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Directional Gradient Vector Flow for Snakes

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Abstract— *Snakes, or active contour models, have been widely used in image segmentation. However, most present snake models cannot discern between positive and negative step edges. In this paper, a new type of dynamic external force for snakes named* directional gradient vector flow *(DGVF) is proposed to solve this problem by incorporating directional gradient information. It makes use of the gradients in both x and y directions and deals with the external force field for the two directions separately. In snake deformation, the DGVF field is utilized dynamically according to the orientation of snake in each iteration. Experiment results demonstrate that the DGVF snake provides a better segmentation than GVF snake in situations when edges of different directions are present and may pose confusion for segmentation.*

Keywords— *Snakes, gradient vector flow, image gradient, segmentation.*

I. Introduction

Snakes was first proposed by Kass *et al.* [1]. Since its publication, deformable models have become one of the most active and successful research areas in image segmentation [2]. As a parametric deformable model, snakes have been widely applied in boundary detection, shape modeling, and motion tracking etc. Various improvements of snakes have been proposed, such as balloons [3], topology adaptive snakes (T-snakes) [4], gradient vector flow (GVF) [5], and generalized GVF (GGVF) [6]. However, very few literature are available on the problem of the direction of gradient. In this paper, a new method incorporating directional gradient information is proposed to improve the effectiveness of snakes.

This paper is organized as follows. In the next section, a revision of the traditional snake and the GVF snake is given. In Section 3, the DGVF algorithm is presented in detail. The experimental results are given in Section 4 and the conclusion Section 5.

II. Background

A. Traditional Snakes

A snake is a curve $\mathbf{x}(s)$ = [$x(s)$, $y(s)$], $s \in$ [0,1], which moves through the spatial domain of an image to minimize the following energy function:

$$E(\mathbf{x}) = \int_0^1 [\frac{1}{2}(\alpha|\frac{\partial \mathbf{x}}{\partial s}|^2 + \beta|\frac{\partial^2 \mathbf{x}}{\partial s^2}|^2) + E_{ext}(\mathbf{x})]\mathrm{d}s \quad (1)$$

In Eqn. (1), the first two terms comprise the internal energy of the snake. The first-order derivative controls stretching and the second-order derivative controls bending. α and β are the weighing parameters controlling the snake's tension and rigidity respectively. The external energy E_{ext} is derived from the image and set to small values at the features of interest. As object boundaries are usually of high gradient in the image I, a typical example of external energy for step edges is $-|\nabla(G_\sigma(x, y) * I(x, y))|^2$ [1].

According to Euler-Lagrange equation, to minimizes E the deformable contour has to evolve dynamically as a function of time t given by:

$$\frac{\partial \mathbf{x}}{\partial t} = \frac{\partial}{\partial s}(\alpha \frac{\partial \mathbf{x}}{\partial s}) - \frac{\partial^2}{\partial s^2}(\beta \frac{\partial^2 \mathbf{x}}{\partial s^2}) + F_{ext}(\mathbf{x}). \quad (2)$$

where the external force F_{ext} is derived from external energy and set so as to attract the snake to strong edges:

$$F_{ext}(\mathbf{x}) = -\nabla E_{ext}(\mathbf{x}). \quad (3)$$

There are two key drawbacks associated with traditional snakes. First, the initial position of the snake must be close enough to the desired contour in the image. Otherwise the snake may be trapped in local minima instead of evolving correctly toward the desired contour. Second, poor convergence may result as the snake has difficulty evolving to concavities or sharp corners.

B. GVF Snakes

To solve the problem of limited capture range and poor convergence, Xu and Prince proposed gradient vector flow (GVF) as a new external force for snakes [5], [7]. Starting from Eqn. (3), the external force $-\nabla E_{ext}(\mathbf{x})$ is replaced with a GVF field $\mathbf{v}(x, y) = [u(x, y), v(x, y)]$ defined as the equilibrium solution of the following system of partial differential equation:

$$\mathbf{v}_t = \mu \nabla^2 \mathbf{v} - (\mathbf{v} - \nabla f)\ |\nabla f|^2, \quad \mathbf{v}_0 = \nabla f \quad (4)$$

where $\mathbf{v}_t$ denotes the partial derivative of $\mathbf{v}$ with respect to time t, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian operator. f is an edge map derived from the image and defined to have large values at the features of interest. A typical choice of f is

$$f(x, y) = -E_{ext}(x, y) = |\nabla(G_\sigma(x, y) * I(x, y))|^2 \quad (5)$$

for step edges.

As the GVF field is calculated as a diffusion of the gradient vectors of an edge map derived from the image, it greatly increases the capture range of the snake and the ability to move into boundary concavities.

III. Directional GVF

In snakes, the role of external force is to attract the deformable contour to the features of interest in an image. Both traditional snakes and GVF snakes define their external energy to be a function of $|\nabla I|$, the gradient magnitude of the image, which is a conventional step edge detector. As the magnitude operator discards the signs of gradients, the snake is unable to distinguish between positive and negative step edges. Park *et al.* proposed a method to improve the active contour model by including gradient direction in the external image force [8]. However, this definition of the external force highly depends on the initial location of the snake in the image.

In this paper, a new approach using directional gradient vector flow (DGVF) is described for snakes to distinguish between positive boundary and negative boundary. For gray-level images, a boundary is defined to be positive if there are positive step edges along its outward normals, i.e. the intensity gradients along the boundary are pointing inward. Contrarily, a boundary is defined to be negative if there are negative step edges along its outward normals.

A. Directional Edge Map

As aforementioned, the solution of GVF field is based on the edge map f in Eqn. (5). In the proposed method, a new edge map is used to preserve the gradient directional information:

$$\mathbf{g}(x,y) = \nabla(G_\sigma(x,y) * I(x,y)) = (g_x(x,y),\ g_y(x,y)) \quad (6)$$

where g_x and g_y are the horizontal and vertical gradients of the image I after it is smoothed by a two-dimensional Gaussian function G_σ. Subsequently the DGVF field is solved in the horizontal and vertical directions separately.

Considering a one-dimensional signal, there are two opposite directions to look through it: x and -x. Suppose in the x direction, d1 is a positive step edge and d2 is negative. Then in the -x direction the situation is reversed: d2 is a positive step edge and d1 is negative. Thus, if only positive (or negative) edges are to be detected, the result obtained will depend on the direction in which the signal is looked through.

Similarly in two-dimensional case, detection of a positive (or negative) boundary is dependent on the direction looked through, which associates with the deformable contour's normal direction at each snaxel (snake element). Since the location of snake is unknown before initialization, image gradients from all directions are considered. For positive boundary:

$$f_x^+(x,y) = max\{g_x(x,y), 0\} \quad (7a)$$
$$f_x^-(x,y) = -min\{g_x(x,y), 0\} \quad (7b)$$
$$f_y^+(x,y) = max\{g_y(x,y), 0\} \quad (7c)$$
$$f_y^-(x,y) = -min\{g_y(x,y), 0\} \quad (7d)$$

and for negative boundary:

$$f_x^+(x,y) = -min\{g_x(x,y), 0\} \quad (8a)$$
$$f_x^-(x,y) = max\{g_x(x,y), 0\} \quad (8b)$$
$$f_y^+(x,y) = -min\{g_y(x,y), 0\} \quad (8c)$$
$$f_y^-(x,y) = max\{g_y(x,y), 0\} \quad (8d)$$

where f_x^+, f_x^-, f_y^+ and f_y^- are the gradients of positive step edges in x, -x, y and -y directions, and they make the directional edge map $\mathbf{f}(x,y) = [f_x^+(x,y), f_x^-(x,y), f_y^+(x,y), f_y^-(x,y)]$.

B. Directional GVF Field

The DGVF field has four components: $\mathbf{v}(x,y) = [u^+(x,y), u^-(x,y), v^+(x,y), v^-(x,y)]$. These components for four directions are found by solving the following partial differential equations:

$$\mathbf{v}_t = \mu\nabla^2\mathbf{v} - (\mathbf{v} - \mathbf{df})\ \mathbf{df}^2, \quad \mathbf{v}_0 = \mathbf{df} \quad (9)$$

where $\mathbf{df} = [df_x^+, df_x^-, df_y^+, df_y^-]$, and

$$df_x^+ = \frac{\partial}{\partial x} f_x^+ \quad (10a)$$
$$df_x^- = \frac{\partial}{\partial x} f_x^- \quad (10b)$$
$$df_y^+ = \frac{\partial}{\partial y} f_y^+ \quad (10c)$$
$$df_y^- = \frac{\partial}{\partial y} f_y^- \quad (10d)$$

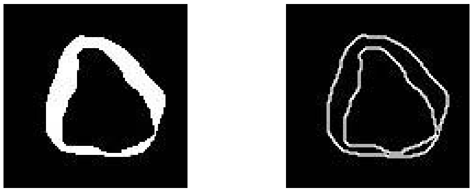
or write Eqn. (9) separately

$$u_t^+ = \mu\nabla^2 u^+ - (u^+ - df_x^+)(df_x^+)^2, \quad u_0^+ = df_x^+ \quad (11a)$$
$$u_t^- = \mu\nabla^2 u^- - (u^- - df_x^-)(df_x^-)^2, \quad u_0^- = df_x^- \quad (11b)$$
$$v_t^+ = \mu\nabla^2 v^+ - (v^+ - df_y^+)(df_y^+)^2, \quad v_0^+ = df_y^+ \quad (11c)$$
$$v_t^- = \mu\nabla^2 v^- - (v^- - df_y^-)(df_y^-)^2, \quad v_0^- = df_y^- \quad (11d)$$

The four equations in Eqn. (11) are decoupled, and therefore can be solved as separate scalar partial differential equations in u^+, u^-, v^+ and v^-. Compared with Eqn. (4), Eqn. (9) uses $\mathbf{df}^2$ instead of $|\nabla f|^2$, ensuring that u^+, u^-, v^+ and v^- are decoupled from each other. The four directions have to be assessed as the snake's orientation cannot be determined at this stage.

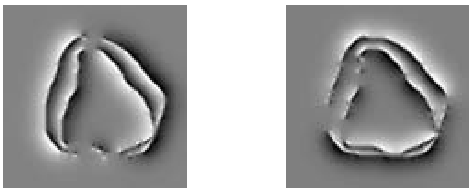
C. Snake Deformation

The external force of snakes can be classified as static or dynamic forces [5]. Static forces are computed from the image data and do not change as the snake deforms. Dynamic forces are associated with the snake and therefore change as the snake deforms. For the traditional snake, external forces and GVF are both static external forces. Parker's method [8] also uses static forces, whose performance is easily influenced by the initial location of the snake. An example of dynamic external force is the pressure force used in balloons [3], which causes the snake to inflate or deflate by a constant force along the normal direction of the contour.



(a) (b)

Fig. 1. (a) The original image of step edges; (b) edge map of GVF: gradient magnitude.



(a) (b)

Fig. 2. (a), (b) GVF field in x and y direction respectively (before normalized with respect to their magnitudes).

The DGVF field **v** is derived from the image as well as GVF field, but it cannot be directly applied to the snake as a static external force. For each snaxel in deformation, the external force which it is subject to depends on its location in the snake and the shape of the snake. Hence the DGVF field is essentially a dynamic external force.

Let θ be the contour's normal direction at a certain snaxel, then $\cos(\theta)$ is the normal vector's component in the x direction, and $\sin(\theta)$ is the normal vector's component in the y direction. If $\cos(\theta)$ is larger/smaller than zero, u^+/u^- should be the horizonal external force F_x at that snaxel. Similarly, If $\sin(\theta)$ is larger/smaller than zero, v^+/v^- should the vertical external force F_y at that snaxel. Hence the snake is deformed under the external force $F_{ext} = [F_x, F_y]$:

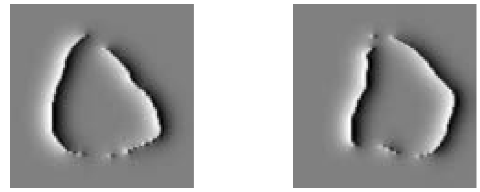
$$F_x = u^+ * max\{\cos(\theta), 0\} - u^- * min\{\cos(\theta), 0\} \quad (12a)$$

$$F_y = v^+ * max\{\sin(\theta), 0\} - v^- * min\{\sin(\theta), 0\} \quad (12b)$$

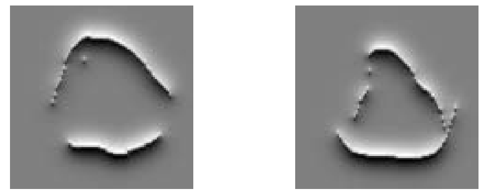
IV. Experimental Results

In this section, the performance of the GVF snake and the DGVF snake are compared. All the edge maps used in snake are normalized to the range [0, 1]. The snakes are dynamically reparameterized during deformation and the distances between neighboring snaxels are maintained within 0.5-1.5 pixels.

The original image is a binary image of an irregular white loop in a black background (Fig. 1(a)). It is noted that in the edge map of GVF (Fig. 1(b)), the inner boundary and the outer boundary of the loop both are of high intensity, because of the magnitude operator. Therefore these two boundaries are indistinguishable in the resultant GVF field (Fig. 2), as well as in the GVF edge map. In comparison, the DGVF field has four components corresponding to the four directions (Fig. 3), and thus makes the inner boundary and outer boundary differentiable for snakes.



(a) (b)



(c) (d)

Fig. 3. The DGVF field in x, -x, y and -y directions for positive step edges.

Snakes, initialized as circles of different radii, deform with the GVF and DGVF fields to detect the boundaries in the original image. When the initial contour is not far away from the positive and negative boundaries, it is found that the GVF snake is confused at those regions where the width of the loop is narrow (Fig. 4(a)). For two boundaries both of high gradient, the snake is attracted to the boundary which is nearer to the initial boundary. When the initial contour is far away from the desired boundaries, the GVF snake converges to the nearer boundary. However at the regions where the two boundaries are too close, the snake is also affected by the GVF field which pulls it to the farther boundary. As a result, the snake stays in the middle of the two boundaries (Fig. 4(b)). On the other hand, the DGVF snake is able to detect the positive boundary (Fig. 4(c), Fig. 4(d)) or negative boundary (Fig. 4(e), Fig. 4(f)) unhesitatingly and independent of the initial contour.

V. Conclusion

A new type of dynamic external force for snakes called directional gradient vector flow (DGVF) is proposed. The method generates the external force field in x and y directions separately to preserve the directional gradient information. Since the DGVF field is utilized dynamically according to the orientation of snake in deformation, the DGVF snake is attracted to positive or negative step edges exclusively, regardless of the proximity of the two types of edges. This algorithm is particularly useful for snake-based image segmentation with complex background.

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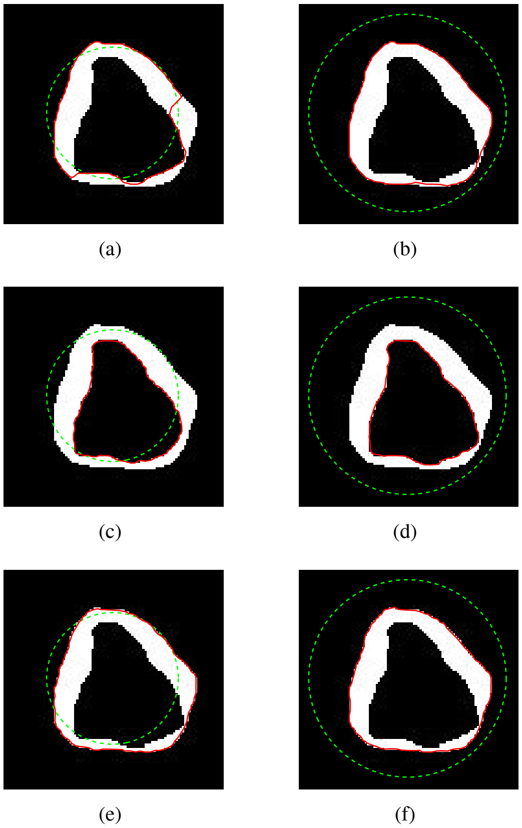

Fig. 4. Boundary detection performance of GVF and DGVF snake. The circles of dashed line are initial snake positions and the contours of solid line are final results of the snakes. (a), (b) result of the GVF snake; (c), (d) results of the DGVF positive boundary-searching snake ; (e), (f) results of the DGVF negative boundary-searching snake.

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