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FLUCTUATIONS OF TURBULENT BED SHEAR STRESS

Nian-Sheng Cheng¹ and Adrian Wing-Keung Law²

Abstract: With the assumption that the bed shear stress fluctuates in a lognormal fashion, the probability density function (pdf) of the standardised bed shear stress is derived as a function of the relative shear stress intensity. The pdf is more skewed with larger relative intensities, but approaches a Gaussian function when the relative intensity is small. The computed pdf agrees well with the reported experimental data for flows over a smooth boundary. The higher-order moments of the bed shear stress, skewness and kurtosis, are shown analytically to be also dependent on the relative intensity. The theoretical dependencies are then compared to a number of measurements available in the literature. The Reynolds number effect on the relative intensity is also discussed.

Keywords: Bed shear stress, Bed shear stress fluctuation, Probability density function, Lognormal distribution, Skewness, Kurtosis, Smooth bed, Reynolds number.

Introduction

Quantifying the characteristics of the hydrodynamic forces exerted by near-bed flows on bed sediment particles is essential to characterise motion of the particles.

¹Asst. Professor, School of Civil and Environmental Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, cnscheng@ntu.edu.sg

²Assoc. Professor, School of Civil and Environmental Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, cwklaw@ntu.edu.sg

Examples include mathematical representations of the Shields diagram, which is used to describe the condition of incipient motion of the bed particles, and theoretical derivations of the bedload transport function. The interaction between the flow and bed particles can be generally investigated either by deterministic or probabilistic methods. The deterministic approach typically utilises the average parameters of the flow dynamics and bed particles. On the other hand, when a probabilistic method is used, both the average characteristics and the probability distributions of all independent variables which affect the mechanism of the interaction have to be known.

In previous studies, the pdf related to the near-bed flow condition was usually assumed to be normal or Gaussian. For example, Einstein and El-Samni (1949) reported that the instantaneous lift force was distributed in the normal fashion. Paintal (1971) simply applied the Gaussian function to depict the pdf of the dimensionless bed shear stress. The common use of the normal function to describe the pdfs of different flow variables is due to its simplicity and its good approximation in fitting experimental data for some cases. However, it is obvious that the Gaussian function does not hold for all cases. When commenting on the Einstein's bedload function, Yalin (1977) stated that the lift force might distribute differently for different bed conditions. Its pdf may be non-Gaussian in the transitional regime, only tending to be normal in the case of hydrodynamic rough beds. As the lift force is proportional to the square of the near-bed velocity, a near-bed velocity that is normally distributed implies that the pdf of the lift force must be non-Gaussian (Cheng and Chiew 1998).

Clearly, further studies are needed to clarify the different pdfs associated with the various variables. Such a clarification, however, is still impossible at this stage because of the lack of systematic measurements of the hydrodynamic forces exerted on the bed particles. In particular, the measurements of the lift or drag forces in the previous studies were only limited to few specified bed configurations with spheres (e.g., Watters and Rao 1970), or a single spherical particle fixed on a smooth bed (e.g., Patnaik et al. 1994, Mollinger and Nieuwstadt 1996). Moreover, the presence of a rough bed with sediment particles almost leads to invalidation of all the techniques currently employed to measure the bed shear stress for the condition of smooth boundaries.

The smooth boundary facilitates the measurement of the bed shear stress using techniques such as hot-film, hot-wire, and LDA and its variants. Much relevant information has been accumulated in the past few decades, which is due to extensive experimental studies done by Eckelmann (1974), Blinco and Simons (1974), Girgis

(1977), Wietrzak and Lueptow (1994), Colella and Keith (1997), Chew et al. (1998), Miyagi et al. (2000), among others. Based on the measurements included in these studies, a preliminary statistical analysis of the bed shear stress is conducted in this study. Following that, the probability density function is proposed and then compared to the experimental data. The high-order moments of the bed shear stress are also addressed. Despite being largely based on the measurements for the hydraulically smooth boundary, the present analysis aims to provide an insight into the statistical characteristics of the bed shear stress for the condition of hydraulically rough boundaries.

Probability Density Distribution of Bed Shear Stress

For a uniform flow over the hydraulically smooth bed, two distinct characteristics of the bed shear stress can be identified based on the previous studies as mentioned above. First, the bed shear stress is always positive, implying that flow reversal does not occur very near the bed. Second, the distribution of the fluctuating bed shear stress is positively skewed. With these two typical characteristics, the pdf of the bed shear stress may be described using the two-parameter lognormal function, which defines a positively skewed frequency curve (Crow and Shimizu 1988). With the lognormal function, the pdf of the bed shear stress τ can be expressed as

$$f(\tau) = \begin{cases} \frac{1}{\sqrt{2\pi}(\ln \tau)_{rms} \tau} \exp \left\{ -\frac{1}{2} \left(\frac{\ln \tau - (\ln \tau)_{mean}}{(\ln \tau)_{rms}} \right)^2 \right\} & \text{for } \tau > 0 \\ 0 & \text{for } \tau \leq 0 \end{cases} \quad (1)$$

where $(\ln \tau)_{mean}$ and $(\ln \tau)_{rms}$ = mean and rms values of the random variable $\ln \tau$, respectively.

The two parameters, $(\ln \tau)_{mean}$ and $(\ln \tau)_{rms}$, in (1) concern the statistical characteristics of the logarithm of the bed shear stress and can be expressed in terms of τ_{mean} and τ_{rms} as follows:

$$(\ln \tau)_{mean} = \ln \left(\frac{\tau_{mean}}{\sqrt{1 + I_\tau^2}} \right) \quad (2)$$

$$(\ln \tau)_{rms} = \sqrt{\ln(1 + I_\tau^2)} \quad (3)$$

where $I_\tau = \tau_{rms}/\tau_{mean}$ = relative intensity of the bed shear stress. With (2) and (3), (1) can be rewritten as

$$f(\tau) = \frac{1}{\sqrt{2\pi \ln(1 + I_\tau^2)} \tau} \exp \left[-\frac{\left(\ln \frac{\tau}{\tau_{mean}} + \ln \sqrt{1 + I_\tau^2} \right)^2}{2 \ln(1 + I_\tau^2)} \right] \quad (4)$$

for $\tau > 0$. Eq. (4) represents the pdf in terms of the mean and rms values of the bed shear stress.

Probability Density Distribution of Standardised Bed Shear Stress

For the purpose of statistical analysis, a random variable is often standardised with its mean and rms values. The standardised bed shear stress is defined as

$$\tau_s = \frac{\tau - \tau_{mean}}{\tau_{rms}} \quad (5)$$

and its pdf can be derived from (4) as

$$f(\tau_s) = \frac{I_\tau}{(1 + I_\tau \tau_s) \sqrt{2\pi \ln(1 + I_\tau^2)}} \exp \left\{ -\frac{[\ln(1 + I_\tau \tau_s) + 0.5 \ln(1 + I_\tau^2)]^2}{2 \ln(1 + I_\tau^2)} \right\} \quad (6)$$

for $\tau_s > -1/I_\tau$.

Eq. (6) shows that the pdf of τ_s depends only on I_τ . It is noted that (6) is different from that obtained simply by replacing τ in (1) with τ_s , as done previously by Blinco and Simons (1974). Being mathematically incorrect, the Blinco and Simons' formulation required the shifting of coordinates in comparing with measurements and also included two unknowns which could not be generally computed. With various values of I_τ , different pdfs of τ_s are plotted in Fig. 1 according to (6). The figure shows that the distribution of the standardised bed shear stress is more skewed for larger I_τ . It also indicates that the distribution approaches the Gaussian function for smaller values of the relative intensity. This can be proven mathematically. Considering that for small values of I_τ ,

$$\ln(1 + I_\tau^2) \approx I_\tau^2 - O(I_\tau^4) \quad (7)$$

(6) can thus be simplified to the Gaussian function:

$$f(\tau_s) = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{\tau_s^2}{2} \right) \quad (8)$$

Comparison with Experimental Data

There are a few experimental or numerical studies available in the literature,

where the pdfs of the bed shear stress were presented in terms of the standardised variable τ_s (Blinco and Simons 1977; Girgis 1977; Kim et al. 1987; and Obi et al. 1996). Comparisons of (6) with these measurements are given in Fig. 2.

Blinco and Simons (1977) determined the boundary shear stress in open channel flows with a hot-film anemometer. They presented the pdf for seven tests with the Reynolds number, R , varying from 4×10^4 to 1.4×10^5 . Two of them, corresponding to the largest and smallest R , are used in this study to compare to (6), as plotted in Figs. 2(a) and 2(b), respectively. The relative intensities used in the computation are those obtained in the experiments, namely 0.395 [Fig. 2(a)] and 0.244 [Fig. 2(b)]. Obi et al. (1996) undertook the measurement of the wall shear stress in a turbulent channel flow using a laser gradient meter which enabled a direct evaluation of the near-wall velocity gradient. They obtained the pdf of the wall shear stress at $I_\tau = 0.35$ as shown in Fig. 2(c). A further comparison of (6) is made with the results predicted by Kim et al. (1987) using the technique of Direct Numerical Simulation (DNS). They performed the computation for a fully developed channel flow at a Reynolds number of 3300, based on the mean centreline velocity and channel half-width. Kim et al.'s results of the pdf at $I_\tau = 0.364$, which were given in Naqwi and Reynolds (1991), are replotted in Fig. 2(d) to compare with (6). Fig. 2 shows that the agreement is very good between the data and computed probability density curves.

Skewness and Kurtosis

The two higher-order moments of the bed shear stress, skewness S and kurtosis or flatness K , are often taken as independent parameters in addition to the relative intensity I_τ in assessing the fluctuations of the bed shear stress. Here, the three parameters, I_τ , S and K , are shown to be interrelated. Using (6), S and K can be related to I_τ , respectively:

$$S = I_\tau^3 + 3I_\tau \quad (9)$$

$$K = I_\tau^8 + 6I_\tau^6 + 15I_\tau^4 + 16I_\tau^2 + 3 \quad (10)$$

From (9) and (10), it follows that the skewness is approximately equal to zero and the kurtosis 3 for small I_τ -values. This is consistent with the earlier finding that for small relative intensities, the distribution of the fluctuating bed shear stress follows the Gaussian function, which is not skewed and has a kurtosis of 3.

Furthermore, from (10), one gets

$$\sqrt{K-3} = \sqrt{I_\tau^8 + 6I_\tau^6 + 15I_\tau^4 + 16I_\tau^2} \approx 4I_\tau + \frac{15}{8}I_\tau^3 + \frac{159}{512}I_\tau^5 + O(I_\tau^7) \quad (11)$$

Ignoring the terms with orders higher than $O(I_\tau^3)$ in (9) and (11) leads to the skewness to be related to K for small I_τ -values in the following simple form:

$$K = \frac{16}{9}S^2 + 3 \quad (12)$$

The measured and computed coefficients of skewness and kurtosis are plotted against I_τ , respectively, in Fig. 3. The measurements were made with different probes including hot-element sensor, laser Doppler anemometry, laser gradient meter, and micro-shear stress sensor, in turbulent boundary-layer and channel flows with water, oil or air as the medium. Fig. 3 shows that the derived S- and K-values are generally larger than the experimental results, particularly for higher shear stress intensities. No specific reasons can be identified for such differences at this stage. However, it is noted that higher uncertainties are associated with the higher-order moments since they are more sensitive to inaccuracies of the experimental data. Moreover, the higher-order moments may also be affected by the stage of the flow development in the test section. This is because the flow, though interpreted to be fully developed based on the time-average velocity, may not be fully developed in terms of the streamwise similarity of higher-order turbulent characteristics. Similar uncertainties may be induced by the side-wall effect.

The relationship between S and K is plotted in Fig. 4 according to (12), with a comparison to the experiment data. The figure displays that for the same value of the skewness, the computed kurtosis is generally larger than the measured results.

Effect of Reynolds Number on Bed Shear Stress Intensity

The fluctuating bed shear stress is not only related to the near-bed flow structure, as expected, but also depends on the bulk flow characteristics. The Reynolds number effect on the bed shear stress intensity is shown in Fig. 5, which provides a sketchy I_τ -R relationship with four series of experimental data given by Blinco and Simons (1974), Screenivan and Antonia (1977), Alfredsson et al. (1988), and Miyagi et al. (1999), respectively. For the non-open-channel data, R is defined based on the channel half-width or boundary-layer thickness, and centreline or free-stream velocity. Despite the scattering of the data, a trendline is superimposed in the figure to represent

the I_τ - R relationship, while possible changes are also indicated for small Reynolds numbers by considering that $I_\tau = 0$ for the limiting case of laminar flows ($R < 500$ for open-channel flows), and then I_τ increases with increasing R for flows changing transitionally to be turbulent. Fig. 5 shows that for turbulent flows, I_τ decreases gradually with increasing R , approaching smaller values only for larger R .

Conclusions

The lognormal function is used to describe the probability density distribution of bed shear stress in the present study. The derived pdf of the standardised bed shear stress is in good agreement with the experimental data reported previously. The higher-order moments of skewness and kurtosis increase with increasing relative intensity of the bed shear stress. The computed coefficients of skewness and kurtosis are found to be larger than those obtained experimentally. The results also show that the relative intensity and thus the pdf of bed shear stress may be generally associated with the Reynolds number.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

f	= probability density function;
I_τ	= τ_{rms}/τ_{rms} = relative intensity of the bed shear stress;
K	= coefficient of kurtosis or flatness;
$(\ln\tau)_{mean}$	= time-average value of the logarithm of the bed shear stress;
$(\ln\tau)_{rms}$	= rms value of the logarithm of the bed shear stress;
R	= Reynolds number

S	= coefficient of skewness;
τ	= bed shear stress;
τ_{mean}	= time-average bed shear stress;
τ_{rms}	= rms value of the bed shear stress; and
τ_S	= $(\tau - \tau_{\text{mean}}) / \tau_{\text{rms}}$ = standardised bed shear stress.

**Fig. 1. Probability Density Distribution of Standardised Bed Shear Stress
Computed Using Eq. (6).**

Fig. 2. Comparison of Eq. (6) (curves) with Data (columns): (a) $I_\tau = 0.395$; (b) $I_\tau = 0.244$; (c) $I_\tau = 0.35$ and (d) $I_\tau = 0.364$.

Fig. 3. Dependence of Skewness and Kurtosis on Relative Intensity of Bed Shear Stress.

Fig. 4. Relationship of Skewness and Kurtosis.

Fig. 5. Dependence of Relative Bed Shear Stress Intensity on Reynolds number.