ratio of the SIF with stress concentration to the SIF for the same crack in a plate without stress concentration [2,22-24], and it is defined as,

$$M_{k,toe} = \frac{K_{toe}}{K_{plate}} \quad (4)$$

where

$$K_{plate} = M f_w M_a \sigma_a \sqrt{\pi a} \quad (5)$$

where K_{plate} is the SIF for a plain plate with a semi-elliptical surface crack at the weld toe, M defines the bulging correction factor, f_w is the finite width correction factor, and M_a is the stress intensity magnification factor for the cracked plate subjected to axial loading. The expressions of M , f_w and M_a are listed in Appendix A [26].

2.3. Validity of the FE mesh generator

The accuracy of the FE mesh generator must be verified before it can be used for the extensive parametric study of load-carrying cruciform fillet-welded joints with multiple cracks. The mesh generator is developed step by step based on the earlier works carried out by the authors [22,23]. At first, a mesh generator for a plain plate containing a semi-elliptical surface crack at the weld toe [23] is created to produce the corresponding FE models. The SIF values obtained are compared with the values obtained from Newman and Raju equation [26] at the crack front deepest point and the crack front ends. A good agreement between one another is observed (Fig. 5) indicating the robustness of mesh design for the weld toe crack. Then, the mesh generator is upgraded to serve for non-load-carrying cruciform fillet-welded joints with a weld toe crack [22] by adding two attachments to the main plate, and the validity is further confirmed by comparing with the results obtained from FEACrackTM software [27]. Subsequently, the mesh generator is developed to generate FE models of load-carrying cruciform fillet-welded joints with a weld toe crack and two weld root cracks by including the non-penetrating region. The mesh strategy used

at the weld root crack needs to be verified because the weld root crack is absent in the previous models [22,23]. It is noted from Fig. 6 that the SIF values obtained from FE analysis correlate very well with the values produced by Frank and Fisher equation [11], confirming the validity of the mesh generator for modelling the weld root crack. Therefore, the present FE mesh generator is robust and accurate for parametric study of load-carrying fillet welded joints with multiple cracks where sufficient elements are used to guarantee good convergence. Convergent tests covering the range of parameters listed in Tables 1 and 2 are performed to determine that the number of elements ranging from 30,000 to 80,000 depending on the crack size and plate geometry is adequate. The most refined model contains around 77,000 20-node brick elements (over 1,000,000 degrees of freedom), and the number of elements for the most coarse model is about 31,000 with over 400,000 degrees of freedom. The extensive validation works mentioned in the above provide confidence in the present FE analysis to attain accurate SIFs at the weld root and weld toe.

3. Extraction of SIF values and effect of various parameters

As indicated in the previous section, the non-penetrating regions at the weld root are modelled as two initial cracks. Hence, there are four weld root crack fronts existed in the analysed model (Fig. 7). The most critical crack front depends upon the largest SIF value as it is usually used to assess the integrity of the cracked structures [15,28]. In the section, the maximum SIF along the four weld root crack fronts and the SIFs at the weld toe crack front deepest point and the crack front ends are extracted at first. Then, the interaction effect of the weld root and weld toe cracks is investigated in detail.

3.1. Extraction of SIF values along crack fronts of weld root cracks

Fig. 8 illustrates a typical variation of SIF values along the Y-axis for the four weld root cracks fronts. Due to the presence of the weld toe crack, the weld roots SIF values are influenced significantly when $Y/B \leq 0.23$. With the increase of Y/B ratio, the effect of the weld toe crack

gradually weakens and the difference of SIF values for the four crack fronts becomes smaller and smaller. The rapid increase of SIF values at the end of the plate width ($Y/B \geq 0.43$) is induced by the boundary effect, and the SIF values in this region are neglected in the extraction procedure. It can be seen clearly from Fig. 8 that the maximum SIF is obtained from the crack front-1, which is closest to the weld toe crack and the most affected by it. Extensive test cases, covering the range of parameters listed in Table 1, demonstrate that the presence of weld toe crack always has the most significant influence on the crack front-1. Therefore, the maximum required SIF value is extracted from the variation curve of SIF versus Y/B at the crack front-1. The Y_{root} can be calculated from Eq. (2) based on the SIF value obtained. For the weld toe crack case, the SIFs at the crack front deepest point and the crack front ends are extracted directly, and the $M_{k,\text{toe}}$ is determined based on Eq. (4).

3.2. Influence of different parameters on Y_{root} and $M_{k,\text{toe}}$ values

The present numerical study is composed of various parameters, and the aim of this sub-section is to illustrate the influence of these parameters on Y_{root} and $M_{k,\text{toe}}$ values. Fig. 9 shows a representative plot depicting the variation of Y_{root} with L/W for different h/T values. Y_{root} values increase with the decrease of h/T , and the increase becomes larger as h/T becomes smaller, which means that a smaller weld size induces a larger Y_{root} value for a constant L/W . The effect of the presence of a weld toe crack on the SIF of the weld root crack is presented in Fig. 10, and it is noted that a larger a/T results in a larger Y_{root} value. Fig. 11 shows the dependence of $M_{k,\text{toe}}$ on L/W by observing that increasing L/W enlarges the $M_{k,\text{toe}}$. In fact, Figs. 10-11 indicate the interaction effect of the weld root and weld toe cracks, conforming the necessity of proposing the SIF equations for the two types of cracks.

4. Y_{root} and $M_{k,\text{toe}}$ estimation equation

In Sub-section 4.1, multiple regression analysis [21,24] is used to develop the Y_{root} and $M_{k,\text{toe}}$ equations for the weld root and weld toe cracks, respectively, whilst the accuracy of the

proposed equations is validated in Sub-section 4.2.

4.1. Multiple regression analysis

The Y_{root} and $M_{\text{k,toe}}$ equations are progressively developed in stages by considering the effects of four parameters (L/W , h/T , a/T and a/c). For the Y_{root} equations, the primary development stage of the equations allows for the effect of varying L/W for a zero weld toe crack depth. Therefore, the Frank and Fisher equation [11] is used as the first function of the Y_{root} equations as shown in Appendix B. The secondary development stage involves the effect of varying a/T for a constant $a/c = 0.1$. The final stage of the equation development takes into account the effect of varying a/c , thereby covering all the Y_{root} data in the regression analysis. For the $M_{\text{k,toe}}$ equations, the first development stage considers the effect of varying a/c for a constant $L/W (= 0.1)$ and $h/T (= 0.45)$. The effect of varying L/W for a constant $h/T (= 0.45)$ is taken into account in the second development stage. In the final stage of the equation development, all the $M_{\text{k,toe}}$ data are included in the regression analysis, making allowance for varying L/W and h/T .

Three basic functions, viz., polynomial, exponential and power functions, are employed to construct the Y_{root} and $M_{\text{k,toe}}$ equations. Based on the basic functions, a trial equation with an initial guess to the coefficients values are given at first [18,19,23,24]. Then, least square fitting method is used to minimise the differences between the FE data and the equation predictions by changing the coefficients in the equation. Finally, the required equations are proposed in three stages mentioned in the above. The development procedures are implemented by a MATLAB code. The new parametric Y_{root} equations for the weld root crack in load-carrying cruciform fillet welded joints with multiple cracks are built up as follows:

$$Y_{\text{root}} = f_{\text{wm}} M_{\text{km}} h_1 \left(\frac{a}{T}, \frac{h}{T} \right) h_2 \left(\frac{a}{T}, \frac{h}{T}, \frac{a}{c} \right) \quad (6)$$

where

$$h_1 \left(\frac{a}{T}, \frac{h}{T} \right) = D_1 \left[1 - \left(\frac{a}{T} \right) \right]^{-0.123596} + D_2 \left(\frac{a}{T} \right)^{D_3} \quad (7)$$

$$D_1 = 0.087604 \left(\frac{h}{T}\right)^2 - 0.146992 \left(\frac{h}{T}\right) + 1.068704 \quad (8)$$

$$D_2 = -1.110122 \left(\frac{h}{T}\right)^3 + 3.848106 \left(\frac{h}{T}\right)^2 - 4.392061 \left(\frac{h}{T}\right) + 1.941125 \quad (9)$$

$$D_3 = 0.641577 \left(\frac{h}{T}\right)^2 + 1.133506 \left(\frac{h}{T}\right) + 0.487384 \quad (10)$$

$$h_2 \left(\frac{a}{T}, \frac{h}{T}, \frac{a}{c}\right) = \left[D_4 \left(\frac{h}{T}\right) + D_5\right] \left(\frac{a}{T}\right)^{D_6} + \left[1 - \left(\frac{a}{T}\right)\right]^{D_7 \left(\frac{h}{T}\right)^2 + D_8 \left(\frac{h}{T}\right) + D_9} \quad (11)$$

$$D_4 = 1.111012 \left(\frac{a}{c}\right)^{-0.188802} - 1.713016 \quad (12)$$

$$D_5 = -0.221411 \left(\frac{a}{c}\right)^2 - 0.034145 \left(\frac{a}{c}\right) + 1.011042 \quad (13)$$

$$D_6 = -0.210891 \left(\frac{a}{c}\right)^2 + 0.541566 \left(\frac{a}{c}\right) + 0.950673 \quad (14)$$

$$D_7 = -0.188517 \left(\frac{a}{c}\right)^{-0.703801} + 0.951512 \quad (15)$$

$$D_8 = 0.510211 \left(\frac{a}{c}\right)^{-0.695732} - 2.525144 \quad (16)$$

$$D_9 = -0.009782 \left(\frac{a}{c}\right)^{-1.728042} + 1.519031 \quad (17)$$

It can be observed clearly that the proposed equations are an extension of the Frank and Fisher equation [11] by considering the effect of the weld toe crack size on the SIF values of weld root crack. The $M_{k,toe}$ equations for the weld toe crack are listed in Appendix C.

4.2. Goodness of fit of proposed Y_{root} and $M_{k,toe}$ equations

The newly proposed Y_{root} and $M_{k,toe}$ equations are complex in nature due to various influencing parameters as well as the large amount of FE data. Therefore, histograms describing error frequency distributions [18,19,23,24] are used to verify the goodness of fit of the equations, and the percentage errors are calculated using the following expression, respectively,

$$\% \text{ Error} = \frac{[Y_{root(equation)} - Y_{root(FEA)}]}{Y_{root(FEA)}} \times 100 \% \quad (18)$$

$$\% \text{ Error} = \frac{[M_{k,toe(\text{equation})} - M_{k,toe(\text{FEA})}]}{M_{k,toe(\text{FEA})}} \times 100 \% \quad (19)$$

In the histogram, the abscissa is denoted by the percentage error and the ordinate is defined as the percentage of values inside a particular error range. Figs. 12-13 shows that the vast majority of regression data for the weld root and weld toe cracks are predicted to within the percentage errors of $\pm 2\%$, illustrating that the proposed equations are a good fit to the FE data obtained. Statistical parameters associated with the goodness of fit are presented in Table 3 to further demonstrate the accuracy of the equations [24]. For the weld root crack, the maximum percentages of over-prediction and under-prediction are 7.93% and 3.96%, respectively. For the weld toe crack front deepest point, the maximum percentages of over-prediction is 7.95% whilst the worst under-prediction is 7.74%. The corresponding percentage values for the weld toe crack front ends are 5.43% and 5.84%, respectively. The mean percentage errors for all the cases are very close to zero, and the percentage error standard deviations are all less than 3%. The above statistical analyses further validate that the proposed Y_{root} and $M_{k,toe}$ equations fit very well with these regression data.

5. Determination of critical crack and weld sizes

For load-carrying cruciform fillet welded joints with multiple cracks, the crack can initiate and propagate from the weld root or the weld toe [9], which depends on the comparison of the SIF values at these two locations [15]. In Section 4, the Y_{root} and $M_{k,toe}$ equations for the weld root and weld toe cracks are proposed by considering the effect of various parameters (L/W , h/T , a/T and a/c), and the corresponding SIF values can be easily calculated according to Eqs. (2) and (4). Based on the above SIF equations attained for the weld root and weld toe cracks, it is easy to determine whether the SIF in the former is higher than that in the latter for given crack and weld parameters (L/W , h/T , a/T and a/c). Fig. 14 shows the change of the crack propagating from the weld root to the weld toe for constant h/T ($= 0.7$) and a/c ($= 0.4$). The red and the black lines are depicted using the following equality relationships:

$$\begin{aligned}
K_{\text{root}} &= K_{\text{toe}}^{\text{deep}} && \text{for red line} \\
K_{\text{root}} &= K_{\text{toe}}^{\text{ends}} && \text{for black line}
\end{aligned}
\tag{20}$$

where $K_{\text{toe}}^{\text{deep}}$ and $K_{\text{toe}}^{\text{ends}}$ are the SIFs for the weld toe crack front deepest point and weld toe crack front ends, respectively. Therefore, the blank area denotes that the crack propagates from the weld root, and the shadow regions on the left and the right of the blue line indicate the crack propagating from the weld toe crack front deepest point and weld toe crack front ends, respectively. The data points in the solid line represent the critical combination of L/W and a/T values at which the crack propagates from the weld root to the weld toe. The SIF equations proposed for the weld root and weld toe cracks provide a convenient way to construct similar figures as Fig. 14, and to determine the critical combination of L/W and a/T values for each group of h/T and a/c . However, for longer cracks ($a/c = 0.1$), the crack propagates from the weld root or the weld toe crack front deepest point as illustrated in Fig. 15 due to the SIF at the crack front deepest point always being higher than the value at the crack front ends. Fig. 15 also indicates that the critical combination values of L/W and a/T are strongly influenced by the variation of h/T and a smaller weld size induces the crack more easily propagating from the weld root, which is consistent with previous experiment tests showing that cracks propagate from the weld root for smaller weld size [16,29].

6. Conclusions

In this study, a FE mesh generator is specially developed to create the 3-D models of load-carrying cruciform fillet welded joints with multiple cracks. Extensive parametric study, covering a wide range of parameters presented in Tables 1 and 2, is carried out to calculate the SIF values of the weld root and weld toe cracks. The Y_{root} and $M_{\text{k,toe}}$ values are obtained according to Eqs. (2) and (4) and the interaction effect of the two types of cracks is investigated in detail. It is observed that a larger a/T results in a larger Y_{root} whilst increasing L/W enlarges the $M_{\text{k,toe}}$. Then, the Y_{root} and $M_{\text{k,toe}}$ equations for the weld root and weld toe cracks are

proposed using multiple regression analysis, and the goodness of fit of the equations is verified by means of percentage error frequency histograms and statistical analyses. Finally, based on the SIF equations established for the weld root and weld toe cracks, the critical combination values of L/W and a/T at which the crack propagates from the weld root to the weld toe for each group of h/T and a/c are determined.

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Appendix A

The expressions of M , f_w and M_a for a plain plate with a surface crack are given in Ref. [26]:

$$M = 1$$

$$f_w = \left\{ \sec \left[\left(\frac{\pi c}{B} \right) \left(\frac{a}{T} \right)^{0.5} \right] \right\}^{0.5}$$

$$M_a = \left[M_1 + M_2 \left(\frac{a}{T} \right)^2 + M_3 \left(\frac{a}{T} \right)^4 \right] \frac{g f_\theta}{\Phi}$$

where

$$M_1 = 1.13 - 0.09 \left(\frac{a}{c} \right)$$

$$M_2 = \left[\frac{0.89}{0.2 + (a/c)} \right] - 0.54$$

$$M_3 = 0.5 - \frac{1}{0.65 + (a/c)} + 14 \left(1 - \frac{a}{c} \right)^{24}$$

Φ is the complete elliptic integral of the second kind, and can be determined from the following solution,

$$\Phi = \left[1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \right]^{0.5}$$

a) At the crack deepest point:

$$g = 1$$

$$f_\theta = 1$$

b) At the crack ends:

$$g = 1.1 + 0.35 \left(\frac{a}{T} \right)^2$$

$$f_\theta = \left(\frac{a}{c} \right)^{0.5}$$

Appendix B

The expressions of f_{wm} and M_{km} are presented below, and these solutions are used to determine the SIF values for load-carrying cruciform fillet welded joints containing only weld root cracks [11].

$$f_{\text{wm}} = \left\{ \sec \left[\frac{\pi}{2} \left(\frac{L}{W} \right) \right] \right\}^{0.5}$$

$$M_{\text{km}} = \lambda_0 + \lambda_1 \left(\frac{L}{W} \right) + \lambda_2 \left(\frac{L}{W} \right)^2$$

where

$$\lambda_0 = 0.956 - 0.343 \left(\frac{h}{T} \right)$$

$$\lambda_1 = -1.219 + 6.210 \left(\frac{h}{T} \right) - 12.220 \left(\frac{h}{T} \right)^2 + 9.704 \left(\frac{h}{T} \right)^3 - 2.741 \left(\frac{h}{T} \right)^4$$

$$\lambda_2 = 1.954 - 7.938 \left(\frac{h}{T} \right) + 13.299 \left(\frac{h}{T} \right)^2 - 9.541 \left(\frac{h}{T} \right)^3 + 2.513 \left(\frac{h}{T} \right)^4$$

Appendix C

The stress intensity magnification factor ($M_{k,toe}^{deep}$ and $M_{k,toe}^{ends}$) equations for the weld toe crack front deepest point and the crack front ends are given in the following.

At the crack front deepest point:

$$M_{k,toe}^{deep} = \left[f_1 \left(\frac{a}{T}, \frac{a}{c} \right) + f_2 \left(\frac{a}{T}, \frac{L}{W} \right) \right] f_3 \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T} \right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{a}{c} \right) = -4.187443 \left(\frac{a}{T} \right)^{A_1} + A_2 \left[1 - \left(\frac{a}{T} \right) \right]^{A_3}$$

$$A_1 = -0.006699 \left(\frac{a}{c} \right)^2 + 0.060199 \left(\frac{a}{c} \right) + 0.132981$$

$$A_2 = 0.243472 \left(\frac{a}{c} \right)^2 - 0.631431 \left(\frac{a}{c} \right) + 4.502722$$

$$A_3 = 0.021592 \left(\frac{a}{c} \right)^2 - 0.082626 \left(\frac{a}{c} \right) - 0.047647$$

$$f_2 \left(\frac{a}{T}, \frac{L}{W} \right) = A_4 \left[1 - \left(\frac{a}{T} \right) \right]^{A_5} + 0.971259 \left(\frac{a}{T} \right)^{A_6}$$

$$A_4 = -2.275102 \left(\frac{L}{W} \right)^3 + 5.099321 \left(\frac{L}{W} \right)^2 - 3.768011 \left(\frac{L}{W} \right) - 0.236142$$

$$A_5 = -0.006393 \left(\frac{L}{W} \right)^{-1.709022} + 0.048061$$

$$A_6 = -3.907304 \left(\frac{L}{W} \right)^3 + 6.919132 \left(\frac{L}{W} \right)^2 - 4.633721 \left(\frac{L}{W} \right) + 0.873854$$

$$f_3 \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T} \right) = \left[A_7 \left(\frac{h}{T} \right)^2 + A_8 \left(\frac{h}{T} \right) + A_9 \right] \left(\frac{a}{T} \right)^{-0.005681}$$

$$- 0.633002 \left[1 - \left(\frac{a}{T} \right) \right]^{A_{10} \left(\frac{h}{T} \right)^2 + A_{11} \left(\frac{h}{T} \right) + A_{12}}$$

$$A_7 = -3.935803 \left(\frac{L}{W} \right)^3 + 4.905402 \left(\frac{L}{W} \right)^2 - 0.692841 \left(\frac{L}{W} \right) + 0.024166$$

$$A_8 = 10.565033 \left(\frac{L}{W} \right)^3 - 12.984031 \left(\frac{L}{W} \right)^2 + 1.674406 \left(\frac{L}{W} \right) - 0.048156$$

$$A_9 = -3.912511 \left(\frac{L}{W}\right)^3 + 4.803722 \left(\frac{L}{W}\right)^2 - 0.603372 \left(\frac{L}{W}\right) + 1.633214$$

$$A_{10} = 3.927331 \left(\frac{L}{W}\right)^2 - 4.092911 \left(\frac{L}{W}\right) + 0.401612$$

$$A_{11} = -4.910425 \left(\frac{L}{W}\right)^2 + 9.111931 \left(\frac{L}{W}\right) - 0.973012$$

$$A_{12} = 1.327904 \left(\frac{L}{W}\right)^2 - 3.213625 \left(\frac{L}{W}\right) + 0.352334$$

At the crack front ends:

$$M_{k,toe}^{ends} = g_1 \left(\frac{a}{T}, \frac{a}{c}\right) g_2 \left(\frac{a}{T}, \frac{L}{W}\right) g_3 \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T}\right)$$

where

$$g_1 \left(\frac{a}{T}, \frac{a}{c}\right) = 0.006645 \left(\frac{a}{T}\right)^{B_1} + B_2 \left[1 - \left(\frac{a}{T}\right)\right]^{B_3}$$

$$B_1 = 0.025504 \left(\frac{a}{c}\right)^{-0.427822} - 1.413056$$

$$B_2 = 0.092443 \left(\frac{a}{c}\right)^{-0.533921} + 1.105046$$

$$B_3 = 0.259243 \exp \left[-0.267633 \left(\frac{a}{c}\right)\right] - 0.431201 \exp \left[-8.547022 \left(\frac{a}{c}\right)\right]$$

$$g_2 \left(\frac{a}{T}, \frac{L}{W}\right) = B_4 \left(\frac{a}{T}\right)^{0.037085} + B_5 \left[1 - \left(\frac{a}{T}\right)\right]^{2.193111}$$

$$B_4 = -2.929901 \left(\frac{L}{W}\right)^3 + 3.708066 \left(\frac{L}{W}\right)^2 - 0.146582 \left(\frac{L}{W}\right) + 0.981334$$

$$B_5 = -3.042231 \left(\frac{L}{W}\right)^3 + 4.955614 \left(\frac{L}{W}\right)^2 - 1.001302 \left(\frac{L}{W}\right) + 0.159388$$

$$g_3 \left(\frac{a}{T}, \frac{h}{T}\right) = B_6 \left(\frac{a}{T}\right)^{B_7} + B_8 \exp \left[\left(\frac{a}{T}\right)^{B_9}\right] + \left[B_{10} \left(\frac{L}{W}\right) + B_{11}\right] \left(\frac{a}{T}\right) \\ + \left[B_{12} \left(\frac{L}{W}\right)^3 + B_{13} \left(\frac{L}{W}\right)^2 + B_{14} \left(\frac{L}{W}\right) + B_{15}\right]$$

$$B_6 = 0.108712 \left(\frac{h}{T}\right)^3 - 0.325564 \left(\frac{h}{T}\right)^2 + 0.330611 \left(\frac{h}{T}\right) + 0.023104$$

$$B_7 = 2.427003 \exp \left[-5.451022 \left(\frac{h}{T} \right) \right] + 1.832011 \exp \left[0.023192 \left(\frac{h}{T} \right) \right]$$

$$B_8 = 0.596244 \left(\frac{h}{T} \right)^2 - 1.505243 \left(\frac{h}{T} \right) + 0.461443$$

$$B_9 = 0.002399 \left(\frac{h}{T} \right)^{-4.387023} + 0.015433$$

$$B_{10} = -0.782511 \left(\frac{h}{T} \right)^3 + 1.669321 \left(\frac{h}{T} \right)^2 - 0.731284 \left(\frac{h}{T} \right) + 0.062355$$

$$B_{11} = -0.327223 \left(\frac{h}{T} \right)^3 + 1.070706 \left(\frac{h}{T} \right)^2 - 1.183403 \left(\frac{h}{T} \right) + 0.336634$$

$$B_{12} = -4.583221 \left(\frac{h}{T} \right)^2 + 10.604032 \left(\frac{h}{T} \right) - 3.838501$$

$$B_{13} = -3.001621 \left(\frac{h}{T} \right)^3 + 13.304051 \left(\frac{h}{T} \right)^2 - 18.651014 \left(\frac{h}{T} \right) + 5.972304$$

$$B_{14} = 0.522173 \left(\frac{h}{T} \right)^3 - 2.046621 \left(\frac{h}{T} \right)^2 + 2.073301 \left(\frac{h}{T} \right) - 0.566133$$

$$B_{15} = -1.654421 \left(\frac{h}{T} \right)^2 + 4.203302 \left(\frac{h}{T} \right) - 0.340781$$

Fig. 1 Load-carrying cruciform fillet welded joints with a semi-elliptical surface crack at the weld toe and two initial cracks at the weld root.

(a) 3-D schematic of half load-carrying cruciform fillet welded joint

Fig. 2 Boundary conditions of load-carrying cruciform fillet welded joints with multiple cracks subjected to axial loading.

Fig. 3 Typical FE mesh of load-carrying cruciform fillet welded joints with multiple cracks for $h/T = 0.7$, $L/W = 0.3$, $a/T = 0.4$ and $a/c = 0.4$.

Fig. 4 Mesh design of weld root and weld toe crack cross-sections.

Fig. 5 Comparison of weld toe crack SIFs obtained from Newman and Raju equation [26] and FE analysis for a plain plate.

Fig. 6 Comparison of weld root crack SIFs obtained from Frank and Fisher equation [11] and FE analysis for load-carrying cruciform welded joints.

Fig. 7 Crack fronts in weld root cracks of load-carrying cruciform fillet welded joint for $h/T = 0.7$, $L/W = 0.3$ and $a/T = 0.4$.

Fig. 8 Variation of SIFs of four weld root cracks fronts for $h/T = 0.7$, $L/W = 0.3$, $a/T = 0.4$ and $a/c = 0.4$.

Fig. 9 Effect of h/T on Y_{root} values for $a/T = 0.3$ and $a/c = 0.2$.

Fig. 10 Effect of a/T on Y_{root} values for $h/T = 0.7$ and $a/c = 0.2$.

Fig. 11 Effect of L/W on $M_{k,toe}$ values for $h/T = 0.7$ and $a/c = 0.2$.

Fig. 12 Error histogram for weld root stress intensity factors of load-carrying cruciform fillet welded joints.

Fig. 13 Error histograms for weld toe stress intensity factors of load-carrying cruciform fillet welded joints.

Fig. 14 Determination of critical crack and weld sizes for $h/T = 0.7$ and $a/c = 0.4$.

Fig. 15 Effect of h/T on determination of critical crack and weld sizes for $a/c = 0.1$.

Table 1 Table of analyses for proposing SIF equations of weld root crack

Loading	h/T	L/W	a/T	a/c
m = axial	0.45	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0
m = axial	0.70	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0
m = axial	0.95	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0
m = axial	1.20	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0

Table 2 Table of analyses for proposing SIF equations of weld toe crack

Loading	h/T	L/W	a/T	a/c
m = axial	0.45	0.1, 0.3, 0.5, 0.7	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0
m = axial	0.70	0.1, 0.3, 0.5, 0.7	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0
m = axial	0.95	0.1, 0.3, 0.5, 0.7	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0
m = axial	1.20	0.1, 0.3, 0.5, 0.7	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0

Table 3 Statistical evaluation of the proposed Y_{root} and $M_{k,\text{toe}}$ equations

Statistical parameters	Weld root crack	Weld toe crack front deepest point	Weld toe crack front ends
Number of FE data used in the analyses	672	576	576
Maximum percentage over- prediction (%)	7.93	7.95	5.43
Maximum percentage under-prediction (%)	-3.96	-7.74	-5.84
$\sum(\text{Residuals})^2$	0.1854	0.9093	0.8471
Mean Percentage Error (%)	-0.05	-0.51	0.04
Percentage Error Standard Deviation (%)	1.71	2.96	1.98