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Determination of surface contour by temporal analysis of shadow moiré fringes

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Abstract

A simple temporal analysis algorithm (phase scanning method) is proposed for 3-D surface profile measurement of a vibrating object based on shadow moiré technique. A grating is positioned close to a low-frequency vibrating object and its shadow is observed through the grating. The moiré fringe patterns, generated by the interference of grating lines and shadow lines, are captured by a high-speed CCD camera with a telecentric gauging lens. Phase values are evaluated point by point using phase scanning method. From the phase values of each point on the object, the high-quality surface profile of the specimen at different instants of vibration can be retrieved. In this application, two specimens are tested to demonstrate the validity of the method, one is a spherical cap with height of 4 mm, the other is a small coin with unevenness of less than 0.2 mm. The experimental results are compared with test results using mechanical stylus method.

Keywords: Shadow moiré; High-speed imaging; Surface contour; Temporal phase analysis.

1. Introduction

A number of noncontact optical techniques are available to obtain topographic information. Among them moiré technique, which was proposed by Meadows et al. [1]

and Takasaki [2] in early 1970s, has been widely used for three-dimensional shape measurement and surface analysis. According to the optical arrangement of the system it is classified into two types of methods [3-5]: shadow moiré and projection moiré. In projection moiré, the fringes, which contain the information of surface profile, are generated by projecting a grating onto the object and viewing it through a second grating in front of the viewer. Comparing with projection moiré, shadow moiré is a relatively cheap and simple technique which uses a single grating placed close to the object and an oblique light source illumination which casts a shadow of the grating on the object surface. The shadow is distorted in accordance with the profile of the test surface. When the shadow is viewed from a different direction through the original grating, the grating and its distorted shadow interfere, generating fringes. These fringes are loci of the surface depth with respect to the plane of the grating. In early years, the fringe patterns are analyzed by tracing the center of the fringes which is an inefficient and inaccurate process. The shortcomings of shadow moiré topography include: (1) the lower resolution of contouring fringes; (2) difficulty to judge whether the surface is convex or concave from one moiré pattern [6]. Due to the progress of computer capacities and image processing techniques in 1990s, different types of phase-shifting methods [7-14] were applied in moiré topography to achieve high resolution measurements. However, phase-shifting approaches also limit the technique to measure fixed surface profiles.

Moiré technique based on time-averaged method has also been applied to dynamic problems [15-18]. All these applications generate images of the object superimposed with moiré fringes from which the vibration amplitude can be determined. However, none of them can be applied to the study of instant movement and contour of an object as a

function of time. In many cases, high resolution 3-D surface profiling of objects under vibration or continual profile changing gives useful information of dynamic response and deformation of the objects concerned.

With the availability of high-speed digital recording, it is now possible to record moiré fringe patterns with rates exceeding 10,000 frames per second (fps). Retrieving instant spatial phase maps from those fringe patterns along time-axis enables obtaining the instant 3-D profiles as well as the dynamic response. Jin et al. [19] developed a novel approach to retrieve point-by-point height information using frequency sweeping method. The moiré patterns are varied by rotating the grating and a series of fringe patterns are obtained. Applying Fourier transform method along the time-axis, the frequency of intensity changing on each point can be obtained, subsequently, the height of each point is determined. The concept of phase evaluation along time-axis [20-22] has also been applied to large continuous deformation measurement using laser interferometry. Joenathan et al. [23,24] applied temporal Fourier analysis [25,26] to measure in-plane and out-of-plane displacement. However, if the object deforms arbitrarily, the frequency spectrum of each point is different. This will make filter process more difficult and less accurate [27].

In this paper, a simple and robust temporal phase evaluation technique, called phase scanning method, is proposed to retrieve the phase value point by point along time-axis. The object having a diffuse surface is positioned behind a grating and subjected to a vibration. The moiré fringe patterns are imaged consecutively by a high-speed CCD camera with a telecentric gauging lens. With proposed temporal fringe analysis technique, the surface profile at any moment, as well as the amplitude and frequency of the vibration,

can be reconstructed. For the purpose of comparison with mechanical stylus method, objects are only subjected to rigid-body motion. However, this algorithm can also be applied to measurement of continuous out-of-plane deformation which varies at on each point on a surface as well as profiles which are varying.

2. Theoretical Analysis

In a typical optical arrangement of shadow moiré shown in Fig. 1, light source and camera are placed at the same distance l from the grating with a pitch p . The mathematical representation of the intensity distribution captured by a CCD camera is governed by the following equation [19]:

$$I(x, y) = a(x, y) + b(x, y) \cos \left\{ \frac{2\pi d h(x, y)}{p[l + h(x, y)]} \right\}, \quad (1)$$

where $a(x, y)$ and $b(x, y)$ are the intensity bias and modulation factor, respectively. d is the distance between the camera axis and the light source, $h(x, y)$ is the distance from the grating plane to a point $P(x, y)$. In normal cases, $l \gg h(x, y)$, Eq. (1) can be simplified as

$$I(x, y) = a(x, y) + b(x, y) \cos[kh(x, y)], \quad (2)$$

where $k = \frac{2\pi d}{pl}$ is a constant related to the optical setup.

When the object is vibrating or deforming along z-axis, Eq. (2) can be rewritten as

$$I(x, y; t) = a(x, y; t) + b(x, y; t) \times \cos\{k[h(x, y; t) + h_0(x, y)]\}, \quad (3)$$

where $h_0(x, y)$ is the initial distance between the grating and object which represents the profile of the object. $h(x, y; t)$ is the time-dependant distance variation due to vibration or deformation. $a(x, y; t)$ and $b(x, y; t)$, which are also functions of time, are both slowly varying functions. Hence, $a(x, y; t)$ and $b(x, y; t)$ can be regarded as constants in one period of intensity change.

If the amplitude of the vibration is large enough, the variation of $kh(x, y; t)$ will be higher than 2π , which implies more than one period intensity change. In most cases, this assumption can be satisfied by proper selection of pitch p of the grating, distance d between camera and light source and distance l between camera and grating. With this assumption, Eq. (3) can be rewritten as

$$I(x_p, y_p; t) = \frac{1}{2}[I_{\max}(x_p, y_p) + I_{\min}(x_p, y_p)] + \frac{1}{2}[I_{\max}(x_p, y_p) - I_{\min}(x_p, y_p)] \times \cos[kH(x, y; t)] \quad (4)$$

where $I_{\max}$ and $I_{\min}$ are detectable maximum and minimum values of intensity within one period of intensity change at a certain point $P(x_p, y_p)$ on the test surface.

$H(x_p, y_p; t)$ is the instant distance between point P on the object and grating.

$$H(x_p, y_p; t) = h(x_p, y_p; t) + h_0(x_p, y_p). \quad (5)$$

Hence $H(x_p, y_p; t)$ can be expressed as

$$H(x_p, y_p; t) = \frac{1}{k} \varphi(x_p, y_p; t) = \frac{1}{k} \arccos \left[\frac{2I(x_p, y_p; t) - I_{\max}(x_p, y_p) - I_{\min}(x_p, y_p)}{I_{\max}(x_p, y_p) - I_{\min}(x_p, y_p)} \right], \quad (6)$$

where $\varphi(x_p, y_p; t)$ is the phase value which is proportional to $H(x_p, y_p; t)$. As the second term in Eq. (5) is a constant on the time-axis, the displacement along z-axis at point P can be obtained as

$$w(x_p, y_p; \Delta t) = H(x_p, y_p; t_2) - H(x_p, y_p; t_1). \quad (7)$$

At a certain time T , the distance between grating and object which represents the instant contour of the specimen can be expressed as

$$H(x_p, y_p; T) = h(x_p, y_p; T) + h_0(x_p, y_p), \quad (8)$$

where $h(x_p, y_p; T)$ denotes the displacement which is related to the vibration amplitude or deformation at instant T .

Phase values $\varphi(x_p, y_p; t)$ given by Eq. (6) are wrapped phase which have values within 0 to π . These values have to be converted from $[0, \pi]$ to $[0, 2\pi]$ for unwrapping process. Table 1 gives the details of the conversion. The phase values after conversion are determined by the direction of vibration and sign of the first derivative of intensity $\frac{\partial I}{\partial t}$ along time axis [27]. Retrieving the displacement and height information from Eq. (7) and Eq. (8) are respectively, 1-D and 2-D phase unwrapping problems.

3. Experimental Illustration

Fig. 2 shows the experimental set up. An object with spherical profile is subjected to a low-frequency vibration using a shaker. The frequency of vibration is controlled by a function generator. A vertical sinusoidal grating is positioned close to the object. A 150W DC light source with optical fiber illuminates the object with certain angle. The fringe

patterns are captured at right angle by a high speed CCD camera with a telecentric gauging lens. With this special lens, magnification error of the object due to vibration or deformation is reduced and only the fringe pattern is varied on the image. As the object is smaller than the telecentric lens, each point on the object is imaged at right angle, in this case, the camera does not have to be placed at the same distance l as the point light source. This will reduce error in alignment. To judge the fringes whether are representing a convex or concave surface, carrier fringes are introduced by rotating grating with a small angle in x-z plane. This will only change the relative distance between the grating and object, but will not introduce error in contouring. However, due to diffraction effect, the fringe contrast reduces with increase in distance. As the angle of rotation is small, it will not seriously affect the results. The variation of grating pitch due to rotation is also omitted.

4. Results and Discussion

Fig. 3(a) shows the initial test object without grating in front. The radius r_s , height h_s and base radius r_b of the spherical cap are 14.5, 4 and 10 mm, respectively, as shown in Fig. 3(b). The frequency of the grating is 2 lines/mm. The distance d and l (shown in Fig. 1) are 195 and 335 mm, respectively. As the grating is placed very close to the object, the assumption of $l \gg h(x, y)$ in Eq. (1) is satisfied. The test object is subjected to a sinusoidal wave vibration with a frequency of approximately 6 Hz. Fringe patterns are captured at intervals of 0.004s with a camera recording rate of 250 fps. Figures 4(a) and (b) shows two typical fringe patterns recorded by high speed CCD camera at different moments. In order to identify whether the surface is convex or concave, one carrier fringe is introduced on the images by rotating grating with a very small angle in x-z plane. One

hundred consecutive images are selected for processing. A 5×5 mean mask is applied to remove high-frequency grating lines and random noise on the images. Fig. 4(c) and (d) are typical fringe patterns after filtering. To determine the direction of vibration or deformation from these fringe patterns, it is necessary to identify the images representing extreme positions of vibration. If all points on object deform in same direction, these images can be readily identified by computer since the first derivative of intensity ($\frac{\partial I}{\partial t}$) of most pixels change sign (either from negative to positive or from positive to negative) in these images. Considering the influence of random noise, a threshold (in this application, 50% of the total pixel number) is set when the extreme positions are identified. Four such images are found from these images.

For each pixel, one hundred data points along the time-axis are obtained. Fig. 5(a) shows the gray value variation of point A (indicated in Fig. 3). The wrapped phase values are presented in Fig. 5(b). After unwrapping along time-axis, the continuous phase profile, as shown in Fig. 5(c), can be obtained. This phase profile represents the displacement of point A along z-axis which is given in Eq. (7). In this application, the displacements of other points on the object are the same as point A. From Fig. 5(c) the frequency of vibration is evaluated as 5.95 Hz, and the amplitude of phase change is 11.52 rad. The amplitude of the vibration is calculated as 1.60 mm. At certain instant, a combination of phase values on each pixel produces a wrapped spatial phase map as shown in Fig. 6(a). After applying conventional unwrapping algorithm [28], the continuous phase map, which is proportional to the surface profile, can be obtained. Fig. 6(b) shows the continuous phase map of the object surface. Divided by coefficient k in Eq. (2), the phase value can be converted into a 3-D surface profile as shown in Fig. 6(c).

To verify the accuracy of the surface profile measurement by the proposed method, a comparison is carried out between mechanical stylus method and the proposed temporal analysis method. Fig. 7 shows a comparison of the profile on cross section B-B (indicated in Fig. 3a). The average discrepancy is 3.1%. Errors are introduced when smooth filter is applied on the images to remove the high frequency grating lines before applying the proposed algorithm. Furthermore, relatively large errors in the profile also occur when the gray value approaches the extreme values. These errors are introduced by: (1) The maximum and minimum values detected by the camera are slightly different from the actual extreme gray scale values in each cycle of phase variation (2π change) along time-axis; (2) For a sine-wave configuration, a slight difference in gray level near the extreme values causes a large change in phase value.

The proposed method is also applied on a coin of 24.5 mm diameter and having a diffuse surface (shown in Fig. 8). A small area of interest containing 256×256 pixels (also indicated in Fig. 8) is cropped. A grating with difference pitch (6 lines/mm) is used to increase the resolution. The distance d and l (shown in Fig. 1) are respectively 245 and 250 mm. The test object is subjected to a triangular wave vibration with a frequency of approximately 5 Hz. The camera recording rate remains at 250 fps. Fig. 9(a) and (b) shows two typical fringe patterns recorded by high speed CCD camera at different moments. Carrier fringes are also introduced on the images by rotating the grating. One hundred consecutive images are selected for processing. The process is similar with the spherical cap mentioned above. Fig. 9(c) and (d) are typical fringe patterns after filtering. Fig. 10 shows the vibration amplitude of point C (indicated in Fig. 8). The frequency and the amplitude of vibration are evaluated as 4.8 Hz and 0.258 mm, respectively. Fig. 11(a)

and (b) shows the wrapped phase map and continuous map after unwrapping. Subsequently the 3-D profile of the interest area is obtained as shown in Fig.11(c). Fig. 12(a) shows a comparison of profile plot on cross section D-D (indicated in Fig.8) using proposed phase scanning method and mechanical stylus method. The average discrepancy is 4.7%. The maximum difference is around $10\mu\text{m}$ which is the same order of error as that of phase shifting method [8-10] where at least three images are required.

As the surface profile of the coin is much smaller than the spherical cap, it is found that increasing the carrier fringe frequency improves the quality of the phase map. However, it is observed that the contrast of the moiré fringes changes with the distance between the object and grating. High contrast fringes are generated when the distance is small. This is due to the diffraction effect of the grating. From Fig. 9(a) and (b) it is observed that the fringe contrast increases from left to right, which implies the distance between the coin and grating decreases in this direction. Higher sensitivity can also be obtained by increasing the distance d , but this will also generate shadow on the object.

As can be seen, a high resolution surface profile of a vibrating object can be retrieved with good accuracy unlike the normal shadow moiré method. However, it is note worthy that the proposed technique is limited to measurement of out-of-plane displacement. Recording rate of the camera should be much larger than the vibration frequency such that enough data points on each pixel can be obtained within one cycle of intensity variation.

5. Concluding Remarks

This paper presents a simple method to retrieve the surface profile from a slow vibrating object based on shadow moiré technique. Normally shadow moiré technique is

well adapted on contouring objects with variation of relief in the order of millimeters or even larger (for example, the spherical cap shown in this paper). With this method, the range of the measurement has been extended to sub-millimeter and accuracy of 10 μm achieved. A high-resolution contour map of a small coin has been obtained. Unlike the conventional phase evaluation techniques, this method analyses the phase value point-by-point along the time-axis. High quality surface profiles are retrieved from a vibrating object, where normal phase-shifting methods are not applicable. With further development, the algorithm may be extended to other optical techniques such as moiré interferometry and electronic speckle pattern interferometry (ESPI) for the study of dynamic problems.

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Fig. 12. A comparison of surface profile of 50-cent coin on cross-section D-D between phase scanning method and mechanical stylus method.

Table 1

Phase Value in [0, π]	Direction of Vibration or Deformation	Slope of Intensity along Time Axis $\frac{\partial I}{\partial t}$	Phase Value in [0, 2π] after conversion
ϕ	Forward	≥ 0	$2\pi - \phi$
		< 0	ϕ
	Backward	≥ 0	ϕ
		< 0	$2\pi - \phi$

Figure 1

Figure 2

(a)

(b)

Figure 3

(a)

(b)

(c)

(d)

Figure 4

(a)

(b)

(c)

Figure 5

(a)

(b)

(c)

Figure 6

Figure 7

Figure 8

(a)

(b)

(c)

(d)

Figure 9

Figure 10

(a)

(b)

Figure 11

Figure 12