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Wavelet analysis of speckle patterns with a temporal carrier

Yu Fu, Cho Jui Tay, Chenggen Quan, and Hong Miao

A novel temporal phase-analysis technique that is based on wavelet analysis and a temporal carrier is presented. To measure displacement on a vibrating object by using electronic speckle pattern interferometry, one captures a series of speckle patterns, using a high-speed CCD camera. To avoid ambiguity in phase estimation, a temporal carrier is generated by a piezoelectric transducer stage in the reference beam of the interferometer. The intensity variation of each pixel on recorded images is then analyzed along the time axis by a robust mathematical tool, i.e., a complex Morlet wavelet transform. After the temporal carrier is removed, the absolute displacement of a vibrating object is obtained without the need for temporal or spatial phase unwrapping. The results obtained by a wavelet transform are compared with those from a temporal Fourier transform. © 2005 Optical Society of America

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1. Introduction

In the past two decades, electronic speckle pattern interferometry (ESPI) has become one of the main experimental techniques for detecting static and dynamic displacement fields of optically rough objects. It is a nondestructive, whole-field technique with excellent sensitivity controlled by the wavelength of laser light. The most common ESPI analysis route has involved the measurement of spatial phase change distributions recorded before and after deformation followed by spatial unwrapping. Generally, phase shifting is the main phase-retrieving method applied with ESPI. It requires several, normally three to five, images together with prescribed phase steps. For this reason, phase-shifting approaches are not easy to use for continuous deformation measurement.¹ Furthermore, spatial phase unwrapping is compulsory to remove the 2π phase discontinuities.² However, it is difficult to accomplish two-dimensional spatial phase unwrapping because of the presence of noise and low modulated pixels in speckle patterns that may produce breaks in a wrapped phase map and generate large phase errors during unwrapping.

Temporal phase analysis and temporal phase-unwrapping techniques^{3–8} were introduced during recent years to overcome these problems. In these techniques, a sequence of speckle patterns is recorded throughout the entire deformation history of the object. Each pixel is then analyzed as a function of time. The phase of the speckles is related to different physical parameters, depending on the optical arrangements. Joenathan *et al.*^{9–11} used this technique to measure large continuous deformation of objects. The reported upper limit on the object deformation measurement when the temporal analysis technique is used is more than 100 μm instead of the 6–7 μm that is typical in conventional spatial techniques.

There are several temporal analysis algorithms.¹² Among them, Fourier transformation^{13,14} is the predominant method in temporal speckle pattern interferometry. The intensity fluctuation that is due to deformation of each pixel is first transformed, and one side of the spectrum is filtered with a bandpass filter. The filtered spectrum is inverse transformed to yield the wrapped phase. The phase values are then unwrapped along the time axis at each pixel independently of other pixels in the image. Fourier-transform analysis gains high accuracy when the temporal frequency is high and the spectrum is narrow. However, in most cases the spectrum is wide because of the nonlinear deformation along the time axis. In recent years, a wavelet transform¹⁵ was introduced in temporal phase analysis to overcome the disadvantages of the Fourier transform. The concept was introduced by Colonna de Lega in 1996, and some preliminary

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results were presented.^{16,17} Cherbuliez and Jacquot¹⁸ and Cherbuliez¹⁹ extended the study by applying various processing algorithms to develop an efficient software for phase retrieving from dynamic fringe or speckle patterns. Previous research^{20,21} also showed the advantages of the wavelet transform compared with the Fourier transform in temporal phase analysis.

The temporal phase-analysis technique has the advantage of eliminating speckle noise, as it evaluates the phase pixel by pixel along the time axis. However, it does have its disadvantages: It cannot analyze a part of an object that is not moving with the rest; neither can it analyze objects whose different parts deform in different directions. Determination of the absolute sign of the phase is impossible by either temporal Fourier or wavelet analysis. This limits the technique to the measurement of deformation in one direction that is already known. Adding a carrier frequency to the image-acquisition process is a method for overcoming these problems. In this study, we applied a temporal carrier by shifting the reference object constantly in one beam of the interferometer. The carrier frequency is higher enough that the phase change on any point is in one direction. When a vibrating object is measured, the temporal intensity variation on each pixel is generally a frequency-modulated signal, where wavelet analysis is more suitable^{20–22} for retrieving the phase. In this paper we describe our application of a wavelet analysis method to measure the absolute out-of-plane displacement of a vibrating cantilever beam. The phase is retrieved by integration and phase unwrapping in time, and knowledge of its spatial domain is not required. The phase variation that is due to a temporal carrier is also measured experimentally. After the effects of the temporal carrier are removed, an absolute displacement of the vibrating beam is obtained. The results are also compared with those from a temporal Fourier transform.

2. Theoretical Analysis

A schematic layout of the ESPI setup for out-of-plane deformation measurement with a temporal carrier is shown in Fig. 1. An expanded laser beam is collimated and separated by a beam splitter into object and reference beams. The temporal carrier is generated by constant shifting of the reference plate with a piezoelectric transducer (PZT) stage. A series of speckle patterns is captured by a high-speed CCD camera with a telecentric lens during deformation. The intensity of each pixel can be expressed as

$$\begin{aligned} I_{xy}(t) &= I_{0_{xy}}(t) + A_{xy}(t)\cos[\phi_{xy}(t)] \\ &= I_{0_{xy}}(t) + A_{xy}(t)\cos[\phi_c(t) + \phi_{xy}(t)] \\ &= I_{0_{xy}}(t)\left\{1 + V\cos\left[2\pi f_c t + \phi_{0_{xy}} + \frac{4\pi z_{xy}(t)}{\lambda}\right]\right\}, \end{aligned} \quad (1)$$

where $I_{0_{xy}}(t)$ is the intensity bias of the speckle pat-

Fig. 1. Schematic layout of the ESPI system.

tern, V is the visibility, $\phi_{0_{xy}}$ is the random phase, f_c is the temporal carrier frequency, $\phi_c(t) = 2\pi f_c t$ is the phase change that is due to the temporal carrier, and $z_{xy}(t)$ is the out-of-plane deformation of the object. At each pixel the temporal intensity variation is a frequency-modulated signal and is analyzed by a continuous wavelet transform (CWT).

The CWT of a signal $s(t)$ is defined as its inner product with a family of wavelet functions $\psi_{a,b}(t)$:

$$W_s(a, b) = \int_{-\infty}^{+\infty} s(t)\psi_{a,b}^*(t)dt, \quad (2)$$

where

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a, b \in R, \quad a > 0. \quad (3)$$

$\psi(t)$ is known as the mother wavelet, and $\psi_{a,b}(t)$ are the basis functions of the transform, known as daughter wavelets. a is the scaling factor related to the frequency, b is the time shift, and $*$ denotes the complex conjugate. The factor $1/\sqrt{a}$ in Eq. (3) is used to keep the energy of $\psi_{a,b}(t)$ constant during scaling and translation.

Signal $s(t)$ can be recovered from wavelet coefficients $W_s(a, b)$ by an inverse wavelet transform, given by¹⁵

$$s(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_0^{+\infty} W_s(a, b)\psi\left(\frac{t-b}{a}\right)\frac{da}{a^2} db, \quad (4)$$

where the constant C_ψ is given by

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < +\infty \quad (5)$$

and $\hat{\psi}(\omega)$ denotes the Fourier transform of $\psi(t)$. Mother wavelet function $\psi(t)$ is a zero mean wiggle (real or complex), localized both in time and in frequency, and it satisfies the admissibility condition expressed by Eq. (5).

An analytic wavelet is selected for analysis of phase related properties of real functions (e.g., determination of instantaneous frequency). The most commonly used mother wavelet for such applications is the complex Morlet wavelet, because it senses local oscillations. The Morlet wavelet is defined by

$$\psi(t) = g(t)\exp(i\omega_0 t), \quad g(t) = \exp\left(-\frac{t^2}{2}\right). \quad (6)$$

Thus ω_0 is the mother frequency, a parameter that has to be chosen properly. To remove negative frequencies and avoid dc signals, the Fourier transform at $\omega = 0$, $\hat{\psi}(0)$, should be numerically negligible. This condition is achievable when $\omega_0 > 5$. However, the condition that $\omega_0 = 2\pi$ was chosen to satisfy the admissibility condition such that the wavelet function is able to remove the negative frequencies as well as the dc term of the signals. One shortcoming of a CWT is the large error generated at the boundary of the signal. To overcome this problem we extended the signal at its left- and right-hand edges. A linear predictive extrapolation method²³ was used in this study. The advantage of this extrapolation method is that the phase and the frequency of intensity variations are maintained. After the CWT of the extended data has been carried out, the wavelet coefficients are truncated appropriately.

The CWT expands a one-dimensional temporal intensity variation of certain pixels into a two-dimensional plane of scaling a and position b (which is related to the time axis). Substituting Eqs. (1) and (6) into Eq. (2), we can express the wavelet transform of temporal intensity variation as²⁴

$$W_{xy}(a, b) = \frac{\sqrt{a}}{2} A_{xy}(b) \left(\hat{g}\{a[\zeta - \varphi_{xy}'(b)]\} + \varepsilon\left(b, \frac{\omega_0}{a}\right) \right) \{\exp[i\varphi_{xy}(b)]\}, \quad (7)$$

where $\zeta = \omega_0/a$ and ε is a corrective term that normally can be negligible. The main purpose of the calculations is to find a trajectory of maximum values of $|W_{xy}(a, b)|^2$ on the a - b plane; called a ridge. As $|\hat{g}(\omega)|$ possesses a maximum at $\omega = 0$, and when ε is negligible, $|W_{xy}(a, b)|^2$ reaches a maximum value for

$$\varphi_{xy}'(b) = \zeta_{rb} = \omega_0/a_{rb}, \quad (8)$$

where $\varphi_{xy}'(b)$ is defined as the instantaneous frequency of the signal and a_{rb} denotes the value of a at

Fig. 2. (a) Reference block, cantilevered beam, and loading device. (b) Typical speckle pattern on a reference block and a cantilever beam and area of interest.

instant b on the ridge. The wavelet transform on the ridge can then be expressed as

$$W_{xy}(a_{rb}, b) \approx \frac{\sqrt{a_{rb}}}{2} A_{xy}(b) \hat{g}(0) \exp[i\varphi_{xy}(b)]. \quad (9)$$

Complex phase value Φ_W of wavelet transform $W_{xy}(a, b)$ on the ridge equals $\varphi_{xy}(b)$. In this study, phase value $\varphi_{xy}(b)$ is calculated by integration of the instantaneous frequency in Eq. (8), and a phase-unwrapping procedure is not needed in the temporal and spatial domains. The phase change between two instants, T_1 and T_2 , at point $P(x, y)$ can be expressed by

$$\Delta\varphi = \varphi_{xyT_2} - \varphi_{xyT_1} = 2\pi f_c(T_2 - T_1) + \frac{4\pi z_{(T_2-T_1)}}{\lambda}, \quad (10)$$

where $z_{(T_2-T_1)}$ is the out-of-plane displacement of point P between two instants, T_1 and T_2 . The phase change that is due to the temporal carrier can also be measured experimentally and removed from Eq. (10).

3. Experimental Illustration

Figure 1 shows the experimental setup. The specimen tested in this study [shown in Fig. 2(a)] is a Perspex cantilever beam with a diffuse surface. The length, width, and thickness of the beam are 400, 20, and 4 mm, respectively. A vibrator produces a sinusoidal vibration at the free end of the beam. The frequency of vibration is controlled by a function generator, and only the area near the clamping end is measured. The beam of a He-Ne laser (30 mW, $\lambda = 632.8$ nm) passes through a beam splitter to illuminate the specimen and a reference plate at a

right angle. To generate a temporal carrier, we mount the reference plate on a computer-controlled PZT. During vibration of the cantilever beam, the reference plate is applied with a linear rigid body motion at a certain velocity. To retrieve the phase change of the temporal carrier, a still reference block with a diffuse surface is mounted above the vibrating beam and is captured together with the beam. The object and reference beams are recorded on a CCD sensor. During deformation of the object, a series of speckle patterns is captured by a high-speed CCD camera (Kodak Motion Corder Analyzer, SR-Ultra) with a telecentric gauging lens. The size of the output image varies at different recording rates. At a recording rate of 250 frames/s, it reaches 512×480 pixels. The highest recording rate is 10,000 frames/s with a reduced image size of 128×34 pixels. The setup is fixed on a vibration isolation table with a transparent cover. As the recording time is short, the influence of environmental vibrations can be ignored.

4. Results and Discussion

Figure 2(b) shows a typical speckle pattern captured from a part of the cantilever beam, together with a motionless reference block above the beam at intervals of 0.004 s and an imaging rate of 250 frames/s. The area of interest on the cantilever beam contains 400×100 pixels and has a length and width of 60.8 and 15.2 mm, respectively. Five hundred speckle patterns were captured during a 2-s period. Among them, 400 consecutive images were selected for processing. For each pixel, 400 sampling points along the time axis were obtained. Figure 3(a) shows the intensity variation of point R [indicated in Fig. 2(b)] on the reference block. The modulus of the Morlet wavelet transform of the intensity variation of point R is shown in Fig. 3(b). The dashed line in Fig. 3(b) shows the ridge of the wavelet transform where the maximum modulus is found. Although a_{rb} on the ridge are fairly constant, some variations of a_{rb} caused by noise on a certain pixel are still observed. To eliminate the effects of noise we calculated the average value of a_{rb} in an area of 50×50 pixels on the reference block [also indicated in Fig. 2(b)]. Figure 3(c) shows the average scaling a_{rb} on the ridge. As the reference block is not moving, the ridge's value represents the effect of a temporal carrier that was applied on the reference plate by a PZT. Little variations of the average a_{rb} are still observed, which implies that the temporal carrier is constant along the time axis. Integration of $2\pi/a_{rb}$ was carried out along the time axis to generate a continuous phase change $\phi_c(t)$ that was due to the temporal carrier.

Figures 4 and 5 show the intensity variations and the modulus of the Morlet wavelet transform on points A and B, respectively [indicated in Fig. 2(b)]. It can be observed that the frequency variation of point A is less than that of point B, implying that the vibration amplitude on point A is less than that of point B because these two points have the same vibration frequency in this case. This assumption is

Fig. 3. (a) Temporal intensity variation of points R on the reference block. (b) Plot of the modulus of the Morlet wavelet transform at point R. (c) The average ridge detected on the reference block.

reasonable because point A is closer to the clamping end than point B. Similarly to what was done for point R, integration was carried out on each pixel to generate a continuous temporal phase change $\phi_{xy}(t) = \phi_c(t)\phi_{xy}(t)$. Figure 6(a) shows the temporal phase change obtained on point B and in the reference block. The difference between these two curves gives the absolute phase change of point B that is due to vibration. In a speckle interferometer, as is shown in Fig. 1, a 2π phase change represents a displacement of $\lambda/2$ ($= 316.4$ nm) in the z direction. Figure 6(b) shows the temporal displacement obtained on point B. Figure 7(a) shows the out-of-plane displacement in cross section C–C [shown in Fig. 2(b)] at three time intervals, $T_1 - T_0$, $T_2 - T_0$, and $T_3 - T_0$, [shown in Fig. 6(b)].

For comparison, temporal Fourier analysis was also applied to the same speckle patterns. On part of a reference block, a narrow-bandpass filter was applied as the spectrum of the signal was concentrated. On cantilever beam, a relatively wider filter was applied as the filtering window should include all fre-

Fig. 4. (a) Temporal intensity variation of points A on the cantilever beam. (b) Plot of the modulus of the Morlet wavelet transform at point A.

quencies. One-dimensional phase unwrapping was then applied along the time axis, as all phase values retrieved by an inverse Fourier transform fall within a $[0, 2\pi)$ range. Figure 7(b) shows the temporal displacements on cross section C–C obtained with a temporal Fourier transform. Figure 8 shows the three-dimensional displacement plot obtained by temporal

Fig. 5. (a) Temporal intensity variations of points B on a cantilever beam. (b) Plot of the modulus of Morlet wavelet transform at point B.

Fig. 6. (a) Phase variation retrieved on the reference block and at point B. (b) Out-of-plane displacement obtained at point B.

wavelet analysis and a Fourier transform. A 3×3 median filter was applied to both phase maps to remove some ill-behaved pixels. We observed that a CWT on each pixel generates a smoother spatial displacement distribution at different instants than does a Fourier transform. The maximum displacement fluctuation that is due to noise is $0.04 \mu\text{m}$ in a

Fig. 7. Displacement distribution on cross section C–C at different time intervals obtained by (a) temporal wavelet transform and (b) temporal Fourier transform.

Fig. 8. Displacement distribution $T_1 - T_0$ on a cantilever beam obtained by (a) a temporal wavelet transform and (b) a temporal Fourier transform.

Fourier transform but is only $0.02 \mu\text{m}$ in wavelet analysis.

From the above comparison between the results of wavelet and Fourier analyses, it can be concluded that wavelet analysis shows better results in displacement measurement of a vibrating object. As wavelet analysis calculates the optimized frequency at each instant, it performs adaptive bandpass filtering of the measured signal and thus limits the influence of various noise sources and increases the measurement resolution significantly. In contrast, the Fourier transform uses a broader filter, which is less efficient in eliminating noise. The maximum displacement fluctuation that is due to noise depends on the width of the bandpass filter and on the quality of the speckle patterns.

A continuous wavelet transform maps a one-dimensional intensity variation signal to a two-dimensional plane of position and frequency and then extracts the optimized frequencies. Obviously this is a time-consuming process and requires high computing speed and a large memory. In our study, the computation time is about 10 times larger than that of a temporal Fourier transform. Similarly to those in other temporal phase-analysis methods, the temporal wavelet transform is also limited by the Nyquist sampling theorem. Analyzing signals with a frequency higher than half of the acquisition rate is impossible. Therefore selecting a temporal carrier frequency that

is suitable for image acquisition, as was mentioned above, is not so easy. The temporal frequency that is due to deformation depends on two parameters: frequency and amplitude of the vibration. The temporal carrier frequency should be high enough that the phase change of each point on the object is in one direction. However, it cannot be too high because of limitations of the Nyquist sampling theorem, as the phase change on some points is the sum of the temporal carrier and displacement. Sometimes a compromise is not easily reached if the capturing rate of the camera is not high enough.

5. Concluding Remarks

In this paper we have presented a novel method for retrieving the transient out-of-plane displacement of a vibrating object that uses a combination of temporal wavelet analysis and a temporal carrier. Introducing a temporal carrier ensures that a phase change of each point on a vibrating object exists in one direction, such that temporal phase-analysis methods can be applied. In this case the variation in temporal intensity of each pixel is a frequency-modulated signal that the frequency is varying along the time axis. As wavelet analysis calculates the optimized frequency at each instant, it has the advantage compared with a Fourier transform of being able to extract instantaneous frequencies. Furthermore, a high-quality deformation map of the object can also be obtained without any phase-unwrapping. A comparison of temporal wavelet and Fourier transforms shows that wavelet analysis can limit the influence of various noise sources and significantly improve the results of displacement measurement. The maximum displacement fluctuation caused by noise was limited to $0.02 \mu\text{m}$.

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