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Vibration measurement by temporal Fourier analyses of a digital hologram sequence

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A method for whole-field noncontact measurement of displacement, velocity, and acceleration of a vibrating object based on image-plane digital holography is presented. A series of digital holograms of a vibrating object are captured by use of a high-speed CCD camera. The result of the reconstruction is a three-dimensional complex-valued matrix with noise. We apply Fourier analysis and windowed Fourier analysis in both the spatial and the temporal domains to extract the displacement, the velocity, and the acceleration. The instantaneous displacement is obtained by temporal unwrapping of the filtered phase map, whereas the velocity and acceleration are evaluated by Fourier analysis and by windowed Fourier analysis along the time axis. The combination of digital holography and temporal Fourier analyses allows for evaluation of the vibration, without a phase ambiguity problem, and smooth spatial distribution of instantaneous displacement, velocity, and acceleration of each instant are obtained. The comparison of Fourier analysis and windowed Fourier analysis in velocity and acceleration measurements is also presented. © 2007 Optical Society of America

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1. Introduction

In many areas of industry, vibration measurement is done to create the data that are necessary to draw meaningful and significant conclusions about the system being tested. It is more important for noise, fatigue, or condition monitoring. Transducers, such as velocity pickups and accelerometers, are generally used for vibration measurement and analyses. However, measurement by a transducer is a contact point-wise method and is difficult to apply on a small object as the mass of the transducer can also change the vibration behavior. Since the 1960s, optical interferometry has been applied for whole-field, noncontact vibration measurement [1]. If the test object vibrates at a sinusoidal frequency whose period is many times smaller than the exposure time, vibrational amplitude related [2] or vibrational amplitude-gradient phase fringes [3] will be reconstructed, depending on the optical layout (time-average method). In many cases, high resolution three-dimensional (3-D) displacement

or surface profiling of objects under vibration or continuous profile change can give useful information about the dynamic response and deformation of the objects concerned. However, it is difficult to achieve this with the time-average method. The use of a twin-cavity double-pulsed laser in interferometry [4,5] has been reported as an alternative to obtain transient deformations. However, this technique has an important limitation. To obtain evolution of the transient deformation, an experiment must be repeated many times with a different interval of two pulses. This means nonrepeatable events cannot be studied in detail.

Because of the rapid development of high-speed digital recording devices, it is now possible to record interferograms with rates exceeding 100,000 frames/s. Retrieval of precise spatial phase maps from these transient interferograms enables measurement of instantaneous 3-D profiles, deformations as well as dynamic responses. Different methods are used to analyze this interferogram sequence. The instantaneous phase distributions can be retrieved by spatial frequency analysis [6] or by fast phase shifting [7], followed by a phase unwrapping process along the time

axis [8]. However, the former method suffers when the spatial carrier does not exist, whereas the latter requires a high-speed phase shifter synchronized with a high-speed camera in the experiment, which is not easily realized. Other methods are based on temporal phase analysis techniques [9,10] in which each pixel in the interferograms is analyzed independently as a function of time. A Fourier transform is typically applied to extract the phase in the temporal domain [11,12]. In recent years, wavelet analysis has also been introduced in temporal phase extraction [13]. Previous research [10,14] has shown that wavelet analysis can limit the influence of various noise sources and improve the result in displacement measurement. In addition, wavelet analysis can extract the instantaneous frequency of the temporal intensity variation from which the velocity of the deformation can be evaluated [10]. The temporal phase analysis technique also has the advantage of eliminating speckle noise, as it evaluates the phase pixel by pixel along the time axis. However, it does have a disadvantage: if only the intensity variations of pixels are analyzed, it cannot extract the phase from a part of an object that is not moving with the rest or from objects that deform in different directions at different parts. Determination of the absolute sign of the phase is impossible by both temporal Fourier and wavelet analysis. This limits the technique to the measurement of deformation in one known direction. Adding a temporal carrier [14–16] to the image acquisition process is a method to overcome these problems. However, the carrier frequency limits the measurement range of phase variation that is due to the constraints of Nyquist sampling and the acquisition speed of a high-speed camera [14].

In the past decade, a novel computer-aided optical technology, digital holography [17], has been successfully applied in different types of measurement. This is due to the development of CCD, or complementary metal oxide semiconductor sensors, and the capabilities of computers. It is now possible to record holograms directly with a camera and to reconstruct the object digitally by computer simulation. The result of digital reconstruction is not only the amplitude of the object such as conventional holography, electronic speckle pattern interferometry (ESPI), and shearography, but the phase of the object as well. This advantage makes digital holography more suitable for dynamic measurement. In addition, no phase ambiguity problem exists in further processing of the reconstructed phase. Different dynamic measurement techniques based on digital holography combined with a pulsed ruby laser [5,18] and time-average method [19–21] have been reported. Previous reports [22,23] described a method for measuring dynamic events in which digital holograms of an object are recorded on a high-speed sensor and the phase of the wavefront recorded at different instants is calculated from the recorded intensity by use of a two-dimensional (2-D) digital Fourier transform method [18,24]. By unwrapping the phase in the temporal domain it is possible to obtain the displacement, in-

cluding the direction of a vibrating object as a function of time.

In vibration analysis, displacement is not the only key parameter to be measured. Velocity and acceleration are two other items in the characteristic equation of vibration. Here we record a sequence of digital holograms of a vibration object by using a high-speed camera, and these holograms are evaluated by the above-mentioned 2-D digital Fourier transform method [24]. The result is a 3-D matrix of complex amplitude. Further processing of this matrix makes it possible to evaluate the displacement, velocity, and acceleration simultaneously. In this investigation, a windowed Fourier transform (WFT) [25,26] is selected as the main processing technique to extract the displacement, velocity, and acceleration from the reconstructed complex amplitudes. The displacement is calculated by 2-D spatial windowed Fourier filtering of the instantaneous phase map followed by temporal phase unwrapping. The velocity and acceleration are evaluated by two methods: (1) extracting the windowed Fourier ridges and (2) numerical differentiation followed by a Fourier low-pass filter along the time axis. A combination of the results of the temporal analysis on each pixel at any instant will give an instantaneous distribution of displacement, velocity, and acceleration. The comparison of Fourier analysis and windowed Fourier analysis in velocity and acceleration measurements is also presented.

2. Theoretical Analysis

A. Image-Plane Digital Holography

A schematic layout of image-plane digital holography that is sensitive to out-of-plane displacement is shown in Fig. 1. The laser light is split into a beam for illumination of the object beam and a reference beam. The object beam illuminates a specimen with a diffuse surface along a direction e_i . Some light is scattered in the observation direction e_o , where an image-plane hologram is formed on the CCD sensor as a result of the interference between the reference beam and the object beam. The aperture serves to limit the spatial frequencies of the interference pattern. The reference beam is introduced by a single-mode optical fiber and diverges toward the detector from a point located

Fig. 1. Schematic layout of the experimental setup.

close to the aperture. Let $R(x, y)$ and $U(x, y)$ denote the reference and object waves, the intensity recorded on the CCD sensor can then be expressed as

$$I_H(x, y) = |R_H(x, y)|^2 + |U_H(x, y)|^2 + R_H(x, y) \times U_H^*(x, y) + R_H^*(x, y)U_H(x, y), \quad (1)$$

where subscript H indicates that these values are in the image plane and the $*$ denotes the complex conjugate. If $\theta_{\max}$ is the maximum angle between the object and the reference beams, the maximum spatial frequency in the hologram is

$$f_{\max} = \frac{2}{\lambda} \sin\left(\frac{\theta_{\max}}{2}\right), \quad (2)$$

where λ is the wavelength of the laser. The experimental setup should be properly designed so that the Nyquist sampling theorem is satisfied over the full area of the detector. In our case it is $f_{\max} < 1/(2\Delta)$, where Δ denotes the pixel size of the camera.

The last two terms in Eq. (1) contain information about the amplitude and phase of the object wave. This information can be obtained by spatial filtering using the Fourier-transform method. The detailed description of Fourier evaluation of a digital image-plane hologram can be found in Ref. [24]. Figure 2(a) shows a typical digital hologram of a vibrating cantilever beam captured by the high-speed camera indicated in Fig. 1. By taking the Fourier transform of the recorded intensity it is possible to filter out one of the last two terms in Eq. (1). The Fourier spectrum and filtering window are shown in Fig. 2(b). After an

inverse Fourier transform, the complex amplitude of the wavefront, $U_H(x, y)$, is obtained. When a series of digital holograms is captured during the deformation or vibration of an object, a sequence of $U_H(x, y; t)$ is obtained. In this investigation, the obtained 3-D complex matrix is the original measurement data from which the displacement, velocity, and acceleration of the testing object are evaluated.

It is well known that deformation of an object involves a change in the phase of the object beam. The relationship between the phase change $\Delta\phi = \phi_{t_2} - \phi_{t_1}$ and the out of displacement z is given by

$$\Delta\phi = \frac{2\pi z}{\lambda} \cdot \mathbf{S}, \quad (3)$$

where $\mathbf{S} = \mathbf{e}_i - \mathbf{e}_o$ is the sensitivity vector given by the geometry of the setup, and $\mathbf{e}_i$ and $\mathbf{e}_o$ are the unit vectors of illumination and observation, respectively. The phase difference between two digital holograms recorded at t_1 and t_n can be calculated by

$$\Delta\phi = \arctan \frac{\text{Im}[U(x, y; t_n)U^*(x, y; t_1)]}{\text{Re}[U(x, y; t_n)U^*(x, y; t_1)]}, \quad (4)$$

where Re and Im denote the real and imaginary parts, respectively, of the complex value. Figure 2(c) shows a typical instantaneous phase difference on a vibrating cantilever beam. The $\Delta\phi$ obtained from Eq. (4) is within $[0, 2\pi)$, and phase unwrapping in the time axis [8] allows for determination of the total displacement since the first frame of the measurement process, as opposed to the displacement relative to some other points in the field of view, is all that can be achieved with spatial unwrapping. If the first hologram is considered as a reference, we will have a series of $U(x, y; t_n)U^*(x, y; t_1)$ whose phase values after temporal unwrapping, $\phi(x, y; t_n)$, are proportional to the displacement at instant t_n and a series of $U(x, y; t_n)U^*(x, y; t_{n-1})$ whose phase values are proportional to the numerical differentiation of the displacement. In this application, two algorithms based on Fourier-transform filtering and windowed Fourier ridge methods are applied temporally to evaluate the velocity and acceleration pixel by pixel.

B. Fourier Analysis and Windowed Fourier Analysis

1. Temporal Fourier Analysis

Fourier transform (FT) with low-pass filtering [27] is one of the most popular filtering methods in image processing. In this investigation, the reconstructed phase value $\phi(x, y; t)$ of point $P(x, y)$ on a vibrating object is unwrapped along the time axis and is followed by a numerical differentiation. The first derivative of the phase, $\vartheta(x, y; t) = d\phi/dt$ is then converted to an exponential signal, $\exp(j \cdot \vartheta)$, where $j = \sqrt{-1}$. The spectrum of this exponential signal will contain values around zero frequency, with certain bandwidth from negative to positive frequencies provided ϑ is within the range of $(-\pi, \pi)$. The width of the spectrum depends on the value of the second derivative of the phase, in this case the acceleration of vi-

Fig. 2. (a) Typical digital hologram obtained on a cantilever beam; (b) spectrum of the digital hologram obtained; (c) typical original wrapped phase map indicating the relative displacement of the beam.

Fig. 3. (a) Simulated phase variation with noise containing two vibration frequencies; (b) first derivative of (a) with and without noise; (c) first derivative of (a) after low-pass filtering; (d) first derivative of (a) obtained with a windowed Fourier ridge.

bration. Low-pass filtering is then applied, followed by an inverse FT. The obtained wrapped phase needs to be unwrapped again along the time axis. Figure 3(a) shows a simulated phase variation with some additive random noise. The signal includes two frequencies, 80 and 120 points/frame. If the time interval between two adjacent frames were 1 s, the maximum velocity measurement range of this method would be within $(-\pi, \pi)$ rad/s because of the constraint of the Nyquist sampling theorem. Figure 3(b) shows the numerical differentiation of the signal and the smooth line that is the first derivative of the ideal value without noise. The result from the above-mentioned Fourier analysis is shown in Fig. 3(c). A similar process is repeated to obtain the acceleration. Figure 4(a) shows the second derivative of the signal

with and without noise. This time the spectrum of the exponential signal is much narrower and a relatively small filtering window size can be selected. Figure 4(b) is the second derivative of the phase variation obtained by Fourier analysis. If the movement of the object is not a rigid body motion, each point will have a different spectrum compared with others. The main problem involved in Fourier analysis will be determination of the filtering window size. For a wider filtering window more noise will be included in the result. However, it is not practical to have an adaptive filtering window for each pixel. This problem is more obvious in velocity measurements, as the spectrum of exponential signal is wider in this case. It can be observed from the simulation that the acceleration curve [Fig. 4(b)] obtained by Fourier analysis is

Fig. 4. Second derivative of Fig. 3(a) (a) with and without noise, (b) after low-pass filtering, (c) obtained with a windowed Fourier ridge.

smoother than the velocity curve [Fig. 3(c)]. Compared with the ideal signal, the relative error [(calculating value – ideal value)/(ideal value)] is more than 10% in the velocity measurement but only 3% in the acceleration evaluation.

2. Windowed Fourier Ridge

A one-dimensional (1-D) windowed Fourier transform (WFT) and the inverse windowed Fourier transform (IWFT) can be written as [28]

$$Sf(u, \xi) = \int_{-\infty}^{+\infty} f(t) g_{u,\xi}^*(t) dt, \quad (5)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} Sf(u, \xi) g_{u,\xi}(t) d\xi du, \quad (6)$$

where $f(t)$ is the original signal, $Sf(u, \xi)$ denotes the spectrum of the WFT, and $g_{u,\xi}(t)$ is the WFT kernel that can be expressed as

$$g_{u,\xi}(t) = g(t - u) \exp(j\xi t). \quad (7)$$

The window $g(t)$ is usually chosen as a Gaussian function:

$$g(t) = \exp(-t^2/2\sigma^2), \quad (8)$$

which permits the best time-frequency localization in analysis. Parameter σ controls the extension of $g(t)$. On point $P(x, y)$ of the object, we calculate a series of phase values $\Delta\phi(x, y; t_n) = \arg[U(x, y, t_n)U^*(x, y, t_1)]$, $n = 1, 2, \dots, N$, where N is the total frame number captured, and we convert the unwrapped phase φ_P to an exponential signal $C_P = \exp(j \cdot \varphi_P)$. The WFT of this complex signal will be [28]

$$SC_p(u, \xi) = \frac{\sqrt{s}}{2} A_{xy}(u) \exp\{j[\varphi(u) - \xi u]\} \times (\hat{g}\{s[\xi - \varphi'(u)]\} + \varepsilon(u, \xi)), \quad (9)$$

where u and ξ represent the time and frequency, respectively, and s is a scaling factor. For a fixed s , $g_s(t) = s^{-1/2}g(t/s)$ has a support size of s . $A_{xy}(u)$ is the modulus of C_p [in our case, $A_{xy}(u) = 1$] and $\varepsilon(u, \xi)$ is a corrective term that can be neglected if $A(u)$ and $\varphi'(u)$ have small relative variations over the support of window g ; $\hat{g}(\omega)$ denotes the FT of $g(t)$. The trajectory of maximum $|SC_p(u, \xi)|^2$ on the u - ξ plane is called a windowed Fourier ridge. Since $|\hat{g}(\omega)|$ is maximum at $\omega = 0$, and if ε is negligible, $|SC_p(u, \xi)|^2$ reaches a maximum when

$$\xi(u) = \varphi'(u), \quad (10)$$

where $\varphi'(u)$ is defined as the instantaneous frequency of the signal, which is proportional to the velocity of $P(x, y)$. Equation (10) is valid based on the assumption that the phase of the signal is linear in the local area determined by the extension of g . However, the real phase of a vibrating object is rarely linear, and hence a linear phase approximation error exists. Obviously the assumption is better satisfied when the window size is small. According to our evaluation, the relative error of $\varphi'(u)$ that is due to linear phase approximation will be less than 1% when $\sigma \leq 3$, but around 10% when $\sigma = 10$. However, a large window size is more robust against noise. The choice of window size should be based on a trade-off between accuracy of the phase approximation and immunity to noise. In our case, the linear phase approximation error is our main concern as the noise is not so obvious in the temporal signal. Figure 3(d) shows the modulus of the WFT of the simulated signal [Fig. 3(a)]. The dashed line pinpoints the ridge where the maximum moduli are found. The $\sigma = 4$ is selected as the window size. An accurate velocity is obtained with a relative error of less than 2%. Similarly, the acceleration is evaluated by extracting the instantaneous frequency of the signal in Fig. 3(b). This time the window size is selected as $\sigma = 8$. Figure 4(c) shows the windowed Fourier ridge. Although it is also a smooth signal, a slight offset in frequency can be observed and a relative error of 7% is found compared with the ideal acceleration.

3. Windowed Fourier Filtering

In digital holography, the displacement can be directly evaluated by Eq. (4). However, the result is a series of wrapped phase maps with speckle noise. In this study, each instantaneous wrapped phase map is filtered by 2-D WFT. Details of the filtering are given in Refs. [25] and [26]. A comparison of the WFT and the FT has already shown that the WFT performs better than the FT in 2-D and 3-D cases [25,29]. These filtered phase maps are then unwrapped temporally to obtain the absolute displacement.

3. Experimental Illustration

Figure 1 shows the experimental setup. The specimen tested in this study is a copper cantilever beam with a diffuse surface. The length, width, and thickness of the beam are 85, 10 and 0.3 mm, respectively. A shaker produces a sinusoidal vibration with a frequency of 26 Hz at the free end of the beam. The frequency of vibration is controlled by a function generator, and only the area near the clamping end is measured. The beam of a diode-pumped solid-state laser (Coherent Verdi V10, 10 W, $\lambda = 532$ nm) is divided into an object beam and a reference beam. The cantilever beam is illuminated at an angle of 40° and is imaged at a right angle using a positive lens by the high-speed camera sensor. With this arrangement, a 1 rad phase change is equivalent to a 45.1 nm displacement in the z direction. The interference be-

tween the reference and the object beams generates an image-plane hologram on the camera sensor. The camera used is a Basler A504k with a pixel size of $12 \mu\text{m} \times 12 \mu\text{m}$. The imaging rate is 2000 frames/s. Two hundred image-plane holograms are captured during a 0.1 s period. The area captured on the cantilever beam contains 1280×256 pixels and has a length and width of 15 and 3 mm, respectively. Figure 2(c) is a typical wrapped phase map obtained by Eq. (4). The area of 900×256 pixels [as shown in Fig. 2(c)] has been selected for FT and WFT processing.

4. Results and Discussion

Figure 5(a) is the original wrapped phase $\Delta\phi$ of frame No. 70 ($t = 0.0345$ s) in the sequence. The phase after 2-D WFT filtering is shown in Fig. 5(b). It is observed that the WFT filter performs well and a smooth wrapped phase is obtained. Similarly it is applied to all the original wrapped phase maps. After temporal phase unwrapping, the absolute displacement at any instant can be evaluated. Figure 5(c) shows the displacement of the cantilever beam at an instant $t = 0.0345$ s.

Figure 6(a) shows the original phase variation of point A [indicated in Fig. 2(c)] after temporal unwrap-

Fig. 5. (a) Typical original wrapped phase at instant $t = 0.0345$ s, (b) wrapped phase after 2-D windowed Fourier filtering, (c) 3-D plot of displacement at instant $t = 0.0345$ s.

Fig. 6. (a) Original phase variation of point A obtained by the proposed digital holography, (b) velocity on point A evaluated with a windowed Fourier ridge, (c) first derivative of (a) obtained by numerical differentiation, (d) velocity on point A obtained by low-pass filtering.

ping. The phase values of $U(x, y; t_n)U^*(x, y; t_1)$ at point A are processed by the windowed Fourier ridge method. Figure 6(b) is the spectrogram of the WFT. As the signal is not so noisy, $\sigma = 2$ is selected as a window size to limit the linear phase approximation error. The velocity is evaluated from the ridge. With the experimental setup mentioned in Section 3, 1 rad/frame is equivalent to a velocity of $90.2 \mu\text{m/s}$. The maximum velocity of point A is around $200 \mu\text{m/s}$. Figure 6(c) shows the numerical differentiation of the phase variation on point A. It is converted to the exponential signal followed by a lower pass filtering in the frequency domain. Figure 6(d) is the velocity obtained by Fourier analysis. Compared with the result from the WFT, the noise effect is more obvious. If the lower pass filtering window is not selected properly, the fluctuation that is due to noise will be more serious. Figures 7(a) and 7(b) show 3-D plots of the

velocity at instant $t = 0.0345 \text{ s}$ obtained by windowed Fourier analysis and Fourier analysis, respectively. A 5×5 median filter was applied to the velocity maps to remove some ill-behaved pixels. A smoother distribution is observed in the WFT case. Similar to above-mentioned simulation case, the spectrum of the signal is broad when the velocity is large, and the effect of a low-pass filter is not so obvious.

Figures 8(a) and 8(b) are 3-D plots of the acceleration distribution at the instant $t = 0.0345 \text{ s}$ using the WFT and FT analysis. As for the velocity measurement, a 5×5 median filter was applied to the acceleration map. In the WFT case, $\sigma = 5$ is selected as the window size as the signal processed is much noisier than that in the velocity evaluation. But, different from the velocity measurement, a similar quality of acceleration distribution is observed in both methods. The advantage of the WFT is not obvious as

(a)

(b)

Fig. 7. 3-D plot of velocity at instant $t = 0.0345$ s obtained by (a) a windowed Fourier ridge and (b) Fourier analysis.

the spectrum of the signal in the Fourier analysis is narrow, and a small low-pass filtering window can be selected. On the other hand, the size of the Gaussian window in the WFT has to be broadened to eliminate the noise. Therefore, more errors that are due to the linear phase approximation assumption will be introduced.

The windowed Fourier ridge method maps a 1-D signal to a 2-D plane of time and frequency and extracts the instantaneous frequencies with the highest energy. Although some fast converging iterative algorithms [30] have been introduced and it is not necessary to explore the whole time-frequency plane, it is still a time-consuming process and requires a high computing speed and large memory. In our investigation, we used the maximum value in the Fourier spectrum of $f(t)g(t - u)$ as an initial estimation of the ridge value at each instant and then searched the actual ridge with a preset accuracy. The computation time depends on the accuracy required, but it is still much longer than that of the Fourier analysis. Because of this drawback of the WFT, it is more suitable for the signal with a broad spectrum, such as the velocity evaluation in our case. In our acceleration measurement, Fourier analysis is recommended to reduce the computation time. However, this disadvantage of the

(a)

(b)

Fig. 8. 3-D plot of acceleration at instant $t = 0.0345$ s obtained by (a) a windowed Fourier ridge and (b) Fourier analysis.

WFT has become inconspicuous because of the rapid improvement in the capacity of computers.

It is worth noting that by use of the proposed technique, the measurement range of the velocity and acceleration is limited due to the constraint of the Nyquist sampling theorem and the imaging rate of the high-speed camera. The maximum phase change between two adjacent frames is $\pm\pi$, which is equivalent to the velocity of $\pm 283 \mu\text{m/s}$ with the experimental setup and imaging rate mentioned above. This is the reason only a portion of the beam is selected to be processed [Fig. 2(c)]. However, digital holography can extract the phase from one hologram, so that no phase ambiguity problem exists. It is not necessary to introduce the temporal carrier. Compared with other optical interferometries such as ESPI and shearography, the vibration measurement range is tremendously enlarged with the absence of temporal carrier frequency.

In previous research, a wavelet was also successfully used in vibration measurements taken by ESPI [14]. The WFT and the wavelet transform are two similar techniques, but the wavelet has an automatically varying window size. In many cases, the wavelet performs better than the WFT. However, it is not suitable for vibration measurements by digital holography. In the wavelet transform, the window size increases when the frequency decreases. When the instantaneous frequency of the signal is close to zero, the window size of

the wavelet along the time axis is extended to be nearly infinite. Thus it is not practical to process the signal when the frequency is around zero. It is more suitable to process a signal whose spectrum is shifted to one size by a carrier frequency. In velocity measurements taken with digital holography, the instantaneous frequency of the signal will always be zero when the movement changes direction. The wavelet transform will introduce large errors around those instants. In contrast, the WFT has a fixed window size and performs much better in low-frequency areas.

5. Concluding Remarks

We have presented a novel method for evaluating the transient displacement, velocity, and acceleration of a vibrating object using spatial and temporal processing of a series of digital holograms. We applied 2-D windowed Fourier filtering to obtain a smooth distribution of displacement, and temporal FT and WFT were applied to calculate the instantaneous velocity and acceleration of the object. In the velocity evaluation, as the spectrum of the signal is broad, the windowed Fourier ridge method shows its capability in instantaneous frequency extraction and noise reduction and thus performs better than Fourier analysis. However in the acceleration evaluation, the spectrum of the signal is narrow, and Fourier analysis also performs well to eliminate noise and is thus recommended because of its fast processing speed. The combination of digital holography and temporal Fourier analysis allows for the simultaneous evaluation of three important vibration parameters at different instants by a whole-field noncontact optical measurement technology.

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