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Nonlinear Diffusion in Laplacian Pyramid Domain for Ultrasonic Speckle Reduction

Fan Zhang, Yang Mo Yoo, Liang Mong Koh, and Yongmin Kim*, *Fellow, IEEE*

Abstract—A new speckle reduction method, i.e., Laplacian pyramid-based nonlinear diffusion (LPND), is proposed for medical ultrasound imaging. With this method, speckle is removed by nonlinear diffusion filtering of bandpass ultrasound images in Laplacian pyramid domain. For nonlinear diffusion in each pyramid layer, a gradient threshold is automatically determined by a variation of median absolute deviation (MAD) estimator. The performance of the proposed LPND method has been compared with that of other speckle reduction methods, including the recently proposed speckle reducing anisotropic diffusion (SRAD) and nonlinear coherent diffusion (NCD). In simulation and phantom studies, an average gain of 1.55 dB and 1.34 dB in contrast-to-noise ratio was obtained compared to SRAD and NCD, respectively. The visual comparison of despeckled *in vivo* ultrasound images from liver and carotid artery shows that the proposed LPND method could effectively preserve edges and detailed structures while thoroughly suppressing speckle. These preliminary results indicate that the proposed speckle reduction method could improve image quality and the visibility of small structures and fine details in medical ultrasound imaging.

Index Terms—Laplacian pyramid, multiscale analysis, nonlinear diffusion, speckle reduction, ultrasound imaging.

I. INTRODUCTION

ULTRASOUND imaging has several advantages over other medical imaging modalities (e.g., X-rays, computed tomography, and magnetic resonance). It is safe, relatively low cost, and allows real-time imaging. However, the diagnostic usefulness of ultrasound imaging is at times limited due to its low image quality. One of the main reasons for this low image quality is the presence of signal-dependent noise known as speckle. Speckle is a granular pattern formed due to constructive and destructive coherent interferences of backscattered echoes from the scatterers that are typically much smaller than the spatial resolution (i.e., wavelength of an ultrasound wave) of medical ultrasound systems [1]. The speckle pattern

depends on the structure of the imaged tissue and various imaging parameters, e.g., the frequency and geometry of an ultrasound transducer. Thus, two images captured under the same condition will show an identical speckle pattern. Because of its dependence on the microstructure of tissue parenchyma, speckle is often used in diagnosis, such as diffuse liver diseases [2]. However, speckle typically shows up as noise since it reduces image contrast and obscures image details. Also, it affects human interpretation of the acquired ultrasound images and degrades the speed and accuracy of ultrasound image processing tasks.

Generally speaking, there are two main purposes for speckle reduction in medical ultrasound imaging. First, speckle reduction can aid human interpretation of ultrasound images. Second, speckle reduction is a preprocessing step for many ultrasound image processing tasks such as segmentation and registration. When applying a speckle reduction technique as an aid for visual diagnosis, we need to keep in mind that certain speckle contains diagnostic information and should be retained. Also, some important details may be smeared or lost when performing speckle reduction. In some cases, clinicians prefer an original image to a despeckled image, because the original image contains more diagnostic information. From this point of view, the despeckled image should be considered a complement to the original image, and not a replacement. When speckle reduction is applied as a preprocessing step for segmentation or registration, any speckle can be considered noise without differentiation. In these applications, speckle prevents image segmentation and registration techniques from generating optimal results. Speckle reduction makes an ultrasound image cleaner with clearer boundaries, and thus significantly improves the speed and accuracy of automatic or semiautomatic image segmentation and registration techniques.

Several speckle reduction techniques based on compounding and postacquisition image processing techniques have been proposed [3]–[23]. With the compounding techniques, a series of ultrasound images of the same target are acquired from different scan directions [3], with different transducer frequencies [4], or under different strains [5]. Those images are then averaged to form a composite image. While the compounding methods can improve target detectability, they suffer from degraded spatial resolution and increased system complexity. On the other hand, the postacquisition image processing techniques do not need many hardware modifications. The postacquisition techniques for speckle reduction can be classified into two categories: single scale spatial filtering and multiscale methods.

Bamber *et al.* [6] developed a speckle reduction filter that changes the amount of smoothing according to the ratio of local

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variance to local mean. In their method, smoothing is increased in homogeneous regions where speckle is fully developed and reduced or even avoided in other regions to preserve details. Dutt *et al.* [7] suggested an unsharp masking filter and adjusted its smoothing level depending on the statistics of log-compressed images. These two spatial filtering methods have difficulty in removing speckle near or on image edges. The adaptive weighted median filter (AWMF) [8] based on the use of pixel replication was proposed to eliminate the requirement in traditional median filtering that speckle must be smaller than half of the filter window size. However, its performance is highly sensitive to some empirically determined parameters, especially when the filter window is small. Czerwinski *et al.* [9] proposed to filter an ultrasound image by utilizing short line segments in different angular orientations and selecting the orientation that is most likely to represent a line in the image. However, this technique poses a tradeoff between effective line enhancement and speckle reduction. Region growing-based spatial filtering methods were proposed in [10]–[13]. In these methods, it is assumed that pixels that have similar gray level and connectivity are contextually related and likely to belong to the same object or region. After all pixels are allocated to different groups, spatial filtering is performed based on the local statistics of adaptive regions whose sizes and shapes are determined by the information content of the image. The main difficulty in applying these methods is how to design appropriate similarity criteria for region growing.

Alternatively, diffusion-based spatial filtering methods have been proposed [14], [15]. Yu and Acton [14] introduced an edge sensitive diffusion method [i.e., speckle reducing anisotropic diffusion (SRAD)] to suppress speckle while preserving edge information. Abd-Elmoniem *et al.* [15] presented a tensor-based anisotropic diffusion method [i.e., nonlinear coherent diffusion (NCD)] for speckle reduction and coherence enhancement. Both of the two diffusion methods can preserve or even enhance prominent edges when removing speckle, but they have one common limitation in retaining subtle features, such as small cysts and lesions in ultrasound images.

For reducing speckle in ultrasound images, several multiscale (e.g., wavelet and pyramid) methods have been proposed. The wavelet-based speckle reduction methods can be classified into three groups: thresholding [16], [17], Bayesian estimation [18]–[20], and coefficients' correlation [21]. In the thresholding methods, the wavelet coefficients smaller than a predefined threshold are regarded as contributed by noise and then removed. These thresholding techniques have difficulty in determining an appropriate threshold. Bayesian estimation methods approximate the noise-free signal based on the distribution model of noise-free signal and that of noise. Thus, reasonable distribution models are crucial to the successful application of these techniques to medical ultrasound imaging. Pizurica *et al.* [21] proposed an undecimated (or overcomplete) wavelet domain denoising method which utilizes the correlation of useful wavelet coefficients across scales. It does not rely on the exact prior knowledge of the noise distribution and, therefore, is more flexible and robust compared to the other wavelet-based methods.

Pyramid transform has also been used for reducing speckle [22], [23]. Unlike sub-band decomposition in wavelet trans-

form, approximation and interpolation filters in pyramid transform have lowpass properties so that pyramid transform does not require quadrature mirror filters. Sattar *et al.* [22] adopted the Feaveau's pyramid transform. In this method, edge detection on interpolated detailed images is used to determine which of the pixels in the detailed images are to be included in the reconstruction stage. However, none of their three proposed edge detection approaches are sophisticated enough to perform well on ultrasound images. Considering the multiplicative nature of speckle, Aiazzi *et al.* [23] introduced a ratio Laplacian pyramid. In this method, the conventional Kuan filter [24] is extended to multiscale domain by processing the interscale layers of the ratio Laplacian pyramid, but it suffers from the need to estimate the noise variance in each interscale layer.

In this paper, a new speckle reduction method based on nonlinear diffusion filtering of bandpass ultrasound images in the Laplacian pyramid domain is proposed to more effectively suppress speckle while preserving edges and detailed features. The proposed method is mainly focused on producing simplified ultrasound images for subsequent computer-assisted image analysis such as automatic or semiautomatic segmentation and registration, although it can also provide a visual diagnostic aid for clinicians to interpret ultrasound images. Part of the work in this paper has been reported in [25].

This paper is structured as follows. In Section II, the theoretical background of the proposed method, including pyramid transform and nonlinear diffusion, is presented. Section III describes the proposed Laplacian pyramid-based nonlinear diffusion (LPND) method in detail and the automatic determination of the gradient threshold used in the nonlinear diffusion process. The implementation stage of the proposed speckle reduction technique in an ultrasound imaging system is also discussed. In Section IV, the numerical implementation scheme of the proposed LPND method is presented. The implementation provides a tunable parameter to adjust the smoothing level of speckle reduction. The computational complexity of the proposed method is also discussed. Section V compares the performance of the LPND with some other speckle reduction methods (e.g., SRAD and NCD) in simulation, phantom, and *in vivo* studies. Section VI concludes the paper.

II. THEORETICAL BACKGROUND

A. Pyramid Transform

Since the introduction of Laplacian pyramid by Burt and Adelson [26], a number of pyramid representations have been developed [22], [23], [27]–[30]. A general structure of pyramid transforms consists of decomposition and reconstruction stages and can be described by approximation and interpolation filtering. In the decomposition stage, a signal is successively decomposed into a decimated approximation signal and a signal containing residual information. This residual signal is computed as the difference between the signal on a finer scale and the interpolated signal from a coarser scale. A finer scale corresponds to a lower pyramid layer. The lowest pyramid layer has the same size as the original image. A specific pyramid is determined by its particular decimation factor and approximation and interpolation filters. In the Laplacian pyramid, two

operators, REDUCE and EXPAND are commonly used. The REDUCE operator performs a two-dimensional (2-D) lowpass filtering followed by a sub-sampling by a factor of two in both directions. The EXPAND operator enlarges an image to twice the size in both directions by up-sampling (i.e., insertion of zeros) and a lowpass filtering followed by a multiplication by a factor of four, which is necessary to maintain the average intensity being reduced by the insertion of zeros. For an input image I , let its Gaussian pyramid at layer l be G_l , and its Laplacian pyramid at layer l be L_l , where $l = 0, 1, 2, \dots, d-1$ and d is the total decomposition layer. Then, the Gaussian and Laplacian pyramid can be defined as

$$G_0 = I \quad (1)$$

$$G_l = \text{REDUCE}[G_{l-1}] \quad (2)$$

$$L_l = G_l - \text{EXPAND}[G_{l+1}]. \quad (3)$$

The Gaussian pyramid consists of a set of lowpass filtered copies of the original image at different sizes, whereas the Laplacian pyramid decomposes the original image into a set of bandpass images and a final lowpass image. Reconstruction of an image from its Laplacian pyramid can be achieved by simply reversing the decomposition steps.

B. Nonlinear Diffusion

Diffusion filtering removes noise from an image by modifying the image via a partial differential equation (PDE). Perona and Malik [31] proposed the nonlinear diffusion as described by the following equation:

$$\frac{\partial I}{\partial t} = \text{div}[c(\|\nabla I\|) \cdot \nabla I], \quad I(t=0) = I_0 \quad (4)$$

where div is the divergence operator, $\|\nabla I\|$ is the gradient magnitude of an image I , $c(\|\nabla I\|)$ is the diffusion coefficient or diffusivity function, and I_0 is the original image. If the function $c(\|\nabla I\|)$ is constant for all image locations, the diffusion process becomes linear. For nonlinear diffusion, the diffusivity function $c(\|\nabla I\|)$ is a monotonically decreasing function of the gradient magnitude. Thus, diffusion is discouraged across regions with large gradient magnitudes and encouraged within regions with small gradient magnitudes. This leads to improved structure preservation and image denoising properties. Perona and Malik [31] suggested two diffusivity functions

$$c_1(\|\nabla I\|) = \frac{1}{1 + (\|\nabla I\|/k)^2} \quad (5)$$

$$c_2(\|\nabla I\|) = \exp[-(\|\nabla I\|/k)^2] \quad (6)$$

where k is a constant called the gradient threshold. It plays an important role in determining the degree of smoothing in the diffusion process.

III. LAPLACIAN PYRAMID BASED NONLINEAR DIFFUSION METHOD

Similar to wavelet-based denoising methods, the proposed Laplacian pyramid-based nonlinear diffusion method consists

Fig. 1. Block diagram of the proposed LPND method, including pyramid decomposition, nonlinear diffusion, and pyramid reconstruction.

of three steps as shown in Fig. 1: 1) transformation of an image into its Laplacian pyramid domain, 2) manipulation of pyramid coefficients by regularized nonlinear diffusion, and 3) finally reconstruction of the diffused Laplacian pyramid. After an image is decomposed into its pyramid structure of decreasing frequencies, the main noise (i.e., speckle in this study) and useful signal components of the image exist in different layers because of their different frequency characteristics. Speckle noise has high frequency so that it mainly exists in low pyramid layers (i.e., fine scales). On the other hand, in the highest pyramid layer (i.e., the coarsest scale), speckle noise is negligible. Thus, performing spatial adaptive filtering in each bandpass layer can effectively suppress speckle without degrading slowly varying signal too much.

The second step is the nonlinear diffusion filtering in each bandpass layer of Laplacian pyramid to suppress speckle while preserving edges, as summarized in Fig. 2. This nonlinear diffusion process differentiates the proposed method from other Laplacian pyramid-based denoising methods as proposed in [27], [28], [32]. The diffusivity functions in (5) and (6) may create problems, such as failure in convergence and high sensitivity to noise. Although these problems can be handled by applying regularizing finite difference discretizations, it would be desirable to have a regularization that does not depend on discretization effects. In the proposed LPND, a Gaussian regularization strategy [33] which estimates the gradient ∇I on a Gaussian lowpass-filtered version of the image, is adopted. Based on this strategy, (4) becomes

$$\frac{\partial I}{\partial t} = \text{div}[c(\|\nabla(G(\sigma) * I)\|) \cdot \nabla I] \quad (7)$$

where σ is the standard deviation of a Gaussian filter G and describes a level of uniform smoothing used to measure the image gradient, and $*$ denotes convolution. The value of the regularization parameter σ is chosen empirically according to the following arguments. While σ should be big enough to diminish

Fig. 2. Flowchart of nonlinear diffusion in the proposed LPND method.

unwanted intensity variations due to noise, it should be small enough to preserve the signal of interest such as edges which are sufficiently small. Thus, this involves a tradeoff between noise suppression and structure preservation.

The diffusivity function can be computed using (5) or (6). It has been found that a greater decay rate of the diffusivity function will create sharper edges that persist over longer time intervals in a narrower range of edge slopes, and that a more gradual decay rate will sharpen edges over a wider range of edge slopes. For the same value of k , the two diffusivity functions in (5) and (6) will lead the diffusion process to significantly different results [34]. To make the two diffusivity functions give similar results for the same value of k , we can modify the diffusivity function $c_2(\|\nabla I\|)$ to be

$$c_2(\|\nabla I\|) = \exp[-(\|\nabla I\|^2 / (2k^2))]. \quad (8)$$

In our simulations, no significant differences were observed between the results obtained from (5) and (8).

The choice of the gradient threshold k plays an important role in determining the parts of an image that will be blurred or enhanced in the diffusion process. If the value of k is set too high, the diffusion will act as a smoothing filter, diffusing across some edge boundaries. If k is too low, some big noise will be preserved or even amplified instead of being reduced. In conventional diffusion methods, the gradient threshold k is mainly determined by experience depending on noise level and edge strength in the processed image. When the noise level is unknown, this parameter needs to be selected by trial-and-error to get an optimal result. This heuristic procedure is unsuitable for clinical applications where multiple frames are dynamically utilized. Perona and Malik [31] used Canny's noise estimator to select the gradient threshold automatically. However, their au-

tomatic threshold selection mechanism requires a bi-peak histogram, which is not common in medical ultrasound images. Currently, the gradient threshold k is estimated using the robust median absolute deviation (MAD) estimator [35]

$$k = \frac{\text{MAD}(\|\nabla I\|)}{0.6745} \quad (9)$$

where the constant is derived from the fact that the MAD of a zero mean normal distribution with unit variance is 0.6745.

Our experiments show that the MAD estimate of the gradient threshold k is close to the optimal value for the conventional nonlinear diffusion process in most cases. However, for the LPND method, this value needs to be adjusted in each pyramid layer. The new decision rule on the gradient threshold k for the proposed LPND method can be represented by

$$k(l) = \frac{1}{0.6745} \cdot \text{MAD}(\|\nabla I(l)\|) \cdot \sqrt{2 \log((l+1)/l)} \quad (10)$$

where l is the pyramid layer. The image gradient ∇I is computed using the pyramid coefficients in the corresponding pyramid layer. In this new gradient threshold decision rule, a relatively large value is used in the lowest pyramid layer where speckle is dominant, in order to remove it more thoroughly. On the other hand, to preserve structure boundaries, small gradient thresholds are applied in higher layers. Thus, this new gradient threshold decision rule given in (10) can help the LPND suppress noise more thoroughly and preserve important image features more effectively.

Fig. 3 shows a functional block diagram of an ultrasound imaging system, including a speckle reduction stage. We suggest applying the proposed LPND on the envelope-detected data to preserve more useful information in an ultrasound image, as illustrated in Fig. 3. In a real ultrasound imaging system, compression is not simply logarithmic operation as usually assumed. Some useful information about the imaged object may be deteriorated or even lost after the compression stage. Consequently, any processing which works with envelope-detected data has more information at its disposal and, thus, may preserve more useful information. Furthermore, compared to processing the scan-converted image, processing the envelope-detected data has fewer pixels and thus incurs lower computational cost. When applied on the envelope-detected data, the LPND is used in a homomorphic manner, i.e., the envelope-detected data are log-compressed prior to applying the LPND. Exponential transform is applied after the LPND. In this way, the multiplicative noise contained in the envelope-detected data is transformed to additive noise and then suppressed by the LPND. Nonlinear diffusion adjusts its smoothing level according to its diffusivity function, which is based on local gradients. It performs well for images corrupted by additive noise. In the case of multiplicative noise, noise in bright regions has higher variance and could be interpreted wrongly as features by the diffusivity function. Consequently, nonlinear diffusion will enhance the noise instead of reducing them. Therefore, nonlinear diffusion should be applied in a homomorphic manner, when used for multiplicative noise reduction.

The proposed LPND can also be applied on the log-compressed data or the scan-converted image. However, slightly dif-

Fig. 3. Block diagram of an ultrasound imaging system showing the preferred location of the proposed speckle reduction method.

ferent results will be produced compared to the results from processing the envelope-detected data, because some information lost after the compression stage cannot be recovered by working with the log-compressed data or the scan-converted image. Applying a speckle reduction technique like the LPND on the scan-converted image is not without advantages, since the scan-converted image is always accessible whereas most commercial ultrasound systems do not output the envelope-detected or log-compressed data. If the LPND is performed after log-compression or scan conversion in an ultrasound imaging system, it is directly applied without a homomorphic operation.

IV. THE SOLUTION OF THE DIFFUSION EQUATION IN PYRAMID DOMAIN

Many numerical schemes can be used to solve the diffusion equation [36]. In most cases, a finite difference scheme is preferred because of its relatively easy implementation for 2-D digital images. With the finite difference scheme and central differencing in spatial domain, the 2-D discrete nonlinear diffusion equation can be expressed as

$$\begin{aligned}
 I(i, j, t + 1) = & I(i, j, t) + \lambda [c_N \\
 & \cdot \nabla I_N(i, j, t) + c_S \\
 & \cdot \nabla I_S(i, j, t) + c_W \cdot \nabla I_W(i, j, t) \\
 & + c_E \cdot \nabla I_E(i, j, t)]
 \end{aligned} \quad (11)$$

where λ is the time step and controls the speed of diffusion. The local image gradients are approximated by directional differences

$$\nabla I_N(i, j) = I(i - 1, j) - I(i, j) \quad (12a)$$

$$\nabla I_S(i, j) = I(i + 1, j) - I(i, j) \quad (12b)$$

$$\nabla I_W(i, j) = I(i, j - 1) - I(i, j) \quad (12c)$$

$$\nabla I_E(i, j) = I(i, j + 1) - I(i, j). \quad (12d)$$

Allowing $\|\nabla I\|^2$ to be discretized as the average of the four squared directional differences, we can compute the gradient magnitude $\|\nabla I\|$ by

$$\begin{aligned}
 \|\nabla I\| = & 0.5 \\
 & \times \sqrt{\|\nabla I_N\|^2 + \|\nabla I_S\|^2 + \|\nabla I_W\|^2 + \|\nabla I_E\|^2}.
 \end{aligned} \quad (13)$$

To solve the PDE in (11), a boundary condition needs to be imposed to fit the solution to an actual problem. Since zero-

padding and periodic boundary conditions may lead to undesirable false discontinuities, the Neumann boundary condition that assumes values beyond an image border are equivalent to values on the border is utilized.

The stability requirements of the explicit discretization scheme in (11) limit the time step to be no larger than $1/(2D)$, where D is the number of dimensions (i.e., $D = 2$ in this study) along which image gradients are calculated. As discussed in [37], iteration and multiscale processing have a similar effect because they both perform local operations to produce global effects. Therefore, a multiscale iterative method requires a much smaller number of iterations as compared to a single scale processing method. The proposed LPND can be considered a multiscale iterative method. Thus, the time step limitation (i.e., $\lambda \leq 0.25$) is not prohibitive for the LPND. When the processed image contains a large amount of noise and the computational time is critical, the additive operator splitting (AOS) scheme [38] can be adopted to allow for a larger time step and consequently a smaller number of iterations. Otherwise, the explicit discretization scheme is preferred because less diffusion time is needed in this case and a larger time step allowed by the AOS scheme may affect the accuracy of solution.

As the diffusion process is iterative, one challenge of diffusion filtering is in deciding when the diffusion process is to be stopped. It can be stopped manually by setting a fixed number of iterations. However, in real applications, different images may need different numbers of diffusion iterations. Thus, a mechanism to stop the diffusion automatically is preferred. Several criteria for estimating the optimal stopping time have been proposed. We have tested Weickert's relative variance [39] and Mrázek's decorrelation [40] criteria in our simulations. Although working well for conventional diffusion methods, they were found to be inappropriate for the LPND. Weickert's criterion is based on the fact that the ratio between the variance of the diffused image at time t and the variance of the original image should decrease monotonically from 1 to 0. Prescribing a threshold for this ratio will stop the diffusion automatically. Since different pyramid layers in the LPND have different noise levels, they need different thresholds for the ratio. In practice, without any *a priori* knowledge of the speckle variance, it is a challenging task to determine the different thresholds based on experience and trial-and-error. Mrázek's criterion is based on the correlation between the filtered image at time t and noise. This correlation should decrease first, meaning that noise is reduced, and increase later, implying the signal is now being

Fig. 4. Convergence of MAE in the proposed LPND method.

affected. Thus, the diffusion should stop when the correlation reaches its minimum. However, this correlation does not have a unique minimum for the LPND since the lowest pyramid layer contains very little signal and is close to the degenerate case. Therefore, the decorrelation criterion is unsuitable for determining the stopping time of the LPND.

The mean absolute error (MAE) between two adjacent diffusion steps can be used to stop the iteration

$$\text{MAE}(I(t)) = \frac{1}{M \times N} \times \sum_{(i,j)=1}^{M,N} \sqrt{(I(i,j,t) - I(i,j,t-1))^2} \quad (14)$$

where $I(i,j,t)$ and $I(i,j,t-1)$ are the filtered values of the pixel (i,j) at time t and $t-1$, respectively, and M and N are the numbers of columns and rows in the processed image, respectively.

As discussed in [41] and the references therein, the convergence condition of a nonlinear diffusion process is dependent on the image size, the diffusivity function, and the numerical scheme. Some diffusion implementations will diverge whereas a convergent nonlinear diffusion process converges exponentially. To study the convergence of the proposed LPND, the MAE values from a four-layer LPND process (i.e., nonlinear diffusion is applied in three bandpass layers) with the above scheme on a real ultrasound liver image are plotted in Fig. 4. The time step is 0.2, and the number of iterations is 100. This number of iterations far exceeds the necessary number of iterations to get a smooth image, and thus is considered enough for examining the convergence of the LPND. As shown in Fig. 4, the MAE value decreases exponentially with the number of iterations, and the diffusion iteration can be stopped by setting a threshold for the MAE value.

When the change between two adjacent diffusion steps is smaller than a prescribed threshold, the diffusion filtering is stopped automatically. Alternatively, this threshold for stopping the diffusion filtering can be adjusted by clinicians according to their personal preferences and the different purposes for speckle reduction. If the speckle reduction is applied as a visual aid for the interpretation of an ultrasound image, a short diffusion time

is suitable to remove speckle without blurring fine structures and fuzzy boundaries. If the speckle reduction technique is used as a preprocessing step before some image processing tasks such as segmentation and registration, a longer diffusion time can be adopted to create more homogeneous regions and thus improve the performance of segmentation and registration.

The computational complexity of the proposed LPND can be analyzed by dividing it into two parts. For a 2-D image of $M \times N$ pixels, the computational complexity of Laplacian pyramid decomposition and reconstruction is $O(M \times N)$ (i.e., the order of $M \times N$). For the same image, the computational complexity of nonlinear diffusion is $i_{\text{diff}} \cdot O(M \times N)$, where i_{diff} is the number of iterations. Using a four-layer decomposition as shown in Fig. 1, we can write the computational complexity of the proposed LPND as the sum of two main parts: one is Laplacian pyramid decomposition and reconstruction of an image of $M \times N$ pixels, and the other is three nonlinear diffusions on three images of different sizes (i.e., $M \times N$, $(M \times N)/4$, and $(M \times N)/16$). Therefore, the overall computational complexity of the LPND is $(1 + i_0 + (i_1/4) + (i_2/16)) \cdot O(M \times N)$, where i_0 , i_1 , and i_2 are the numbers of iterations for nonlinear diffusion in the first, second, and third layers of Laplacian pyramid (i.e., L_0 , L_1 , and L_2 in Fig. 1), respectively.

In our current implementation using the explicit discretization scheme and time step $\lambda = 0.2$, and stopping diffusion automatically by the MAE criterion where the threshold for MAE is 0.005, it takes around 6 s (15, 27, and 45 iterations in the first, second, and third layers of Laplacian pyramid, respectively) for the LPND to process a 266×512 unfiltered envelope-detected ultrasound image in Matlab (Mathworks Inc., Natick, MA, USA) on a Pentium 4 (2.4 GHz) PC. By adopting the AOS scheme and time step $\lambda = 2$, we can reduce the number of iterations for nonlinear diffusion in each layer of the Laplacian pyramid to two. This number of iterations is comparable to the one discussed in [15], where a real-time implementation is achieved using the AOS scheme and three iterations of their diffusion process.

V. EXPERIMENT DESIGN AND RESULTS

To evaluate the performance of the LPND, simulation, phantom, and *in vivo* studies were conducted. The simulation and phantom studies give quantitative performance analysis, and the *in vivo* study demonstrates the feasibility and usefulness of our proposed method in real applications.

In each study, the performance of the LPND was compared with those of Laplacian pyramid-based Wiener filter (LPW), Gaussian regularized nonlinear diffusion (ND), SRAD [14], and NCD [15]. LPW uses Wiener filters in all bandpass layers of the Laplacian pyramid of an image. The Wiener filter was implemented using the built-in function “wiener2” in Matlab, and its filter window was set to be 7×7 , 5×5 , and 3×3 from the lowest pyramid layer to the second highest one, respectively. For both LPND and LPW, four pyramid layers and 9×9 binomial filters for both REDUCE and EXPAND operators were used. Using more layers did not guarantee improved performance. A smaller binomial filter window would blur image edges too much, whereas a larger binomial filter window would increase the computational complexity and did not improve performance

Fig. 5. Simulated B -mode image and its filtered results. (a) Underlying echogeneity map. (b) Simulated speckled image. (c)–(g) Images filtered by LPND, LPW, ND, SRAD, and NCD.

much. The ND utilizes (7) for speckle reduction. Its gradient threshold was determined by (9). The diffusivity functions for LPND and ND were computed using (8), the standard deviation of Gaussian filter in (7) was 1.0, and the time step was 0.2. The stopping time of LPND, ND, and NCD was determined by the MAE criterion given in (14), and the thresholds for the MAE were all set to 0.005. The SRAD needed a relatively small time step λ , which was set to 0.02. Its stopping time was also determined automatically by the MAE criterion, and the threshold for the MAE was 0.05, 0.1, and 0.2 to give optimal results for simulation, phantom, and *in vivo* studies, respectively. The coherence stopping level in NCD was set to 0.07 to achieve an optimal coherence enhancement without introducing artifacts. Other parameters of SRAD and NCD were same as those used in [14] and [15]. The LPND, LPW, ND, and NCD were implemented in homomorphic manners on envelope-detected images, which were formed by computing $B = \sqrt{R_I^2 + R_Q^2}$, where R_I and R_Q are in-phase and quadrature echo signals respectively. We took the logarithm of the envelope-detected images, then processed them using LPND, LPW, ND, or NCD method, and finally performed exponential transform of the processed data. The SRAD was applied directly on envelope-detected images, as adopted in [14].

A. Results From Simulation Study

The five different speckle reduction methods (i.e., LPND, LPW, ND, SRAD, and NCD) were applied on simulated B -mode images, which were generated by considering a radio-frequency (RF) image to be the convolution of a 2-D

point spread function with a 2-D underlying echogeneity map, as adopted in [14]. Images with different simulating parameters were tested, and similar results were obtained in all these cases. Thus, we only present the result obtained with the following parameters: the center frequency was 5 MHz, the pulse width was 1.2, the lateral beam width was 1.5, and the variance of the underlying complex Gaussian random field was 1.0.

The echogeneity map of 256×256 pixels was generated as follows. A ventricular cavity, an artery interior and a vascular wall, a rectangular target, a bright circular target, a dark circular target, an elliptical target, three small cysts, and four point targets, whose mean values of ultrasound cross sections are 2, 2.5, 20, 25, 18, 3.5, 3, 1.5, and 40, respectively, were placed at different locations in the image. The mean value of the background was 10.

Fig. 5(a) and (b) shows the underlying echogeneity map and a simulated envelope-detected image, respectively. Both images are log-compressed and then normalized with the same function for better visualization. The echogeneity map was used as the reference image for calculating quantitative measures.

To quantify the achieved performance improvements, three metrics were computed on the reference and filtered images. One measure is mean squared error (MSE) defined as

$$\text{MSE} = \frac{1}{M \times N} \sum_{(i,j)=1}^{M \times N} (\hat{S}(i,j) - S(i,j))^2 \quad (15)$$

where S and $\hat{S}$ are the reference and filtered images, respectively.

To evaluate the performance of edge preservation, the parameter β as used in [17], [18], [22] was computed by

$$\beta = \frac{\Gamma(\Delta S - \overline{\Delta S}, \Delta \hat{S} - \overline{\Delta \hat{S}})}{\sqrt{\Gamma(\Delta S - \overline{\Delta S}, \Delta S - \overline{\Delta S}) \cdot \Gamma(\Delta \hat{S} - \overline{\Delta \hat{S}}, \Delta \hat{S} - \overline{\Delta \hat{S}})}} \quad (16)$$

and where ΔS and $\Delta \hat{S}$ are the highpass filtered versions of S and $\hat{S}$, respectively, obtained with a 3×3 standard approximation of the Laplacian operator, $\overline{\Delta S}$ and $\overline{\Delta \hat{S}}$ are mean intensity of pixels in the region of ΔS and $\Delta \hat{S}$, respectively, and

$$\Gamma(S_1, S_2) = \sum_{(i,j)=1}^{M \times N} (S_1(i, j) \cdot S_2(i, j)). \quad (17)$$

The range of β value is [0 1], with unity for ideal edge preservation.

The structure similarity (SSIM) [42] was used to evaluate the overall processing quality

$$\text{SSIM} = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (18)$$

where μ_i and σ_i are the mean and standard deviation of intensities of pixels in a local window, respectively, and C_i is the constant to avoid instability. All involved parameters were set as suggested in [42]. The value of SSIM varies between 0 and 1, with unity for ideal processing quality, i.e., when the filtered image is equal to the reference image.

Besides, the contrast-to-noise-ratio (CNR), which is sometimes referred as lesion signal-to-noise ratio [43], [44], was computed by

$$\text{CNR} = \frac{|\mu_1 - \mu_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}} \quad (19)$$

where μ_1 and σ_1^2 are the mean and variance of intensities of pixels in a region of interest (ROI), and μ_2 and σ_2^2 are the mean and variance of intensities of pixels in a background region that has the same size as the ROI to be compared with.

The obtained values of MSE, β , and SSIM from all the methods are summarized in Table I. To reduce the sample bias in the random process of noise generation, the experiment was repeated 30 times under the same parameter settings except that the noise was simulated using different random seeds. Table I shows the mean and standard deviation values obtained from the 30 samples. The results in Table I show that the proposed LPND outperforms all the other four methods in terms of MSE and SSIM, while ND, SRAD, and NCD achieve comparable β values to LPND. The two-sided paired t -tests were conducted under the null hypothesis that there is no statistically significant difference between the β values achieved by LPND, ND, SRAD, and NCD. A value of $p < 0.05$ was considered statistically significant. The β values achieved by LPND are significantly different from those achieved by ND ($p < 10^{-5}$). However, there are no significant difference between the β

TABLE I
MSE, β , AND SSIM VALUES ON 30 SIMULATED IMAGES WITH THE SAME SPECKLE LEVEL

	MSE	β	SSIM
Noisy	260.46 $\pm$ 7.15	0.0285 $\pm$ 0.0057	0.3922 $\pm$ 0.0038
LPND	76.88 $\pm$ 3.11	0.1009 $\pm$ 0.0119	0.8107 $\pm$ 0.0038
LPW	93.05 $\pm$ 3.53	0.0317 $\pm$ 0.0053	0.7535 $\pm$ 0.0049
ND	86.38 $\pm$ 3.52	0.0805 $\pm$ 0.0189	0.7934 $\pm$ 0.0043
SRAD	144.06 $\pm$ 4.68	0.0955 $\pm$ 0.0169	0.7290 $\pm$ 0.0039
NCD	82.49 $\pm$ 2.96	0.1003 $\pm$ 0.0126	0.7992 $\pm$ 0.0039

TABLE II
CNR VALUES ON 30 SIMULATED IMAGES WITH THE SAME SPECKLE

	ROI 1	ROI 2	ROI 3	ROI 4	ROI 5	ROI 6
Noisy	1.37 $\pm$ 0.06	1.18 $\pm$ 0.06	0.71 $\pm$ 0.08	1.06 $\pm$ 0.07	1.04 $\pm$ 0.09	1.15 $\pm$ 0.05
LPND	4.73 $\pm$ 0.34	4.14 $\pm$ 0.33	2.29 $\pm$ 0.36	3.58 $\pm$ 0.49	2.68 $\pm$ 0.39	3.17 $\pm$ 0.27
LPW	3.07 $\pm$ 0.18	2.69 $\pm$ 0.21	1.51 $\pm$ 0.20	2.32 $\pm$ 0.29	2.00 $\pm$ 0.22	2.38 $\pm$ 0.19
ND	3.44 $\pm$ 0.25	3.03 $\pm$ 0.23	1.77 $\pm$ 0.25	2.64 $\pm$ 0.35	2.29 $\pm$ 0.29	2.43 $\pm$ 0.27
SRAD	3.34 $\pm$ 0.25	3.02 $\pm$ 0.25	1.97 $\pm$ 0.27	2.60 $\pm$ 0.31	3.62 $\pm$ 0.36	2.15 $\pm$ 0.22
NCD	3.65 $\pm$ 0.27	3.16 $\pm$ 0.24	1.85 $\pm$ 0.26	2.74 $\pm$ 0.35	2.38 $\pm$ 0.28	2.67 $\pm$ 0.28

Fig. 6. Six ROIs for calculating CNR in the simulation study.

values achieved by LPND and those by SRAD ($p = 0.1612$), and no significant difference between the β values achieved by LPND and those by NCD ($p = 0.8555$).

The mean and standard deviation of CNR values from each method on the 30 simulated images are summarized in Table II. The six ROIs in the reference image [i.e., Fig. 5(a)] are marked in Fig. 6. The two-sided paired t -tests were conducted under the null hypothesis that there is no statistically significant difference between the CNR values obtained from LPND and those from other four methods for each ROI. For ROIs 1, 2, 3, 4, and 6, the hypothesis was rejected with $p < 0.001$. For ROI 5, LPND was also significantly different from LPW, ND, and NCD ($p < 0.0011$). On the other hand, no significant difference between LPND and SRAD was found for ROI 5 ($p = 0.5401$).

Fig. 5(c)–(g) gives a visual comparison of their performance on the simulated image in Fig. 5(b). All these filtered images are log-compressed and then normalized with the same functions, so that they have the same dynamic range. Compared to the other approaches, LPND shows the most noise reduction while preserving small structures such as the three small cysts and the four point targets. LPW does not give satisfactory speckle suppression. ND enhances edges, but it does not keep correct edge locations. SRAD dilates bright regions and erodes dark regions. With the SRAD, the boundaries of bright regions are broadened and those of dark regions are shrunk. NCD enhances the edge coherence but cannot suppress enough noise. The three small cysts and the four point targets are blurred, as shown in Fig. 5(g).

Fig. 7. Phantom image and its filtered results. (a) Original noisy image. (b)–(f) Image filtered by LPND, LPW, ND, SRAD, and NCD.

Fig. 8. Real ultrasound liver image and its filtered results. (a) Original noisy image. (b)–(f) Image filtered by LPND, LPW, ND, SRAD, and NCD.

B. Results From Phantom Study

The five speckle reduction methods including the proposed LPND were applied on a commercial gray scale phantom image (i.e., 403GS LE, Gammex RMI, Middleton, WI, USA). A sequence of in-phase and quadrature (R_I and R_Q) data containing 30 frames was acquired using a commercial ultrasound machine (i.e., EUB-6000, Hitachi Medical Corp., Japan) equipped with a special programmable board for data acquisition. The programmable board was connected with a PC, and the acquired in-phase and quadrature data were transferred to the PC for processing. The envelope of the acquired data was used for testing all the five methods.

Fig. 7 shows their results. For quantitative analysis, the CNR values were computed on the filtered images. The results on the 30 frames are summarized in Table III. ROIs 1–4 are the four contrast regions from left to right in the reference image, as shown in Fig. 7(a). They are a high scatter region relative to

TABLE III
CNR VALUES ON A PHANTOM IMAGE SEQUENCE CONTAINING 30 DIFFERENT FRAMES

CNR	ROI 1	ROI 2	ROI 3	ROI 4
Noisy	1.80 ± 0.10	0.51 ± 0.08	1.52 ± 0.07	3.81 ± 0.13
LPND	6.34 ± 0.81	2.17 ± 0.45	6.19 ± 0.57	6.86 ± 0.34
LPW	4.45 ± 0.40	1.35 ± 0.26	4.28 ± 0.29	6.61 ± 0.33
ND	5.57 ± 0.55	1.72 ± 0.33	5.06 ± 0.31	8.61 ± 0.56
SRAD	5.93 ± 0.50	1.77 ± 0.32	5.11 ± 0.38	7.33 ± 0.74
NCD	5.59 ± 0.58	1.76 ± 0.33	5.12 ± 0.31	8.07 ± 0.57

the background (ROI 1), a gray scale target of +6 dB (ROI 2), a gray scale target of −6 dB (ROI 3), and an anechoic cyst (ROI 4), respectively.

For each ROI, the two-sided paired t -tests were conducted under the null hypothesis that there is no statistically significant difference between the CNR values achieved by LPND and the other four methods. For ROIs 1–3, the hypothesis between the CNR values from LPND and those from SRAD in ROI 1 was

Fig. 9. Profiles along the highlight line in the liver image. (a) Original noisy image showing the highlight line. (b)–(g) Profiles along the highlight line in the images filtered by LPND, LPW, ND, SRAD, and NCD.

Fig. 10. Real ultrasound carotid artery cross-sectional image and its filtered results. (a) Original noisy image. (b)–(f) Images filtered by LPND, LPW, ND, SRAD, and NCD.

rejected by $p < 0.023$, and all the other hypotheses were rejected by $p < 10^{-3}$. However, for ROI 4, which is an anechoic cyst, pyramid-based methods gave lower CNR values compared to diffusion-based methods.

C. Results From in Vivo Study

Two sequences of R_I and R_Q data were acquired from liver and carotid artery of a volunteer using 3.5-MHz linear and 7.5-MHz convex array transducers. The results from different frames are similar, thus only one frame from each set is used here for illustration. Fig. 8 shows the results on liver images obtained using a 3.5-MHz convex array transducer. To show the differences between them more clearly, a line presenting the pixel values on row 175 is highlighted, and the pixel values

on this line from the original and filtered images are shown in Fig. 9. Fig. 10 shows the result on common carotid artery cross-sectional images using a 7.5-MHz linear array transducer operating at 10 MHz. The pixel values on column 180 from the original and filtered images are compared in Fig. 11. All the five methods were applied on the envelope-detected data.

As can be seen from Figs. 8–11, LPND preserves image edges and small structures (e.g., the regions numbered 4, 5, and 6 in Figs. 9 and 11) while effectively suppressing speckle in background (e.g., the regions numbered 1, 2, and 3 in Figs. 9 and 11). Due to its optimal noise reduction, LPND outperforms the other four methods, resulting in increased contrast and improved visibility of small structures in each image. All the other four methods have limited noise reduction performance. ND shows

Fig. 11. Profiles along the highlight line in the carotid artery cross-sectional image. (a) Original noisy image showing the highlight line. (b)–(g) Profiles along the highlight line in the images filtered by LPND, LPW, ND, SRAD, and NCD.

sharper but jagged edges and gives a relatively low contrast. SRAD broadens the boundaries of bright regions (the regions numbered 4, 5, and 6 in Fig. 11) and shrinks those of dark regions (the regions numbered 4 and 5 in Fig. 9). Although NCD enhances the coherence of organ surfaces, it also causes blurring of small structures, and the contrast of its output image is not as good as that of LPND.

VI. CONCLUSION

A new nonlinear diffusion method in Laplacian pyramid domain for ultrasonic speckle reduction has been investigated. It consists of the following three steps. First, the input image is decomposed into its Laplacian pyramid domain. Then, a regularized nonlinear diffusion process is performed in each pyramid layer except the highest one to remove the speckle. Finally, the diffused Laplacian pyramid is reconstructed to get the despeckled image. An automatic estimation of the gradient threshold by MAD is also integrated in the LPND to make the algorithm practical.

The proposed LPND preserves edges and small structures while maximally removing speckle. Thus, it has the potential to improve the diagnostic capability of current ultrasound imaging and to enhance the performance of some high level image processing tasks in the ultrasound imaging systems. It may be further improved by using information from coarse scales to control diffusion filtering at finer scales. Steidl *et al.* [45] have proved that a nonlinear diffusion on the Laplacian pyramid of a one-dimensional signal is equivalent to wavelet shrinkage on multiple scales. Based on their work, another possible future work is to derive a wavelet shrinkage function that may achieve similar result as the LPND.

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