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Digital image correlation for whole field out-of-plane displacement measurement using a single camera

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Abstract

Digital image correlation (DIC) using a single camera has been widely used for in-plane displacement and strain measurement. However, in order to obtain out-of-plane displacement, one should employ stereo vision systems which enable multiple directions detection. In this paper, we develop a simple method for whole field out-of-plane displacement measurement using only one camera. The proposed method employs digital image correlation to calculate an apparent in-plane displacement field which is introduced by magnification change due to an unknown out-of-plane displacement. The unknown out-of-plane displacement is subsequently determined from a pinhole camera model and the apparent in-plane displacement. Experiments conducted on a plate, a cantilever beam and a surface with height variation demonstrate that the method is simple, accurate and suitable for measurement of non-structured objects.

Keywords: Digital image correlation (DIC); Out-of-plane displacement; Magnification change; Pinhole camera model

1. Introduction

Digital image correlation is a non-contact optical method for displacement and strain measurement. This method utilizes a natural or artificial surface speckle pattern as an information carrier and cross correlates two slightly different images captured before and after a deformation, to obtain whole field displacement of an object. In the literature, Peter and Ranson [1] first employed digital image correlation method for surface displacement and strain computation. The earliest version of digital image correlation algorithm is computation intensive. By adopting bi-cubic interpolation and Newton-Raphson iteration methods, the searching accuracy and speed were greatly improved [2–4]. However, the iterative algorithms suffer from the limitations that it may cause non-convergence or converge to a wrong point. This problem can be overcome by giving a proper initial searching position, normally within several pixels of the true matching

position, before initiating the iterative searching [5]. To further improve the computation speed, Zhou and Goodson [6] proposed an iterative algorithm which requires only calculation of the first-order spatial derivatives. In the past two decades, digital image correlation method has been well developed [7–9] and accurate, fast and robust commercial software, like VIC2D [5] and ARAMIS, are available. The method of digital image correlation has been widely applied to various fields [10–15].

Conventionally, digital image correlation (DIC) utilizing a single camera is mainly used to obtain in-plane displacement and strain while DIC using multiple cameras will provide full three-dimensional displacement measurement. There are, however, some key limitations on a multiple camera system including relatively complicated optical system, mismatch in triangulation of corresponding points, defocus problem when the magnification is high and a laborious calibration process. Hence, a single camera system would be more desirable.

In the literature, Dai and Su [16] had proposed a method using a single camera for shape identification, and thus is able to determine out-of-plane displacement. However, the proposed method requires speckle projection and needs quite a lot pre-captured speckle images. Furthermore, the proposed method requires a large amount of computation for shape determination. In this study we have developed a simple alternative method which does not need speckle projection and requires much less computation, yet provide accurate results.

In a system designed for in-plane displacement measurement of an object, any out-of-plane displacement will cause a change of magnification of the imaging system, and thus introduce an apparent in-plane displacement. This apparent in-plane displacement would cause measurement error in normal sense. It however provides useful information for the retrieval of out-of-plane displacement. Asundi and co-worker [17,18] had used white light speckle method to measure the apparent in-plane displacement. However, the method needs laborious laser reconstruction and fringe evaluation. In this paper, we apply digital image correlation technique on the speckle images captured before and after an unknown out-of-plane displacement and calculate the apparent in-plane displacement. The unknown out-of-plane displacement is subsequently recovered by using a pinhole camera model.

The basic principle for the proposed method is presented in this paper. Experiments conducted on a uniformly displaced plate, a non-uniformly deformed cantilever beam and a surface with a step demonstrate that the proposed method is accurate and suitable for measuring non-structured objects.

2. Theory

2.1. The method of digital image correlation

Digital image correlation is a non-contact optical method for displacement and strain measurement. This method uses the cross-correlation function or its alternatives to

compare images captured before and after a small deformation, and obtain the whole field in-plane displacement quantitatively [3–6].

In digital image correlation, a set of neighboring points in an undeformed state is assumed to remain as neighboring points after deformation. Fig. 1 illustrates schematically the deformation process of a planar object. Quadrangle S (dash-line) is a reference (or undeformed) subimage and quadrangle S_1 (solid-line) is the corresponding deformed subimage. In order to obtain in-plane displacements u_m and v_m of point M , subimage S is matched with subimage S_1 using a correlation operation. If subset S is sufficiently small, the coordinates of points in S_1 can be approximated by first-order Taylor expansion as follows:

$$x_{n1} = x_m + u_m + \left(1 + \frac{\partial u}{\partial x}\bigg|_M\right) \cdot \Delta x + \frac{\partial u}{\partial y}\bigg|_M \cdot \Delta y, \quad (1)$$

$$y_{n1} = y_m + v_m + \frac{\partial v}{\partial x}\bigg|_M \cdot \Delta x + \left(1 + \frac{\partial v}{\partial y}\bigg|_M\right) \cdot \Delta y, \quad (2)$$

where the coordinates are as shown in Fig. 1.

Let $f(x,y)$ and $f_d(x,y)$ be the gray value distribution of the undeformed and deformed image, respectively. For a subset S , a correlation coefficient C is defined as

$$C = \frac{\sum_{N \in S} [f(x_n, y_n) - f_d(x_{n1}, y_{n1})]^2}{\sum_{N \in S} [f(x_n, y_n)]^2}, \quad (3)$$

where (x_n, y_n) is a point in subset S in the undeformed image, and (x_{n1}, y_{n1}) is a corresponding point (defined by Eqs. (1) and (2)) in subset S_1 in the deformed image. It is clear that if parameters u_m, v_m are the real displacements and $\frac{\partial u}{\partial x}\bigg|_M, \frac{\partial v}{\partial x}\bigg|_M, \frac{\partial u}{\partial y}\bigg|_M, \frac{\partial v}{\partial y}\bigg|_M$ are the displacement derivatives of point M , the correlation coefficient C would be zero. Hence, minimization of the coefficient C would provide the best estimates of the parameters. Minimization of the correlation coefficient C is a non-linear optimization process, and Newton–Raphson or Levenburg–Marquardt iteration methods [3,4] are usually used in the implementation of the process.

To achieve subpixel accuracy, interpolation schemes should be implemented to reconstruct a continuous gray value distribution in the deformed images. Normally higher order interpolation would provide more accurate results, but with the limitation of requiring more computation time. The choice of different schemes depends on different requirements; bi-cubic and bi-quintic spline interpolation schemes are widely used.

2.2. Retrieval of out-of-plane displacement

A commonly used first-order representation of typical optical system [19] is shown in Fig. 2. The system has a few properties. One is that rays parallel to the optical axis from the left side will refract at the second principal plane and go through the focal point. Another is that rays directed at the front nodal point will exit the lens from the second nodal point at exactly the same angle with respect to the optical axis. From these properties, an image can be uniquely determined from the main and marginal rays. The object distance s , image distance s_1 and focal length of the optical system f have the relation of

$$1/s + 1/s_1 = 1/f. \quad (4)$$

For a thin lens, the first and second principal planes become one and the model is further simplified. Consider a thin lens L as shown in Fig. 3. An object of radius h is placed at a distance b from the thin lens L . From Eq. (4), the image distance a can be uniquely determined. When a sensor is placed at a distance a from the lens, a clear image of radius h' is recorded. When the object undergoes a small out-of-plane displacement d_b and the sensor is kept unchanged, another image of radius H is recorded (assuming that d_b is smaller than the depth of field of the optical system, the recorded image would be clear). From Fig. 3, we have

$$H - h' = \frac{d_b}{b - d_b} h'. \quad (5)$$

If the out-of-plane displacement d_b is much smaller than b , Eq. (5) can be written as

$$d_b = \frac{b(H - h')}{h'}. \quad (6)$$

By correlating the two recorded images using DIC algorithm, the apparent in-plane displacement field $(H - h')$ can be determined. The parameter b can be obtained by a calibration process described as follows.

Rearrange Eq. (6), we have

$$\frac{H - h'}{h'} = \frac{1}{b} d_b. \quad (7)$$

The slope $\frac{H - h'}{h'}$ corresponding to a particular prescribed displacement d_b can be obtained by processing the apparent in-plane displacement data $(H - h')$. Hence, by prescribing a series of out-of-plane displacements of the object, an average value of the parameter b can be obtained through a linear regression process. Hence, the whole field out-of-plane displacement d_b can be readily obtained from Eq. (6) after calibration.

It should be noted that the above calibration method is based on the assumption

of planar objects for which the value of b is a constant. For objects with non-planar profiles, but the surface height variation is small compared to the object distance, the proposed calibration method can also be used to obtain an average object distance. For objects with large height variation on the surface, a different calibration method which determines the value of b at each point of the image should be proposed for more accurate measurement.

3. Experiments

The experimental setup is shown in Fig. 4. A flat plate is mounted on a linear translation stage with a resolution of 10 μm and placed perpendicularly to a CCD camera of 768 $\times$ 576 pixels. The object is imaged onto the camera's image plane via a Nikon lens (Model: Micro Nikkor, $f=55$ mm) and images are recorded through a frame grabber and stored in a computer. The f/number is set to 22 to obtain a small aperture size for large depth of field.

By translating the plate by a series of prescribed out-of-plane displacements, and computing the corresponding slopes of the apparent in-plane displacement, the measurement system is calibrated for out-of-plane displacement measurement.

For non-rigid body out-of-plane displacement measurement, the flat plate is replaced by a cantilever beam (length = 153 mm, width = 23 mm, height = 2.96 mm). One end of the beam is fully clamped, and the free end of the beam is loaded by a point load as shown in Fig. 4. Images of the beam of initial and loaded states are captured and stored for further processing.

To testify the possibility of the proposed method for measurement of non-structured object, a plate with a step of height 1.4 mm on its surface is used. Images of the step before and after an out-of-plane displacement are captured and the proposed method is used to determine the unknown out-of-plane displacement.

4. Results and discussion

Fig. 5 shows an apparent in-plane displacement field obtained by correlating two images before and after an out-of-plane displacement. It can be seen that the displacement field seems to be radial, agrees well with the theoretical prediction shown in Fig. 3. When the apparent in-plane displacement field is separated to displacement in x -axis and displacement in y -axis, the typical u and v displacement fields show flat planes as shown in Fig. 6, which is in agreement with theoretical modeling using Eq. (7). The slope of the apparent displacement can be calculated from Fig. 6 by fitting the experimental data using linear function. Fig. 7 shows a plot of the slope of the apparent in-plane displacement u of the plate specimen against the prescribed out-of-plane displacement. It is seen that the slope of the apparent in-plane displacement is proportional to the prescribed out-of-plane displacement, which agrees well with theoretical modeling again. The experimental data can be linearized using the least squares method by the equation $y = 0.00368x$ with a

correlation coefficient $R = 0.9999$. The initial object distance b is found to be 271.74 mm. It can be seen from the correlation coefficient $R = 0.9999$ that the line is linear and that out-of-plane displacement of the magnitude of 0.1 mm can be measured accurately (as seen from Fig. 7).

After calibration, the plate is given a series of out-of-plane displacements and images are captured. The apparent in-plane displacements ($H - h'$) are obtained by correlating the initial and displaced images. The parameter b is obtained from the calibration process described above. And the parameter h' is the distance from the image center. The whole field out-of-plane displacement is thus obtained from Eq. (6). Fig. 8(a) shows the experimental results when the prescribed out-of-plane displacement is 800 μm . The mean value is 796.1 μm with a standard deviation of 4.3 μm . Compared with the prescribed displacement, the maximum discrepancy is 14.2 μm . Fig. 8(b) shows the experiment result when the prescribed out-of-plane displacement is 60 μm . The mean value is 60.2 μm with a standard deviation of 1.7 μm . The maximum discrepancy is 5.4 μm .

The above study shows that the proposed method can accurately measure rigid-body out-of-plane displacement. For non-rigid body measurement, the flat plate is replaced by a cantilever beam. The free end of the beam is loaded by a point load as shown in Fig. 4 and the deflection of the cantilever beam is determined by the proposed method.

When loaded at the free end, the cantilever beam undergoes non-linear deflection. Speckle images of the beam surface are captured and processed. Fig. 9 shows the apparent in-plane displacement field when the out-of-plane displacement of the free end of the cantilever beam is 2 mm. Since the out-of-plane displacement is not rigid-body, the apparent in-plane displacement is no longer a flat plane like in Fig. 6. It is now some curve plane which contains the information of the non-rigid-body out-of-plane displacement. Fig. 10(a) shows the out-of-plane displacement of the cantilever obtained from experiment. Since the material properties of the beam, the loading and boundary condition are known, the theoretical out-of-plane displacement can be obtained from simple beam theory as shown in Fig. 10(b). Fig. 10(c) shows a comparison between theoretical and experimental out-of-plane displacement at the middle section of the beam. It can be seen that the experimental results agree well with theory, and the maximum discrepancy is less than 8%.

It is noted that loading at the free end of a cantilever beam would also introduce an in-plane displacement component, though it is much smaller than the out-of-plane component. This would however contribute to the apparent in-plane displacement, and thus introduce discrepancy in the reconstructed out-of-plane displacement. The “real in-plane displacement effect” can be eliminated by calculating the theoretical in-plane displacement according to the material properties, loading and boundary condition, and subtracting this value from the experimentally obtained apparent in-plane displacement. Fig. 11(a) shows the out-of-plane displacement after subtraction of the “real in-plane displacement effect” and Fig. 11(b) shows the theoretical out-of-plane displacement. Fig. 11(c) shows a comparison of theoretical and experimental out-of-plane displacement at the middle section of the cantilever. It is seen

that agreement is better and the maximum discrepancy is less than 5%.

The experiments on the flat plate and cantilever beam described above contain flat surfaces. However, for a non-planar surface, the value of b in Eq. (5) is not a constant and should be determined for each point on the object. In most cases, the height variation of the object surface is small compared to b . Hence, in general the value of b could be assumed constant. This is illustrated in the following example.

Fig. 12 shows the image of an object with a step height of 1.4 mm and Fig. 13 shows the apparent in-plane displacement field with a prescribed out-of-plane displacement of 2 mm. Fig. 14 shows the out-of-plane displacement on the object computed using Eq. (6). The mean value is 2.003 mm with a standard deviation of 0.01 mm. Compared with the prescribed displacement of 2 mm, the discrepancy is less than 1%.

The accuracy of the proposed method for out-of-plane displacement is dependent on many factors including environmental vibration level, signal to noise ratio of the CCD, optical system aberration, speckle quality and accuracy of digital image correlation algorithm. The influence of the environment, CCD and speckle quality, and correlation algorithm accuracy can be quantified by capturing a sequence of consecutive images without giving the test sample any deformation, and calculating the image shift using specific digital image correlation algorithm. In the above experiments, the total noise effect is found to be under 0.04 pixels. Lens aberration would affect the results when the object undergoes a displacement and the calibration for the lens aberration depends on the optical system. In the experiment, the magnification is not large and the out-of-plane displacement is small, thus lens aberration is assumed to be small enough to be ignored. Besides the above factors, the resolution of the proposed method also varies with the magnification of the imaging system. The resolution increases with the magnification, however, the field of view (measurement area) decreases with the magnification. In our study, the present method indicates a resolution of 5 μm for a magnification of 1. However, when the magnification decreases to 0.1, the resolution falls to 20 μm .

The measuring range for out-of-plane displacement is subjected to focusing range of the image, which is related to the pixel and aperture size as well as magnification of the imaging system. For a f/number of 22, a magnification of 0.2 and a pixel size of 11 μm , the depth of field (measuring range) of the imaging system is 14.5 mm. In the experiments described above, a measuring range of 10 mm has been verified.

It should be noted that at the central region of the image, due to the small values of h' , errors resulted from using the digital image correlation analysis will be greatly amplified. To minimize the effect the experimental data would need to be smoothed before processing. This is achieved by first filtering the raw experimental data and then fitting it with a polynomial curve.

The present method has an obvious advantage that high accuracy can be achieved with a very simple optical setup. Another advantage is that the method is insensitive to rigid-body in-plane displacement which means that out-of-plane displacement can be

separated from rigid-body in-plane displacement. However, this method also has some uncertainty in the measurement accuracy due to the somewhat arbitrary data processing method. The choice of filtering and curve fitting method would depend on the experience and knowledge of the displacement forms. The speckle method and choice of subset size in digital image correlation algorithm for best accuracy also depend on the user's experience. Another drawback is that the measuring range and measuring area decreases with a higher magnification. This method is however most suitable for out-of-plane displacement measurement when knowledge of the displacement form is known, as a proper curve fitting method can be used.

5. Concluding remarks

This paper presents a new method using digital image correlation for out-of-plane displacement measurement. The proposed method employs digital image correlation technique to calculate an apparent in-plane displacement introduced by a change in magnification due to an unknown out-of-plane displacement, and subsequently retrieve the whole field out-of-plane displacement. Experiments conducted on a uniformly translated plate, a cantilever loaded at the free end and a step show that the method is accurate and suitable for measurement of both structured and slightly non-structured objects. In addition, the proposed method utilizes only a single camera for two-dimensional out-of-plane displacement measurement and the measuring system is much simpler than that of multiple camera systems used in most previous methods.

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Fig. 1

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