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Optimum Masking Levels and Coefficient Sparseness for Hilbert Transformers and Half-Band Filters Designed Using the Frequency-Response Masking Technique

Yong Ching Lim, *Fellow, IEEE*, Ya Jun Yu, *Member, IEEE*, and Tapio Saramäki, *Fellow, IEEE*

Abstract—Hilbert transformers and half-band filters are two very important special classes of finite-impulse response filters often used in signal processing applications. Furthermore, there exists a very close relationship between these two special classes of filters in such a way that a half-band filter can be derived from a Hilbert transformer in a straightforward manner and vice versa. It has been shown that these two classes of filters may be synthesized using the frequency-response masking (FRM) technique resulting in very efficient implementation when the filters are very sharp. While filters synthesized using the FRM technique has been characterized for the general low-pass case, Hilbert transformers and half-band filters synthesized using the FRM technique have not been characterized. The characterization of the two classes of filter is a focus of this paper. In this paper, we re-develop the FRM structure for the synthesis of Hilbert transformer from a new perspective. This new approach uses a frequency response correction term produced by masking the frequency response of a sparse coefficient filter, whose frequency response is periodic, to sharpen the bandedge of a low-order Hilbert transformer. Optimum masking levels and coefficient sparseness for the Hilbert transformers are derived; corresponding quantities for the half-band filters are obtained via the close relationship between these two classes of filters.

Index Terms—Finite-impulse response (FIR) digital filter, frequency-response masking (FRM), Hilbert transformer, half-band filter, sparse coefficient filter.

I. INTRODUCTION

THROUGHOUT this paper, the phrase “a filter $H(z)$ ” refers to a filter whose z -transform transfer function is $H(z)$. We shall also denote the frequency response of $H(z)$ by $H(e^{j\omega})$. Furthermore, by the phrase “a filter $H(e^{j\omega})$,” we refer to the filter whose frequency response is $H(e^{j\omega})$.

The conventional frequency-response masking (FRM) structure shown in Fig. 1 was first introduced in [1]. It has been used very successfully for the synthesis of very sharp low-pass,

Fig. 1. Structure of a filter synthesized using the FRM technique. All the subfilters are assumed to be zero phase. Causality can be restored by introducing appropriate delays into the subfilters.

high-pass, and bandpass filters with extremely low complexity. Further improvements on the structure, powerful specialized optimization algorithms, and interesting applications have been developed by many authors for the FRM [2]–[24] technique.

It was shown in [2] that the FRM technique can also be used to synthesize half-band filters. Since every other coefficient value of a half-band filter is zero, the various estimation formulae derived in [4] are no longer applicable.

A Hilbert transformer with odd length may be derived from a low-pass half-band filter by discarding the centre coefficient, multiplying its n th coefficient by $e^{jn\pi/2}$, and scale up the coefficient values by a factor of two. Making use of this close relationship between a Hilbert transformer and a half-band filter, [3] illustrated the synthesis of a Hilbert transformer using the FRM technique. Although [2] and [3] illustrated the application of the FRM technique for the synthesis of Hilbert transformer and half-band filter, important information on the coefficient sparseness (M) for the bandedge shaping filter and the optimum masking levels (K_{opt}) were not presented.

In this paper, we adopted a new approach toward the synthesis of filters using the FRM technique. In this new approach, the Hilbert transformer is synthesized as a parallel connection of two filters. One of these two filters is a low-order Hilbert transformer. The other one is a series connection of a masking filter and a sparse coefficient bandedge shaping filter producing a correction term that sharpens the bandedge of the low-order Hilbert transformer.

A Hilbert transformer with odd length can be derived from one with even length by replacing each z^{-1} in the z -transform transfer function of the even length Hilbert transformer by z^{-2} ; this will cause the transition width to shrink by a factor of two. As a result of this straightforward relationship between the odd length and even length Hilbert transformers, the odd length Hilbert transformer will not be discussed in this paper.

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Fig. 2. Structure for the synthesis of a Hilbert transformer using the FRM technique.

Our new FRM approach is presented in Section II. In the FRM technique, each delay of a prototype bandedge shaping filter is replaced by M delays. The optimum estimate for M is presented in Section III together with the estimates for the resulting complexity and effective filter length. The application of our new FRM technique to the design of a Hilbert transformer for suppressing acoustic feedback is presented in Section IV; a factor of 9 reduction in the number of nontrivial coefficients is achieved when compared to the Chebyshev optimum design. Further reduction in the complexity of the filter may be achieved by considering a multi-level masking structure; this is presented in Section V. A two level-masking design is presented in Section VI to meet the requirement of the example of Section IV. The two-level masking approach produces a factor of 17 reduction in the number of nontrivial coefficients comparing to the Chebyshev optimum design.

II. HILBERT TRANSFORMER SYNTHESIZED USING FRM TECHNIQUE

The z -transform transfer function $H(z)$ of a Hilbert transformer with length N , where N is an even integer, is given by

$$H(z) = e^{j(\frac{\pi}{2} - \frac{N-1}{2}\omega)} \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\left(\left(n - \frac{1}{2}\right)\omega\right). \quad (1)$$

In (1), $h(n)$ is the n th impulse response. Since N is even, its group delay, given by $(N-1)/2$, is not an integer and the filter is said to have “half-sample delay”.

Fig. 2 shows the structure for implementing the Hilbert transformer using the FRM technique. The z -transform transfer function for the overall filter is given by

$$H(z) = H_1(z^M)H_M(z) + H_b(z). \quad (2)$$

It is a system of subfilters consisting of a parallel connection of two branches. One of the parallel branches consists of a basic filter $H_b(z)$. The other branch consists of a cascade of a band-edge sharpening filter $H_1(z^M)$ and a masking filter $H_M(z)$. The basic filter $H_b(z)$ provides a low-order approximation (with wide transition width) to the desired specification. The cascade of $H_1(z^M)$ and $H_M(z)$ provides a transfer function correction term for sharpening the transition band.

Let the lengths of $H_b(z)$, $H_1(z)$, and $H_M(z)$ be N_b , N_1 , and N_M , respectively. The length of $H_1(z^M)H_M(z)$ will be $MN_1 + N_M - M$. The delay introduced by $H_b(z)$ and that introduced by $H_1(z^M)H_M(z)$ must be the same; otherwise, pure delays must be introduced into the shorter one to equalize

Fig. 3. Frequency responses of the subfilters for even length $H_b(z)$. Note that $\theta_1/M = 2\pi\Delta$. $2\pi\Delta$ is the transition width of the desired transfer function.

them. In order to avoid inserting half-sample delay in the implementation, the parity of N_b and that of $MN_1 + N_M - M$ must be the same. Furthermore, $H_1(z^M)H_M(z)$ must have anti-symmetrical impulse response.

Consider a wide transition band Hilbert transformer $H_b(e^{j\omega})$ as shown in Fig. 3(a) where N_b is even. The computational complexity of $H_b(z)$ is low since its transition band is wide. Now consider a transition band correction filter $H_e(e^{j\omega})$ as shown in Fig. 3(b). When $H_e(e^{j\omega})$ is added to $H_b(e^{j\omega})$, a sharp transition band Hilbert transformer, $H(e^{j\omega})$, as shown in Fig. 3(c) is obtained. Our objective is to design a very low complexity filter $H_e(e^{j\omega})$ using the FRM technique. Consider a bandedge shaping filter $H_1(e^{j\omega})$ as shown in Fig. 3(d). The complexity of $H_1(z)$ is low because it has wide transition width. Replacing each delay of $H_1(z)$ by M delays, a filter $H_1(e^{jM\omega})$ as shown in Fig. 3(e) is obtained. A masking filter $H_M(e^{j\omega})$ as shown in Fig. 3(f) is used to mask the unwanted passbands of $H_1(e^{jM\omega})$ to produce the frequency response $H_e(e^{j\omega})$ as shown in Fig. 3(b). $H_M(e^{j\omega})$ has low complexity because its frequency response has a wide transition band. Since the length of $H_b(z)$ is even, the length of $H_1(z^M)H_M(z)$, i.e., $MN_1 + N_M - M$, must also be even. If M is even, N_M must also be even. If M is odd, N_M and N_1 must have different parities. By considering the gain of $H_M(e^{j\omega})$ in the vicinity of $\omega = 0$, it is clear that $H_M(z)$ has symmetrical impulse response. Thus, $H_1(z)$ must have anti-symmetrical impulse response to satisfy the condition that $H_1(z^M)H_M(z)$ must have anti-symmetrical impulse response.

The bandedges of $H_b(z)$ and $H_M(z)$ are the same. Let the bandedge of $H_b(z)$ be θ_b as shown in Fig. 3. Let the bandedge of $H_1(z)$ be θ_1 . As can be seen from Fig. 3, $\theta_b \leq (2\pi - \theta_1)/M$.

III. OPTIMUM M FOR EVEN LENGTH HILBERT TRANSFORMERS

Joint simultaneous optimization of $H_1(z^M)$, $H_M(z)$, and $H_b(z)$ is a nonlinear optimization problem and can be solved by general purpose or specialized optimization packages. Nevertheless, regardless of the optimization packages used, it is necessary to determine the value of M before initiating the optimization process.

Let the transition width of the desired Hilbert transformer be Δf_s where f_s is the sampling frequency and let its passband ripple magnitude be δ . It is shown in Appendix I that, for $\Delta < 0.2$, the filter length, N_{opt} , of the Chebyshev optimum design is given by

$$N_{\text{opt}} \approx \frac{\Phi_H(\delta)}{\Delta} + 1 \approx \frac{\Phi_H(\delta)}{\Delta}. \quad (3)$$

The length of $H_M(z)$ may be estimated from the Chebyshev polynomial

$$P_N(x) = \begin{cases} \cos\left(\frac{N-1}{2} \cos^{-1}(x)\right), & |x| \leq 1 \\ \cosh\left(\frac{N-1}{2} \cosh^{-1}(x)\right), & |x| > 1 \end{cases} \quad (4a)$$

where

$$x = \frac{X_0 + 1}{2} \cos(\omega) + \frac{X_0 - 1}{2}. \quad (4c)$$

Equation (4c) maps ω from 0 to π into x from X_0 to -1 . Let $H_M(z)$ has unity gain at $z = 1$ and let the peak stopband ripple magnitude of $H_M(z)$ be δ . It can be shown that

$$\frac{3 - \cos(\theta_b)}{1 + \cos(\theta_b)} = \cosh\left(\frac{2}{N-1} \cosh^{-1}\left(\frac{1}{\delta}\right)\right). \quad (5)$$

Let $\Delta_b = (\theta_b/2\pi)$. For $\Delta_b < 0.2$

$$\begin{aligned} N_M &\approx \frac{\frac{1}{\pi} \cosh^{-1}\left(\frac{1}{\delta}\right)}{\Delta_b} + 1 \approx \frac{\frac{1}{\pi} \cosh^{-1}\left(\frac{1}{\delta}\right)}{\Delta_b} \\ &\approx \frac{\ln(2) - \ln(\delta)}{\pi} \approx \Delta_b \approx \frac{\Phi_M(\delta)}{\Delta_b} \end{aligned} \quad (6a)$$

where

$$\Phi_M(\delta) = 0.22064 - 0.73294 \log_{10}(\delta). \quad (6b)$$

Since the desired transition width is Δf_s , the transition width of $H_1(z)$ is $M\Delta f_s$. Thus, its length is

$$N_1 = \frac{\Phi_H(\delta)}{M\Delta}. \quad (7)$$

The transition width of $H_b(z)$ is $(1 - M\Delta)f_s/M$ and N_b is given by

$$N_b = \frac{\Phi_H(\delta)}{\frac{1}{M} - \Delta}. \quad (8)$$

The transition width of $H_M(z)$ is the same as that of $H_b(z)$. Thus

$$N_M = \frac{\Phi_M(\delta)}{\frac{1}{M} - \Delta}. \quad (9)$$

The total number of nontrivial coefficients, $N_{\text{total}}(\delta)$, is given by

$$N_{\text{total}}(\delta) = N_1 + N_b + N_M$$

TABLE I
VALUES OF $\Phi_H(\delta)$, $\Phi_M(\delta)$ AND $\Phi_M(\delta)/\Phi_H(\delta)$

δ	0.1	0.01	0.001	0.0001	0.00001
$\Phi_H(\delta)$	0.534	1.166	1.830	2.510	3.189
$\Phi_M(\delta)$	0.939	1.687	2.419	3.152	3.885
$\frac{\Phi_M(\delta)}{\Phi_H(\delta)}$	1.76	1.45	1.32	1.26	1.22

$$= \frac{\Phi_H(\delta)}{M\Delta} + \frac{\Phi_H(\delta)}{\frac{1}{M} - \Delta} + \frac{\Phi_M(\delta)}{\frac{1}{M} - \Delta}. \quad (10)$$

It can be shown by differentiating (10) with respect to M and equating the derivative to zero that, for small Δ , the value of M , denoted by M_{opt} , corresponding to the minimum value of $N_{\text{total}}(\delta)$ is given by

$$M_{\text{opt}} \approx \sqrt{\frac{\Phi_H(\delta)}{(\Phi_H(\delta) + \Phi_M(\delta))\Delta}}. \quad (11a)$$

Several values of $\Phi_H(\delta)$, $\Phi_M(\delta)$ and $\Phi_M(\delta)/\Phi_H(\delta)$ for δ ranging from 0.1 to 0.00001 are tabulated in Table I. As can be seen from Table I, for $0.01 > \delta > 0.00001$, $\Phi_M(\delta)/\Phi_H(\delta) \approx 1.32$. This further simplifies (11a) to

$$M_{\text{opt}} \approx \sqrt{\frac{0.43}{\Delta}}. \quad (11b)$$

For M_{opt} given by (11a)

$$N_1 \approx \sqrt{\frac{\Phi_H(\delta)(\Phi_H(\delta) + \Phi_M(\delta))}{\Delta}}. \quad (12a)$$

For M_{opt} given by (11a) and $1/M_{\text{opt}} \gg \Delta$

$$N_b + N_M \approx \sqrt{\frac{\Phi_H(\delta)(\Phi_H(\delta) + \Phi_M(\delta))}{\Delta}}. \quad (12b)$$

For $\Phi_M(\delta)/\Phi_H(\delta) = 1.32$, it can be easily shown from (3) and (12) that $N_1 + N_b + N_M < N_{\text{opt}}$ for $\Delta < 0.108$. Thus, our new FRM technique will produce a design with a smaller number of nontrivial coefficients than that of the Chebyshev optimum design if the transition width of the desired Hilbert transformer is less than $0.108f_s$. For an audio signal sampled at 48 kHz (44.1 kHz), our new FRM technique will produce a saving in complexity comparing to the Chebyshev optimum design if frequency components below 5.2 kHz (4.8 kHz) must be faithfully transformed.

The effective length N_{eff} of the filter designed using our new FRM technique is given by

$$N_{\text{eff}} \approx M_{\text{opt}}N_1 + N_M. \quad (13)$$

From (3), (7), (9), (11), and (13)

$$\begin{aligned} N_{\text{eff}} &\approx N_{\text{opt}} \left(1 + \frac{\Phi_M(\delta)}{\Phi_H(\delta)} \sqrt{\frac{\Phi_H(\delta)\Delta}{\Phi_H(\delta) + \Phi_M(\delta)}} \right) \\ &\approx N_{\text{opt}}(1 + 0.86\sqrt{\Delta}). \end{aligned} \quad (14)$$

In (14), the term $0.86\sqrt{\Delta}$ represents the fractional increase in filter length when compared to the Chebyshev optimum design. The fractional increase is less than 10% for $\Delta < 0.013$.

Fig. 4. Frequency shifter is inserted to suppress oscillation due to acoustic feedback.

 TABLE II
COEFFICIENT VALUES OF $H_b(z)$

$h_b(-47) = 0.00014858 = -h_b(47)$	$h_b(-23) = 0.01266332 = -h_b(23)$
$h_b(-45) = 0.00016629 = -h_b(45)$	$h_b(-21) = 0.01582046 = -h_b(21)$
$h_b(-43) = 0.00028316 = -h_b(43)$	$h_b(-19) = 0.01970981 = -h_b(19)$
$h_b(-41) = 0.00042792 = -h_b(41)$	$h_b(-17) = 0.02461925 = -h_b(17)$
$h_b(-39) = 0.00161558 = -h_b(39)$	$h_b(-15) = 0.03073442 = -h_b(15)$
$h_b(-37) = 0.00175315 = -h_b(37)$	$h_b(-13) = 0.03856408 = -h_b(13)$
$h_b(-35) = 0.00295337 = -h_b(35)$	$h_b(-11) = 0.04904002 = -h_b(11)$
$h_b(-33) = 0.00374368 = -h_b(33)$	$h_b(-9) = 0.06315979 = -h_b(9)$
$h_b(-31) = 0.00488766 = -h_b(31)$	$h_b(-7) = 0.08529546 = -h_b(7)$
$h_b(-29) = 0.00615065 = -h_b(29)$	$h_b(-5) = 0.12344694 = -h_b(5)$
$h_b(-27) = 0.00805283 = -h_b(27)$	$h_b(-3) = 0.21037431 = -h_b(3)$
$h_b(-25) = 0.01006458 = -h_b(25)$	$h_b(-1) = 0.63529816 = -h_b(1)$

IV. EXAMPLE

A problem often encountered in an acoustic feedback system is oscillation due to feedback. Such oscillation can be suppressed by inserting a frequency shifter into the system as shown in Fig. 4. The frequency shifter shifts the frequency by an adjustable amount ranging from 0 to 5 Hz. A shift of 2 Hz produces no noticeable distortion to most people. The implementation of the shifter requires a sharp Hilbert transformer if low frequency components must also be faithfully shifted.

In a particular system, the application required a Hilbert transformer with the following specifications. Sampling rate: 32 kHz. Lower-bandedge: 20 Hz. Peak ripple magnitude: 0.0001.

The above requirements correspond to $\Delta = 0.000625$. In order to reduce the arithmetic complexity of the Hilbert transformer further, we chose an odd length filter with pass-band spanning from 20 to 15 980 Hz. The design was done by first designing an even length Hilbert transformer with $\Delta = 0.000625 \times 2 = 0.00125$. The even length Hilbert transformer was then converted to an odd length Hilbert transformer by replacing the variable z of its z -transform transfer function by z^2 . The transition width shrank from 0.00125 to 0.000625 when z was replaced by z^2 . The estimated length of a direct form even length Chebyshev optimum design meeting the specification was 2000. Using our FRM technique, the value of M suggested by (11a) was 18.8. The values of N_1 , N_b , and N_M estimated from (7), (8), and (9) for $M = 19$ were 106.6, 48.8, and 61.4, respectively. The optimization algorithm reported in [19] was used to optimize the subfilters and a design with $N_1 = 106$, $N_b = 48$, and $N_M = 59$ was obtained. The total number of nontrivial coefficient was $106 + 48 + 59 = 213$. It was only about 10.7% of the 2000 nontrivial coefficients of the

 TABLE III
COEFFICIENT VALUES OF $H_M(z)$

$h_M(-58) = 0.00076333 = h_M(58)$	$h_M(-28) = 0.01630893 = h_M(28)$
$h_M(-56) = 0.00069048 = h_M(56)$	$h_M(-26) = 0.01880688 = h_M(26)$
$h_M(-54) = 0.00131371 = h_M(54)$	$h_M(-24) = 0.02113358 = h_M(24)$
$h_M(-52) = 0.00160856 = h_M(52)$	$h_M(-22) = 0.02372854 = h_M(22)$
$h_M(-50) = 0.00208650 = h_M(50)$	$h_M(-20) = 0.02391805 = h_M(20)$
$h_M(-48) = 0.00255934 = h_M(48)$	$h_M(-18) = 0.02720104 = h_M(18)$
$h_M(-46) = 0.00335705 = h_M(46)$	$h_M(-16) = 0.02762120 = h_M(16)$
$h_M(-44) = 0.00409132 = h_M(44)$	$h_M(-14) = 0.02990513 = h_M(14)$
$h_M(-42) = 0.00503851 = h_M(42)$	$h_M(-12) = 0.03173558 = h_M(12)$
$h_M(-40) = 0.00612590 = h_M(40)$	$h_M(-10) = 0.03375364 = h_M(10)$
$h_M(-38) = 0.00738429 = h_M(38)$	$h_M(-8) = 0.03471639 = h_M(8)$
$h_M(-36) = 0.00892903 = h_M(36)$	$h_M(-6) = 0.03611482 = h_M(6)$
$h_M(-34) = 0.01057801 = h_M(34)$	$h_M(-4) = 0.03701503 = h_M(4)$
$h_M(-32) = 0.01242610 = h_M(32)$	$h_M(-2) = 0.03760300 = h_M(2)$
$h_M(-30) = 0.01455075 = h_M(30)$	$h_M(0) = 0.03787069$

 TABLE IV
COEFFICIENT VALUES OF $H_1(z^{19})$

$h_1(-1995) = 0.00012105 = -h_1(1995)$	$h_1(-969) = 0.00532561 = -h_1(969)$
$h_1(-1957) = 0.00001606 = -h_1(1957)$	$h_1(-931) = 0.00593894 = -h_1(931)$
$h_1(-1919) = 0.00011230 = -h_1(1919)$	$h_1(-893) = 0.00660967 = -h_1(893)$
$h_1(-1881) = 0.00009540 = -h_1(1881)$	$h_1(-855) = 0.00734753 = -h_1(855)$
$h_1(-1843) = 0.00015646 = -h_1(1843)$	$h_1(-817) = 0.00815578 = -h_1(817)$
$h_1(-1805) = 0.00017722 = -h_1(1805)$	$h_1(-779) = 0.00904647 = -h_1(779)$
$h_1(-1767) = 0.00023753 = -h_1(1767)$	$h_1(-741) = 0.01002558 = -h_1(741)$
$h_1(-1729) = 0.00028299 = -h_1(1729)$	$h_1(-703) = 0.01110894 = -h_1(703)$
$h_1(-1691) = 0.00035519 = -h_1(1691)$	$h_1(-665) = 0.01230689 = -h_1(665)$
$h_1(-1653) = 0.00042427 = -h_1(1653)$	$h_1(-627) = 0.01364168 = -h_1(627)$
$h_1(-1615) = 0.00051575 = -h_1(1615)$	$h_1(-589) = 0.01513135 = -h_1(589)$
$h_1(-1577) = 0.00061115 = -h_1(1577)$	$h_1(-551) = 0.01680929 = -h_1(551)$
$h_1(-1539) = 0.00072803 = -h_1(1539)$	$h_1(-513) = 0.01870760 = -h_1(513)$
$h_1(-1501) = 0.00085411 = -h_1(1501)$	$h_1(-475) = 0.02088073 = -h_1(475)$
$h_1(-1463) = 0.00100256 = -h_1(1463)$	$h_1(-437) = 0.02338879 = -h_1(437)$
$h_1(-1425) = 0.00116478 = -h_1(1425)$	$h_1(-399) = 0.02632959 = -h_1(399)$
$h_1(-1387) = 0.00135144 = -h_1(1387)$	$h_1(-361) = 0.02982433 = -h_1(361)$
$h_1(-1349) = 0.00155626 = -h_1(1349)$	$h_1(-323) = 0.03407189 = -h_1(323)$
$h_1(-1311) = 0.00178839 = -h_1(1311)$	$h_1(-285) = 0.03934913 = -h_1(285)$
$h_1(-1273) = 0.00204323 = -h_1(1273)$	$h_1(-247) = 0.04613329 = -h_1(247)$
$h_1(-1235) = 0.00232900 = -h_1(1235)$	$h_1(-209) = 0.05519160 = -h_1(209)$
$h_1(-1197) = 0.00264243 = -h_1(1197)$	$h_1(-171) = 0.06800972 = -h_1(171)$
$h_1(-1159) = 0.00299119 = -h_1(1159)$	$h_1(-133) = 0.08754305 = -h_1(133)$
$h_1(-1121) = 0.00337321 = -h_1(1121)$	$h_1(-95) = 0.12115935 = -h_1(95)$
$h_1(-1083) = 0.00379594 = -h_1(1083)$	$h_1(-57) = 0.19117149 = -h_1(57)$
$h_1(-1045) = 0.00425837 = -h_1(1045)$	$h_1(-19) = 0.40604989 = -h_1(19)$
$h_1(-1007) = 0.00476822 = -h_1(1007)$	

Chebyshev optimum design. The effective length of the Chebyshev optimum design (after replacing z by z^2) was 3999. The effective length of our FRM design (after replacing z by z^2) was 4107; it was only about 2.7% longer than the Chebyshev optimum design. The coefficient values of the subfilters (after replacing z by z^2 or z^{2M} which ever is appropriate) of the odd length Hilbert transformer are tabulated in Tables II–IV. The passband ripple of the overall filter is shown in Fig. 5 and the frequency responses of the various subfilters are shown in Fig. 6.

V. MULTI-LEVEL FRM

If the transition width of the required filter is very narrow, the lengths of $H_1(z^M)$, $H_M(z)$ and $H_b(z)$ may still be very long. In

Fig. 5. Passband ripples of the Hilbert transformer.

Fig. 6. Frequency responses of the individual subfilters. (a) $H_b(e^{j\omega})$, (b) $H_M(e^{j\omega})$, (c) $H_1(e^{jM\omega})$, (d) $H_M(e^{j\omega})H_1(e^{jM\omega})$, (e) $H_b(e^{j\omega}) + H_M(e^{j\omega})H_1(e^{jM\omega})$.

this case, a multi-level FRM technique may be employed to further reduce the complexity. In the multi-level FRM technique, we define

$$H^{(K)}\left(z^{\prod_{i=1}^K M_i}\right) = H^{(K+1)}\left(z^{\prod_{i=1}^{K+1} M_i}\right) H_M^{(K+1)}\left(z^{\prod_{i=1}^K M_i}\right) + H_b^{(K+1)}\left(z^{\prod_{i=1}^K M_i}\right) \quad (15)$$

where $\prod_{i=1}^0 M_i$ is defined as unity.

Consider a K -level masking structure. The case for $K = 3$ is shown in Fig. 7. Let the lengths of $H^{(K)}(z)$, $H_M^{(i)}(z)$, and $H_b^{(i)}(z)$ be $N^{(K)}$, $N_M^{(i)}$, and $N_b^{(i)}$, respectively. Estimates for

$N^{(K)}$, $N_M^{(i)}$, and $N_b^{(i)}$ for even values of $N^{(K)}$ and $N_b^{(i)}$ are given in

$$N^{(K)} = \frac{\Phi_H(\delta)}{\Delta \prod_{i=1}^K M_i} \quad (16a)$$

$$N_b^{(k)} = \frac{\Phi_H(\delta)}{\frac{1}{M_k} - \Delta \prod_{i=1}^{k-1} M_i}, \quad k = 1, \dots, K \quad (16b)$$

$$N_M^{(k)} = \frac{\Phi_M(\delta)}{\frac{1}{M_k} - \Delta \prod_{i=1}^{k-1} M_i}, \quad k = 1, \dots, K. \quad (16c)$$

The total number of nontrivial coefficients, $N_{\text{total}}^{(K)}(\delta)$ of a K -level masking design is given by

$$N_{\text{total}}^{(K)}(\delta) = N^{(K)} + \sum_{i=1}^K N_b^{(i)} + \sum_{i=1}^K N_M^{(i)} = \frac{\Phi_H(\delta)}{\Delta \prod_{i=1}^K M_i} + \sum_{k=1}^K \frac{\Phi_H(\delta) + \Phi_M(\delta)}{\frac{1}{M_k} - \Delta \prod_{i=1}^{k-1} M_i}. \quad (17)$$

The optimum values of M_i , $i = 1, 2, \dots, K$ are equal to $M_{\text{opt}}(K)$ where

$$M_{\text{opt}}(K) \approx \left(\frac{\Phi_H(\delta)}{(\Phi_H(\delta) + \Phi_M(\delta))\Delta} \right)^{\frac{1}{K+1}} \approx \left(\frac{0.43}{\Delta} \right)^{\frac{1}{K+1}} \quad \text{for } \frac{\Phi_M(\delta)}{\Phi_H(\delta)} = 1.32. \quad (18)$$

For $M_i = M_{\text{opt}}(K)$ given in (18), we have

$$N^{(K)} = N_M^{(K)} + N_b^{(K)} \approx \left(\frac{\Phi_H(\delta)(\Phi_H(\delta) + \Phi_M(\delta))^K}{\Delta} \right)^{\frac{1}{K+1}} \approx \frac{2.32^{\frac{K}{K+1}} \Phi_H(\delta)}{\Delta^{\frac{1}{K+1}}}. \quad (19)$$

The total number of nontrivial coefficients, $N_{\text{total}}^{(K)}(\delta)$, of a K -level masking design is given by

$$\begin{aligned} N_{\text{total}}^{(K)}(\delta) &= N^{(K)} + \sum_{i=1}^K (N_b^{(i)} + N_M^{(i)}) \\ &= (K+1) \left(\frac{\Phi_H(\delta)(\Phi_H(\delta) + \Phi_M(\delta))^K}{\Delta} \right)^{\frac{1}{K+1}} \\ &= \Phi_H(\delta)(K+1)2.32^{K/(K+1)}\Delta^{-1/(K+1)} \quad \text{for } \Phi_M(\delta)/\Phi_H(\delta) = 1.32. \end{aligned} \quad (20a) \quad (20b)$$

The values of $(K+1)2.32^{K/(K+1)}\Delta^{-1/(K+1)}$ for several values of Δ and K are tabulated in Table V. It can be seen from Table V that, for a given value of Δ , $(K+1)2.32^{K/(K+1)}\Delta^{-1/(K+1)}$ decreases initially with increasing K until it reaches a minimum and then increases with increasing K . The value of K that will yield the minimum

TABLE V
VALUES OF $(K+1)2.32^{K/(K+1)}\delta^{-1/(K+1)}$ FOR SEVERAL VALUES OF δ
AND K . RESULTS FOR $K = 0$ CORRESPOND TO THOSE WITHOUT
USING OUR PROPOSED FRM TECHNIQUE

	$\Delta = 0.01$	$\Delta = 0.001$	$\Delta = 0.0001$
$K = 0$	100	1000	10000
$K = 1$	30.5	96.3	304.6
$K = 2$	24.4	52.6	113.3
$K = 3$	23.8	42.3	75.2
$K = 4$	24.6	39.0	61.9
$K = 5$	26.1	38.3	56.2
$K = 6$	27.8	38.6	53.7
$K = 7$	29.7	39.6	52.8
$K = 8$	31.7	41.0	52.9
$K = 9$	33.8	42.6	53.6

Fig. 7. Three-level masking structure.

value of $(K+1)2.32^{K/(K+1)}\Delta^{-1/(K+1)}$ depends on the value of Δ .

It can be shown by manipulating (20a) that $N_{\text{total}}^{(K+1)}(\delta) > N_{\text{total}}^{(K)}(\delta)$ if

$$\Delta > \beta(\delta, K) \quad (21a)$$

where

$$\begin{aligned} \beta(\delta, K) &= \frac{\Phi_H(\delta)}{\Phi_H(\delta) + \Phi_M(\delta)} \left(\frac{K+1}{K+2} \right)^{(K+1)(K+2)} \\ &\approx 0.43 \left(\frac{K+1}{K+2} \right)^{(K+1)(K+2)}. \end{aligned} \quad (21b)$$

The value of K that minimizes $N_{\text{total}}^{(K)}(\delta)$, denoted by K_{opt} , satisfies the constraint

$$\beta(\delta, K_{\text{opt}} - 1) > \Delta > \beta(\delta, K_{\text{opt}}). \quad (22)$$

The optimum values of K for various transition widths are tabulated in Table VI. If K is selected to minimize $N_{\text{total}}^{(K)}(\delta)$ based on (20), it can be shown by considering (20a) and (18) that $M_{\text{opt}}(K)$ is bounded by

$$\left(\frac{K_{\text{opt}} + 1}{K_{\text{opt}}} \right)^{K_{\text{opt}}} < M_{\text{opt}}(K_{\text{opt}}) < \left(\frac{K_{\text{opt}} + 2}{K_{\text{opt}} + 1} \right)^{K_{\text{opt}} + 2}. \quad (23)$$

TABLE VI
RANGES OF Δ CORRESPONDING TO VARIOUS VALUES OF K_{opt} FOR
MINIMUM $N_{\text{total}}^{(K_{\text{opt}})}(\delta)$

K_{opt}	$\Delta = \frac{\text{Transition width}}{\text{Sampling frequency}}$
1	0.108 ~ 0.038
2	0.038 ~ 0.0136
3	0.0136 ~ 0.0050
4	0.0050 ~ 0.00181
5	0.00181 ~ 0.00066
6	0.00066 ~ 0.000243
7	0.000243 ~ 0.000089
8	0.000089 ~ 0.000033
9	0.000033 ~ 0.0000120
10	0.0000120 ~ 0.0000044

Note that both

$$\left(\frac{K_{\text{opt}} + 1}{K_{\text{opt}}} \right)^{K_{\text{opt}}} \quad \text{and} \quad \left(\frac{K_{\text{opt}} + 2}{K_{\text{opt}} + 1} \right)^{K_{\text{opt}} + 2}$$

approach e (the base of the natural logarithm) as K_{opt} approaches infinity. Thus, $M_{\text{opt}}(K_{\text{opt}})$ approaches e as Δ approaches zero. It is also interesting to note that $\beta(\delta, K_{\text{opt}} - 1)/\beta(\delta, K_{\text{opt}})$ also approaches e as K_{opt} approaches infinity.

The effective length of the filter $N_{\text{eff}}(K)$ is given by

$$\begin{aligned} N_{\text{eff}}(K) &= N^{(K)} \prod_{i=1}^K M_i + \sum_{k=1}^K N_M^{(k)} \prod_{i=1}^{k-1} M_i \\ &= N_{\text{opt}} \left(1 + \Delta \frac{M_{\text{opt}}^{K+1}(K) - M_{\text{opt}}(K)}{M_{\text{opt}}(K) - 1} \right). \end{aligned} \quad (24)$$

In (24), the term $\Delta(M_{\text{opt}}^{K+1}(K) - M_{\text{opt}}(K))/(M_{\text{opt}}(K) - 1)$ represents the fractional increase in filter length when compared to the Chebyshev optimum design. For $\Delta \rightarrow 0$, we have $K_{\text{opt}} \rightarrow \infty$, $M_{\text{opt}}^{K_{\text{opt}}+1}(K_{\text{opt}}) - M_{\text{opt}}(K_{\text{opt}}) \rightarrow M_{\text{opt}}^{K_{\text{opt}}+1}(K_{\text{opt}})$, $M_{\text{opt}}(K_{\text{opt}}) \rightarrow 2.7$, and applying the result of (18)

$$N_{\text{eff}}(K_{\text{opt}}) \rightarrow N_{\text{opt}} \left(1 + \frac{\Phi_H(\delta)}{1.7(\Phi_H(\delta) + \Phi_M(\delta))} \right). \quad (25)$$

For $\Phi_M(\delta)/\Phi_H(\delta) = 1.32$, $(\Phi_H(\delta))/(1.7(\Phi_H(\delta) + \Phi_M(\delta))) \approx 0.25$. Thus, as $K_{\text{opt}} \rightarrow \infty$ for the case where $\Delta \rightarrow 0$, the FRM technique will produce a Hilbert transformer whose length is about 25% longer than that of the Chebyshev optimum design.

VI. TWO-LEVEL MASKING EXAMPLE

For the acoustic feedback suppression example in Section IV, the optimum value of K as listed in Table VI was 5 (corresponding to $\Delta = 0.00125$). The value of $N_{\text{total}}^{(5)}(0.0001)$ predicted from (20a) was 90; it is smaller than the Chebyshev optimum design of 2000 by a factor of 22! However, the actual selection of the value of K depends on several factors. One of the most important factors influencing the selection of K is the availability of user friendly and reliable optimization packages. The difficulty faced in the design process increases

TABLE VII
COEFFICIENT VALUES OF $H_b^{(1)}(z)$

$h_b^{(1)}(-17) = 0.00035664 = -h_b^{(1)}(17)$	$h_b^{(1)}(-7) = 0.05116141 = -h_b^{(1)}(7)$
$h_b^{(1)}(-15) = 0.00112735 = -h_b^{(1)}(15)$	$h_b^{(1)}(-5) = 0.09558504 = -h_b^{(1)}(5)$
$h_b^{(1)}(-13) = 0.00571987 = -h_b^{(1)}(13)$	$h_b^{(1)}(-3) = 0.19152228 = -h_b^{(1)}(3)$
$h_b^{(1)}(-11) = 0.01278547 = -h_b^{(1)}(11)$	$h_b^{(1)}(-1) = 0.63050716 = -h_b^{(1)}(1)$
$h_b^{(1)}(-9) = 0.02680092 = -h_b^{(1)}(9)$	

TABLE VIII
COEFFICIENT VALUES OF $H_M^{(1)}(z)$

$h_M^{(1)}(-20) = 0.00204909 = h_M^{(1)}(20)$	$h_M^{(1)}(-8) = 0.06345628 = h_M^{(1)}(8)$
$h_M^{(1)}(-18) = 0.00468652 = h_M^{(1)}(18)$	$h_M^{(1)}(-6) = 0.07736226 = h_M^{(1)}(6)$
$h_M^{(1)}(-16) = 0.01000542 = h_M^{(1)}(16)$	$h_M^{(1)}(-4) = 0.09310812 = h_M^{(1)}(4)$
$h_M^{(1)}(-14) = 0.01825014 = h_M^{(1)}(14)$	$h_M^{(1)}(-2) = 0.10265599 = h_M^{(1)}(2)$
$h_M^{(1)}(-12) = 0.03019052 = h_M^{(1)}(12)$	$h_M^{(1)}(0) = 0.10639832$
$h_M^{(1)}(-10) = 0.04503649 = h_M^{(1)}(10)$	

TABLE IX
COEFFICIENT VALUES OF $H_b^{(2)}(z^7)$

$h_b^{(2)}(-105) = 0.00077414 = -h_b^{(2)}(105)$	$h_b^{(2)}(-49) = 0.04783235 = -h_b^{(2)}(49)$
$h_b^{(2)}(-91) = 0.00533037 = -h_b^{(2)}(91)$	$h_b^{(2)}(-35) = 0.08969270 = -h_b^{(2)}(35)$
$h_b^{(2)}(-77) = 0.01192049 = -h_b^{(2)}(77)$	$h_b^{(2)}(-21) = 0.17271575 = -h_b^{(2)}(21)$
$h_b^{(2)}(-63) = 0.02492091 = -h_b^{(2)}(63)$	$h_b^{(2)}(-7) = 0.40522743 = -h_b^{(2)}(7)$

TABLE X
COEFFICIENT VALUES OF $H_M^{(2)}(z^7)$

$h_M^{(2)}(-140) = 0.00215243 = h_M^{(2)}(140)$	$h_M^{(2)}(-56) = 0.06351735 = h_M^{(2)}(56)$
$h_M^{(2)}(-126) = 0.00451527 = h_M^{(2)}(126)$	$h_M^{(2)}(-42) = 0.07706411 = h_M^{(2)}(42)$
$h_M^{(2)}(-112) = 0.00940580 = h_M^{(2)}(112)$	$h_M^{(2)}(-28) = 0.09314295 = h_M^{(2)}(28)$
$h_M^{(2)}(-98) = 0.01720234 = h_M^{(2)}(98)$	$h_M^{(2)}(-14) = 0.10409780 = h_M^{(2)}(14)$
$h_M^{(2)}(-84) = 0.02935234 = h_M^{(2)}(84)$	$h_M^{(2)}(0) = 0.10870994$
$h_M^{(2)}(-70) = 0.04519465 = h_M^{(2)}(70)$	

with increasing K ; optimization algorithms become less user friendly as K increases. Instead of choosing $K = 5$, we choose $K = 2$ to illustrate the advantage that can be gained in multi-level masking. For two-level masking, the estimated values were $M_1 = M_2 = 7, N^{(2)} = 41, N_M^{(1)} = 23.5, N_M^{(2)} = 22.3, N_b^{(1)} = 18.7, N_b^{(2)} = 17.6$. A design with $M_1 = M_2 = 7, N^{(2)} = 42, N_M^{(1)} = N_M^{(2)} = 21$, and $N_b^{(1)} = 18$, and $N_b^{(2)} = 16$ was obtained. The total number of nontrivial coefficient was $42 + 21 + 21 + 18 + 16 = 118$. This is only 6% of the 2000 nontrivial coefficients of the Chebyshev optimum design; the saving was a factor of 17. The effective length of our FRM design (after replacing z by z^2) was 4339; it was only 8.5% longer than the Chebyshev optimum design. The coefficient values of the subfilters (after replacing z by z^2, z^{2M_1}, z^{2M_2} or $z^{2M_1M_2}$ which ever is appropriate) are tabulated in Tables VII–XI. The passband ripple of the overall filter is shown in Fig. 8 and the frequency responses of the various subfilters are shown in Fig. 9.

TABLE XI
COEFFICIENT VALUES OF $H^{(2)}(z^{49})$

$h^{(2)}(-2009) = 0.00017441 = -h^{(2)}(2009)$	$h^{(2)}(-931) = 0.01548193 = -h^{(2)}(931)$
$h^{(2)}(-1911) = 0.00024335 = -h^{(2)}(1911)$	$h^{(2)}(-833) = 0.02027347 = -h^{(2)}(833)$
$h^{(2)}(-1813) = 0.00052237 = -h^{(2)}(1813)$	$h^{(2)}(-735) = 0.02636264 = -h^{(2)}(735)$
$h^{(2)}(-1715) = 0.00086057 = -h^{(2)}(1715)$	$h^{(2)}(-637) = 0.03422554 = -h^{(2)}(637)$
$h^{(2)}(-1617) = 0.00140905 = -h^{(2)}(1617)$	$h^{(2)}(-539) = 0.04463223 = -h^{(2)}(539)$
$h^{(2)}(-1519) = 0.00215043 = -h^{(2)}(1519)$	$h^{(2)}(-441) = 0.05904263 = -h^{(2)}(441)$
$h^{(2)}(-1421) = 0.00319015 = -h^{(2)}(1421)$	$h^{(2)}(-343) = 0.08044706 = -h^{(2)}(343)$
$h^{(2)}(-1323) = 0.00456441 = -h^{(2)}(1323)$	$h^{(2)}(-245) = 0.11620845 = -h^{(2)}(245)$
$h^{(2)}(-1225) = 0.00638009 = -h^{(2)}(1225)$	$h^{(2)}(-147) = 0.18938801 = -h^{(2)}(147)$
$h^{(2)}(-1127) = 0.00871393 = -h^{(2)}(1127)$	$h^{(2)}(-49) = 0.41325457 = -h^{(2)}(49)$
$h^{(2)}(-1029) = 0.01170195 = -h^{(2)}(1029)$

Fig. 8. Passband ripples of the Hilbert transformer. The frequency axis is normalized against the sampling frequency.

VII. HALF-BAND FILTER

An odd length low-pass half-band filter may be derived from an even length Hilbert transformer by scaling all the coefficient values of the Hilbert transformer by a factor of half ($1/2$), inserting a zero value coefficient between two coefficients, multiplying its n th coefficient by $je^{-jn\pi/2}$, and replacing the centre coefficient by $1/2$. The factor of two reduction in transition width due to inserting a zero between every two coefficients is nullified by the factor of two increase in transition width when the centre coefficient (value = $1/2$) is inserted to raise the frequency response of the Hilbert transformer to become that of the half-band filter. Since the coefficient values have been scaled by half, the ripple magnitude of the resultant half-band filter is half that of the original Hilbert transformer. Taking this into consideration, the set of equations for the half-band filter may be obtained from that for the Hilbert transformer by replacing δ by 2δ . Specifically, for a half-band filter, (18) becomes

$$M_{\text{opt}}(K) \approx \left(\frac{\Phi_H(2\delta)}{(\Phi_H(2\delta) + \Phi_M(2\delta))\Delta} \right)^{\frac{1}{K+1}} \approx \left(\frac{0.43}{\Delta} \right)^{\frac{1}{K+1}} \quad \text{for } \frac{\Phi_M(2\delta)}{\Phi_H(2\delta)} = 1.32 \quad (26)$$

and (22) becomes

$$\beta(2\delta, K_{\text{opt}} - 1) > \Delta > \beta(2\delta, K_{\text{opt}}). \quad (27)$$

VIII. $H_b(z)$ WITH SPECIAL PROPERTIES

In the examples presented in Sections IV and VI, $H_b(z)$ was optimized to minimize the overall complexity of the Hilbert transformer. However, it should be noted that $H_b(z)$ can be any low-order Hilbert transformer implemented using any structure

Fig. 9. Frequency responses of the individual subfilters and the overall filter.

such as those in [25]–[27] and can be designed independent on $H_1(z^M)$ and $H_M(z)$. The frequency responses of $H_1(z^M)$ and $H_M(z)$ are then optimized as a correction term to sharpen the frequency response of the overall filter. In this case, the frequency response of the overall filter at the vicinity of $\omega = 0$ is determined mainly by $H_1(z^M)$ and $H_M(z)$. If the attenuation of $H_M(z)$ is very high for frequencies far away from $\omega = 0$, the characteristics of the frequency response of the overall filter will be similar to that of $H_b(z)$ for frequencies far away from $\omega = 0$. Nevertheless, the overall complexity of the filter will be higher than that where $H_b(z)$, $H_1(z^M)$, and $H_M(z)$ are jointly optimized.

IX. CONCLUSION

A set of equations for characterizing the Hilbert transformer based on the implementation structure shown in Fig. 7 have been derived. It is also demonstrated that, in an acoustic feedback oscillation suppression example, the FRM technique produced a Hilbert transformer with reduction in the number of nontrivial coefficients by a factor of 17 when compared to the Chebyshev optimum design.

APPENDIX I

The expression [28] relating the length (N_0), of a low-pass filter to its passband and stopband ripples, δ_p and δ_s , respectively, and its normalized transition width (β) is

$$N_0 \approx \frac{\Phi_1(\delta_p, \delta_s)}{\beta} - \Phi_2(\delta_p, \delta_s)\beta + 1$$

$$\Phi_1(\delta_p, \delta_s) = [a_1(\log_{10} \delta_p)^2 + a_2 \log_{10} \delta_p + a_3] \times \log_{10} \delta_s + [a_4(\log_{10} \delta_p)^2 + a_5 \log_{10} \delta_p + a_6]$$

$$a_1 = 5.309 \times 10^{-3}$$

$$a_2 = 7.114 \times 10^{-2}$$

$$a_3 = -4.761 \times 10^{-1}$$

$$a_4 = -2.66 \times 10^{-3}$$

$$a_5 = -5.941 \times 10^{-1}$$

$$a_6 = -4.278 \times 10^{-1}$$

$$\Phi_2(\delta_p, \delta_s) = 11.01217 + 0.51244 \times (\log_{10} \delta_p - \log_{10} \delta_s).$$

A Hilbert transformer may be derived from a low-pass half-band filter by discarding the centre coefficient, multiplying its n th coefficient by $-je^{jn\pi/2}$, discarding trivial coefficients, and scale up the coefficient values by a factor of two. Replacing $\delta_p = \delta_s = \delta/2$ we have

$$N_{\text{opt}} \approx \frac{\Phi_H(\delta)}{\Delta} + 1$$

where $\Phi_H(\delta) = 0.002655(\log_{10}(\delta))^3 + 0.031843(\log_{10}(\delta))^2 - 0.554993 \log_{10}(\delta) - 0.049788$.

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