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Temporal wavelet analysis for deformation measurement of small components using micro-ESPI

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ABSTRACT

Measuring continuous deformation of specimens whose dimensions are in the range of sub-millimeter introduces a number of difficulties using laser speckle interferometry. During deformation, the speckle patterns recorded on a camera sensor change constantly. These time-dependent speckle patterns would provide the deformation history of the object. However, compared to large objects, noise effect is much more serious due to the high magnification. In this study, a series of speckle patterns on small objects are captured during deformation by high speed camera and the temporal intensity variation of each pixel is analyzed by a robust mathematical tool --- complex Morlet wavelet transform instead of conventional Fourier transform. The transient velocity and displacement of each point can be retrieved without the need for temporal or spatial phase unwrapping process. Displacements obtained are compared with those from temporal Fourier transform, and the results show that wavelet transform minimize the influence of noise and provide better results.

Keywords: high-speed imaging, continuous wavelet transforms (CWT), temporal phase analysis, speckle interferometry, instantaneous frequency, micro components.

1. INTRODUCTION

In the last decades, electronic speckle pattern interferometry (ESPI) was successfully used for detecting static and dynamic deformation and strain on specimens and components with dimensions on the macroscopic level. It is a nondestructive, whole-field technique with the excellent sensitivity controlled by the wavelength of laser light. However, when the object size scales down to one millimeter or even below, a number of problems arises when speckle techniques are to be applied. One of them is that the noise effect is more serious due to the high magnification, especially in continuous deformation measurement.

Temporal phase analysis and temporal phase unwrapping techniques have been widely used to study continuously deformed objects since late 1990's. Among various temporal phase analysis algorithms[1-6], Fourier transform is the most popular method. The intensity fluctuation due to deformation of each pixel is first transformed, and one side of the spectrum is filtered with a bandpass filter. The filtered spectrum is inverse-transformed to obtain the wrapped phase. The phase values are then unwrapped along the time axis at each pixel. It is well known that the accuracy of the Fourier-transform analysis increases for higher temporal frequency with a narrow spectrum. However, in most cases the deformation of each point on the object is different, and the deformation of each pixel may also be nonlinear along the time axis. As mentioned above, the noise effect will also affect the result when small components are measured. Automatic filtering process becomes difficult as the width of the bandpass filter has to be broadened, and introduces further errors in phase extraction. In this paper, a new temporal phase analysis method ------ temporal wavelet transform is introduced to process a series of speckle patterns obtained by ESPI to measure the instantaneous displacement and velocity of a continuously deforming small object. Displacements obtained are compared with those from temporal Fourier transform, and the advantages and drawbacks of temporal wavelet analysis are also discussed.

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2. THEORETICAL ANALYSIS

The schematic layout of Micro ESPI setup for out-of-plane measurement is shown in Fig. 1. An expanded laser beam is collimated and separated by a beam splitter into object and reference beams. A series of speckle patterns is captured by a high-speed CCD camera during deformation. The intensity of each pixel can be expressed as

$$\begin{aligned} I_{xy}(t) &= I_{0xy}(t) + A_{xy}(t)\cos[\varphi_{xy}(t)] \\ &= I_{0xy}(t)\left\{1 + V\cos\left[\phi_{0xy} + \frac{4\pi z_{xy}(t)}{\lambda}\right]\right\} \end{aligned} \tag{1}$$

where $I_{0xy}(t)$ is the intensity bias of the speckle pattern, V is the visibility of the speckle modulation, ϕ_{0xy} is a random phase, and $z_{xy}(t)$ is the out-of-plane deformation of the object. A series of speckle patterns are recorded during the deformation and at each pixel, the intensity variation is analyzed by continuous wavelet transform (CWT).

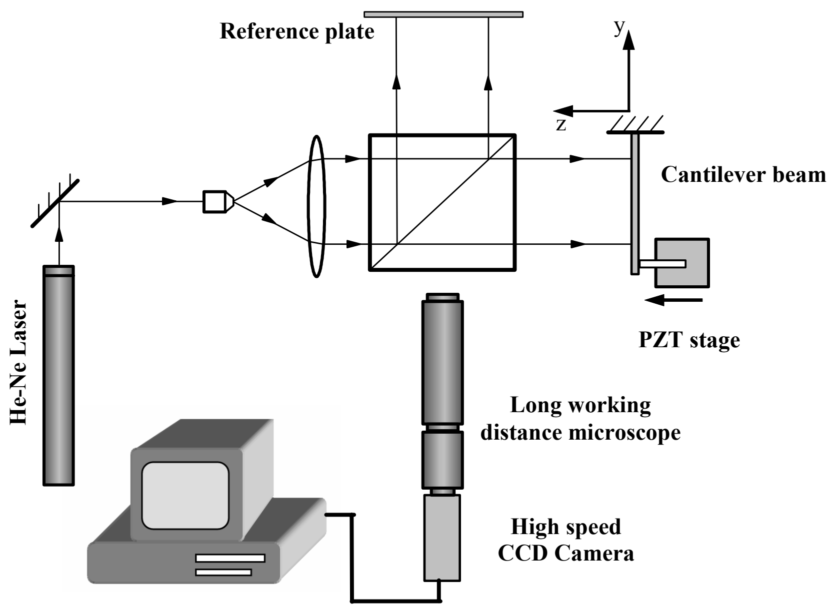


Fig. 1 The experimental set up of micro-ESPI

Wavelet transform is a new and robust mathematic tool for signal analysis. CWT was also used for phase extraction on different types of fringe patterns with spatial carriers. These applications use one-dimensional CWT along a spatial axis. The instantaneous frequency of gray value variation can be obtained by extracting the 'ridge' of the wavelet coefficients, followed by an integration process to retrieve the phase. Wavelet transform has also been applied on temporal phase analysis of speckle interferometry. The concept was first introduced by Colonna de Lega[7] in 1996 and some preliminary results were presented[8-9]. In this study, temporal wavelet analysis is applied to retrieve the instantaneous velocity and displacement from a series of speckle patterns.

The continuous wavelet transform of a signal *s(t)* is defined as its inner product with a family of wavelet function $\Psi_{a,b}(t)$.

$$W_S(a,b) = \int_{-\infty}^{+\infty} s(t)\Psi^*{}_{a,b}(t)dt \tag{2}$$

where * denotes the complex conjugate. $\Psi(t)$ is in general called the mother wavelet, and the basis functions of the transform, $\Psi_{a,b}(t)$, called daughter wavelets can be constructed by elementary operations consisting of *time shifts* and *scaling* (i.e. dilation or contraction). In this study, the complex Morlet wavelet is selected as mother wavelet because it gives the smallest Heisenberg box:

$$\Psi(t) = g(t)\exp(i\omega_0 t) \tag{3}$$

where $g(t) = \exp\left(-\frac{t^2}{2}\right)$. Here $\omega_0 = 2\pi$ is chosen to satisfy the admissibility condition so that the wavelet function is able to remove the negative frequencies as well as avoid the DC contribution of the signals. Substituting Eqs. (1) and (3) into Eq.(2), the wavelet transform of temporal intensity variation on pixel *P*(*x, y*) can be expressed as[10]

$$W_{xy}(a,b) = \frac{\sqrt{a}}{2} A_{xy}(b)\left\{\hat{g}\left[a\left(\zeta - \varphi'_{xy}(b)\right)\right] + \varepsilon(b,\zeta)\right\}\left\{\exp\left[i\varphi_{xy}(b)\right]\right\} \tag{4}$$

where $\zeta = \frac{\omega_0}{a}$, and ε is a corrective term which normally can be negligible. The trajectory of maximum $|W_{xy}(a,b)|^2$ on the *a*-*b* plane is called a 'ridge'. Since $|\hat{g}(\omega)|$ is maximum at $\omega = 0$, and if $\varepsilon(b,\zeta)$ is negligible, $|W_{xy}(a,b)|^2$ reaches maximum when

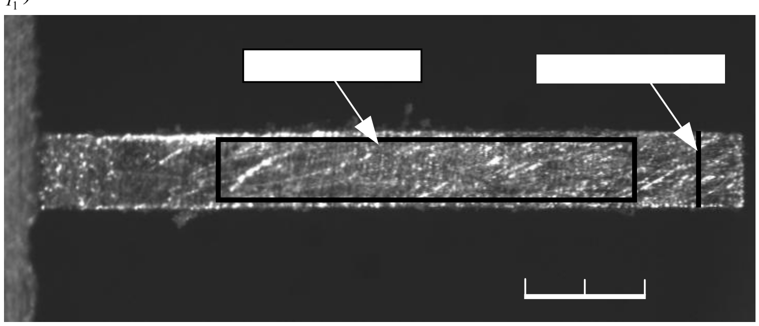
$$\varphi'_{xy}(b) = \zeta_{rb} = \frac{\omega_0}{a_{rb}} \tag{5}$$

where $\varphi'_{xy}(b)$ is defined as instantaneous frequency of signal, and a_{rb} denotes the value of *a* at instant *b* on the ridge. The instantaneous velocity of point *P*(*x*, *y*) at instant *b* can be retrieved directly from $\varphi'_{xy}(b)$.

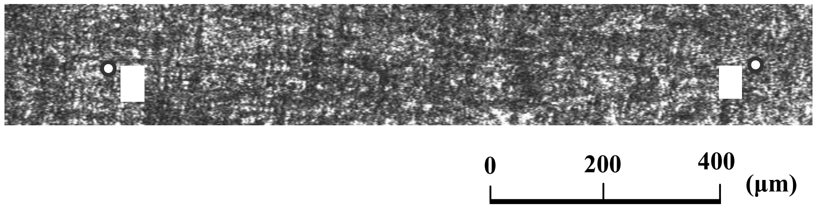
The wavelet transform on the ridge can then be expressed as

$$W_{xy}(a_{rb},b) \approx \frac{\sqrt{a_{rb}}}{2} A_{xy}(b)\hat{g}(0)\exp\left[i\varphi_{xy}(b)\right] \tag{6}$$

The complex phase value of wavelet transform $W_{xy}(a,b)$ on the ridge equals to $\varphi_{xy}(b)$. For deformation measurement, the phase value $\varphi_{xy}(b)$ is calculated by integration of the instantaneous frequency in Eq. (5), and phase unwrapping procedure is avoided in both temporal and spatial domain. Combination of phase values of each pixel at certain instant *T* generates a phase map of φ_T, and deformation between two instants T_1 and T_2 can be obtained by $(\varphi_{T_2} - \varphi_{T_1})$.



(a)



(b)

Fig. 2 (a) Specimen: a tiny cantilever beam and area of interest; (b) typical speckle patterns captured by CCD camera.

3. EXPERIMENTAL ILLUSTRATION

Figure 2(a) shows a tiny steel cantilever beam being tested. The length, width and thickness of the beam are 2.5 mm, 0.25 mm and 0.25 mm, respectively. The beam is continuously loaded with a linear displacement by a PZT stage. During deformation of the beam, a series of speckle patterns is captured by a high speed CCD camera (KODAK Motion Corder Analyzer, SR-Ultra) with a long working distance microscopic lens (shown in Fig. 1). Fig. 2(b) shows a typical speckle pattern captured at intervals of 0.008 s with imaging rate of 125 frames/second (fps). Five hundred speckle patterns are captured during a four-second period. Among them, Two hundred and fifty six consecutive images are selected for processing.

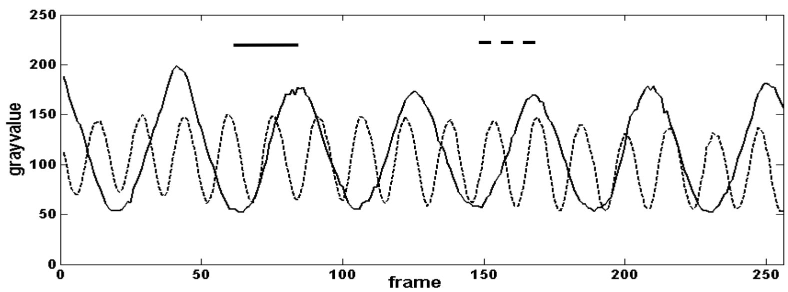
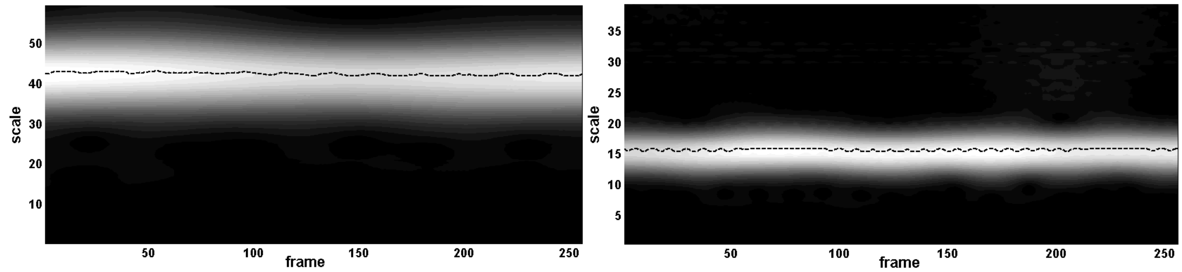


Fig. 3 Gray values of points A and B.

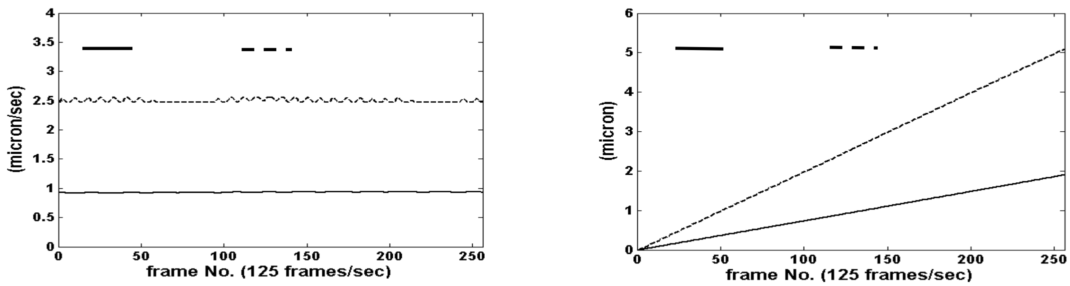
4. RESULTS AND DISCUSSIONS

Figure 3 shows the gray value variation of points A and B [indicated in Fig. 2(b)]. A difference in amplitude is observed, however, only temporal frequencies are considered as they contain information on velocity and displacement. The temporal frequency of point A is lower than that of point B. This implies that Point B deforms faster than point B. The modulus of the Morlet wavelet transform of intensity variation of points A and B are shown in Figs. 4(a) and 4(b) respectively. The dashed line shows the ridge of the wavelet transformation where the maximum modulus are found. Little variation of a_{rb} is observed on the ridge, which implies the velocities are almost constant along time axis. Figure 5(a) shows the velocities on points A and B where the transient velocities at each pixel can be retrieved. Integration of $\frac{2\pi}{a_{rb}}$ is carried out on each pixel to generate a continuous temporal phase φ. At a certain instant *T*, the phase change due to the displacement can be obtained by $(\varphi_T - \varphi_{T_0})$, where φ_{T_0} is the phase at instant $T_0 = 0$. In an ESPI set-up as shown in Fig. 1, a 2π phase change represents a displacement of $\lambda / 2$ (=316.4 nm) in the z direction. The transient displacements of points A and B are shown in Fig. 5(b). Figure 6(a) shows a 3-D plot of the out-of-plane displacement of the beam between two instants $T_1 = 0.4s$ and $T_2 = 1.2s$. A 3×3 median filter has been used on the phase map to remove several ill-behaved pixels.



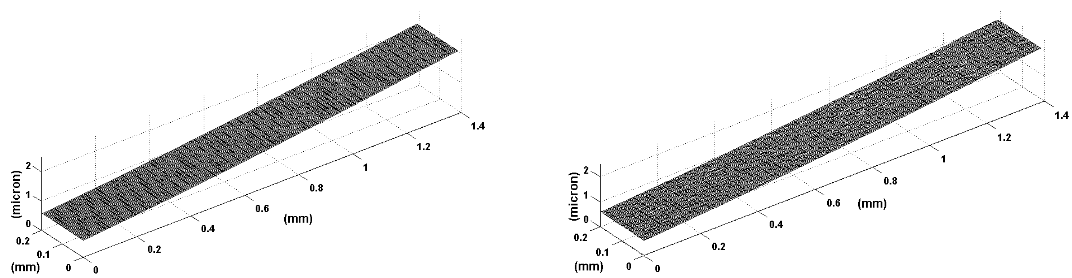
(a) (b)

Fig. 4(a) Modulus of the Morlet wavelet transform at point A. (b) Modulus of the Morlet wavelet transform at point B.



(a) (b)

Fig. 5 (a) Instantaneous velocity and (b) displacement at points A and B.



(a) (b)

Fig. 6 Displacement of the beam between two instants $T_1 = 0.4s$ and $T_2 = 1.2s$ by use of
(a) temporal wavelet analysis and (b) temporal Fourier analysis.

For comparison, temporal Fourier analysis is also applied on the same speckle patterns. Bandpass filters with different width are applied. As the temporal frequencies of each pixel are different, a relatively wider filter provides the best result as it includes all frequencies. A one-dimensional phase unwrapping is then applied along the time axis, as all phase values obtained by inverse Fourier transformation are within $[0, 2\pi)$. Figure 6(b) shows the 3-D displacement plot from temporal Fourier analysis. As in wavelet transform, a 3×3 median filter is also applied on the phase map. Figure 7 shows a comparison of displacement on central line of the cantilever beam at different instants. A smoother spatial distribution of displacement is observed using wavelet analysis. The maximum displacement fluctuation due to noise is around 0.06 μm in Fourier transform, but only 0.02 μm in wavelet analysis.

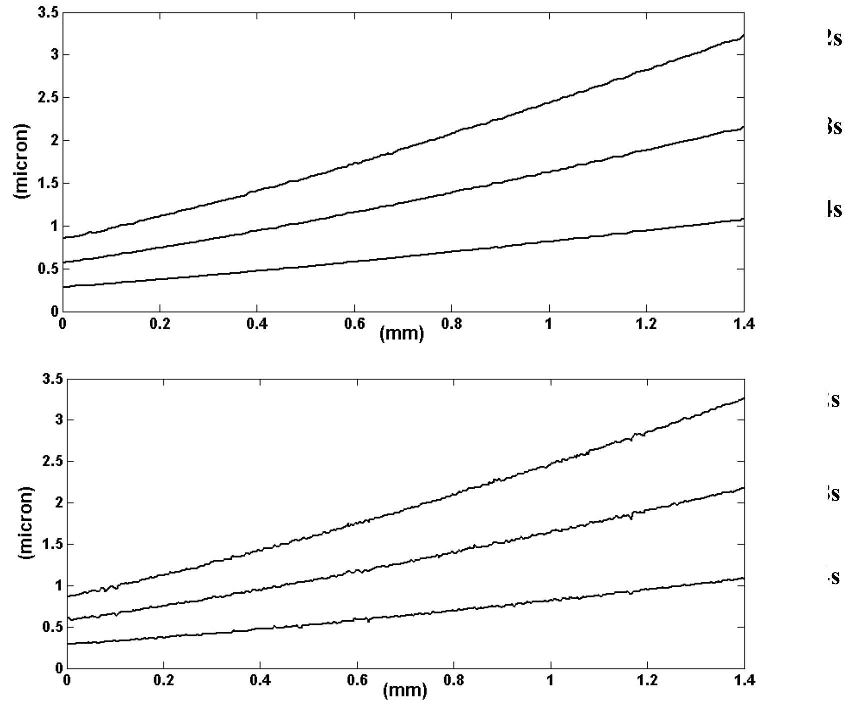


Fig. 7 Comparison of displacements at central line of the cantilever beam between (a) wavelet and (b) Fourier analysis.

From the above comparison between the results of wavelet and Fourier analysis, it can be observed that wavelet analysis shows better results in continuous displacement measurement. As wavelet analysis calculates the optimized frequency at each instant, it performs an adaptive band-pass filtering of the measured signal, thus limits the influence of various noise sources and increases the resolution of measurement significantly. In contrast, Fourier transform uses a broader filter which is less efficient in eliminating noise effect. The maximum displacement fluctuation due to noise depends on width of the bandpass filter and quality of the speckle patterns. Continuous wavelet transform maps a one-dimensional intensity variation signal to a two-dimensional plane of position and frequency, and extracts the optimized frequencies. Although some fast-converging iterative algorithms are introduced and it is not necessary to explore the whole time-frequency plane, CWT is still a time-consuming process and requires high computing speed and memory. The computation time is about 10 times larger than that of temporal Fourier transform. This is the main drawback of CWT in temporal phase analysis. Similar to other temporal phase analysis methods, wavelet transform is also limited by Nyquist sampling theorem. It is impossible to analyze signals with a frequency higher than half of the acquisition rate. However, these two disadvantages become inconspicuous due to the rapid improvement in capacity of computers and high speed CCD cameras.

5. CONCLUSION

This paper presents a novel method to retrieve the transient velocity and displacement on a continuously deforming cantilever beam using temporal wavelet analysis of speckle patterns obtained from micro-ESPI. Unlike conventional phase evaluation techniques such as Fourier transform, this method analyses the phase values point-by-point along the time axis using a complex analytical wavelet --- Morlet wavelet transform. Comparing to Fourier transform, wavelet has advantages on extracting the instantaneous frequencies, from which the velocity can be retrieved directly. High quality deformation map of the object can also be obtained without any phase unwrapping processes. A comparison between temporal wavelet transform and Fourier transform shows that wavelet analysis can limit the influence of various noise sources, and significantly improve the results in displacement measurement.

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