Investigation of transverse mode instability suppression in large mode area fibre

Xia, Nan

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INVESTIGATION OF TRANSVERSE MODE INSTABILITY SUPPRESSION IN LARGE MODE AREA FIBRE

XIA NAN

School of Electrical and Electronics Engineering

2019

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XIA NAN

School of Electrical and Electronics Engineering

A thesis submitted to the Nanyang Technological University in
partial fulfilment of the requirements for the degree of Doctor of
Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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[Seongwoo Yoo]

Authorship Attribution Statement

This thesis contains material from [two] paper(s) published in the following peer-reviewed journal(s) / from papers accepted at conferences in which I am listed as an author.

Chapter 3 is published as N. Xia and S. Yoo, "Mode instability in ytterbium-doped non-circular fibres," Opt. Express 25(12), 13230-13251 (2017).

The contributions of the co-authors are as follows:

- I did the theoretical investigation and prepared the manuscript drafts.
- Prof. Yoo checked my theoretical investigation and edited the manuscript drafts.

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The contributions of the co-authors are as follows:

- I did the theoretical investigation and prepared the manuscript drafts.
- Prof. Yoo checked my theoretical investigation and edited the manuscript drafts.

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.....
Date

.....
[Xia Nan]

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Abstract

In the past decades, fibre laser and amplifier systems have seen a rapid power increase due to the efficient heat dissipation and robust single-mode operation. The fibre lasers and amplifiers have become attractive sources for applications spanning from industry and defence to scientific research. In particular, the fibre lasers and amplifiers have penetrated for material processing industry including marking, cutting, welding and drilling. It had been known that the highest achievable output power in fibre laser and amplifier systems is largely limited by the nonlinear scatterings such as stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS). However, a new phenomenon under high power operation, the so-called transverse mode instability (TMI), was experimentally observed in a large mode area (LMA) ytterbium (Yb)-doped fibre amplifier in 2011. The TMI can occur much earlier than the SBS and SRS in average power scaling, hence becoming the first limitation in high-power fibre operations till now.

The TMI refers to the output beam degradation, caused by the power transfer from the fundamental mode (FM) to the higher-order mode (HOM), for input pump power above a certain threshold. The origin of TMI is attributed to the stimulated thermal Rayleigh scattering (STRS) that provokes a phase shift, induced by the thermal diffusion time across fibre core, between the irradiance grating and thermally induced refractive index grating. The interference between two populated modes produces the irradiance grating along the fibre, which has a period equal to the beating length between two modes. Then the thermally induced refractive index grating is generated by irradiance grating through a quantum defect (QD) in fibre core. The QD created by transition from pump to signal frequency is recognized as the main heat source to modify the refractive index. Recently, it was found that photodarkening (PD) leads additional heating and contributes to the thermal grating. Hence, PD suppressed fibre becomes more important for high power scaling. As the TMI has been discovered very recently, approaches to suppressing the TMI have been very limited. HOM suppression by fibre bending has been the most popular method to suppress the TMI. TMI suppression by pump modulation was reported, but it is not a preferred method as it could lead to damage of the system. Therefore, the motivation of this thesis is to address the TMI in a fibre level through investigation of fibre designs suitable for power scaling with high TMI suppression.

Firstly, I developed a four-dimensional (4D) (x,y,z,t) highly numerical model, with the fast Fourier transform (FFT) based beam propagation method (BPM), to simulate and investigate the TMI behaviours in an arbitrary fibre design. Secondly, I theoretically study the TMI in non-circular ytterbium-doped fibres to investigate the heat sink arrangement and TMI suppression. Thirdly, I investigated maximum achievable power in a PD fibre through parametric optimization. A PD fibre provides high pump absorption and high fabrication yield, which would benefit designing laser or amplifier configurations when the TMI is suppressed. Lastly, a TMI suppressed design is proposed based on annular core fibres. TMI effects are theoretically investigated in Yb-doped ring-core and multicore fibres. The annular core is found effective to reduce thermal loss, thus suppressing the TMI. The ring shaped near field beam can be converted to a Gaussian-like beam profile in a far-field with up to 88% conversion efficiency. In the end, the ring-core fibre with outside circular diameter of 16 μm and core thickness of 2 μm was fabricated. I made the experimental lasing setup for the ring-core fibre and tested the performance of it.

The thesis contributes to thorough investigation of the TMI behaviour in a fibre level, leading to new insights of fibre design aspects. The thesis concludes that power scaling in fibre lasers and amplifiers can be continuously progressive by investigating alternative fibre designs that could considerably suppress the TMI.

Symbols and Acronyms

SBS	stimulated Brillouin scattering
SRS	stimulated Raman scattering
TMI	transverse mode instability
LMA	large mode area
Yb	ytterbium
FM	fundamental mode
HOM	higher-order mode
STRS	stimulated thermal Rayleigh scattering
QD	quantum defect
PD	photodarkening
4D	four-dimensional
FFT	fast Fourier transform
BPM	beam propagation method
Nd	neodymium
CW	continuous-wave
NA	numerical aperture
FM	fundamental mode
MOPA	master oscillator power amplifier
STRS	stimulated thermal Rayleigh scattering
AR.	aspect ratio
CCAR	cladding-to-core area ratio
MCVD	modified chemical vapor deposition
RIP	refractive index profile
HR	highly reflection
LD	laser diodes
DM	dichroic mirrors

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Chapter 1 Introduction

1.1 Development of high-power fibre laser

In the past decades, high power fibre lasers with good beam quality have experienced a significant progress in the average power scaling. Currently, the high-power fibre lasers are attractive sources in a wide range of industrial processes, including marking, cutting, welding and drilling. New applications in the scientific research [1-4] and defence [5, 6] promote a strong motivation in further output power scaling of fibre lasers. Figure 1.1 presents examples of the applications of high-power lasers in the industrial processing and defence system [7].

Figure 1.1: Applications of high-power lasers in the (a) cutting, (b) welding and (c) defence system [7].

The optical fibres initially were developed to replace the conventional cables in telecommunications, which is mainly due to the low-attenuation, immunity to electromagnetic interference, radiation-free and high-bandwidth in optical frequency [8-10]. A new perspective of fibre technology has arisen with the advent of active fibres doped with rare-earth ions. In 1985, the first neodymium (Nd)-doped fibre with low loss was fabricated and its application in a fibre laser was demonstrated by the B. S. Poole and N. D. Payne et al. [11, 12], which completely unlocked the potential of optical fibre communications. Subsequently, the demonstration of Nd-doped fibre laser with double cladding was firstly proposed in 1988, which marks the starting of high-power fibre lasers [13]. In 1992, averaged output power of several watts in a diode-pumped continuous-wave (CW) Nd-doped fibre laser was presented in [14]. The double-clad CW ytterbium (Yb)-doped fibre laser with a large area and numerical aperture (NA) inner cladding achieves 110 W output power in 1999 [15]. Soon after that, the output power from CW Yb-doped fibre amplifier has exponentially grown to more than 1 kW

in 2004, which utilized the counter-pumping configuration, larger inner cladding area and NA than previous fibre lasers [16]. In 2009, 10 kW output power CW Yb-doped fibre laser, which used a novel tandem pumping approach, was reported with a diffraction-limited beam [17].

Figure 1.2 presents the evolution of average output power of nearly diffraction limited fibre lasers in the past 25 years [18]. It is seen that the output power of ultrashort pulsed fibre lasers nearly follows the similar increasing rate as the CW fibre lasers. The ultrashort pulsed laser reached 100 W and 1 kW of average power in 2005 [19] and 2009 [20], respectively. The green square represents a 20-kW continuous wave commercial fibre laser, but its beam quality has yet to be precisely specified. Hence, both the operation regimes will experience similar limitation of average power scaling.

Figure 1.2: The evolution of average output power of nearly diffraction limited fibre lasers in the past 25 years [18].

1.2 Limitations of average power scaling in high-power fibre operation

In the past two decades, the fibre laser and amplifier systems have experienced a significant growth in scaling of output average power compared to the solid-state lasers. The high-power fibre lasers and amplifiers are recognized as the attractive sources in provoking the commercial interest and driving the industrial rapid progress, which are attributed to the following features [21-24]:

- (a) Fundamental mode operation – the waveguide structure of fibre allows the variation of core and cladding index difference, which enables the selection of a fundamental mode (FM) even in a large mode area (LMA) fibre.
- (b) Alignment-free systems – Access to fibre components evolved from optical telecommunication technology enables compact high-power fibre lasers or amplifiers designs.
- (c) High gains – The fibre waveguide promotes a high single-pass gain, which is due to the long interaction path between the light and active medium. Furthermore, the master oscillator power amplifier (MOPA) system (including the low power master oscillator and high gain fibre amplifier) allows control of gain through multiple stages of gain chain.
- (d) Efficient heat dissipation – The large surface-to-active-volume ratio of fibre geometry efficiently extracts heat and makes robust single-mode operation.

However, some critical limitations hinder the further scaling of average output power in fibre lasers and amplifiers. The limitations are originated from various aspects of thermal issue, nonlinearities, damage and pump power. The limitations are often combined and induce even in lower power scaling as compared to the individual limitation. Recently, transverse mode instability (TMI) related to the thermal issue has been observed as a critical limitation for beam quality control as well as power scaling. These limitations are analysed in the following sections.

1.2.1 Thermal limitation

For the high-power fibre lasers and amplifiers, the thermal limitations to the power scaling become serious problems, which have been detailly studied in [25, 26]. Firstly, the maximum average output power of fibre laser and amplifier, which corresponds to the thermal fracture, is presented as [25-28]:

$$P_{\text{out}}^{\text{rupture}} = \frac{\eta_{\text{laser}}}{\eta_{\text{heat}}} \cdot \frac{4\pi \cdot R_m}{1 - \frac{a^2}{2b^2}} \cdot L \quad (1.1)$$

where L is the fibre laser and amplifier length, a and b are the fibre core and cladding radii, respectively. R_m is the rupture modulus of the glass [29], η_{laser} is the optical-to-optical conversion efficiency and η_{heat} is the portion of the heat converted by the absorbed pump power in fibre.

Secondly, the highest output power when the fibre core becomes melt is shown as [25-28]:

$$P_{out}^{melting} = \frac{\eta_{laser}}{\eta_{heat}} \cdot \frac{4\pi \cdot k \cdot (T_m - T_c)}{1 + \frac{2k}{b \cdot h} + 2 \ln(\frac{b}{a})} \cdot L \quad (1.2)$$

where k is the thermal conductivity [30], T_m is the melt temperature for fibre core, T_c is the cooling temperature at the fibre cladding boundary and h is the convective coefficient depending on the heat dissipation mechanism [31].

Lastly, the high power in the lasers or amplifiers generates a heat-induced temperature pattern that creates a thermal lens, which has competitive index to the fibre core. Hence, the maximum output power in fibre lasers or amplifiers is given as [25-28]:

$$P_{out}^{lens} = \frac{\eta_{laser}}{\eta_{heat}} \cdot \frac{\pi \cdot k \cdot \lambda^2}{2 \frac{dn}{dT} \cdot a^2} \cdot L \quad (1.3)$$

where λ is the signal wavelength and dn/dT is the thermo-optic coefficient. It is seen from Eq. (1.1) – (1.3) that the maximum achievable output power before thermal limitation can increase with high lasing efficiency or low heat generation in a fibre. Furthermore, better heat dissipation mechanism at fibre boundary or larger thermal conductivity for fibre core and cladding material promotes the increase of output power.

1.2.2 Nonlinear limitation

When considering the high-power CW fibre lasers and amplifiers with broad linewidth, the stimulated Raman scattering (SRS) plays a key role in limiting the power scaling. The SRS refers to the progressive power conversion from signal to the unwanted longer wavelengths. Consequently, the critical output power at which the SRS becomes serious is presented as [25, 32, 33]:

$$P_{out}^{SRS} = \frac{16\pi \cdot a^2}{g_R L} \cdot \Gamma^2 \cdot \ln(G) \quad (1.4)$$

where g_R is the Raman gain coefficient, Γ is the ratio of signal mode field radius to the core radius, and G is the power gain of signal.

For the optical signals with narrow linewidth, compared to the Brillouin linewidth, in pulse fibre lasers and amplifiers, the stimulated Brillouin scattering (SBS) clamps the maximum output power. The critical output power when SBS becomes significant is expressed as [25, 32, 33]:

$$P_{\text{out}}^{\text{SBS}} = \frac{17\pi \cdot a^2}{g_B(\Delta\nu) \cdot L} \cdot \Gamma^2 \cdot \ln(G) \quad (1.5)$$

where $g_B(\Delta\nu)$ is the Brillouin gain coefficient. It is seen from Eq. (1.4) and (1.5), increasing the signal effective mode area (core size) or reducing the fibre length are two effective ways to improve the maximum output power when SRS and SBS are significant in high power fibre lasers and amplifiers. Otherwise, the nonlinear effects can be suppressed through the fibre design or a longitudinal temperature gradient [34, 35].

1.2.3 Damage limitation

As the fibre damage usually happens at the end facet, so the output peak intensity from fibre lasers and amplifiers is limited, which has been studied in [36-39]. The highest damage-limited average output power of fibre lasers and amplifiers is given by [25]:

$$P_{\text{out}}^{\text{damage}} = \Gamma^2 \cdot I_{\text{damage}} \cdot \pi a^2 \quad (1.6)$$

where I_{damage} is the highest intensity allowed in the fibre. It is shown from Eq. (1.6) that larger signal effective mode area can improve the output average power limited by the fibre damage.

1.2.4 Pump power limitation

The laser diodes play a very important role in influencing the fibre lasers and amplifiers output power, which is mainly attributed to the absorbed pump power in fibre. The highest average output power due to the laser diode limitation is presented as [25]:

$$P_{\text{out}}^{\text{pump}} = \eta_{\text{laser}} \cdot I_{\text{pump}} \cdot (\pi \cdot b^2) \cdot (\pi \cdot NA^2) \quad (1.7)$$

where I_{pump} is the output light intensity from laser diode [40], b and NA are the radius and numerical aperture of pump cladding. It is clear to see from Eq. (1.7) that the increasing brightness of laser diode, larger fibre cladding size and NA can enhance the maximum output power, which is limited by the laser diode.

By considering the thermal, nonlinear, damage and pump power limitations together, the achievable highest output power from CW and pulse fibre lasers and amplifiers are presented in Fig. 1.3 [25]. It is shown that the maximum output power of the CW and pulse fibre lasers or amplifiers are 36.6 kW and 1.86 kW, respectively, which are limited by the nonlinear effects (SRS or SBS) and the thermal lensing effects. Additionally, it is seen that for the core diameter below 90 μm , the provision of pump power by the laser diode limits the highest output power

in both the CW and pulse fibre lasers or amplifiers. Therefore, larger core diameter is essential for the further power scaling in high power fibre operations as presented in Fig. 1.3.

Figure 1.3: Contour plot of the minimum of seven physical power limits in the (a) CW fibre laser and (b) pulse fibre laser (power units are in kW) [25].

1.2.5 Transverse mode instability limitation

In 2011, transverse mode instability (TMI) was reported as a limitation for fundamental mode power scaling. The TMI can occur well before other limitations hence becoming the first obstacle to overcome for power scaling. The TMI refers to the output beam degradation, caused by power transfer from the fundamental mode (FM) to the higher order mode (HOM), for power above a certain threshold, and this phenomenon was firstly observed in a large mode area (LMA) ytterbium (Yb)-doped fibre amplifier [41, 42]. Figure 1.4 shows the experimental observation of TMI effect in high power LMA fibre amplifier [42]. It is clear to see that when the pump power is 380 W, the output beam is single mode. However, when the pump power is slightly increased to 390 W, the output beam degrades.

Figure 1.4: Experimental observation of TMI effect when the input pump power below and above a certain threshold [42].

H. J. Otto et al. analytically calculates the highest output power under the consideration of TMI in high power LMA fibre lasers, and compares it with the limitations by thermal lensing, nonlinearity and pump power [43]. Figure 1.5 shows the maximum output power of a fibre laser system as a function of core diameter and fibre length when considering the TMI [43]. It is seen that the TMI sets earlier power scalability limitations compared to other effects, suggesting the TMI must be suppressed for high power scaling. Additionally, M. N. Zervas has analysed that the combined limitation of thermal lensing, TMI and mechanical reliability (bending radius) on the power scalability in high power fibre operation as presented in Fig 1.6 [44]. It indicates that higher output power can be achieved when the bending is considered in fibre, for the large bending loss of HOM helps reduce the TMI and increase threshold power.

Figure 1.5: Maximum average output power of a fibre laser system as a function of fibre core diameter and length [43].

Figure 1.6: Power limits in high power fibre lasers [44].

1.3 Transverse mode instability limitation

The newly reported TMI effect in fibre is recognized as the major limitation for high power scalability in fibre lasers and amplifiers till now. Hence, it is very essential to investigate the TMI in the high-power fibre operation. In the following, I briefly introduce the origin, forming conditions of TMI, origins of the phase shifted seed source and the proposed methods to suppress the TMI.

1.3.1 Origin of transverse mode instability

The TMI in optical fibres is viewed as a manifestation of Stimulated Thermal Rayleigh Scattering (STRS), which was first observed in the absorbing liquids with giant-pulse Ruby lasers in 1960s [45-55]. The STRS refers to the power coupling to scattered lights, which is caused by the travelling temperature wave, generated from the interference between pump and scattered lights, via absorbing the heat [45]. The analytical solutions presented in [56] help to interpret the STRS phenomenon observed in the bulk media and liquid. However, it cannot

accurately predict the TMI behaviour in optical fibres for without considering the transverse dependence of electric field and thermal gradient.

The principles behind the TMI and STRS are very similar, while still exists some differences. Firstly, the heating for TMI is generated from the quantum defect, which is different to the absorptive heating in STRS. Additionally, the interference between two transverse modes is another cause for the TMI in optical fibres, while two interfering beams of scattered light and pump induce the STRS.

1.3.2 Forming conditions of transverse mode instability

Soon after the observation of TMI effect in the high-power fibre amplifier in 2011, C. Jauregui et al. suggested that the TMI is caused by the changed refractive index in core, which is induced by variations in the population of upper level lasing ions [57]. Shortly afterwards, A. Smith et al. considered the temporal evolution of the upper level ions population and temperature-induced index grating [58]. Now it is widely agreed that the TMI is caused by the following three conditions [59]:

(a) FM and HOM interfere each other to produce an intensity grating that has a period equal to the beating length between the modes. Figure 1.7 (a) shows the evolution of signal intensity along the active fibre [18].

(b) The formed intensity grating provokes different population inversion levels, thus different heat generation through a quantum defect. The generated heat variation leads to the formation of thermal grating via the thermo-optic effect, which subsequently accounts for refractive index variation, leading to power transfer between the modes. Figure 1.7 (b - d) represents the evolution of ion inversion level, thermally-induced temperature grating and its consequent refractive index variation along the active fibre length [18].

(c) A phase shift between the signal intensity grating and the temperature-induced refractive index grating is induced by thermal diffusion time across the core, frequency offset and different propagation constants between the involved modes. Figure 1.8 (a) presents the evolution of normalized signal intensity and changed refractive index along the active fibre length without considering the thermal diffusion time [59]. It is seen that if the thermal diffusion time is not accounted, the signal intensity grating, and the temperature-induced refractive index grating are in phase, indicating no power transfer between the modes. Figure 1.8 (b) shows the phase shift between the intensity pattern and changed refractive index grating

along fibre length when considering the thermal diffusion time [59]. The phase shift allows unidirectional power transfer from the FM to the HOM.

Figure 1.7: The evolution of (a) signal intensity pattern, (b) ion inversion level, (c) thermally-induced temperature grating and (d) changed refractive index along the active fibre length [18].

Figure 1.8: The evolution of normalized signal intensity and changed refractive index along the active fibre length when (a) without considering the thermal diffusion time and (b) considering the thermal diffusion time at a fixed point across fibre transverse [59].

1.3.3 Sources of higher order mode seeded power

For the generation of TMI, the HOM must be seeded at a nonzero power level, thus being amplified along fibre to reach the threshold power of TMI. In the following, four possible sources for the seeded HOM are presented.

(a) Amplitude modulation of the injected signal – When the input signal is injected into the amplifier and amplitude modulated, the signal intensity grating oscillates in time with a modulation frequency. Additionally, if this modulation frequency falls into the range of STRS gain band, then the seeded HOM power is amplified together with the power transfer from FM along fibre length, which causes the TMI [59-63].

(b) Amplitude modulation of the pump light – When the pump light is amplitude modulated in time, then the injected signal can also be modulated through the laser amplification. If the modulation frequency of pump light is in the STRS gain band, then the HOM power significantly increases along the fibre as described in (a) [59-63].

(c) STRS – The thermodynamic fluctuations in temperature across the fibre core generate the amplitude modulation for the injected signal and scatter the power from FM to HOM. It is noted that the scattered power is proportional to the input FM power and the temperature across fibre core [64]. The seeded HOM power is expected to be mainly contributed by the STRS if the amplitude modulation of signal and pump are not present [59, 61].

(d) Quantum noise seed – The quantum noise, or shot noise, serves to seed the STRS process, which has been presented in [65]. As the level of quantum noise is two to three orders less than the STRS, hence it is not expected to dominate the HOM seed source [59, 65].

1.3.4 Proposed methods to suppress the transverse mode instability

In this section, I briefly introduce the methods proposed by many researchers around world to suppress the TMI in a fibre and increases the TMI threshold power for high power fibre operations.

(a) Efficient thermal management – Improve the thermal management of fibre amplifier to promote the heat dissipation can suppress the TMI and increase the threshold power. Figure 1.9 compares the measured M^2 value versus the output signal power between the convective cooling and conductive cooling [66]. It is seen that as the conductive cooling is more efficient in heat dissipation than the convective cooling, which makes the output signal power at

threshold increases from 320 W in convective cooling to 460 W in conductive cooling. Hence, efficient thermal management is good for the TMI suppression [66].

Figure 1.9: Beam quality versus the output signal power in the fibre amplifier at convective cooling and conductive cooling [66].

(b) Increase the optical loss of HOM – The amplification of seeded HOM power can be compensated by introducing additional loss to HOM. Then, the loss can be higher than its gain, thus suppressing the TMI and improve the threshold power. The Influence of HOM loss on the relative threshold pump power is shown in Figure 1.10 [62]. It is noted that when the HOM specific loss increased from 0 dB/m to about 75 dB/m, the threshold power improved by 1.5 times [59, 62].

Figure 1.10: Threshold power versus the HOM specific loss in fibre amplifier [62].

(c) Increase the pump saturation effect in a fibre – When the fibre core size remains unchanged, the larger cladding size produces the stronger pump saturation effect in fibre. A strong saturation effect breaks the similar spatial profiles of the signal intensity grating and the changed refractive index grating, hence obstructing the essential conditions for TMI as

presented in section 1.3.1 (b) [59, 63]. Consequently, the TMI effect can be suppressed and the threshold power increases. Figure 1.11 presents an example of threshold power improvement by a confined doping scheme and the saturation effect [63]. It is clear to see that when increasing the cladding diameter from 50 μm to 500 μm in the co-pumped fibre amplifier with doping diameter of 50 μm , the threshold power significantly increases from 500 W to about 1400 W. Therefore, increasing the cladding size or saturation effect is an efficient way to suppress the TMI and increase its threshold power.

Figure 1.11: Threshold versus the cladding diameter in fibre amplifier at different pumping schemes and doping diameters [63].

(d) Confined doping – Reducing the doping area helps to decrease the overlap between the doping and HOM profile, thus prohibiting the amplification of HOM and suppressing the TMI effect in fibre as analysed in [67]. It is shown by Figure 1.11 that when the doping diameter decreases from 50 μm to 30 μm in the co-pumped fibre amplifier with cladding diameter of 100 μm , the threshold power increases from 500 W to 750 W.

(e) Tandem pumping – When the wavelength difference between signal and pump becomes smaller, the QD-induced thermal load reduces, thus diminishing the changed refractive index and suppressing the TMI [68-70]. Figure 1.12 (a) presents the TMI threshold power versus the changed signal wavelength when pumping at 976 nm [68]. It is seen that when the signal wavelength gets closer to the pump wavelength from 1064 nm to 1000 nm, the TMI threshold power increased from 140 W to 470 W, which is due to the reduced generation of thermal load. The TMI threshold power versus the changed pump wavelength when the signal wavelength fixed at 1030 nm is shown in figure 1.12 (b) [68]. It is observed that the TMI is suppressed and

threshold power increased when the pump wavelength becomes larger, which makes it get close to the signal wavelength and generate less thermal load [68].

Figure 1.12: TMI threshold power versus the (a) signal wavelength for the pumping at 976 nm and (b) pump wavelength for the fixed signal wavelength emitting at 1030 nm in the fibre amplifier [68].

(e) Reducing the core NA – The reduction of core NA for the same core size decreases the overlap between the FM and HOM, thus prohibiting the power transfer from FM to HOM even at the presence of TMI. Figure 1.13 presents the threshold power versus the core NA for the fixed fibre cladding and core size [71]. It is shown that for the core and cladding diameter of 20 μm and 400 μm , respectively, when the core NA reduced from 0.07 to 0.045, the TMI threshold power increased from 1000 W to about 1550 W. This is due to that smaller core NA decreases the overlap between modes, thus suppressing the TMI and increasing the threshold power [71].

Figure 1.13: TMI threshold power versus the core NA in the fibre amplifiers with different cladding and core size [71].

1.4 Motivation

The proposed methods to suppress the TMI and increase the threshold power including efficient thermal management, large specific loss of HOM, strong pump saturation effect, confined doping, tandem pumping and the reduction of core NA in high power fibre lasers and amplifiers have been discussed in the previous section. Additionally, in recent years, the photodarkening (PD)-induced heat from absorption of both signal and pump is recognized as another heat source contributing to the TMI except QD. However, it has not been thoroughly investigated the relationship of heating, PD, saturation effect and TMI simultaneously. [70, 72-74]. In particular, an average heat load was suggested as an alternative quantity for TMI threshold [75]. However, it is still unclear if the average heat load is universally applicable to any fibre design. It is essential to have further insight of the TMI with the consideration of all the influencing parameters. Thus, the main motivations of the thesis include:

- (a) I developed a four-dimensional (4D) (x,y,z,t) highly numerical model, with the fast Fourier transform (FFT) based beam propagation method (BPM), to simulate and investigate the TMI behaviours in an arbitrary fibre design. Additionally, the model intrinsically includes pump saturation effect and a thermal lensing effect that influence the TMI behaviour. Furthermore, it allows incorporating additional physical effects, such as photodarkening effect and bending effect.
- (b) I presented a study of TMI in non-circular ytterbium-doped fibres including the rectangular core in a circular or D-shaped cladding. The D-shaped cladding is found efficient to suppress the TMI due to better heat dissipation, as compared to the circular cladding. However, the rectangular core does not suppress the TMI despite its better heat dissipation than a circular core counterpart.
- (c) I theoretically investigated the fibre parametric studies for power scaling in a PD fibre through the suppression of TMI and nonlinearity. We examine the fibre design parameter dependence of TMI in aspects of thermal load and threshold power. It is suggested that 1.7 kW output power is achievable in a PD fibre within practical design parameters.
- (d) I studied the TMI effect in the non-PD Yb-doped ring-core and multicore fibres. It is found that the TMI effect is highly suppressed in the ring-core and multicore fibres in comparison with the circular core fibre, which is due to the less heat generated in fibre core for the fixed signal effective mode area.

(e) The ring-core fibre with outside circular diameter of 16 μm and core thickness of 2 μm was fabricated. I made the experimental lasing setup for the ring-core fibre and tested the performance of it.

1.5 Outline of the thesis

In Chapter 2, I describe the 4D, FFT based BPM model in detail, which is used to simulate the TMI behaviour in fibre. It includes solving of beam propagation with a help of a split-step method, calculation of Green's function for steady-periodic heating and temperature distribution. Additionally, the calculated evolutions of FM, HOM and pump power along the fibre length are also present.

In Chapter 3, non-circular fibre is investigated for the TMI behaviours, leveraging on the developed simulation code. The non-circular fibre provides unique geometry for efficient heat dissipation. The effect of the better heat dissipation for TMI suppression is discussed.

In Chapter 4, I present the fibre parametric dependent of TMI in a PD fibre. The influences of fibre length, core NA, ion concentration, fibre core and cladding size on TMI behaviours are investigated. Furthermore, the TMI suppressed PD fibre within practical design parameters, which achieves 1.7 kW output power, is suggested.

In Chapter 5, a unique ring-core and multicore fibre designs for the TMI suppression are proposed. Consequently, I compare the TMI behaviours and threshold power between the circular core and optimized annular core fibres.

In Chapter 6, I summarize the thesis and some potential future research works.

Chapter 2 Theoretical model of the transverse mode instability

The TMI behaviours in high-power fibre operation can be modelled via the numerical [76-80], analytical or semi-analytical models [81-87]. The numeric models presented in [77, 79] can simulate a variety of shapes of the core, refractive index and doping profiles. Additionally, it is simple to add additional physical effects, such as the pump saturation, thermal lensing, mode coupling, photodarkening and bending` effects. However, the highly numeric model needs longer computing time compared with other analytical or semi-analytical models. The advantages of analytical or semi-analytical models include shorter running time and mathematical representation of TMI mechanism and behaviours but with assumed simplification of other parameters. For instance, the analytical models presented in [81, 82] do not consider the pump saturation and thermal lensing effect despite their great influence on the TMI in larger core fibres [88]. In this study, the fully numerical model is developed. In the following, I introduce the theoretical model of TMI using the split-step fast Fourier transform (FFT) beam propagation method (BPM) in detail, including why the selection of BPM, running process of model, solvement of beam propagation equation, temperature distribution and time-averaged signal and pump power.

2.1 Why the selection of beam propagation method

In this thesis, I develop a split-step FFT-based BPM to simulate the TMI behaviour in fibre, and the reasons why I select this method are listed below:

- (a) BPM – The accurate and general BPM to calculate the electric field in optical fibre is proposed in the 1970s [89, 90], for it can solve the Helmholtz equation, which describes the electric field propagation in fibre, by accurately calculating the spatial (near-field) and angular (far-field) properties of the electric field.
- (b) Split-step – The split-step method is proposed since the seventies to numerically simulate the CW optical diffraction, which includes the guiding and electric field propagating in optical fibres [91, 92]. One split-step usually includes three sub-step, two half-step corresponds to the linear diffractive propagation of electric field in a homogeneous medium, and one full-step propagation without considering the diffraction [77].

(c) FFT – The FFT method is usually used to model the propagation of electric field in the optical fibre [93], for it provides an accurate description of the field by providing the detailed information about confinement of the beam, energy losses and the beam's angular properties [90-92].

2.2 Running process of the model

Figure 2.1: Flow chart diagram of the TMI numerical model running process.

In this part, I introduce the running process of highly numerical TMI model. Figure 2.1 presents the flow chart diagram of the TMI model. Firstly, the input signal field (including the FM and HOM fields and powers) and pump intensity profile are constructed at the input end of fibre. Secondly, for one split-step of fibre, the signal field experiences the linear diffractive propagation at first half step length, then the step length non-linear diffractive propagation, and linear diffractive propagation half step length in the last. For the linear diffractive propagation, the FFT method is used to update the signal field, which adds the calculated propagation phase in k space. The thermo-optic index, which is related to the increased temperature distribution, is added to the phase of signal field in time domain for the non-linear diffractive propagation. The increased temperature distribution is calculated with the utilization of steady-periodic

Green's functions. Lastly, if this one split-step of fibre is not the last step of whole fibre, then proceeds to the next step, otherwise, the running stops. It is noted that as the thermal diffusion time, which causes the phase shift between intensity and changed thermo-optic index profiles, is one essential factor for the TMI. Therefore, the running process mentioned above should be repeated for each time point in one cycle and then splice them in time to construct the updated signal field and pump power.

2.3 Beam propagation equation

In the TMI numerical model, only the FM LP₀₁ and HOM LP₁₁ are considered to define the TMI because the power transfer is strongest from LP₀₁ to LP₁₁ as discussed in [77]. An input signal field is presented by [77]:

$$E_s(x, y, t) = \sqrt{P_{01}} E_{01}(x, y) + \sqrt{P_{11}} E_{11}(x, y) e^{-i\Delta\omega t} \quad (2.1)$$

where P_{01} is the LP₀₁ mode input power and P_{11} is the LP₁₁ mode input power, E_{01} and E_{11} are the normalized LP₀₁ and LP₁₁ mode amplitudes, respectively, $\Delta\omega$ is the frequency offset between modes. The beam propagation equation considering the PD-induced loss is described as follows:

$$\frac{\partial E_s(x, y, z, t)}{\partial z} = -\frac{i}{2k_c} \nabla_{\perp}^2 E_s(x, y, z, t) - \frac{i[k^2(x, y, z, t) - k_c^2]}{2k_c} E_s(x, y, z, t) + [g(x, y, z, t) - \frac{1}{2} \frac{\ln 10}{10} PD_{loss}(x, y, z, t)] \cdot E_s(x, y, z, t) \quad (2.2)$$

where $\nabla_{\perp}^2$ is the Laplacian operator in the x and y dimensions, $g(x, y, z, t)$ is the laser gain, $PD_{loss}(x, y, z, t)$ (in dB/m) is the PD-induced loss at a saturable level for the pump and signal at about 1 μm [94, 95], $k(x, y, z, t)$ and k_c are carrier wave numbers in the core and cladding, respectively. Equation (2.2) is the scalar, paraxial beam propagation equation for an isotropic medium, and the derivation process is presented in [90-92]. Additionally, it is appropriate for the small index contrasts fibre, such as the step-index fibre [90-92]. The wave number in the core, laser gain [77, 90, 91] and PD-induced loss are expressed as:

$$k(x, y, z, t) = \frac{\omega_c n_{core}(x, y, z, t)}{c} \quad (2.3)$$

$$g(x, y, z, t) = \frac{1}{2} [-\sigma_s^a + (\sigma_s^a + \sigma_s^e) n_u(x, y, z, t)] N_{yb}(x, y) \quad (2.4)$$

$$PD_{loss}(x, y, z, t) = [175 \times (\frac{N_{yb}(x, y)}{8.74 \times 10^{25}})^{2.09}] \cdot \frac{n_u(x, y, z, t)}{0.46 \cdot \gamma} \quad (2.5)$$

where ω_c is the carrier frequency, σ_{sa} and σ_{se} are the signal absorption and emission cross-sections, $N_{Yb}(x,y)$ is the doping profile. $n_{core}(x,y,z,t)$ and $n_u(x,y,z,t)$ are the core refractive index during amplification and upper state population, respectively. Equation (2.5) represents a PD-induced loss at the saturation level, ignoring temporal evolution to the saturation level over operation time of an active fibre, and it is obtained from a fit of the experimental data as presented in [87, 94, 95]. The wavelength conversion factor γ from 630 nm to about 1 μ m in photonic crystal fibre (PCF) is 24.5 [87] and 70.9 in step index fibre [96]. For our simulation centers on the step index fibre design in this thesis, we select the conversion factor of 70.9 in the following simulations. $n_{core}(x,y,z,t)$ and $n_u(x,y,z,t)$ are described through [77]:

$$n_{core}(x, y, z, t) = n_{core} + \frac{dn}{dT} \Delta T(x, y, z, t) \quad (2.6)$$

$$n_u(x, y, z, t) = \frac{\frac{P_p(z, t) \sigma_p^a}{h \nu_p A_p} + \frac{I_s(x, y, z, t) \sigma_s^a}{h \nu_s}}{\frac{P_p(z, t) (\sigma_p^a + \sigma_p^e)}{h \nu_p A_p} + \frac{I_s(x, y, z, t) (\sigma_s^a + \sigma_s^e)}{h \nu_s} + \frac{1}{\tau}} \quad (2.7)$$

where n_{core} is a pristine refractive index of core, dn/dT is the thermo-optic coefficient, σ_{pa} and σ_{pe} are the pump absorption and emission cross-sections, $I_s(x,y,z,t)$ is the signal intensity, A_p is the pumping area, ν_s and ν_p are the signal and pump frequencies, h is the Planck constant and τ is the excited dopant ion lifetime. The derivation of Eq. (2.7) is presented in [77], and it is appropriate for an Yb-doped fibre amplifier, which is a three-level system. Figure 2.2 (a) shows the signal intensity pattern along fibre in the transversal xz plane, which has been calculated by Eq. (2.2) [96]. It is seen that a beating length, generated by interference between FM and HOM, exists in the signal intensity pattern. The ion inversion level pattern, which follows the signal irradiance pattern as presented by Eq. (2.7), is shown in Fig. 2.2 (b) [96].

Figure 2.2: (a) Signal intensity pattern and (b) Ion inversion level along the fibre in transversal plane [96].

2.4 Solve of temperature distribution

In this section, I introduce the contributions to the thermal load in high power fibre operation and the steady-periodic Green's function to calculate the Temperature distribution.

2.4.1 Contributions to the thermal load

In order to get the changed refractive index $n_{core}(x,y,z,t)$ in Eq. (2.6), I calculate the temperature distribution $T(x,y,z,t)$ using the following equation:

$$\rho C \frac{\partial T(x, y, z, t)}{\partial t} = Q(x, y, z, t) + K \left(\frac{\partial^2 T(x, y, z, t)}{\partial x^2} + \frac{\partial^2 T(x, y, z, t)}{\partial y^2} \right) \quad (2.8)$$

where K is the thermal conductivity, ρ is the density, C is the heat capacity. The derivation process of Eq. (2.8) is presented in [97, 98], and it is applicable for the Dirichlet boundary condition as the thermal boundary, implying that temperature at the fibre surface is set at a fixed value. Because the mode coupling depends only on the temperature variations in the core relative to the mean fibre temperature, so this thermal model can be used to simulate the TMI behavior, which has been verified in [59]. It is noted that the temperature distribution $T(x,y,z,t)$ in the core calculated by this thermal model just represents the temperature variations in the core relative to the mean fibre temperature. The thermal load $Q(x,y,z,t)$ in fibre core is contributed by the QD-induced heat, and the heat generated by PD from the absorption of signal and pump. The expressions for the individual thermal load are described as [73]:

$$Q_{QD}(x, y, z, t) = N_{yb}(x, y) \left[\frac{V_p - V_s}{V_p} \right] [\sigma_p^a - (\sigma_s^a + \sigma_s^e) \cdot n_u(x, y, z, t)] \frac{P_p(z, t)}{A_p} \quad (2.9)$$

$$Q_{PD}^s(x, y, z, t) = \frac{Ln10}{10} \times PD_{loss}(x, y, z, t) I_s(x, y, z, t) \quad (2.10)$$

$$Q_{PD}^p(x, y, z, t) = \frac{Ln10}{10} \times PD_{loss}(x, y, z, t) \frac{P_p(z, t)}{A_p} \quad (2.11)$$

$$Q(x, y, z, t) = Q_{QD}(x, y, z, t) + Q_{PD}^s(x, y, z, t) + Q_{PD}^p(x, y, z, t) \quad (2.12)$$

It should be noted that the PD-induced loss is insignificant until it reaches a saturable level. In this regime, only the QD contributes to the heat source. This is to make a clear contrast before and after the PD saturates. When the PD is saturated, QD combined PD work as the heat source.

2.4.2 Steady-periodic Green's function

For the solvement of temperature distribution $T(x, y, z, t)$ shown in Eq. (2.8), I use the Green's function method to solve it, which is presented in [97-102]. Additionally, I use a Dirichlet boundary condition for our thermal boundary, implying that temperature at the fibre surface is set at a fixed value, which is the same consideration as in [58, 59, 77, 79]. When the temperature is fixed at the thermal boundary, the Green's function can be presented as [77, 98]:

$$G(x, y, \omega | x', y') = \sum_{n=0}^{\infty} F_n(y, y') P_n(x, x', \omega) \quad (2.13)$$

$$F_n(y, y') = \frac{1}{WK} \sin\left(\frac{n\pi}{W} y\right) \sin\left(\frac{n\pi}{W} y'\right) \quad (2.14)$$

$$P_n(x, x', \omega) = \left\{ \exp[-\sigma_n(2H - |x - x'|)] - \exp[-\sigma_n(2H - x - x')] \right. \\ \left. + \exp[-\sigma_n |x - x'|] - \exp[-\sigma_n(x + x')] \right\} / \sigma_n (1 - \exp[-2\sigma_n H]) \quad (2.15)$$

$$\sigma_n^2 = \left(\frac{n\pi}{W}\right)^2 + i\omega \frac{\rho C}{K} \quad (2.16)$$

where $0 \leq x \leq H$, $0 \leq y \leq W$, and ω is the frequency of heat, and it is composed of the stationary part $\omega=0$ and the moving part $\omega=\Delta\omega$. The temperature can be calculated over the entire (x,y) grid with the contribution of periodically heated point (x',y') via the Green's function. The use of this set of Green's functions to compute the temperature distribution is described as follows [98]:

$$T(x, y, t) = \text{Real} \left[\sum_{m=0}^1 \sum_{x', y'} q(x', y', \omega_m) G(x, y, \omega_m | x', y') \exp(i\omega_m t) \right] \quad (2.17)$$

where $q(x', y', \omega_m)$ is a temporal Fourier transform of the thermal load $Q(x', y', t)$, and it is presented as [98]:

$$q(x', y', \omega_m) = \Delta x \Delta y \sum_{i=0}^{N_t-1} Q(x', y', t_i) \exp(-i\omega_m t_i) \quad (2.18)$$

The process for the temperature distribution $T(x, y, z, t)$, which represents the temperature variations in the core relative to the mean fibre temperature, calculation is summarized as follows:

- (a) As I select a Dirichlet boundary condition for our thermal boundary, implying that the temperature at the fibre surface is set at a fixed value, so the Eqs. (2.13) – (2.18) are used to calculate the temperature distribution $T(x, y, z, t)$.
- (b) I calculate the Green's function $G(x, y, x', y')$ distribution (Eq. (2.13)) within the fibre cladding domain $0 \leq x \leq H$, $0 \leq y \leq W$, $0 \leq x' \leq H$ and $0 \leq y' \leq W$, based on the Eq. (2.14) – (2.16).
- (c) The total thermal load $Q(x, y, z, t)$ (Eq. (2.12)) is obtained from the Eqs. (2.9) – (2.11), which relates to the doping distribution, ion inversion level, signal and pump intensity. Then the $q(x', y')$ distribution (Eq. (2.18)) in the range of $0 \leq x' \leq H$ and $0 \leq y' \leq W$ is calculated from the Eq. (2.12).
- (d) Lastly, the temperature distribution $T(x, y, z, t)$ is obtained from the Eq. (2.13) and (2.17).

Figure 2.3 (a) presents the temperature distribution along fibre length in the transversal x-z plane, and it is calculated using the Eq. (2.13) – (2.18) [103]. It is seen that the changed refractive index distribution shown in Fig. 2.3 (b) follows the pattern of temperature distribution [103], which can be explained by the Eq. (2.6). This changed refractive index pattern accounts for the power transfer between FM and HOM.

Figure 2.3: (a) Temperature distribution pattern and (b) changed refractive index pattern along the fibre in the transversal x-z plane [103].

2.5 Solve of beam propagation equation

In this section, I have a brief introduction to the split-step FFT method used to solve the beam propagation equation and the calculation of time related FM, HOM and pump powers along fibre length.

2.5.1 Split-step Fast Fourier Transform method

I utilize the split-step FFT method to solve the beam propagation equation as shown in Eq. (2.2), which has been proposed in [89-92, 104-107]. In the TMI simulation, each step size Δz consists of three sub-steps, and in the first and third sub-step, the field is advanced by $\Delta z/2$ in a homogeneous medium with a refractive index equal to the cladding index. In these two sub-steps, the BPM keeps only the diffractive term on right-hand-side as shown below and it is solved in the k space [77, 90 - 92].

$$\frac{\partial E_s(x, y, z, t)}{\partial z} = -\frac{i}{2k_c} \nabla_{\perp}^2 E_s(x, y, z, t) \quad (2.19)$$

where the field $E_s(x, y, z, t)$ is transformed to the k space by using the FFT presented as:

$$E_s(x, y, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_s(k_x, k_y, z, t) e^{ik_x x} e^{ik_y y} dk_x dk_y \quad (2.20)$$

By substituting Eq. (20) into Eq. (19), we obtain:

$$\frac{\partial E_s(k_x, k_y, z, t)}{\partial z} = i \frac{k_x^2}{2k_c} E_s(k_x, k_y, z, t) + i \frac{k_y^2}{2k_c} E_s(k_x, k_y, z, t) \quad (2.21)$$

Subsequently, the k space field $E_s(k_x, k_y, z, t)$ is advanced by $\Delta z/2$ consisting of a phase shift at each (k_x, k_y) plane as [77, 90 - 92]:

$$\phi(k_x, k_y) = -\frac{\Delta z}{2} \frac{(k_x^2 + k_y^2)}{2k_c} \quad (2.22)$$

Lastly, the IFFT is used to convert the k space field $E_s(k_x, k_y, z, t)$ back to $E_s(x, y, z, t)$.

In the second sub-step, the field is advanced by Δz , keeping only the phases induced by the thermally generated index plus guiding index, together with a signal gain on the right-hand-side of Eq. (2.2). Then the BPM for this step becomes:

$$\frac{\partial E_s(x, y, z, t)}{\partial z} = -\frac{i[k^2(x, y, z, t) - k_c^2]}{2k_c} E_s(x, y, z, t) + g(x, y, z, t) E_s(x, y, z, t) \quad (2.23)$$

with the phase of the field advanced over Δz described as [77, 90 - 92]:

$$\phi(x, y, t) = \Delta z \frac{k^2(x, y, t) - k_c^2}{2k_c} \quad (2.24)$$

2.5.2 Calculation of time-averaged signal and pump powers

The time related pump power, $P_p(z, t)$, FM power, $P_{0I}(z, t)$ and HOM power, $P_{1I}(z, t)$, are calculated at each fibre length z and time t using the following equations [77]:

$$\frac{dP_p(z, t)}{dz} = \frac{P_p(z, t)}{A_p} \iint [(\sigma_p^a + \sigma_p^e) n_u(x, y, z, t) - \sigma_p^a] N_{Yb}(x, y) dx dy \quad (2.25)$$

$$P_{0I}(z, t) = \left| \frac{\iint E_s(x, y, z, t) E_{0I}(x, y) dx dy}{\iint [E_{0I}(x, y)]^2 dx dy} \right|^2$$

$$P_{1I}(z, t) = \left| \frac{\iint E_s(x, y, z, t) E_{1I}(x, y) dx dy}{\iint [E_{1I}(x, y)]^2 dx dy} \right|^2 \quad (2.26)$$

Thus, the temperature distribution, pump power and individual modal power can be calculated along the fibre length. Figure 2.4 presents the time-averaged FM power, $P_{0I}(z, t)$, HOM power, $P_{1I}(z, t)$ and pump power evolution along the fibre. It is seen that when the TMI is considered in high power fibre amplifier, the power transfer from the FM to the HOM happens.

With reference to the Eqs. (2.1), (2.3) and (2.4), the TMI model can simulate an arbitrary shape of core, refractive index profile and doping profile. Additionally, the pump saturation

effect is reflected by Eqs. (2.4) and (2.7), and it is influenced by the pump intensity and total signal intensity. The thermal lensing effect is also included in the first frequency term in Eq. (2.17).

Figure 2.4: Evolution of the time-averaged FM, HOM and pump powers along the fibre.

Chapter 3 Mode instability in ytterbium-doped non-circular fibres

A rectangular (or ribbon) core fibre design has been proposed for the effective FM area scaling above $1000 \mu\text{m}^2$ [108-113]. The asymmetric core offers bending insensitivity [110], robust higher-order mode area scaling [112], and better heat dissipation [108] than a circular core fibre, suggesting better TMI performance. However, there has been no investigation on TMI in this type of fibre, to the best of our knowledge. In this chapter, I use the split-step FFT-based BPM to present theoretical investigation of TMI in a rectangular core fibre. The influence of core shapes on the TMI is investigated by varying the core aspect ratio (AR.) and I present important parameters governing the TMI including the frequency offset between FM and HOM, TMI threshold pump power and temperature distribution in the non-circular fibre geometries. The benefit of better heat dissipation in the non-circular fibres is examined against the TMI threshold pump power, and we also employ a power coupling coefficient to attain more insight of the TMI behaviours in non-circular fibres. Hence, this chapter contributes to the investigation of TMI in non-circular fibres, which hasn't been examined in prior studies, with an emphasis on the influence of temperature, pump saturation effect, thermal lensing effect and power coupling between modes which is associated with frequency offset.

3.1 Calculation of frequency offset for transverse mode instability

The phase shift between moving irradiance pattern and temperature grating is essential to provoke the power transfer from FM (LP_{01}) to HOM (LP_{11}). This phase shift is induced by the thermal diffusion time across the core and frequency offset between the modes. The most efficient power transfer in circular cores occurs at the phase shift where the frequency offset matches to the inversion of thermal diffusion time, thus the core radius [59], and such frequency offset agrees with the experimental result [114]. Now I present theoretical results on the most efficient frequency offset in rectangular cores which are asymmetric. I investigate the frequency offset in rectangular cores with different core AR. and this can be served as a good guideline to estimate significance of TMI in different core geometries. In the following, I compute the output power content of LP_{11} versus the frequency offset in rectangular cores with AR. of 1:1, 4:1 and 10:1 to find out the frequency offset that makes the most efficient power transfer from LP_{01} to LP_{11} .

Table 3.1: Fibre parameters used in simulation

Parameter	Fibre 1	Fibre 2	Fibre 3	Fibre 4
Core radius	25 μm	-	-	-
Core width	-	44.3 μm	88.8 μm	140 μm
Core height	-	44.3 μm	22.2 μm	14 μm
AR.	-	1:1	4:1	10:1
Cladding radius	200 μm	200 μm	200 μm	200 μm
Core area	1963 μm^2	1963 μm^2	1963 μm^2	1960 μm^2
Cladding index	1.45	1.45	1.45	1.45
Core NA	0.06	0.06	0.06	0.06
LP ₀₁ effective mode area	1167 μm^2	1083 μm^2	1232 μm^2	1368 μm^2
LP ₁₁ effective mode area	1062 μm^2	1015 μm^2	1196 μm^2	1345 μm^2
Signal effective mode area	1167 μm^2	1083 μm^2	1232 μm^2	1368 μm^2
LP ₀₁ refractive index	1.45117	1.45116	1.45110	1.45099
LP ₁₁ refractive index	1.45105	1.45104	1.45107	1.45097
Ion concentration	$3.5 \times 10^{25} \text{ m}^{-3}$	$3.5 \times 10^{25} \text{ m}^{-3}$	$3.5 \times 10^{25} \text{ m}^{-3}$	$3.5 \times 10^{25} \text{ m}^{-3}$
Doping area	Uniform core doping	Uniform core doping	Uniform core doping	Uniform core doping
Fibre length	2 m	2 m	2 m	2 m
LP ₀₁ overlap with doping	0.9826	0.9811	0.9627	0.9229
LP ₁₁ overlap with doping	0.9525	0.9508	0.9619	0.9223
LP ₀₁ overlap with LP ₁₁	0.8576	0.8598	0.8645	0.8692

The parameters of circular core and rectangular core fibres used in this paper are listed in Table 3.1. Fibre 1 is a circular core fibre with the FM area of 1167 μm^2 , while Fibre 2 to Fibre 4 are rectangular core fibres with AR. of 1:1, 4:1 and 10:1, respectively, with the FM area slightly different from Fibre 1 as presented in Table 3.1. Additionally, the doping profiles in all four fibres are uniform core doping. The core area is kept nearly the same around 1960 μm^2 for all fibres, which promises nearly the same total signal output power, pump power and pump efficiency (absorption). The pump and signal wavelengths are assumed at 976 and 1064 nm, respectively. We assume the Yb-doped aluminosilicate fibre, thus using signal emission and absorption cross-sections of $3.58 \times 10^{-25} \text{ m}^2$, and $6.00 \times 10^{-27} \text{ m}^2$, pump emission and absorption cross-sections of $1.87 \times 10^{-24} \text{ m}^2$, and $1.53 \times 10^{-24} \text{ m}^2$, respectively. Other fibre parameters used in the simulations are presented in the Table 1. For material properties, we use mass density of 2201 kg/m³, heat capacity of 702 J/(kg×K) and thermal conductivity of 1.38 W/(m×K). In the simulation, launched signal powers at LP₀₁ and LP₁₁ mode are set at 10 W, and 10⁻³ W,

respectively, with a co-propagating pump beam (pumping radius fixed at $100\text{ }\mu\text{m}$) in the cladding. I selected a bit higher LP_{11} mode input power than the quantum noise level. However, it does not affect the conclusions although it results in a bit low TMI threshold power. I note that even higher input power of 0.5 W was employed in [76]. Both the x and y grid spacings, Δx and Δy , are $1.56\text{ }\mu\text{m}$. And a step size, Δz , is set at $10\text{ }\mu\text{m}$. We select 64 time grids points for one time cycle (defined as $2\pi/\Delta\omega$). Figure 3.1 shows the intensity profiles of FM (LP_{01}) and HOM (LP_{11}) in the circular core and rectangular core fibres. These intensity profiles represent the eigenmodes of the corresponding core structures, which are obtained from the COMSOL. The dashed lines surrounding the circular cladding indicate the thermal boundary used to calculate the temperature distribution in fibre.

Figure 3.1: Intensity of FM (LP_{01}) and HOM (LP_{11}) in (a) $50\text{ }\mu\text{m}$ circular core (fibre 1), (b) square core $44.3\text{ }\mu\text{m} \times 44.3\text{ }\mu\text{m}$ (AR. 1:1) (fibre 2), (c) rectangular core $88.8\text{ }\mu\text{m} \times 22.2\text{ }\mu\text{m}$ (AR. 4:1) (fibre 3) and (d) rectangular core $140\text{ }\mu\text{m} \times 14\text{ }\mu\text{m}$ (AR. 10:1) (fibre 4). The dashed lines indicate the thermal boundary.

I run simulation with the defined fibre parameters, following the aforementioned model, with varying the frequency offset between LP_{01} and LP_{11} modes. Figure 3.2 represents the calculated LP_{11} mode power content in the output power at various frequency offset values and I scale the peak power to a unity value for easy comparison. The calculation process for the curve shown in figure 3.2 is as follows: I firstly select the frequency offset point shown in the x-axis of figure 3.2 as the $\Delta\omega$ presented in Eq. (2.1) to get the input signal intensity profile. Then I follow the TMI model running process shown in Fig. 2.1 to get the output total signal, pump, LP_{01} and LP_{11} modes power (Please see Eqs. (2.25) and (2.26)). Lastly, the output power content of LP_{11} mode is calculated. In the circular core fibre (Fibre 1), the LP_{11} power peaks at 1160 Hz , which is nearly the inversion of thermal diffusion time across the core ($\tau \approx 1.12r^2$, where r is the core

radius in micrometres and τ is the thermal diffusion in microseconds [59]). In the rectangular cores, the peaked frequency offset shifts to lower values with the increasing AR. (or increasing longer axis and correspondingly decreasing shorter axis). Hence, the results suggest that the thermal diffusion time in the rectangular cores is determined by the longer axis rather than the shorter axis. In the rectangular cores, the modes are elongated along the longer axis [See Fig. 3.1], and their interaction is more intense along the longer axis. Therefore, the thermal diffusion time that accounts for the TMI follows the longer axis. Consequently, the core with AR. of 10 shows the lowest matched frequency offset at 260 Hz while the square core with AR. of 1 has the highest frequency offset of 1580 Hz due to its smallest core width among the investigated fibres [See Table 3.1]. The identified matched frequency offsets are used in the following simulations.

Figure 3.2: Normalized output LP_{11} mode power content versus frequency offset for 50 μm circular core fibre (fibre 1), 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (fibre 2), 88.8 $\mu\text{m} \times 22.2 \mu\text{m}$ rectangular core (AR. 4:1) fibre (fibre 3) and 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4).

3.2 Transverse mode instability in rectangular core fibres

The theoretical frequency offset for obtaining the maximum power transfer from LP_{01} to LP_{11} mode is in good agreement with the measured frequency in experiment [114]. Hence, we assume that the TMI becomes most efficient at the identified peak frequency offsets [see Fig. 3.2] even in the rectangular cores.

I firstly evaluate the efficiency of heat dissipation in rectangular cores. Figure 3.3 shows the increased temperature distribution (at $t=0$) at the launch end when input pump power is

assumed at 160 W within a 100 μm pumping radius. The signal powers are set to 10 W at LP_{01} , and 10^{-3} W at LP_{11} . The circular core fibre (fibre 1) has its peak temperature of 1.58 K in the core, and the peak temperature reduces with larger AR. rectangular core, e.g. 1.486 K with AR. of 4, and 1.253 K with AR. of 10. It is noted that as I select the Dirichlet boundary condition as the thermal boundary, implying that temperature at the fibre surface is set at a fixed value. Hence, the temperature distribution shown in Fig. 3.3 represents the temperature variations in the core relative to the mean fibre temperature, which mainly influences the power coupling between modes, thus the TMI behaviour as presented in [59]. In actual, the proximity to the cladding outer thermal boundary benefits the rectangular core with larger AR. for better heat dissipation. With the same reason, the square core 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ fibre (fibre 2) is found to have the highest peak temperature of 1.644 K.

Figure 3.3: Increased temperature distribution (at $t=0$) at the launch end in (a) 50 μm circular core fibre (Fibre 1), (b) 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (Fibre 2), (c) 88.8 $\mu\text{m} \times 22.2 \mu\text{m}$ rectangular core (AR. 4:1) fibre (Fibre 3) and (d) 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (Fibre 4).

Figure 3.4 represents the increased temperature profile (at $t=0$) evolution along the fibre in a x-z plane (showing the longer axis of rectangular cores) with the same launched pump and signal powers as in Fig. 3.3. I calculate the temperature profile across the whole fibre structure, but only the core temperature is shown, as the increased temperature in fibre core can change the index profile as shown in Eq. (2.6), and then promotes the TMI. The temperature grows until the length at about 0.6 m, and gradually decreases along the fibre, and as expected, the

larger AR. rectangular core builds lower temperature along the fibre. The fibre 4 with AR. of 10 has 3.618 K of maximum temperature while the circular core fibre (fibre 1) builds higher temperature of 4.103 K. I also note that the fibre 2 (AR. of 1) builds the highest temperature of 4.241 K among the tested fibres, and the fibre 3 with AR. of 4 reaches 3.957 K of maximum temperature. The results again manifest that rectangular core fibre with larger AR. has better thermal dissipation than circular core fibre. It is mainly because that larger AR. makes the fibre core (acts as heat source) edge closer to the cladding, therefore the heat can be more efficiently dissipated and the increased temperature in the core is suppressed. Hence, lower TMI is expected in the larger AR. rectangular core as compared to the circular core.

Figure 3.4: Increased temperature profile (at $t=0$) evolution in the core along fibre length for (a) 50 μm circular core fibre (fibre 1), (b) 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (fibre 2), (c) 88.8 $\mu\text{m} \times 22.2 \mu\text{m}$ rectangular core (AR. 4:1) fibre (fibre 3) and (d) 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4). The same launched pump and signal powers are assumed as in Fig. 3.3.

Figure 3.5 shows the TMI behaviours in the investigated fibres. Time averaged LP_{01} mode power, LP_{11} mode power, total signal power and pump power evolution are compared. All the tested fibres mark similar total signal output power around 150 W, however, the LP_{11} mode power grows differently. Interestingly, the larger AR. rectangular core suffers more significant TMI as evidenced with the higher output power of LP_{11} mode in Fig. 3.5, and this seems

contradicting to the heat dissipation results. The output powers of pump and signal are listed in Table 3.2 for easy comparison. It is found that the LP_{11} mode constitutes 26.6% of the total signal power in fibre 4 (AR. of 10) whereas the LP_{11} mode content is only 0.7% in the circular core (fibre 1). Thus, the benefit of temperature reduction does not appear in the TMI. The pump efficiency indicates pump absorption, defined by $(1 - P_{\text{pout}}/P_{\text{pin}})$ where P_{pout} and P_{pin} denote input and output pump powers, respectively. It is also noted from Table 3.2 that for the circular core (fibre 1), the sum of LP_{01} and LP_{11} power is 149.64 W, which is consistent with total output power of 149.71 W. However, for the rectangular core (AR. 10:1) (fibre 4), the sum of LP_{01} and LP_{11} power is only 147.05 W, less than total signal power of 148.6 W. We believe that this is mainly caused by a stronger thermal lensing effect in rectangular core (AR. 10:1) (fibre 4). Due to the thermal lensing effect, other higher order modes could be populated [115] and some power of the LP_{01} mode is transferred to them. The thermal lensing effect on the TMI is presented in section 3.2.3.

Figure 3.5: Time averaged powers versus fibre length for (a) 50 μm circular core fibre (fibre 1), (b) 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (fibre 2), (c) 88.8 $\mu\text{m} \times 22.2 \mu\text{m}$ rectangular core (AR. 4:1) fibre (fibre 3) and (d) 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4). The same launched pump and signal powers are assumed as in Fig. 3.3.

Table 3.2: Output powers in the investigated fibres

	Fibre 1	Fibre 2	Fibre 3	Fibre 4
Output LP ₀₁ power	148.62 W	147.71 W	141.59 W	107.6 W
Output LP ₁₁ power	1.02 W	0.46 W	7.29 W	39.45 W
Output total signal power	149.71 W	148.51 W	149.34 W	148.6 W
Pump efficiency	> 95%	> 95%	> 95%	> 95%

I further investigate the TMI behaviours in the tested fibres while increasing the launched pump power, to evaluate the TMI threshold power. I consider the threshold pump power when output LP₁₁ mode power grows to 10% of the total signal power. Figure 3.6 shows the output LP₁₁ power content versus the input pump power. Again, the threshold pump power in fibre 4 with AR. of 10 is much lower than in fibre 1 despite the lower temperature in fibre 4. The threshold power for fibre 4 is only 61.7% (145 W) of fibre 1 (238 W). In fact, the threshold pump power gets reduced with the increasing AR. despite better heat dissipation. I also note that fibre 2 with AR. of 1 exhibits larger threshold by 14.9%, from 238 W to 273 W, than the circular core (fibre 1), which contradicts to the temperature results again. Modes overlap or overlap between dopant and the modes might affect the TMI behaviour. However, as shown in Table 3.1, the overlaps are nearly the same for all the fibres and would have negligible influence on the TMI. Other parameters that can influence the TMI include signal effective mode areas which are substantially different in the tested fibres. In the following, we analyse the influence of different effective mode areas under the context of pump saturation effect on the TMI.

Figure 3.6: The output LP₁₁ power content versus the input pump power for 50 μm circular core fibre (fibre 1), 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (fibre 2), 88.8 $\mu\text{m} \times 22.2 \mu\text{m}$ rectangular core (AR. 4:1) fibre (fibre 3) and 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4). The same launched signal powers are assumed as in Fig. 3.3.

3.2.1 Pump saturation effect

In the Table 3.1, the fibre 4 has the largest signal effective mode area of $1368 \mu\text{m}^2$, which is 17.2% larger than fibre 1 having $1167 \mu\text{m}^2$ of the signal mode area. I also note that fibre 2 has the smallest signal mode area of $1083 \mu\text{m}^2$ and the signal mode area of fibre 3 is positioned between the fibre 1 and fibre 4. A higher intensity of signal or lower intensity of pump expedites depletion of excited Yb ions and brings in the pump saturation effect more significantly. The stronger pump saturation is found efficient to suppress the TMI, hence a fibre with smaller signal effective mode area will provide better TMI suppression [63]. In our simulation, fibre 4 contains the largest signal mode area, thus being expected for lower signal intensity and weaker pump saturation effect. I test the pump saturation effect as shown in Fig. 3.7. The upper state population (n_2) profiles and the normalized total signal intensity (scaled to unity at peak) at the input end are presented for fibre 1, fibre 2 and fibre 4 when pump power is assumed at 150 W. The stronger pump saturation effect, which breaks the similar pattern between the signal intensity pattern and the temperature distribution, is efficient to suppress the TMI as presented in [63]. For the temperature distribution is in a linear relationship with the ion inversion level n_2 (Please see Eqs. (2.8) and (2.9)), so I consider the overlap between n_2 and signal intensity profiles to evaluate the influence of pump saturation effect on the TMI. The smaller overlap

means the suppression of TMI and vice versa. The overlap calculated for fibre 1, fibre 2 and fibre 4 are 0.9962, 0.9937 and 0.9987, respectively, when a pumping radius is 100 μm the same as in previous plots. Therefore, the overlap is calculated at the maximum in fibre 4 and the minimum in fibre 2, and I also note that the fibre 1 has the overlap in-between the other two fibres. Therefore, it is expected that the significance of the TMI follows the significance of the overlap, i.e. lower TMI threshold power for fibre 4 and higher threshold power for fibre 2. This seems consistent with our simulation results shown in Fig. 3.6.

Figure 3.7: The normalized signal intensity and upper state population (n_2) profiles at the input end for (a) 50 μm circular core fibre (fibre 1), (b) 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (fibre 2), (c) 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4). Insets enlarge the selected portion to emphasize the overlap differences by adjusting pump intensities. The same launched pump and signal powers are assumed as in Fig. 3.3. PR represents the pumping radius.

It is then interesting to examine the TMI when the pump saturation effect becomes equivalent in all fibres. To achieve the same level of saturation effect in all tested fibres, I adjust the pump intensity by changing the pumping radius in fibre 2 and fibre 4 as illustrated in Figs. 3.7(b) and 3.7(c). The pump intensity adjustment leads to the changes in overlap to 0.9965 and 0.9961 in fibre 2 and fibre 4, respectively, and the overlap in fibre 1 is maintained as 0.9962. Then, all the fibres offer nearly the same overlaps, thus the same level of saturation effect.

Consequently, the TMI behaviour at the same level of saturation effect is presented in Fig. 3.8. As expected, the TMI threshold power is reduced in fibre 2 when difference in the

saturation effect is lifted [See Fig. 3.6 for comparison]. fibre 2 follows the same TMI behaviour as the circular core fibre (fibre 1), albeit not completely the same. Its TMI threshold pump power drops from 273 W to 245 W, which is quite close to the threshold power of fibre 1 at 238 W. Therefore, we conclude that the stronger pump saturation in fibre 2 plays a key role in suppressing the TMI as compared to the fibre 1. In contrast, fibre 4 does not follow the expectation even when the difference of pump saturation effect is removed. Although the threshold pump power is improved from 145 W to 182 W with the enhanced saturation effect, the fibre still exhibits the strongest TMI despite its lowest temperature even at the same level of saturation.

Figure 3.8: The output LP_{11} power content versus the input pump power for 50 μm circular core fibre (fibre 1), 44.3 $\mu\text{m} \times 44.3 \mu\text{m}$ square core (AR. 1:1) fibre (fibre 2), and 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4). The same launched pump and signal powers are assumed as in Fig. 3.3.

3.2.2 Power coupling strength

The TMI is viewed as the STRS that induces a power coupling between supported modes in an optical fibre, and the power coupling strength can be evaluated by introducing a power coupling coefficient. It is reported that the coupling coefficient of STRS is determined by a few factors including the doping area in a core, temperature built by a quantum defect, an overlap of the supported modes and the frequency offset between two modes [59,67]. For instance, the power coupling coefficient can be described as [67]:

$$\chi_{1,2}(\Delta\omega) = \frac{\frac{dn}{dT}k^2}{K\beta_{1,2}} \text{Im}[A(\Delta\omega)](1 - \frac{\lambda_s}{\lambda_p}) \quad (3.1)$$

where k is the vacuum wavenumber, β_1 and β_2 are the propagation constant for LP₀₁ mode and LP₁₁ mode. Therefore, the coupling coefficient is mainly determined by the quantity $\text{Im}(A)$ because other terms are related to material constants and waveguide parameters. The $\text{Im}(A)$ is described as

$$\begin{aligned} \text{Im}(A(\Delta\omega)) = & \iint_{\Omega} E_{01}(x, y)E_{11}(x, y) \times \\ & (\iint_{\Omega_d} \text{Im}(G(x, y, x', y', \Delta\omega))E_{01}(x', y')E_{11}(x', y')dx'dy')dx dy \end{aligned} \quad (3.2)$$

where Ω and Ω_d denote the whole fibre cross section and doped cross section, G is the Green's function used to solve the temperature distribution shown in Eq. (2.17), and the expression of G is presented in Eqs. (2.13) - (2.16).

The power coupling coefficient (χ_1 nearly equals to χ_2) is calculated as 0.4213 W⁻¹ for fibre 4, which is much larger than 0.1167 W⁻¹ for fibre 1. The larger χ implies a stronger power coupling between modes, which manifests the stronger TMI. Thus, I conclude that fibre 4 suffers more significant TMI as shown in Fig. 3.6 due to its 3.6 times stronger coupling strength than fibre 1. The benefit of the lower temperature in fibre 4 is overshadowed by the strong coupling coefficient and does not help to suppress the TMI. Interestingly, the coupling coefficient is a function of the frequency offset, and I have shown that the frequency offset is mainly determined by the longer axis in a rectangular core fibre. Therefore, the coupling strength in a rectangular core fibre is similar to the circular core with a core diameter same as the longer axis of the rectangular core if the differences in pump saturation effect and increased temperature distribution are insignificant. Additionally, the quantity power coupling coefficient in fibre 2 is 0.1042 W⁻¹, which is a little smaller than 0.1167 W⁻¹ in fibre 1, hence implying that the TMI is a little weaker in fibre 2 compared with that in fibre 1 as shown in Fig. 3.8.

3.2.3 Thermal lensing effect

Figure 3.9: The signal effective area as a function of propagation length, z , for (a) 50 μm circular core fibre (fibre 1) and (b) 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4) when the same launched pump and signal powers are assumed as in Fig. 3.3.

When the signal power is mostly confined to the fundamental mode, the temperature develops its profile following the fundamental mode, resulting in temperature peak at the centre of fibre core. This in turn causes the thermal lensing effect in a fibre [114]. Figure 3.9 presents the signal effective area as a function of propagation length, z , when the same launched pump and signal powers are assumed as in Fig. 3.3 and pumping radius fixed at 100 μm . As the thermal lensing effect relates to both the increased temperature distribution and core size [114]. It is seen from fig. 3.39 that the thermal lensing is very weak in the circular core fibre, while quite strong in the rectangular core fibre with AR. 10:1. This is due to that the increased temperature (for instance smaller than 2 K at the input end) is quite low and cannot make strong thermal lensing in the circular core fibre. However, the large core size (width along the x-axis) in the rectangular core fibre with AR. 10:1 makes it generate quite strong thermal lensing effect. Additionally, it is clear to see that in both fibres, the signal effective area decreases until about $z=0.5$ m, and subsequently increases to recover the original effective area at the output end. This is attributed to the thermal lensing effect, which corresponds to the temperature profile evolution as shown in Fig. 3.4. The static temperature becomes its maximum at ~ 0.5 m, hence most significant thermal lensing effect. The maximum signal effective area reduction in a circular core (fibre 1) amounts to 2.23%, which is much less than 9.58% in fibre 4 with AR. of 10, implying that rectangular core with larger AR (or wider core) generates stronger thermal lensing. As the thermal lensing effect mainly pushes the fundamental mode to the core centre, the modal overlap gets lowered, which leads to an increase in TMI threshold power [88]. Additionally, the reduced signal effective mode area caused by thermal lensing can enhance

the pump saturation effect, thus leading to TMI suppression again. Therefore, the stronger thermal lensing effect in a rectangular core fibre with larger AR. helps increase the TMI threshold power as compared to a circular core fibre. Even so, the stronger power coupling between modes in rectangular core fibre with larger AR. overshadows the thermal lensing effect, to make the TMI stronger in the rectangular cores. We also note that fibre 4 exhibits more significant TMI along the entire fibre than fibre 1 even at the point of the maximum thermal lensing effect as presented in Fig. 3.5.

3.2.4 Transverse mode instability threshold power

The relationship between threshold pump power and core area is presented in Fig. 3.10. We assume the core areas of $1960 \mu\text{m}^2$, $2940 \mu\text{m}^2$, $3920 \mu\text{m}^2$, $5880 \mu\text{m}^2$ and $7840 \mu\text{m}^2$, for circular, square core (AR. 1:1), rectangular core (AR. 4:1) and rectangular core (AR. 10:1) fibres. As expected from the previous results, it is clear to see that when the core area is same, the square core (AR. 1:1) fibre has the highest threshold pump power due to the strongest pump saturation effect and the weakest power coupling strength. On the other hand, the rectangular core (AR. 10:1) fibre suffers the smallest threshold power, which is mainly caused by the strongest power coupling and then the weakest pump saturation effect among the tested fibres. It is also noted that in all the tested fibres, the threshold power decreases with the increasing core area by weakening the pump saturation effect.

Figure 3.10: The threshold pump power versus core area for circular core, square core (AR. 1:1), rectangular core (AR. 4:1) and rectangular core (AR. 10:1) fibres. The same launched signal powers are assumed as in Fig. 3.3.

Figure 3.11 shows the threshold pump power (left axis) and power coupling coefficient χ (right axis) for circular core, square core (AR. 1:1), rectangular cores with AR. of 4:1 and AR. of 10:1 when the input signal effective mode area is fixed at $1170 \mu\text{m}^2$, $1750 \mu\text{m}^2$, $2300 \mu\text{m}^2$, $3500 \mu\text{m}^2$ and $4650 \mu\text{m}^2$. As seen in Fig. 3.11, when the effective mode area is fixed, the threshold power in a circular core fibre is very close to its square core counterpart, and the slightly higher TMI threshold power in the square fibre is attributed to the slightly smaller coupling strength, χ . That is to say, the stronger pump saturation effect in square core (AR. 1:1) fibre is the main factor contributing to the higher threshold power than the circular core fibre shown in Fig. 3.10. Therefore, we confirm that with the same pump saturation effect, the TMI threshold power can be mainly decided by the power coupling strength. Similarly, the TMI threshold power is decreasing with the increasing AR. in rectangular cores. Even with the same effective mode area (thus nearly same saturation effect), the larger AR. core sees stronger TMI due to the stronger coupling strength. Apparently, the stronger coupling is caused by the lower frequency offset in rectangular cores with larger AR. Furthermore, the coupling strength gets stronger with the increasing effective mode area in all the tested fibres because of the increasing core size.

Figure 3.11: The threshold pump power (left axis) and power coupling coefficient χ (right axis) versus effective mode area for circular core, rectangular core (AR. 1:1), rectangular core (AR. 4:1) and rectangular core (AR. 10:1) fibres. The same launched signal powers are assumed as in Fig. 3.3.

3.3 Transverse mode instability in D-shaped cladding fibres

I now move on to investigate the TMI in a D-shaped cladding fibre when the core is in a circular or rectangular shape. The purpose of this work is to confirm the influence of temperature alone on the TMI when differences in the pump saturation effect and the power coupling strength become negligible. We compare the TMI between D-shaped cladding and circular cladding fibres and select the core designs of fibre 1 and fibre 4 for the investigation. Figure 3.12 represents the investigated fibre structures having a D-shaped cladding with 400 μm and 200 μm in long and short axes, respectively. The dashed lines represent the thermal boundary position in D-shaped cladding fibres, and the Dirichlet boundary condition is assumed as in the circular cladding fibres. In both the circular and D-shaped cladding fibres, we assume the same pumping radius of 100 μm to keep the pump saturation effect the same. The input signal powers at LP_{01} and LP_{11} mode are 10 W and 10^{-3} W, respectively. The parameters of fibre core in D-shaped cladding are kept the same as listed in Table 3.1. As the frequency offset is mainly determined by the thermal diffusion time (heat transfer from core centre to core edge) [59] or signal effective mode area [67], hence the circular core and rectangular core in D-shaped cladding fibres have the nearly same frequency offset as in circular cladding fibres at 1160 Hz and 260 Hz, respectively.

Figure 3.12: Intensity of (a) LP_{01} and (b) LP_{11} mode in the 50 μm circular core fibre (fibre 1) with D-shaped cladding, (c) LP_{01} and (d) LP_{11} in the 140 $\mu\text{m} \times 14 \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4) with D-shaped cladding. The same core parameters are used as listed in Table 3.1. The dashed lines indicate the thermal boundary position.

3.3.1 Transverse mode instability in circular core D-shaped cladding fibre

The increased temperature distributions (at $t=0$) at the launch end of fibre 1 with the circular and D-shaped cladding are shown in Fig. 3.13. Input pump power of 250W is assumed while the modal input powers are kept the same as in Fig. 3.3. Clearly, the D-shaped cladding helps to reduce the temperature from 1.681 K in a circular cladding to 1.384 K in the D-shaped cladding. The temperature suppression is achieved by the cladding geometry, i.e. the circular core (acts as a heat source) gets closer to the cladding boundary (acts as a heat sink) in the D-shaped cladding. The corresponding increased temperature profile evolution (at $t=0$) along the fibre is presented in Fig. 3.14. The maximum increased temperature is reduced by about 14.6% from 6.217 K in the circular cladding to 5.31 K in the D-shaped cladding, which reconfirms that the D-shaped cladding fibre provides more efficient heat dissipation. Hence, better TMI suppression is expected in the D-shaped cladding.

Figure 3.13: Increased temperature distribution (at $t=0$) within the whole core and cladding region at the launch end in 50 μm circular core (fibre 1) with (a) circular cladding and (b) D-shaped cladding. The same launched signal powers are assumed as in Fig. 3.3. The input pump power is 250 W with 100 μm pumping radius.

Figure 3.14: Increased temperature profile evolution (at $t=0$) within the core along fibre in 50 μm circular core (fibre 1) with (a) circular cladding and (b) D-shaped cladding. The same launched signal powers, pump power and pumping radius are assumed as in Fig. 3.13.

Our numerical simulation results of time averaged LP_{01} and LP_{11} mode power, total signal power and pump power evolution are shown in Fig. 3.15. It is noted from fig. 3.14 that the peak temperature exists at around $z=0.6$ m. This is because that the temperature distribution is related to the thermal load distribution, thus total signal and pump power as reflected in Eqs. (2.7) and (2.9). Therefore, the temperature at around $z=0.6$ m, where the signal and pump power become the same as seen from Fig. 3.15, generates the highest value along the fibre length. The higher output power of LP_{11} mode in the circular cladding fibre is due to the higher temperature compared with the D-shaped cladding fibre. Hence, the results confirm that better heat dissipation helps suppress the TMI when the same pump saturation effect and power coupling strength are applied. The TMI threshold pump power together with the maximum increased core temperature of both cladding shapes are examined by increasing the pump power, and the results are shown in Fig. 3.16. The threshold pump power in the D-shaped cladding is reduced by 17% from 275 W to 235 W as compared to the circular cladding. This improvement is attributed to the temperature decrease by about 15% as indicated in the right axis in Fig. 3.16.

Figure 3.15: Time averaged powers versus fibre length for 50 μm circular core (fibre 1) with (a) circular cladding and (b) D-shaped cladding. The same launched signal powers, pump power and pumping radius are assumed as in Fig. 3.13.

Figure 3.16: The output LP_{11} power content (left axis) and maximum increased temperature in the core (right axis) versus the input pump power for 50 μm circular core (fibre 1) with circular cladding and D-shaped cladding. The same launched signal powers and pumping radius are assumed as in Fig. 3.13.

3.3.2 Transverse mode instability in rectangular core D-shaped cladding fibre

We extend our investigation to the rectangular core with AR. of 10 (fibre 4) in a circular cladding and a D-shaped cladding. Figure 3.17 shows the increased temperature distribution (at $t=0$) at the launch end for both fibres when the input pump power is 140 W with 100 μm pumping radius. As before, the D-shaped cladding facilitates the heat dissipation, resulting in

lower temperature of 0.96 K than 1.23 K in the circular cladding. The increased temperature profile evolution (at $t=0$) within the fibre core is also presented for both fibres in Fig. 3.18. The simulation result confirms that D-shaped cladding reaches lower temperature of 2.05 K than 2.72 K in the circular cladding, which is about 24.7% reduction. The larger reduction than circular core (fibre 1) suggests that the rectangular core (fibre 4) offers more efficient heat dissipation than circular core in the D-shaped cladding.

Figure 3.17: Increased temperature distribution (at $t=0$) at the launch end within the whole core and cladding region in $140\text{ }\mu\text{m} \times 14\text{ }\mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4) with (a) circular cladding and (b) D-shaped cladding. The same launched signal powers are assumed as in Fig. 3.3. The input pump power is 140 W with 100 μm pumping radius.

Figure 3.18: Increased temperature profile evolution (at $t=0$) in the core along fibre for $140\text{ }\mu\text{m} \times 14\text{ }\mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4) with (a) circular cladding and (b) D-shaped cladding. The same launched signal powers and pumping radius are assumed as in Fig. 3.17.

Figure 3.19 presents the time averaged LP_{01} and LP_{11} mode power, total signal power and pump power evolution in both cladding shapes. The output LP_{11} mode power in the D-shaped cladding fibre grows to 1.494 W shown in Fig. 3.19 (b), which is much less than 10 W in the circular cladding fibre presented in Fig. 3.19 (a). Thus, the efficient heat dissipation is found

helpful to suppress the TMI when other factors such as the saturation effect and coupling strength are same in both fibres.

Figure 3.19: Time averaged powers versus fibre length for $140\ \mu\text{m} \times 14\ \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4) with (a) circular cladding and (b) D-shaped cladding. The same launched signal powers and pumping radius are assumed as in Fig. 3.17.

Figure 3.20: The output LP_{11} power content (left axis) and maximum increased temperature in the core (right axis) versus the input pump power for $140\ \mu\text{m} \times 14\ \mu\text{m}$ rectangular core (AR. 10:1) fibre (fibre 4) with circular cladding and D-shaped cladding. The same launched signal powers and pumping radius are assumed as in Fig. 3.17.

Figure 3.20 shows the output LP_{11} power content (left axis) and maximum increased temperature in the core (right axis) versus input pump power. Apparently, the maximum increased temperature in the D-shaped cladding fibre drops by about 24.7% compared to the circular cladding fibre. Consequently, the threshold pump power in the D-shaped cladding fibre

increases by 29% compared to the circular cladding fibre, from 145 W to 187 W. I note that the threshold power of the rectangular core (AR. 10:1) D-shaped cladding fibre is much lower than the circular core circular cladding fibre (fibre 1) (187 W versus 235 W) due to the stronger power coupling and weaker pump saturation effect in the rectangular core. I also note that the temperature reduction by introducing the D-shaped cladding in the rectangular core amounts to 24.7%, which is larger than the temperature drop of 14.6% in the circular core counterparts. Despite the larger temperature reduction in the rectangular core induced by the D-shaped cladding, the TMI does not follow the temperature reduction, but is mainly determined by the pump saturation effect and power coupling strength.

3.4 Conclusion

In this chapter, I present theoretical investigation of the TMI in non-circular fibres including rectangular core fibres and D-shaped cladding fibres for the first time, to the best of our knowledge. The results reveal that the temperature built in a fibre is not solely responsible for the TMI, but the pump saturation effect, coupling strength and thermal lensing effect also play a significant role in governing the TMI behavior. Furthermore, the coupling strength is a function of the frequency offset, which is mainly determined by the longer axis in a rectangular core fibre. Thus, the power coupling strength in a rectangular core fibre can be nearly as strong as a circular core fibre with the same core diameter as the longer axis of the rectangular core. On the other hand, when the same core area (or signal effective area) is applied to both the circular and the rectangular cores, the longer axis in the rectangular core is larger than the core diameter in the circular core. Consequently, the TMI threshold power becomes higher in the circular core because of the weaker power coupling strength due to higher frequency offset. Temperature reduction by shaping the core does not always suppress the TMI, although shaping the cladding can suppress the TMI via better heat dissipation.

In the investigation, at the fixed core area, the rectangular core (AR. 10:1) fibre offers the most efficient heat dissipation (24.6% better than a circular core) among the circular core, square core (AR. 1:1), and rectangular cores with AR. of 4 and 10. However, rectangular core (AR. 10:1) fibre suffers the worst TMI among the tested fibres, which drops the threshold pump power by 38.3%, from 238 W to 145 W, in comparison with circular core fibre. I find this is mainly caused by the stonger power coupling strength and then the weaker pump saturation effect than other types of fibres. Additionally, we also simulate the TMI in D-shaped cladding fibres, and find that such cladding structure offers more efficient heat dissipation, which suppresses the TMI and increases the threshold pump power when pump saturation effect and

power coupling strength are the same. For instance, the D-shaped cladding circular core and rectangular core (AR. 10:1) fibres reduce the temperature by 15% and 25%, respectively, and increase the threshold pump power by 17% and 35.2% when compared to circular cladding counterparts. Despite this better heat dissipation, the rectangular core (AR. 10:1) D-shaped cladding fibre suffers much more significant TMI with 189 W threshold power than its circular core circular cladding counterpart having 235 W threshold power, which is opposite to the temperature increase. Our investigation reconfirms that the significance of TMI is not simply predicted by temperature, as the influence of temperature on TMI can be overshadowed by pump saturation effect and power coupling strength between modes.

Chapter 4 Power scaling in photodarkening fibre by transverse mode instability suppression

Since the first experimental observation of TMI in the Yb-doped fibre amplifier, the QD is recognized as the main heat source to modify the refractive index, which accounts for the power transfer from FM to HOM. Recently, the PD-induced heat from absorption of both signal and pump is recognized as another heat source contributing to the TMI [75, 80]. The TMI behaviours at the presence of PD has been experimentally investigated in [75] as well as theoretically studied in [74, 75, 85]. However, the prior theoretical investigations were somewhat limited to portray the various interactive effects on TMI such as pump saturation, thermal lensing, and mode coupling effects. For instance, the thermal lensing effect has been overlooked although its role becomes significant in a LMA fibre or under a high heat load, because it promotes the pump saturation effect for TMI suppression [88]. In addition, the PD-induced heat load was often assumed as a constant fraction of total heat load, which needs verification. On the other hand, a strong pump saturation effect breaks the similar profile distributions between the signal irradiance and heat load, thus contributing to suppression of the TMI [63]. The saturation effect can enhance the TMI threshold, which could allow a fibre to hold higher thermal load before the onset of TMI. This relation has not been examined in the prior studies. Furthermore, a constant average heat load was suggested as an indicator to predict the TMI threshold [72]. This average heat load for the TMI threshold was experimentally found in a range of 15 - 34 W/m [75, 116, 117]. This wide range indicates that the average heat load at TMI threshold is dependent on the fibre parameters. Therefore, more rigorous theoretical investigation for TMI behaviour including influence of fibre parameters is required to give insights for TMI suppressed fibre designs. Additionally, mitigation strategies to suppress the TMI in a PD fibre were mainly focused on a device level such as changes of pump and signal wavelengths [75, 85]. In a fibre level, the TMI suppression can be achieved by reducing the Yb concentration via reduction of the PD-induced heat load [75, 118]. However, the reduced concentration means longer fibre absorption length for efficient operation, which subsequently triggers undesired nonlinearities such as stimulated Raman scattering (SRS) or stimulated Brillouin scattering (SBS). Alternatively, the absorption length can be maintained by reducing the fibre cladding-to-core area ratio (CCAR) [75]. However, this approach hampers the pump saturation effect and expedites the TMI [63]. Then, it is the motivation of

this contribution to examine parametric dependence of TMI, using a numerical model, to serve as an additional guidance for TMI suppressed fibre designs.

In this chapter, we use the 4D split-step FFT-based BPM presented in chapter 2 to calculate the TMI behaviour in a PD fibre. The numerical study considers the TMI before the PD-induced loss saturates, and after PD-induced loss reaches a saturable level. The numerical model allows us to investigate the combined effects of heat source as well as the saturation effect in a scope of suppressing the TMI, with an arbitrary choice of core shapes, refractive index and doping profiles. Moreover, pump saturation, thermal lensing and mode coupling effects are all intrinsically included at a cost of longer running time. In the following, we present the TMI suppressed fibre design particularly when two heat sources, QD and PD, are present. The simulation considers fibre length, cladding radius (pumping radius), core numerical aperture (NA), core radius and ion concentration as design parameters to investigate the TMI as well as nonlinearity for power-scaling in a PD fibre.

4.1 Influence of fibre parameters on transverse mode instability

In this section, I first compare the frequency offset and thermal lensing effect in fibre amplifiers before and after PD saturates, then examine the parametric influence on TMI.

The HOM can be seeded through quantum noise, spontaneous thermal Rayleigh scattering and imperfect coupling of a seed source to the amplifier [59, 61]. The HOM input power can be set in a wide range of power levels from the quantum noise level of 10^{-16} W [58, 59, 77] to above 10^{-10} W [61] or even higher powers of 0.1 W [119], 0.5 W [76] and 1.5 W [78]. In this chapter, I set the HOM input power P_{11} at 1 μ W and FM input power P_{10} at 10 W. We define the TMI threshold power when the HOM takes 5% of total output signal power. Important simulation parameters are listed in Table 4.1. An Al:Yb co-doped fibre is assumed.

Table 4.1: Simulation parameters

Symbol	Quantity	Value
R_{core}	core radius	25-50 μm
NA	Core numerical aperture (NA)	0.03-0.06
λ_s	signal wavelength	1064 nm
λ_p	pump wavelength	976 nm
σ_s^a	signal absorption cross section	$6.00 \times 10^{-27} \text{ m}^2$
σ_s^e	signal emission cross section	$3.58 \times 10^{-25} \text{ m}^2$
σ_p^a	pump absorption cross section	$1.53 \times 10^{-24} \text{ m}^2$
σ_p^e	pump emission cross section	$1.87 \times 10^{-24} \text{ m}^2$
ρ	mass density	2201 kg/m^3
C	heat capacity	702 J/(kg $\times$ K)
K	thermal conductivity	1.38 W/(m $\times$ K)
dn/dT	thermo-optic coefficient	$1.2 \times 10^{-5} \text{ K}^{-1}$
P_{01}	input FM power	10 W
P_{11}	input HOM power	1 μW
R_{pump}	cladding radius	25 μm -200 μm
N_{Yb}	Yb ion concentration	$0.5 \times 10^{25} \text{ m}^{-3}$ - $3.5 \times 10^{25} \text{ m}^{-3}$
τ	excited ion lifetime	850 μs
R_{dope}	doping radius	Uniform doping
Δx	x grid spacing	1.56 μm
Δy	y grid spacing	1.56 μm
Δz	step size along fibre	10 μm
N_t	number of time grids	64 grids for a time cycle ($1/\Delta\omega$)

4.1.1 Frequency offset with photodarkening

The frequency offset between FM and HOM is essential for generating the phase shift, accounting for power transfer from the FM to HOM. In the following, I numerically calculate the frequency offset, at which the strongest power transfer from FM to HOM happens before and after the PD saturation. As defined earlier, *before PD saturation* refers to the undeveloped PD (QD as a heat source), and *after PD saturation* refers to the saturated PD (QD combined PD as a heat source).

Figure 4.1: Normalized output HOM power content versus the frequency offset between FM and HOM in the fibre before and after PD saturation. The parameters used are listed in table I. Key parameters include core and cladding radius of 25 μm and 100 μm , respectively, Yb ion concentration of $3.5 \times 10^{25} \text{ m}^{-3}$ and core NA of 0.06.

Figure 4.1 shows the power content of LP_{11} mode in the total output signal power at different frequency offsets. It is found that the PD reduces the frequency offset for the maximum power transfer. The frequency offset lies at 1160 Hz before PD saturation, while it shifts to 1130 Hz after PD saturation. This shift accounts for a change of a spatial profile of Yb absorption in the presence of the PD. When the PD is present, the Yb absorption becomes considerably stronger near the edge of core, which increases the thermal diffusion time, thus lower the maximum frequency offset. The results are consistent with [120, 121]. We use the frequency offsets at the maximum power transfer in the following simulations.

4.1.2 Thermal evolution and thermal lensing effect

Figure 4.2: Signal effective mode area along the propagation fibre length for (a) before PD saturation and (b) after PD saturation. The parameters are assumed as in Fig. 4.1 and the input pump power is 300 W.

When the signal power is mostly confined to the FM, the QD and PD-induced temperature profile follows the FM profile. This induces peak temperature at the centre of core and causes the thermal lensing or self-focusing of the signal light [88]. It is expected that in a PD fibre, more significant thermal lensing effect is generated after PD saturation, which is due to the larger heat generated by PD, thus more reduction of effective signal area and larger pump saturation effect. We calculate the signal effective mode area consisted of LP_{01} and LP_{11} to verify this. Figure 4.2 presents the signal effective mode area along the propagation length, z , when the same parameters are assumed as in Fig. 4.1. It is clear to see that in both before and after PD saturation, the thermal lensing effect is prominent until the signal propagates around 0.6 m. Subsequently, the mode area recovers to the original effective area at the end of output. The evolution of the mode area is attributed to the temperature profile evolved along the fibre as shown in Fig. 4.3. Apparently, the mode area is directly influenced by the thermal profile. The mode area in the fibre after PD saturation decreases from $1173 \mu\text{m}^2$ at $z=0$ m to about $1060 \mu\text{m}^2$ at $z=0.6$ m, which is more significant than the before PD saturation counterpart ($1173 \mu\text{m}^2$ to $1075 \mu\text{m}^2$). This agrees to our expectation from the additional PD-induced thermal load as described in Eq. (2.9) – (2.12). Hence, the thermal lensing effect is stronger in a PD fibre after PD saturation. However, the reduced mode area, or larger pump saturation effect, by the thermal lensing is not sufficient to counterbalance the PD-induced additional heat load on TMI. This is discussed in the following sections.

Figure 4.3: Temperature distribution in the x-z plane (at $t=0$) along the propagation fibre length for (a) before PD and (b) after PD saturation. The parameters are assumed as in Fig. 4.2.

4.1.3 Fibre length

I numerically investigate the length dependence on TMI in this section. Two fibre lengths are considered for 95% and nearly 100% of pump absorption, respectively. A fibre length was reported to be insignificant to affect the TMI, when pump absorption above 95%, in a non-PD

fibre [116]. In the following, I test this in a PD fibre. Firstly, I start with the frequency offset calculation as done in Fig. 4.1. We found that the frequency offsets are invariant with the fibre lengths, and the same frequency offsets in Fig. 4.1 are assumed in this section.

Table 4.2: Output powers before and after PD saturation

	Before PD (1.6 m)	Before PD (5 m)	After PD (1.6 m)	After PD (5 m)
Input pump power	300 W	300 W	300 W	300 W
Output LP ₀₁ power	264.30 W	276.68 W	253.12 W	186.13 W
Output LP ₁₁ power	0.21 W	0.22 W	4.53 W	79.73 W
Total output power	264.73 W	277.71 W	258.12 W	267.21 W
Output pump power	15.12 W	1.57×10^{-4} W	15.78 W	1.67×10^{-4} W
Pump absorption	95%	Nearly 100%	95%	Nearly 100%

Figure 4.4: Time averaged powers versus fibre length for (a) 1.6 m and (b) 5 m before PD saturates, (c) 1.6 m and (d) 5 m after PD saturates. The parameters are assumed as in Fig. 4.2.

Figure 4.4 presents the TMI behaviors of the time averaged FM mode power, LP₀₁, HOM mode power, LP₁₁ and pump power evolution along fibres before and after PD saturation. The fibre lengths are assumed as 1.6 m and 5 m for 95% and nearly 100% of pump absorption, respectively. It is interesting to see that the power transfer from LP₀₁ to LP₁₁ nearly stops after 95% of pump absorption in fibres before PD saturation, confirming the length independence of TMI when PD is not presented [115]. However, when PD kicks in, the power transfer continues even after the 95% pump absorption, indicating PD as an additional cause for TMI.

The output powers presented in the Figure 4.4 are listed in Table 4.2. We can see a clear contrast in the LP_{11} output power between before and after PD saturation cases, with different fibre lengths. Furthermore, the total output power is generally reduced in the fibres after PD saturates. This is mainly attributed to the PD-induced loss as observed in Fig. 4.5. Additionally, the slightly higher total output power than the plus of two modal powers is caused by other higher-order mode excitation due to the thermal grating [79, 115].

Figure 4.5: PD-induced loss in the x-z plane (at $t=0$) along the propagation fibre length for (a) 1.6 m long and (b) 5 m long fibres. The parameters are assumed as in Fig. 4.2.

Figure 4.6: Output HOM power content, LP_{11} versus the input pump power in the PD fibres that before and after PD saturation, at the fibre length of 1.6 m and 5 m, respectively. The other fibre parameters are the same as that in Fig. 4.1.

In the following I investigate the TMI threshold power in fibres. As shown in Fig. 4.6, the threshold power is not influenced by the fibre length when PD is ignored. This is clearly portrayed by Eq. (2.9) in that the QD-induced thermal grating becomes nearly zero when the fibre length is extended. It is also shown that with the same fibre length, the threshold power

is significantly reduced after PD saturation (315 W) as compared to before PD saturation (415 W). This is well explained by the additional heat source from the PD [87]. More interestingly, unlike the fibre before PD, the fibre after PD shows fibre length dependent TMI threshold. Indeed, the threshold power keeps being reduced by 15.9% to 265 W in 5 m from 315 W in 1.6 m. Pump power is nearly exhausted after the 95% absorption length, and contribution of the QD heat source becomes negligible. Hence, the reduced TMI threshold power with fibre length after PD should be related to the PD contribution. To verify this speculation, thermal grating profiles are plotted in fibres after PD saturation and presented in Fig. 4.7. I calculate the PD-induced thermal grating from Eq. (2.10) and (2.11) and separate the pump and signal contributions in the investigation.

Figure 4.7: Evolution of the PD-induced thermal grating (at $t=0$) from pump absorption in (a) 1.6 m and (b) 5 m PD fibres, and from signal absorption in (c) 1.6 m and (d) 5 m PD fibres. The same parameters as that in Fig. 4.2 are assumed.

The PD-induced thermal grating from pump absorption (Fig. 4.7 (a) and (b)) is determined by the absorption profile, as reflected in Eq. (2.5) and (2.1). The pump-induced thermal grating becomes weaker and gets nearly invisible after the 95% absorption length (1.6 m). Hence, the pumping contribution to the PD-induced thermal grating has little influence on the length dependent TMI threshold. In contrast, the PD-induced heat from signal absorption continuously propagates along the entire fibre even after depletion of the n_u , and promotes the

TMI even in the absence of QD-induced heat (Fig. 4.7 (c) and (d)). Therefore, it is concluded that the signal-induced PD accounts for the length dependent TMI threshold. A fibre length is often determined for maximum output power extraction and longer than 95% absorption length usually generates more power in an efficient fibre (For example, see Table 4.2). However, our result shows that the length optimization in a PD fibre should be more carefully conducted to avoid the onset of TMI by the signal- induced PD.

It is of great interest to correlate the TMI threshold power with a thermal load. To gain an insight of the relation, I analyse the thermal load along fibre length as presented in Fig. 4.8. The time averaged thermal load distribution in Fig. 4.8 reveals evolution of thermal loads of each heat contributors. I keep the input pump power at 300 W for all the cases, which promises a fair comparison. It is found that the total thermal load is mainly contributed by the QD regardless of the presence of PD. As discussed, the signal-induced PD heat load persists along the entire fibre (Fig. 4.8(d)) and makes the length dependent TMI threshold. I compare thermal load contributions of PD and QD in Table 4.3. It is found that the total heat load, or total combined Q , is similar for the fibres before PD while increases for the fibres after PD regardless of fibre lengths. Hence, the total Q seems to be useful to estimate the TMI behaviour in a PD fibre. I will further investigate this in the following sections.

Table 4.3: Averaged and total thermal load in PD fibres

	Before PD (1.6 m)	Before PD (5 m)	After PD (1.6 m)	After PD (5 m)
Input pump power	300 W	300 W	300 W	300 W
Averaged QD-induced Q	14.57 W/m	4.90 W/m	14.53 W/m	4.91 W/m
Total QD-induced Q	23.31 W	24.53 W	23.24 W	24.53 W
Averaged PD-Pump-induced Q	/	/	0.81 W/m	0.26 W/m
Total PD-Pump-induced Q	/	/	1.30 W	1.30 W
Averaged PD-Signal-induced Q	/	/	3.85 W/m	2.05 W/m
Total PD-Signal-induced Q	/	/	6.15 W	10.23 W
Averaged PD-induced Q	/	/	4.66 W/m	2.31 W/m
Total PD-induced Q	/	/	7.45 W	11.53 W
Averaged combined Q	14.57 W/m	4.90 W/m	19.18 W/m	7.21 W/m
Total combined Q	23.31 W	24.53 W	30.69 W	36.07 W

Figure 4.8: Time averaged transversal thermal load (including QD: QD-induced thermal load; PD-Pump: PD-induced thermal load from pump absorption; PD-Signal: PD-induced thermal load from signal absorption; Total-Q: sum of all thermal loads in fibre) versus fibre length for (a) 1.6 m and (b) 5 m before PD saturation, (c) 1.6 m and (d) 5 m after PD saturation. The parameters are assumed same as in Fig.4.7.

4.1.4 Cladding Size

When the fibre core size is fixed, the cladding radius (pumping radius) influences the pump saturation effect and subsequently the TMI threshold [63, 119]. In the following, I study the influence of cladding radius on TMI threshold power in PD fibres. My aim is to understand whether the saturation effect can compensate the PD-induced TMI. I start with investigating the frequency offset that makes most power transfer between the modes at the cladding radius changed from 100 μm to 200 μm . It is calculated that the frequency offsets are at 1130 Hz and 1060 Hz before and after PD saturation, respectively. With 100 μm cladding radius, the frequency offset for most power transfer is red shifted compared to 200 μm cladding radius as presented in Fig. 4.1.

Figure 4.9 presents the LP_{11} output power content versus input pump power in PD fibres before and after PD, with the cladding radius of 100 μm and 200 μm . Here I make the pump absorption same at about 95% for all the fibres by adjusting fibre length, just as done in [63], so that I can directly examine the TMI threshold power and its corresponding thermal loads.

As shown in Fig. 4.9, the threshold power increased by about 67.5%, from 415 W to 695 W when the cladding radius increased from 100 μm to 200 μm before PD saturation. This verifies that the stronger pump saturation effect is a very efficient way to suppress the TMI. However, after PD saturates, the threshold power only has a 22.3% improvement, from 315 W to 385 W, indicating the saturation effect is not sufficient to compensate the PD-induced TMI.

Figure 4.9: Output HOM power content, LP_{11} versus the input pump power in fibres before PD and after PD, with the pumping radius of 100 μm and 200 μm . The fibre parameters are same as that in Fig. 4.1 except the change of cladding radius.

Figure 4.10 presents the evolution of time averaged thermal load along fibre length before and after PD at the TMI thresholds. It is noted that fibre length is adjusted for 95% pump absorption in all the cases. Table 4.4 lists the detailed values of averaged and total thermal loads. Firstly, it is found that the total thermal load, total Q , at the threshold increases with cladding radius, hence indicating that the large cladding size helps the fibre bear larger thermal load before the onset of TMI, with a help of the pump saturation effect. On the other hand, when the averaged thermal load, averaged Q , is considered, my simulation shows that the averaged Q is lower for higher TMI threshold fibres (or the larger cladding fibres). In other words, fibre experiences TMI although the averaged thermal load is lower. Hence, we conclude that using a total Q is more intuitive to interpret TMI behaviour, e.g. a fibre with higher TMI threshold means the fibre withstand higher thermal load before TMI, thanks to other mechanism such as the saturation effect working to help the fibre tolerate higher thermal load. The total Q will be continuously examined in relationship with TMI threshold in the following sections. I also note that the total Q as well as the averaged Q at TMI threshold are constant

when fibre parameters remain same regardless of the presence of PD. However, I cannot observe a constant averaged Q as a TMI threshold indicator for all fibres.

Figure 4.10: Time averaged transversal thermal load (including QD: QD-induced thermal load; PD-Pump: PD-induced thermal load from pump absorption; PD-Signal: PD-induced thermal load from signal absorption; Total-Q: sum of all thermal loads in a fibre versus fibre length for (a) 100 μm and (b) 200 μm cladding radius before PD, (c) 100 μm and (d) 200 μm cladding radius after PD. The parameters are assumed same as in Fig. 4.9. The input pump power is at the TMI threshold power and fibre length is adjusted to make 95% pump absorption.

Table 4.4: Averaged and total thermal load in PD fibres at TMI thresholds

	Before PD (25/100 μm)	Before PD (25/200 μm)	After PD (25/100 μm)	After PD (25/200 μm)
TMI threshold pump power	415 W	695 W	315 W	385 W
Average QD-induced Q	19.89 W/m	11.20 W/m	15.22 W/m	7.64 W/m
Total QD-induced Q	31.82 W	50.40 W	24.35 W	34.38 W
Average PD-Pump-induced Q	/	/	0.86 W/m	0.14 W/m
Total PD-Pump-induced Q	/	/	1.38 W	0.63 W
Average PD-Signal-induced Q	/	/	4.03 W/m	3.13 W/m
Total PD-Signal-induced Q	/	/	6.45 W	14.09 W
Average PD-induced Q	/	/	4.89 W/m	3.27 W/m
Total PD-induced Q	/	/	7.83 W	14.72 W
Average Q	19.89 W/m	11.20 W/m	20.11 W/m	10.91 W/m
Total Q	31.82 W	50.40 W	32.18 W	49.10 W

A trend of threshold power versus the cladding radius are presented in Fig. 4.11 with a fixed pump absorption of 95% for all fibres. It is noted that the threshold power linearly increases with the cladding radius due to the stronger pump saturation effect in the fibre before PD saturation. On the contrary, when PD exists, the pump saturation effect cannot compensate the PD-induced Q and no benefit from the saturation effect is expected after a certain size of cladding. We attribute this to the fact that when the cladding radius becomes larger, the PD-signal-induced Q becomes more significant (See Table 4.4), which is not linear to the threshold power unlike the QD-induced Q and PD-pump-induced Q . Such differences are reflected in Eq. (2.9) - (2.11). Hence, when the cladding radius becomes larger, the threshold total Q is reached quicker in a fibre after PD. Therefore, the saturation effect becomes not sufficient to suppress the TMI after the PD saturates.

Figure 4.11: Threshold power versus the cladding radius in the fibres before and after PD saturation. The same parameters as that in Fig. 4.9 except the cladding radius is changed from 25 μm to 200 μm .

4.1.5 Core NA

Transverse mode profiles are shaped by a core V parameter, which determines overlaps of mode profiles. The TMI threshold was found inversely proportional to the V parameter, thus modal overlap [122]. I further investigate this dependence through individual studies of a core size and a core NA influence. This section examines core NA influence. I set our lowest core NA at 0.03 that is practical in the state-of-the-art fibre fabrication [116, 123]. The frequency offsets with the 0.03 NA are calculated as 1070 Hz and 1050 Hz in the fibres before and after

PD saturation, respectively, which are blue shifted as compared to the core NA of 0.06 (See Fig. 4.1.). The reduced frequency offset is expected as the modes extend into the cladding with lower core NA contributing to larger effective area, which is consistent with [114, 122].

In Fig. 12 I present the output HOM power content versus the input pump power in PD fibres that before and after PD saturation with the core NA of 0.06 and 0.03. The pump absorption is kept at ~95% by adjusting a fibre length in all cases. It is clearly seen that the impact of core NA is more prominent in fibres after PD because expanded mode profiles into cladding helps reduce the PD-induced loss, thus PD-induced thermal loads. More specifically, the threshold power increases by 4.8% from 415 W to 435 W in the fibres before PD, and 9.5% from 315 W to 345 W after PD when the core NA decreases from 0.06 to 0.03. The overall trend is related to the reduced modal overlap, thus suppressing the TMI effect [67].

Figure 4.12: Output HOM power content, LP_{11} versus the input pump power in fibres before and after PD saturation, with the core NA of 0.06 and 0.03. The fibre parameters are same as that in Fig. 4.1. except that the core NA is changed.

Figure 4.13 presents the evolution of time averaged thermal load along the fibres. Fibre lengths are adjusted for 95% pump absorption. The detailed values of averaged and total thermal loads are listed in Table 4.5. It is shown that the decreased core NA (from 0.06 to 0.03) slightly increases fibre tolerance for thermal load, making TMI threshold higher albeit insignificant. In addition, the total thermal loads at TMI thresholds are nearly same for all the cases. I also note that in this case the averaged Q is also consistent, which contrasts to the cladding size dependence of TMI in the previous section.

Table 4.5: Averaged and total thermal load in PD fibres at TMI thresholds

	Before PD (NA 0.06)	Before PD (NA 0.03)	After PD (NA 0.06)	After PD (NA 0.03)
TMI threshold pump power	415 W	435 W	315 W	345 W
Averaged QD-induced Q	19.89 W/m	21.00 W/m	15.22 W/m	16.52 W/m
Total QD-induced Q	31.82 W	33.60 W	24.35 W	26.43 W
Averaged PD-Pump-induced Q	/	/	0.86 W/m	0.93 W/m
Total PD-Pump-induced Q	/	/	1.38 W	1.49 W
Averaged PD-Signal-induced Q	/	/	4.03 W/m	4.36 W/m
Total PD-Signal-induced Q	/	/	6.45 W	6.98 W
Averaged PD-induced Q	/	/	4.89 W/m	5.29 W/m
Total PD-induced Q	/	/	7.83 W	8.47 W
Averaged Q	19.89 W/m	21.00 W/m	20.11 W/m	21.80 W/m
Total Q	31.82 W	33.60 W	32.18 W	34.89 W

Figure 4.13: Time averaged transversal thermal load versus fibre length for (a) NA 0.06 and (b) NA 0.03 before PD saturation, (c) NA 0.06 and (d) NA 0.03 after PD saturation. The parameters are assumed same as in Fig. 4.12. The input pump power is at the threshold power and fibre length is adjusted to make 95% pump absorption.

Figure 4.14 shows TMI threshold variation with core NA in 0.03-0.06 at 95% pump absorption lengths. The threshold power linearly increases with the reduced core NA both in after and before PD saturation, and with a bit higher rate in the fibre after PD saturation. It is noted that bending is a practical concern in the lower core NA fibre. Nonetheless, a 50 cm

bending diameter was reported for efficient lasing in a LMA fibre with 0.03 core NA and 50 cm of core diameter [123].

Figure 4.14: Threshold power versus the core NA in fibres before and after PD saturation. The same parameters as that in Fig. 4.12 except the core NA is changed from 0.03 to 0.06.

4.1.6 Core radius

In this section, I investigate the influence of core radius as a part of a V number on TMI effect. We start with frequency offsets calculation for the most power transfer. The frequency offsets are found at about 370 Hz in both before and after PD saturation when the core radius increases to 50 μm , which is blue shifted as compared to 25 μm core size (See Fig. 4.1). This is attributed to longer thermal diffusion time across the larger fibre core, hence reduced the frequency offset. Interestingly, the larger core size makes the frequency offset difference before and after PD saturation smaller because the mode profile becomes flatter and produces similar inversion profile. This observation agrees with the results in [124].

In Fig. 4.15 I present the output HOM power content versus the input pump power with the core radius of 25 μm and 50 μm , at about 95% pump absorption lengths. Increasing the core size is expected to reduce the TMI thresholds due to weakened pump saturation effect. My simulation confirms this prediction. The threshold power decreased by 4.8% from 415 W to 313 W before PD saturation, and 9.5% from 315 W to 253 W after PD saturates when the core size increased from 25 μm to 50 μm . Therefore, my simulation confirms the relationship of larger V parameter to lower the TMI threshold. However, the underlying mechanisms are different in that the core NA affects modal overlap while the core size affects the pump saturation effect.

Figure 4.15: Output HOM power content, LP_{11} versus the input pump power in the fibres before and after PD, with the core radius of 25 μm and 50 μm . The fibre parameters are the same as that in Fig. 4.1 except the core radius is changed.

Figure 4.16 presents the evolution of time averaged thermal load along fibres at the threshold power. Table 4.6 presents the detailed values of the thermal loads in the fibres. The 25 μm core fibres tolerate larger total thermal load, thus leading to higher TMI thresholds as compared to the 50 μm core counterparts. It should be noted that both the total Q and averaged Q at the threshold vary when core sizes are different. Hence, the results indicate it not a universal constant value emerging for the TMI threshold at different core size.

Table 4.6: Averaged and total thermal load in PD fibres at TMI thresholds

	Before PD (25/100 μm)	Before PD (50/100 μm)	After PD (25/100 μm)	After PD (50/100 μm)
TMI threshold pump power	415 W	315 W	315 W	255 W
Average QD-induced Q	19.89 W/m	28.87 W/m	15.22 W/m	21.40 W/m
Total QD-induced Q	31.82 W	27.43 W	24.35 W	20.33 W
Average PD-Pump-induced Q	/	/	0.86 W/m	3.75 W/m
Total PD-Pump-induced Q	/	/	1.38 W	3.56 W
Average PD-Signal-induced Q	/	/	4.03 W/m	4.88 W/m
Total PD-Signal-induced Q	/	/	6.45 W	4.64 W
Average PD-induced Q	/	/	4.89 W/m	8.63 W/m
Total PD-induced Q	/	/	7.83 W	8.20 W
Average Q	19.89 W/m	28.87 W/m	20.11 W/m	30.03 W/m
Total Q	31.82 W	27.43 W	32.18 W	28.53 W

Figure 4.16: Time averaged transversal thermal load versus fibre length for (a) 25 μm and (b) 50 μm core radius before PD saturation, (c) 25 μm and (d) 50 μm core radius after PD saturation. The parameters are assumed same as in Fig. 4.15. The input pump power is at the threshold power and fibre length is adjusted to make 95% pump absorption.

Figure 4.17: Threshold power versus the core radius in fibres before and after PD saturation. The same parameters as that in Fig. 4.15 are assumed except the core radius is changed from 25 μm to 50 μm .

Figure 17 presents the TMI threshold power versus the core radius at 95% pump absorption lengths. Clearly, the TMI threshold power increases with the decreased core radius for both before and after PD saturation because of the stronger pump saturation effect in larger core size.

Interestingly, the TMI threshold gap between before and after PD becomes reduced with increasing core size. This accounts for the reduced contribution of PD-signal-induced Q in the 50 μm core radius fibre. The PD-signal-induced Q takes 56.59% of total PD-induced Q , which is much smaller than 82.38% in the 25 μm core radius fibre. In contrast, the QD-induced Q and PD-pump-induced Q are nearly linear with the pump power. Therefore, I conclude that the reduced gap of the TMI threshold with increasing core size is attributed to the PD-signal-induced Q in the considered fibre parameters.

4.1.7 Ion concentration

Figure 4.18: Output HOM power content, LP_{11} versus the input pump power in fibres before and after PD saturation, with the ion concentration of $1.5 \times 10^{25} \text{ m}^{-3}$ and $3.5 \times 10^{25} \text{ m}^{-3}$. The fibre parameters are the same as that in Fig. 4.1 except the ion concentration is changed.

In the following, I numerically compare the influence of ion concentration on TMI before and after PD saturates. Apparently, higher concentration results in more PD, thus more PD-induced thermal load. It was already pointed out that reducing ion concentration is effective to suppress the TMI [73]. On the other hand, it was also reported that TMI thresholds are independent of ion concentration [124] when the PD is absent. Hence, it becomes interesting to understand the thermal load contribution from the QD and PD, accounting for the TMI. This section will examine the individual thermal load generated in different ion concentrations. As before we start with identifying the frequency offset for concentrations of $1.5 \times 10^{25} \text{ m}^{-3}$ (lower concentration) and $3.5 \times 10^{25} \text{ m}^{-3}$ (higher concentration, the concentration used in the previous sections). It is calculated that before PD the frequency offset is kept at 1160 Hz for both concentrations. However, after PD saturation, the frequency offset increases from 1130 Hz to

1140 Hz with the lower concentration. This is attributed to lower absorption at the core edge due to lower PD effect in a lower concentration. Figure 4.18 presents the output HOM power content versus the input pump power at the two concentrations. Clearly, the TMI threshold remains same for the fibres before PD, and the TMI suppression is clearly visible in the fibres after PD when the concentration is reduced.

Figure 4.19 presents the evolution of time averaged thermal load along the fibres at the threshold power. All the fibre lengths are adjusted for 95% pump absorption. The fibres with lower concentration are, thus, longer as compared to the higher concentration fibres. The detailed values of averaged and total thermal loads are provided in Table 4.7. It is found that the total thermal load (around 32 W) at the TMI threshold remains constant despite the concentration difference and the presence of PD. It is noted that the fibres under comparison consist of the same fibre parameters such as core, cladding sizes and V parameter. Hence, we can conclude that the total Q can serve as a TMI threshold as long as the parameters remain same. Unfortunately, the averaged Q varies with different concentrations even the parameters remain same. It is also shown that the total PD-induced Q decreases from 7.83 W to 3.69 W with the reduced concentration in fibres after PD saturation, confirming the influence of ion concentration effect on the TMI threshold.

Table 4.7: Averaged and total thermal load in PD fibres at TMI thresholds

	Before PD ($3.5 \times 10^{25} \text{m}^{-3}$)	Before PD ($1.5 \times 10^{25} \text{m}^{-3}$)	After PD ($3.5 \times 10^{25} \text{m}^{-3}$)	After PD ($1.5 \times 10^{25} \text{m}^{-3}$)
TMI threshold pump power	415 W	415 W	315 W	385 W
Average QD-induced Q	19.89 W/m	8.38 W/m	15.22 W/m	7.41 W/m
Total QD-induced Q	31.82 W	32.26 W	24.35 W	28.53 W
Average PD-Pump-induced Q	/	/	0.86 W/m	0.18 W/m
Total PD-Pump-induced Q	/	/	1.38 W	0.69 W
Average PD-Signal-induced Q	/	/	4.03 W/m	0.78 W/m
Total PD-Signal-induced Q	/	/	6.45 W	3.00 W
Average PD-induced Q	/	/	4.89 W/m	0.96 W/m
Total PD-induced Q	/	/	7.83 W	3.69 W
Average Q	19.89 W/m	8.38 W/m	20.11 W/m	8.36 W/m
Total Q	31.82 W	32.26 W	32.18 W	32.19 W

Figure 4.19: Time averaged transversal thermal load versus fibre length for (a) $1.5 \times 10^{25} \text{ m}^{-3}$ and (b) $3.5 \times 10^{25} \text{ m}^{-3}$ ion concentration before PD saturation, (c) $1.5 \times 10^{25} \text{ m}^{-3}$ and (d) $.5 \times 10^{25} \text{ m}^{-3}$ ion concentration after PD saturation. The parameters are assumed same as in Fig. 4.18. The input pump power is at the threshold power and fibre length is adjusted to make 95% pump absorption.

Figure 4.20: Threshold power versus the ion concentration in fibres before and after PD saturation. The same parameters as that in Fig. 4.18 are assumed except the ion concentration is changed from $0.5 \times 10^{25} \text{ m}^{-3}$ to $3.5 \times 10^{25} \text{ m}^{-3}$.

In Fig. 4.20 we show the threshold power versus the ion concentration. As discussed, the threshold power remains nearly same at 415 W at the investigated concentrations in the fibres

before PD. This is in consistent with the experimental result presented in [124]. However, the threshold power continuously decreases from 405 W to 315 W, with increasing ion concentration from $0.5 \times 10^{25} \text{ m}^{-3}$ to $3.5 \times 10^{25} \text{ m}^{-3}$ as deduced from Eq. (2.5) in that PD-induced loss from absorption of signal and pump is related to the square of ion concentration. This phenomenon can also be explained by the semi-analytical model proposed by B. Ward in [74], that ion concentration influences the power coupling between modes. Furthermore, it is interesting to note that when the ion concentration is at the very low level of $0.5 \times 10^{25} \text{ m}^{-3}$, the threshold powers in the PD fibre that after and before the PD saturation becomes closer. This again verifies that the PD-induced Q heavily relies on the ion concentration. It is a desirable choice to select low ion concentration to increase the threshold power in the PD fibre although the low concentration requires longer length and promotes undesired nonlinearities.

Figure 4.21: Time averaged transversal thermal load versus fibre length for (a) 25 μm/100 μm, NA-0.06 and (b) 50 μm/200 μm, NA-0.03 fibres before PD saturation, (c) 25 μm/100 μm, NA-0.06 and (d) 50 μm/200 μm, NA-0.03 after PD saturation. The input pump power is at the threshold power and fibre length is adjusted to make 95% pump absorption.

In previous sections, I have suggested that the total Q as a more intuitive quantity to be related to TMI behaviours. Larger total Q means the fibre can tolerate higher thermal load, thus indicating higher TMI threshold. The total Q at TMI threshold is consistent at various ion concentrations while the averaged Q varies with different concentrations as reflected in table 4.7. In Section 4.1.5 and 4.1.6, I also learnt that the total Q (as well as the averaged Q) is

dependent of fibre parameters that determine the saturation effect and the modal overlap. To confirm such influence and gain further insight, I calculate the thermal load evolution in two fibres. One fibre has 50 μm core radius, 200 μm cladding radius and core NA of 0.03, and the other fibre has 25 μm core radius, 100 μm cladding radius and core NA of 0.06. The two fibres have the same CCAR and the same V parameter, to keep the saturation effect and the modal overlap the same. Figure 4.21 presents the thermal load evolution along a fibre at the threshold power. The fibre lengths are adjusted for 95% pump absorption. The detailed values of averaged and total thermal loads in the fibres are listed in Table 4.8. It confirms that the total Q at the TMI threshold is indeed nearly same (around 32 W) when the CCAR and the V parameter are kept constant. Therefore, our observation leads us to the conclusion that the total Q can be used as an indicator for the TMI threshold if the CCAR and the V-parameter remains unchanged even at different ion concentrations whereas the average Q does not bring out a constant value when ion concentration varies as observed in Table 4.7.

Table 4.8: Averaged and total thermal load in PD fibres at TMI thresholds

	Before PD (25 / 100 μm NA-0.06)	Before PD (50 / 200 μm NA-0.03)	After PD (25 / 100 μm NA-0.06)	After PD (50 / 200 μm NA-0.03)
TMI threshold pump power	415 W	415 W	315 W	315 W
Averaged QD-induced Q	19.89 W/m	18.83 W/m	15.22 W/m	14.41 W/m
Total QD-induced Q	31.82 W	32.01 W	24.35 W	24.50 W
Averaged PD-Pump-induced Q	/	/	0.86 W/m	0.77 W/m
Total PD-Pump-induced Q	/	/	1.38 W	1.31 W
Averaged PD-Signal-induced Q	/	/	4.03 W/m	3.64 W/m
Total PD-Signal-induced Q	/	/	6.45 W	6.19 W
Averaged PD-induced Q	/	/	4.89 W/m	4.41 W/m
Total PD-induced Q	/	/	7.83 W	7.50 W
Averaged Q	19.89 W/m	18.83 W/m	20.11 W/m	18.82 W/m
Total Q	31.82 W	32.01 W	32.18 W	32.00 W

4.2 Transverse mode instability suppression in the presence of PD

Based on the parametric studies in previous sections, this section examines improved fibre design for TMI suppression when PD is unavoidable. We adopt commercial LMA fibre parameters for a PD fibre [125] and the parameters from [73] for a PD resistant fibre, for comparison with our improved TMI-suppressed PD fibre. The TMI-suppressed PD fibre design uses the following strategies: I first reduce the ion concentration to suppress the PD-induced loss and thermal load, which results in a longer fibre to keep the same pump absorption. As the longer fibre length promotes nonlinearities, I increase the core radius to compensate it. The increased core radius then needs lower core NA to keep the same V parameter as the

commercial fibre for acceptable beam quality. Finally, a CCAR is kept same as the commercial fibre to provide the same pump saturation effect, but not less, and keep pump absorption length for high nonlinearity threshold. The parameters of these fibres are listed in Table 4.9.

Table 4.9: Averaged and total thermal load in PD fibres at TMI thresholds

	LMA PD fibre [125] (Type 1)	PD-resist fibre [73] (Type 2)	PD-resist fibre [73] (Type 3)	TMI suppressed PD fibre (Type 4)
Core radius	10 μm	10 μm	10 μm	20 μm
Cladding radius	200 μm	200 μm	90 μm	400 μm
Core NA	0.06	0.06	0.06	0.03
Ion concentration	$6 \times 10^{25} \text{ m}^{-3}$	$1.5 \times 10^{25} \text{ m}^{-3}$	$1.5 \times 10^{25} \text{ m}^{-3}$	$1.5 \times 10^{25} \text{ m}^{-3}$
Fibre length	9 m	36 m	9 m	36 m
TMI threshold (Before PD)	2080 W	2060 W	900 W	2160 W
TMI threshold (After PD)	920 W	1640 W	780 W	1760 W
Nonlinear threshold	49.3 X W	12.3 X W	42.1 X W	50 X W
Pump absorption	95%	95%	95%	95%
Averaged Q at TMI threshold (After PD)	17.9 W/m	4.4 W/m	7.7 W/m	4.3 W/m
Total Q at TMI threshold (After PD)	161.1 W	158.4 W	69.3 W	154.8 W

Figure 4.22: The signal effective mode area versus the fibre bending curvature for type 1 to type 4 fibres. The fibre parameters are the same as that in table 4.9.

When considering LMA with low NA, a bending effect can be a concern. I investigate the bending effect of the considered fibres in Figure 4.22. It is found that the TMI suppressed PD fibre (Type 4) could be bent to a 25 cm bending radius with only 9.9 % of mode area reduction and the guidance is greatly compromised when the bending radius becomes smaller. Indeed, bending of similar mode area fibre in the 25 cm bending radius was experimentally verified in

[123]. With the assumption of the 25 cm bending radius and its corresponding mode area, a nonlinear threshold power for SBS is presented as follows [25, 63]:

$$P_{out}^{SBS} \approx \frac{17 \cdot A_{eff} \cdot X}{g_B(\Delta\nu) \cdot L} \cdot \ln(G) \quad (4.1)$$

where A_{eff} is the effective mode area, $g_B(\Delta\nu)=5 \times 10^{-11}$ m/W is the Brillouin gain coefficient for silica, X is the SBS gain reduction factor, L is the fibre length and G represents the fibre amplifier power gain factor. From Eq. (4.1), it is shown that the nonlinear threshold power is influenced by the signal effective mode area and fibre length. The SBS gain reduction factor X can be increased by phase modulation [93], by using a temperature [126] or strain gradient [127]. The SBS gain reduction factor (M) of 25 is achieved by introducing a strain gradient of 1% in [127]. Higher reduction factor can be achieved by introducing higher strain or other SBS suppression methods [128], The nonlinear threshold power calculation is shown in Table 4.9.

The TMI threshold, nonlinear threshold and total thermal loads at the TMI thresholds of the considered fibres are provided in Table 4.9. According to our calculation, the commercial LMA fibre (type 1) has a TMI threshold power at 2080 W when the PD is absent, which is close to the experimental result of 1900 W in [125]. Under the presence of PD, TMI threshold is at 920 W. This considerably high TMI threshold is attributed to the large CCAR of fibre, which makes the pump saturation effect strong. In order to enhance the TMI threshold power at the presence of PD, a lower ion concentration in a longer fibre length (Type 2) for a fixed CCAR or a lower ion concentration in a smaller cladding (pumping) radius (Type 3) for a fixed fibre length are considered [73]. The TMI threshold power increases to 1640 W after PD in Type 2 fibre as compared to Type 1 after PD saturation. However, fibre length should be longer to compensate the low concentration for a desired pump absorption. The long fibre reduces the SBS onset at $12.3X$ W. Type 3 fibre uses a smaller cladding size to keep the fibre length short for the unchanged pump absorption, which, as a result, weakens the saturation effect, thus reducing the TMI threshold power to 780 W after PD and 900 W before PD. Consequently, it is essential to keep the CCAR same as the commercial fibre and address the length related nonlinearity issue by increasing the core size. Then, the effect of the increased core size has to be counterbalanced by a low core NA. This approach is applied to our TMI suppressed PD fibre (Type 4). Its TMI threshold power is 2160 W before PD and 1760 W after PD with the same nonlinearity threshold as the Type 1 fibre. Hence, I suggest such fibre design strategy for high TMI and nonlinearity threshold within the practically tolerable bending effect. We also note that the TMI threshold in Type 4 is a bit larger than the Type 1 when the PD is ignored. This is mainly caused by the stronger thermal lensing effect in a large core fibre [88].

4.3 Conclusion

In this chapter, I numerically investigate the TMI behaviour, threshold power, averaged and total thermal load, under the influence of fibre length, cladding (pumping) radius, core NA, core radius and ion concentration in the PD fibres. In a PD fibre, the main contribution to the TMI is found to be the QD-induced thermal load. It is also found that the PD-induced thermal load from signal absorption has more influence on TMI than the PD induced thermal load from pump absorption. The observation of parametric study can be summarized as follows:

(a) A larger cladding size is always better for the TMI suppression if PD is absent. However, there is no benefit of increasing the cladding size above a certain value when the PD exists in fibre.

(b) A lower core NA is always better for the PD fibre that both before and after the PD is saturated, while a fibre with PD is more sensitive to the core NA variation. In practice, core NA of 0.03 seems to be the limitation in a step-index fibre considering fabrication tolerance and mode confinement.

(c) A smaller core size is beneficial in increasing the TMI threshold due to the stronger pump saturation effect in both before and after PD saturation. Together with the above second point, a larger V parameter generally reduces the TMI threshold. However, its underlying mechanism is different. Core NA influences modal overlap while core size is related to a saturation effect.

(d) Ion concentration influences the PD-induced thermal load but not the QD-induced thermal load. Hence, lower ion concentration is beneficial to suppress the TMI in a PD fibre that after the PD is saturated.

(e) The total thermal load (including QD and PD) at the TMI threshold remains consistent when PD fibres hold the same CCAR (accounting for the saturation effect) and V parameter (for the modal overlap). However, the averaged thermal load varies with the ion concentration even in the PD fibres with same CCAR and V parameter, which is misleading in predicting the TMI behaviour at some conditions (for instance, at higher ion concentration shown in table 4.7).

Therefore, total thermal load is more intuitive to interpret the TMI behaviour compared to the averaged thermal load. Higher total thermal load for the onset of TMI means a fibre can tolerate more heat, thus possessing higher TMI threshold power.

Lastly, I propose the optimized fibre parameters for suppressing the TMI as well as not promoting the nonlinearity in PD fibres. The simulation suggests that 1.7 kW output power is

achievable in the optimized PD fibre, and simultaneously it does not worsen the nonlinear threshold power, within practical design parameters.

Chapter 5 An annular core fibre for transverse mode instability suppression

In the non-PD Yb-doped fibres, the QD is recognized as the main heat source to modify the refractive index. In the PD fibres, apart from the QD, PD-induced heat from absorption of signal and pump is recognized as another heat source contributing to the TMI. The models including numerical ones [59, 77, 79, 96] and semi-analytical ones [73, 76, 78] were proposed to simulate the TMI behaviour in fibre, and mitigation strategies to suppress the TMI were studied theoretically. For instance, in the non-PD fibres, tandem pumping is an efficient way to suppress the TMI due to the reduced thermal load generated by QD [68]. In the PD fibres, diluting the ion concentration, which reduces the PD-induced loss from absorption of signal and pump, is another efficient method to increase the TMI threshold power as presented in [73]. Therefore, the previous approaches suggest reducing the thermal load generated from QD or PD is an efficient strategy to mitigate the TMI although they require longer fibre length.

A ring-core fibre design was proposed as a transmission platform for mode-division multiplexing [129, 130] in the optical fibre communication systems to satisfy the ever-increasing demand in transmission capacity. The ring-core structure can make the same effective mode area with much smaller core area compared to a circular core counterpart. Hence, the ring-core is expected to generate much less thermal load in a fibre, suggesting reduced TMI. The ring core structure can be realized in a step-index ring-core design or an annularly arranged multicore design. Compared to the multicore fibre, the step-index ring core requires less rigorous fabrication efforts, but its reachable mode area would be smaller.

In this chapter, I use the highly numerical 4D split-step FFT-based BPM to theoretically investigate TMI behavior and its threshold power in the step-index ring-core and multicore fibres. The numerical model intrinsically incorporates pump saturation, thermal lensing and mode coupling effects, with an arbitrary choice of core shapes, refractive index and doping profiles as presented in Chapter 2. The comparison of TMI threshold power, thermal load evolution along fibre length and averaged heat load are presented among the ring-core, multicore and circular core fibres. It is found that the thermal load as well as the averaged heat load are much smaller in both ring core designs. This is due to the reduced heat generated in the ring-core designs. Therefore, the TMI threshold power in ring-core and multicore fibres becomes much larger than that in circular core fibres. The ring cores support a ring mode which is converted to a Gaussian-like beam profile in a far-field with a conversion efficiency of >85%

for the step-index ring core and 70% for the multicore fibre. Despite the power loss in the conversion, the increased TMI threshold makes the fibres much more power scalable.

In this chapter, I first start with the investigation of reachable mode area for both ring core and multicore fibres within practical parameters, and use the design for TMI study. Additionally, we present the influence of bending on ring-core and multicore fibres.

5.1 Transverse mode instability in a step-index ring-core fibre

In this section, I first determine the largest signal effective mode area that a ring-core fibre achieves by considering the effective index difference between modes and the overlap between FM and a doping area. Then the frequency offset for most efficient power transfer between modes, thermal load evolution along fibre length, averaged heat load and TMI threshold power are calculated. Lastly, I investigate achievable output power in the ring core design after considering the conversion of donut-like intensity profile (near-field) to the Gaussian intensity profile (far-field). In the following, I present the main parameters used in simulations.

The contributions to the HOM input power, P_{11} , include the quantum noise, spontaneous thermal Rayleigh scattering, amplitude modulation of the injected signal and pump [59, 61]. The HOM input power is assumed at the quantum noise level of 10^{-16} W in [58, 59, 77], which represents an ideal case and leads to the highest TMI threshold power. In practice, the HOM input power may be higher than the quantum noise level because of the aforementioned contributions, which justifies the P_{11} input power above 10^{-10} W although it is difficult to obtain the accurate value [61]. In addition, the values of HOM input power are set at 0.1 W in [119], 0.5 W in [76] and 1.5 W in [78], which are several orders higher than the other studies. In this paper, we set the HOM input power P_{11} at 1 μ W and FM input power at 10 W. We define the TMI threshold power when the HOM takes 5% of total output signal power. Important simulation parameters are listed in Table 5.1.

Table 5.1: Simulation parameters

Symbol	Quantity	Value
λ_s	signal wavelength	1064 nm
λ_p	pump wavelength	976 nm
σ_s^a	signal absorption cross section	$6.00 \times 10^{-27} \text{ m}^2$
σ_s^e	signal emission cross section	$3.58 \times 10^{-25} \text{ m}^2$
σ_p^a	pump absorption cross section	$1.53 \times 10^{-24} \text{ m}^2$
σ_p^e	pump emission cross section	$1.87 \times 10^{-24} \text{ m}^2$
ρ	mass density	2201 kg/m ³
C	heat capacity	702 J/(kg×K)
K	thermal conductivity	1.38 W/(m×K)
dn/dT	thermo-optic coefficient	$1.2 \times 10^{-5} \text{ K}^{-1}$
P_{0I}	input FM power	10 W
P_{1I}	input HOM power	1 μ W
P_p	input pump power	varies
τ	excited ion lifetime	850 μ s
Δx	x grid spacing	1.56 μ m
Δy	y grid spacing	1.56 μ m
Δz	step size along fibre	10 μ m
N_t	number of time grids	64 grids for a time cycle ($1/\Delta\omega$)

5.1.1 Determination of step-index ring-core parameters

Figure 5.1 presents the structure of a step-index ring core fibre and the intensity profiles of FM and HOM in it. The intensity profiles, which represent the eigenmodes of this ring core structure, are obtained from the COMSOL software. The radius of outer edge of the ring core is denoted as a and the inner diameter as d . The effective index difference between the modes, which influences the power coupling and mode degeneracy, is determined by the value of $d/2a$ [129]. Additionally, the $d/2a$ value also influences the overlap between the FM and a doping area that impacts lasing or amplification efficiency. Therefore, the largest signal effective mode area is based on the acceptable effective index difference between modes and the overlap between FM and doping area.

Figure 5.1: Structure of the ring-core fibre and intensity profile for (a) FM and (b) HOM. The a is the radius of outside circle and the d is the diameter of inside circle.

Figure 5.2 shows the values of $d/2a$ versus effective index difference between modes, overlap between FM and doping area, and signal effective mode area at $a=14\text{ }\mu\text{m}$, $15\text{ }\mu\text{m}$ and $16\text{ }\mu\text{m}$, respectively. It is seen from Figure 5.2 (a) that the effective index difference between modes becomes smaller with the increasing $d/2a$ and the decreasing values of a . To overcome the mode degeneracy and minimize the power coupling between modes, the value of effective index difference between modes should be larger than 1×10^{-4} [131]. Therefore, the value of $d/2a$ ranges from 0.1 to 0.4 satisfies that the index difference requirement as shown in Figure 5.2 (a). Figure 5.2 (b) presents that the overlap between FM and doping area gradually decreases when the $d/2a$ changes from 0.1 to 0.4. The decreasing rate gets larger after the $d/2a$ of 0.4. Hence, the range of $d/2a$ from 0.1 to 0.4 keeps the overlap ratio above 0.85. In Figure 5.2 (c), the signal effective mode area is apparently proportional to the values of a or $d/2a$. Within the constraints of the modal index difference and the overlap, we can conclude that the FM effective mode area can scale to $\sim 784\text{ }\mu\text{m}^2$ in the step-index ring-core fibre with $d/2a$ of 0.4 and a of $16\text{ }\mu\text{m}$. Table 5.2 compares the parameters of ring-core fibre with the achievable effective mode area of $784\text{ }\mu\text{m}^2$, and a circular core fibre with nearly the same effective mode area ($789\text{ }\mu\text{m}^2$). Both fibres have 0.06 core NA. The doping profile is uniform core doping and here I assume that it has no influence on the core refractive index. Additionally, the core area A shown in Table 5.2 is calculated using the $\pi(a^2 - d^2/4)$. As presented, the step-index ring core requires only $\sim 50\%$ of the physical core area as compared to the circular core counterpart.

Figure 5.2: (a) Effective index difference between modes, (b) Overlap between FM and doping area and (c) Signal effective mode area versus the $d/2a$ in a ring core fibre with $a=14\mu\text{m}$, $15\mu\text{m}$ and $16\mu\text{m}$. (RCF: ring core fibre).

Table 5.2: Ring core and circular core fibre parameters

	Ring core fibre	Circular core fibre
outside circle radius a	$16\ \mu\text{m}$	-
inside circle diameter d	$12.8\ \mu\text{m}$	-
core radius R	-	$20\ \mu\text{m}$
core area A	$675.2\ \mu\text{m}^2$	$1256\ \mu\text{m}^2$
signal effective mode area A_{eff}	$784\ \mu\text{m}^2$	$789\ \mu\text{m}^2$
cladding index n_{clad}	1.45	1.45
core numerical aperture NA	0.06	0.06
cladding radius R_{clad}	$200\ \mu\text{m}$	$200\ \mu\text{m}$
Yb ion concentration N_{Yb}	$3.5 \times 10^{25}\ \text{m}^{-3}$	$3.5 \times 10^{25}\ \text{m}^{-3}$
doping area	core doping	core doping

5.1.2 Frequency offset for transverse mode instability

The frequency offset between FM and HOM is essential for the phase shift between moving signal irradiance pattern and temperature grating, which provokes the power transfer from FM to HOM. In the following, I numerically calculate the frequency offset, at which the strongest power transfer from FM to HOM happens in the ring-core (with $a=16\ \mu\text{m}$ and $d=12.8\ \mu\text{m}$) and circular core (with $R=20\ \mu\text{m}$) fibres. The offset will be assumed for the TMI simulation in the following sections.

Figure 5.3 shows a scaled fraction of output LP₁₁ power in the total output signal power at different frequency offsets for the ring-core and circular core fibres. It is seen that the frequency offset for the maximum power transfer from FM to HOM lies at about 1800 Hz in the circular core (with $R=20\text{ }\mu\text{m}$) fibre, which is much smaller than 3000 Hz in the ring-core (with $a=16\text{ }\mu\text{m}$ and $d=12.8\text{ }\mu\text{m}$) fibre. This is attributed to the core size difference. In the step-index ring core, the distance between the centre of core and the core edge, a , is shorter than that in circular core fibre, R . The shorter distance decreases the thermal diffusion time, thus increasing the frequency offset [59, 120].

Figure 5.3: Normalized output LP₁₁ power content versus the frequency offset in the ring-core (with $a=16\text{ }\mu\text{m}$ and $d=12.8\text{ }\mu\text{m}$) and circular core (with $R=20\text{ }\mu\text{m}$) fibres. (RCF: ring core fibre; CCF: circular core fibre).

5.1.3 Transverse mode instability in ring-core fibre

In this section, I numerically investigate the modes power evolution, thermal load distribution along fibre length in the ring-core fibre and compare it with the circular core fibre. Furthermore, the TMI threshold pump power and maximum signal output power are calculated in both fibres. First of all, we verify the expected reduced thermal load in the ring core. The select fibre parameters are listed in Table 5.2 and the input pump power is assumed 940 W. Both the fibres provide $\sim 95\%$ pump absorption with 6.5 m for the circular core fibre and 15 m for the ring-core fibre. In Fig. 5.4(a) and (b), I show the evolution of thermal grating profile (at $t=0$) along the fibres in a x - z plane. Accumulated thermal loads across fibre are also presented in Fig. 5.4(c) and (d). It is found from Fig. 5.5 (a) and (b) that the peak thermal load is nearly same at about $2.2 \times 10^{10}\text{ W/m}^3$ in both the circular core and ring-core fibres. However, the smaller core area

in the ring-core fibre makes thermal load in a fibre core reduced. This is clearly observed in Figure 5.5 (c) and (d) where the thermal load in the circular core is nearly twice as the thermal load in the ring-core fibre. Hence, the reduced thermal load in the ring-core fibre is expected to suppress the TMI.

Figure 5.4: Evolution of the thermal grating profile (at $t=0$) along a fibre in a x-z plane for the (a) circular core and (b) ring-core fibres, and the accumulated thermal load across fibre for the (c) circular core and (d) ring-core fibres. Input pump power is assumed at 940 W. The fibre length is adjusted for $\sim 95\%$ pump absorption.

In Fig. 5.5, I present the TMI behaviours of the time averaged LP_{01} mode power, LP_{11} mode power and pump power evolution along a fibre using the same conditions as in Fig. 5.4. The output powers are also summarized in Table 5.3. It is clearly shown that the output LP_{11} power in the ring-core fibre is only 0.0048 W, much smaller than 39.6952 W in the circular core fibre. It implies that the TMI is much weaker in the ring-core fibre due to the reduced thermal load in the ring core. Consequently, the FM output power is higher in the ring core fibre. I note that both the fibres experience the same amount of saturation effect because of the same mode area. Hence, I attribute the suppressed TMI to the reduced thermal load in the ring core.

Figure 5.5: Time averaged powers versus fibre length for (a) circular core (with $R=20\text{ }\mu\text{m}$) and (b) ring-core (with $a=16\text{ }\mu\text{m}$ and $d=12.8\text{ }\mu\text{m}$) fibres, at the input pump power at 940 W.

Table 5.3: Output powers for the ring-core and circular core fibres

	Ring core fibre	Circular core fibre
output LP_{01} power	790.5022 W	762.2094 W
output LP_{11} power	0.0048 W	39.6952 W
output pump power	51.8592 W	42.7159 W
pump absorption	94.48%	95.46%

Figure 5.6: Output LP_{11} power content versus the input pump power in the ring-core and circular core fibres. The fibre parameters are same as that in Fig. 5.4.

I increase the pump power to investigate TMI threshold in both fibres. Figure 5.6 illustrates the output LP_{11} power content versus the input pump power in the fibres with parameters listed in Table 5.2. The Fibre length is determined for 95% pump absorption. It is apparent that the threshold power significant increases in the ring core fibre. The TMI threshold at which the LP_{11} mode carries 5% of total output power becomes 115% larger (2060 W) in the ring-core fibre from 960 W in the circular core. This verifies that the reduction of thermal load in the

ring-core structure is a very efficient way to suppress the TMI and increase the TMI threshold power compared to the circular core fibre.

The ring core supports a donut-shaped mode, which is converted to a far-field Gaussian-like beam after a certain propagation distance. As the doping profile in ring core fibre is a uniform core doping, so the ring mode generated in the core is amplified along the fibre. Additionally, with the calculated near-field mode profile as represented in Fig. 5.1(a), the corresponding far field intensity distribution after a propagation distance can be obtained by numerically solving the Fresnel diffraction integral using a standard fast Fourier transform (FFT) method [132]. The calculated power in the central peak is 88% of the total output power. In most cases, only the central Gaussian-like beam is useful. Then, the outer concentric circle is considered loss. Figure 5.7 presents the total output power versus input pump power in the circular core and ring-core fibres before the TMI threshold. Consequently, the output power in the near-field at the TMI threshold is 1640 W in the ring-core fibre (square-solid line), which becomes 1440 W in the far-field (triangle-solid line). Despite this loss, the achievable power is much higher than the maximum output power from the circular core fibre before the TMI threshold (circle-solid line). Therefore, the step-index ring-core structure is found useful for average power scaling by TMI suppression.

Figure 5.7: Total output power versus the input pump power in circular core ($R=20\mu\text{m}$) and ring core ($a=16\mu\text{m}$ and $d=12.8\mu\text{m}$) fibres, for the near field (donut-like) and far field (Gaussian-like) intensity profile output. The fibre parameters are same as that in Fig. 5.4.

5.1.4 Bending performance of ring-core fibre

In this section, I compare the bending performance between ring-core (with $a=16\text{ }\mu\text{m}$ and $d=12.8\text{ }\mu\text{m}$) and circular core (with $R=20\text{ }\mu\text{m}$) fibres. Fig. 5.8 presents the signal effective mode area versus bending curvature in the ring-core and circular core fibres. It is seen that the signal effective mode area decreases with the increasing bending curvature in both ring-core and circular core fibres. However, the ring-core fibre is more sensitive to bending compared to circular core fibre. For instance, the effective mode area in ring-core structure decreased from $784\text{ }\mu\text{m}^2$ in straight fibre to $380\text{ }\mu\text{m}^2$ in bent fibre at the bending curvature of 10 m^{-1} , which is much smaller than $585\text{ }\mu\text{m}^2$ in circular core fibre. Therefore, tightly bending is not good for the high-power laser or amplifier operation in the ring-core fibre.

Figure 5.8: Signal effective mode area versus the bending curvature for circular core ($R=20\mu\text{m}$) and ring core ($a=16\mu\text{m}$ and $d=12.8\mu\text{m}$) fibres.

5.2 Transverse mode instability in a step-index multicore fibre

Based on the analysis of TMI comparison between circular core and ring-core fibres, I find that the TMI effect is highly suppressed in the ring-core fibre. This is because less heat is generated in ring-core fibre compared to circular core fibre with same signal effective mode area. However, the largest achievable signal effective mode area in ring-core fibre is about $784\text{ }\mu\text{m}^2$ as presented in previous section. The ring core structure can be realized by arranging multiple cores in an annular configuration. The multiple cores can form a supermodel whose mode area can be larger [133]. In this section, I theoretically study the TMI effect in a multicore fibre.

The simulation suggests that a signal effective mode area can reach $\sim 1800 \mu\text{m}^2$, which makes it applicable for pulsed high-power fibre amplifiers.

5.2.1 Determination of multicore parameters

Figure 5.9 presents the structure of a step-index multicore fibre and the intensity profiles of FM and HOM in it, D is the distance between core to core centre and r is the radius of small core. The intensity profiles, which represent the eigenmodes of this multi core structure, as well as the effective index for each mode are obtained from the COMSOL software. The effective index difference between modes and the overlap between FM and doping area are determined by the value of $D/2r$. In the following, we firstly calculate the largest achievable signal effective mode area in circular core fibre based on the acceptable effective index difference between modes and the overlap between FM and doping area.

Figure 5.9: Structure of the multicore fibre and intensity profile for (a) FM and (b) HOM. r is the radius of small core and D is the distance between core to core centre.

For the sake of discussion, I first set a target mode area for the multicore fibre. I repeat the procedure in Fig. 5.2 to set the target mode area first in the circular step-index core. Figure 5.10 presents the signal effective index difference between LP_{01} and LP_{11} in the circular step-index fibre, overlap between FM and doping area, and signal effective mode area versus the circular core radius. It is seen from figure 5.10 (a) that the effective index difference between modes gets smaller when the circular core radius increases. As I want to keep the modal index difference larger than 1×10^{-4} , I set the largest achievable core radius as $26 \mu\text{m}$. The core NA is assumed at 0.06. Figure 5.10 (b) illustrates that the overlap between FM and doping area increases with the larger circular core radius, and it is about 0.995 at the core radius of $26 \mu\text{m}$. Therefore, I set the achievable FM effective mode area in a circular step-index fibre is $1250.8 \mu\text{m}^2$ at the core radius of $26 \mu\text{m}$ as shown in Fig. 5.10 (c).

Figure 5.10: (a) Effective index difference between modes, (b) Overlap between FM and doping area and (c) Signal effective mode area versus the circular core radius. (CCF: circular core fibre).

In the following, I obtain the parameters of a multicore fibre for the nearly same signal effective mode area of $1250 \mu\text{m}^2$ as that in circular core fibre. Figure 5.11 shows the effective index difference between modes, overlap between FM and doping area, and signal effective mode area versus the values of $D/2r$ at $r=8 \mu\text{m}$, $9 \mu\text{m}$ and $10 \mu\text{m}$, respectively. It is noted in Fig 5.11 (a) that the effective index difference between modes decreases with the increasing values of $D/2r$ or r . To satisfy the required modal index gap, the $D/2r$ falls in the range between 2 and 2.2 at $r=8 \mu\text{m}$, $9 \mu\text{m}$ and $10 \mu\text{m}$. Figure 5.11 (b) shows that the overlap between FM and doping area slightly decreases with the increasing $D/2r$ or reducing r . The values of $D/2r$ from 2 to 2.2 make the overlap ratio above 0.85, which should be sufficient for the efficient lasing and amplification. It is seen from Fig 5.11 (c) that to obtain the signal effective area around $1250 \mu\text{m}^2$, the values of r and $D/2r$ are $8 \mu\text{m}$ and 2.1, respectively. I will use these values for TMI suppression comparison. Because the mode area and the cladding size are kept consistent, I can rule out influence of saturation effect in our study. I also note that the maximum achievable effective mode area in the multicore fibre is $\sim 1800 \mu\text{m}^2$ when $D/2r$ is 2.1 and r is $10 \mu\text{m}$. The selected parameters of circular core and multicore fibres with a nearly same signal effective mode area around $1250 \mu\text{m}^2$ are listed in Table 5.4.

Figure 5.11: (a) Effective index difference between modes, (b) Overlap between FM and doping area and (c) Signal effective mode area versus the $D/2r$ in a multicore fibre with $r=8 \mu\text{m}$, $9 \mu\text{m}$ and $10 \mu\text{m}$. (MCF: multicore fibre).

Table 5.4: Multicore and circular core fibre Parameters

	Multicore fibre	Circular core fibre
core to core center D	$16.8 \mu\text{m}$	/
small core radius r	$8 \mu\text{m}$	/
core radius R	/	$26 \mu\text{m}$
core area A	$1205.8 \mu\text{m}^2$	$2122.6 \mu\text{m}^2$
signal effective mode area A_{eff}	$1285 \mu\text{m}^2$	$1250.8 \mu\text{m}^2$
cladding index n_{clad}	1.45	1.45
core numerical aperture NA	0.06	0.06
cladding radius R_{clad}	$200 \mu\text{m}$	$200 \mu\text{m}$
Yb ion concentration N_{Yb}	$3.5 \times 10^{25} \text{ m}^{-3}$	$3.5 \times 10^{25} \text{ m}^{-3}$
doping area	core doping	core doping
fibre length L	varies	varies

5.2.2 Transverse mode instability in multicore fibre

In this section, the evolution of modes power and thermal load along fibre, the TMI threshold power and maximum output power are numerically compared between the multicore and circular core fibres. Firstly, I calculate that the frequency offset for most power transfer from FM to HOM in circular core and multicore fibres are at 1000 Hz and 2100 Hz, respectively, as done in Fig. 5.3. The increased frequency offset for most power transfer between modes in multicore fibre is due to the shorter distance between the centre of core to the core edge, thus

decreasing the thermal diffusion time, in comparison with the circular core fibre. We use these frequency offsets to conduct the following simulations in circular core and multicore fibres.

Figure 5.12 shows the TMI behaviours of the time averaged LP_{01} , LP_{11} and pump power evolution along fibre in the circular core (with $R=26\text{ }\mu\text{m}$) and multicore (with $D=16.8\text{ }\mu\text{m}$ and $r=8\text{ }\mu\text{m}$) fibres, and the input pump power is at 680W. Table 5.5 lists the output powers shown in the Fig. 5.12 and the pump absorption is defined as the absorbed pump power to the input pump power. It is seen from Fig. 5.12 that the output HOM LP_{11} power in the circular core fibre is 20.2252 W, much larger than $4.1497 \times 10^{-4}\text{ W}$ in the multicore fibre. It indicates that the TMI effect is much stronger in circular core fibre compared to the multicore fibre at the same pump absorption of 94.6%. Furthermore, it is noted from Fig. 12 that the fibre length in circular core and multicore fibres is adjusted to 4.5 m and 6.5 m, respectively, to make nearly same pump absorption. This is due to the smaller core area of $1205.8\text{ }\mu\text{m}^2$ in multicore fibre, compared to $2122.6\text{ }\mu\text{m}^2$ in circular core fibre for the same pumping radius (cladding radius). Hence, multicore fibre needs longer fibre length to achieve the same pump absorption as in circular core fibre.

Figure 5.12: Time averaged powers versus fibre length for (a) circular core (with $R=26\text{ }\mu\text{m}$) and (b) multicore (with $D=16.8\text{ }\mu\text{m}$ and $r=8\text{ }\mu\text{m}$) fibres, at the input pump power at 680 W.

Table 5.5: Multicore and circular core fibre Parameters

	Multicore fibre	Circular core fibre
output LP_{01} power	556.1401 W	568.0217 W
output LP_{11} power	$4.1497 \times 10^{-4}\text{ W}$	20.2252 W
output pump power	36.7875 W	36.7421 W
pump absorption	94.59%	94.60%

Figure 13 presents the evolution of accumulated thermal load at each fibre transverse along fibre in the circular core and multicore fibres. It is noted that the peak value of accumulated thermal load in circular core fibre is about 20.8 W/m, larger than 15 W/m in the multicore fibre,

which is due to the larger core area in circular core fibre as shown in table 5.4. Therefore, the smaller accumulated thermal load in multicore fibre suppresses the TMI effect, in comparison with the circular core fibre as reflected in Fig. 5.13. The multicore structure has similar principle in suppressing the TMI effect as in the circular core fibre, which is due to the less heat generated compared to the circular core fibre.

Figure 5.13: Evolution of the accumulated thermal load at each fibre transverse for the (a) circular core and (b) multicore fibres along the fibre length. The same parameters as that in Fig. 5.12. (CCF: circular core fibre; MCF: multicore fibre).

Figure 5.14: Output LP₁₁ power content versus the input pump power in the multicore and circular core fibres. The fibre parameters are same as that in Fig. 5.12.

The output LP₁₁ power content versus the input pump power in the multicore (with $D=16.8\mu\text{m}$ and $r=8\mu\text{m}$) and circular core (with $R=26\mu\text{m}$) fibres is illustrated in Fig. 5.14, and the fibre length is adjusted to obtain nearly same pump absorption at about 95% for different pump powers. It is seen that the threshold power increased by about 130% from 680 W in the circular core fibre to 1560 W in the multicore fibre. Therefore, the multicore structure is efficient to

suppress the TMI and increase the TMI threshold power due to the reduced generation of thermal load, in comparison with the circular core fibre.

The total output power versus input pump power in the circular core and multicore fibres are presented in Fig. 5.15. Additionally, with the calculated near-field mode profile as represented in Fig. 5.9(a), the corresponding far field intensity distribution after a propagation distance can be obtained by numerically solving the Fresnel diffraction integral using a standard fast Fourier transform (FFT) method [132]. The calculated power in the central peak is 70% of the total output power. The power lost in intensity profile conversion in multicore fibre is larger compared to the ring-core fibre. This is because that the near-field intensity profile distribution in multicore fibre is not so uniform as that in the ring-core fibre. Furthermore, it is noted from Fig. 5.15 that the total output power reduced from 1120 W for the near field intensity profile to 810 W for the far field intensity profile, at the TMI threshold power of 1560 W in the multicore fibre. Additionally, the total output power in circular core fibre is 600 W at the TMI threshold power of 680 W, 25.9% less than 810 W in the ring-core fibre with far field intensity profile. Hence, the multicore structure is better to suppress the TMI compared to circular core fibre, although it is not efficient as in the ring-core fibre due to the higher power lost in the intensity profile conversion.

Figure 5.15: Total output power versus the input pump power in circular core ($R=26\ \mu\text{m}$) and multicore ($D=16.8\ \mu\text{m}$ and $r=8\ \mu\text{m}$) fibres, for the near field (donut-like) and far field (gaussian-like) intensity profile output. The fibre parameters are same as that in Fig. 5.12.

5.2.3 Bending performance of multicore fibre

In the following, I investigate the bending performance of multicore (with $D=16.8\ \mu\text{m}$ and $r=8\ \mu\text{m}$) and circular core (with $R=26\ \mu\text{m}$) fibres. Figure 5.16 shows the signal effective mode area versus bending curvature in the multicore and circular core fibres. It is clearly to see that although the signal effective mode area decreases with the increasing bending curvature in both ring-core and circular core fibres, the bending performance in multicore fibre is very bad. The effective mode area in multicore fibre reduces from $1285\ \mu\text{m}^2$ in straight fibre to $420\ \mu\text{m}^2$, much smaller than $705\ \mu\text{m}^2$ in circular core fibre, at the bending curvature of $10\ \text{m}^{-1}$. Additionally, even at the small bending curvature of $1.6\ \text{m}^{-1}$, the signal effective mode area in multicore fibre is only $700\ \mu\text{m}^2$. Hence, straight fibre with short length is applicable for the multicore structure to suppress the TMI, for instance, the non-PD fibre with high ion concentration.

Figure 5.16: Signal effective mode area versus the bending curvature for circular core ($R=26\ \mu\text{m}$) and multicore ($D=16.8\ \mu\text{m}$ and $r=8\ \mu\text{m}$) fibres. The fibre parameters are same as that in Fig. 5.12.

5.3 Conclusion

In this paper, I numerically analyze the TMI behavior, TMI threshold power and output power in the non-PD Yb-doped ring-core and multicore fibres. The TMI threshold power increased by about 115%, from 960 W in the circular core fibre to 2060 W in the ring-core fibre, which is due to the less heat generated in ring-core structure for the same effective mode area. Furthermore, although 20% power lost after the signal intensity profile conversion from far-

field (donut-like) to near-field (Gaussian-like) in the ring-core fibre, the output power is 1440 W, much larger than 810 W in the circular core fibre. Additionally, the largest achievable signal effective mode area in ring-core fibre is about $784 \mu\text{m}^2$, which makes it only applicable for the CW high-power fibre operation that needs small effective mode area to overcome the nonlinear effect. On the other hand, the multicore fibre has a TMI threshold power of 1560 W, 130% larger than 680 W in the circular core fibre. However, the output power in multicore fibre is 810 W at the TMI threshold power of 1560 W, only 35% increase of the output power at 600 W in circular core fibre. This is due to the 30% power lost after the signal intensity profile conversion from far-field (donut-like) to near-field (Gaussian-like) in the multicore fibre. As the multicore fibre provides a maximum effective mode area of $1800 \mu\text{m}^2$, which surpassed the largest achievable mode area of $1250 \mu\text{m}^2$ in circular core fibre. Therefore, the multicore fibre is applicable for the pulsed high-power fibre operation needing large effective mode area to suppress the nonlinear effect.

Lastly, we provide the bending performance in both the ring-core and multicore fibres. The effective mode area in a ring-core structure reduced from $784 \mu\text{m}^2$ in straight fibre to $380 \mu\text{m}^2$ in bent fibre at the bending curvature of 10 m^{-1} . Therefore, tightly bending is not good for the high-power laser or amplifier operation in the ring-core fibre. However, the multicore fibre has a significant reduction of effective mode area even at a small bending curvature, for instance, $700 \mu\text{m}^2$ at the bending curvature of 1.6 m^{-1} , much smaller than $1285 \mu\text{m}^2$ in straight fibre. Hence, the straight non-PD fibre with high ion concentration is useful for the multicore structure.

Chapter 6 Conclusion and Recommendations

6.1 Experimental lasing setup for the Yb-doped ring-core fibre

With reference to the simulation results presented in Chapter 5, a ring-core fibre appears promising for TMI suppression thanks to the less heat generated in such core structure. Preliminary experimental trials are presented in this chapter including fabricated fibre characteristics and lasing efficiency. Due to deviation in the fabricated fibre parameters from the design, the current ring core fibre, which supports the FM only, is not able to reach a TMI threshold. Nonetheless, I decided to provide this preliminary exploration to serve as a first iteration for future development. The required fibre parameters of my design are compared to the fabricated fibre parameters in Table 6.1.

Table 6.1: Parameters of the ring-core fibres fabricated and my design

	Design	Fabricated
outside circle radius a	16 μm	8 μm
inside circle diameter d	12.8 μm	12 μm
cladding index n_{clad}	1.45	1.45
core numerical aperture NA	0.06	0.06
cladding radius R_{clad}	200 μm	120 μm
Yb ion concentration N_{Yb}	$3.5 \times 10^{25} \text{ m}^{-3}$	$3.5 \times 10^{25} \text{ m}^{-3}$
doping area	core doping	core doping

The Yb-doped ring-core fibre preform was fabricated with a conventional modified chemical vapor deposition (MCVD) process in conjunction with a solution doping technique. The fabricated preform was milled to an octagonal shape for better pump absorption. The shaped preform was subsequently pulled with a low index coating into a double clad fibre. The bare fibre diameter is 240 μm , measured in flat to flat edge. The outside circular diameter is 16 μm (that is $2a = 16 \mu\text{m}$ as shown in Fig. 5.1) and the inner diameter of the ring core is 12 μm (that is $d = 12 \mu\text{m}$ as shown in Fig. 5.1). Hence, the ratio of $d/2a$ is 0.75. The ring core is doped with Yb/Ce/Al. The Ce was doped to suppress PD. Its corresponding core NA is measured about 0.06, and the Yb ion concentration is 4500 - 5000 ppm-wt. Figure 6.1 represents the measured refractive index profile (RIP) of the ring-core fibre preform. The absorption of the fibre is measured using a cut-back method. Firstly, I used the white light source to pass through the 5 m long ring-core fibre and recorded it in the optical spectrum analyser. Then, I cut 1 m ring-core fibre off and repeated the previous procedure to test its spectrum. The measured spectrum

is shown in Fig. 6.2. It is observed that the small signal absorption is about 3 dB/m at the 976 nm absorption peak.

Figure 6.1: Refractive index profile of the Yb-doped ring-core fibre along the diagonal.

Figure 6.2: Absorption spectrum of the Yb-doped ring-core fibre along the diagonal.

Figure 6.3: Schematic diagram of the experimental lasing setup for the ring-core fibre with signal light outputs at both ends of fibre. DM: dichroic mirror; LD: laser diode; RCF: ring-core fibre.

Figure 6.3 presents the schematics of the free-running lasing setup employed for testing the fibre laser efficiency. Two laser diodes (LDs) with the highest output power of 200 W at 976 nm, are combined through the beam combiner. The pump light from the beam combiner output travels through the lens 1 (with NA of 0.22 and focal length of 22 mm), and then two dichroic mirrors (DMs) (transmission at 1064 nm and reflection at 976 nm). The delivered pump light is launched into the ring-core fibre under test through the lens 2 with NA of 0.5 and a focal length of 8 mm. The DM 3 separates the signal and pump beams at the forward output end. It is noted that the ring-core fibre is 9 m.

The measured output power versus the launched pump power is shown in Fig. 6.4. The measured data is in black circle and red dashed line represents the fitting curve. The laser starts lasing at the input pump power of about 20 W and the total output signal power is 0.8 W. The highest output power of the laser is about 82 W at a pump power of 200 W. It is also noted that the slope efficiency of laser with respect to the LD pump power is $\sim 47.2\%$. The low laser efficiency is mainly caused by the ring-core with the value of $d/2a$ is 0.75. With reference to the Fig. 5.2 (b), such high $d/2a$ gives rise to a low overlap between the FM and the doping area at about 0.5, which is the reason that makes such low laser efficiency. From the simulation results shown in Chapter 5, the value of $d/2a$ at 0.4 is suggested for the efficient ring-core lasing setup based on the considerations discussed there.

Figure 6.4: The measured output power versus the pump power for the experimental lasing setup of ring-core fibre with signal light outputs at both ends of fibre.

In this section, I present the parameters of fabricated ring-core fibre, experimental lasing setup and the test of laser slope efficiency. However, as the value of $d/2a$ is 0.75 in the fabricated ring-core fibre, very different to the optimized value of 0.4 as presented in Chapter 5. Therefore, the relative smaller overlap between FM and doping makes the lasing efficiency very low in the test of laser efficiency. To confirm the TMI suppression in a ring core design, my proposed ring-core fibre design, with $d = 12.8 \mu\text{m}$ and $2a = 32 \mu\text{m}$, will be fabricated and tested in the future work.

6.2 Conclusion

The newly reported TMI effect becomes the key factor that hinders the further power scalability of high-power fibre laser and amplifier till now. Hence, it is very essential to suppress the TMI and increase the threshold power for high power fibre operation. In this thesis, I investigate unique fibre designs, such as the rectangular core fibre, TMI-suppressed PD fibre, ring-core and multicore fibres, to suppress the TMI effect. The contributions of this thesis are summarized as follows:

- (a) I developed a highly numerical 4D split-step FFT-based BPM to mimic the TMI behaviours in a fibre amplifier, which intrinsically incorporate the additional physical effects, including the pump saturation, thermal lensing, mode coupling, photodarkening and bending effects.

Additionally, it can simulate a variety of shapes of the core, refractive index and doping profiles.

- (b) I presented the theoretical investigation of the TMI in non-circular fibres including rectangular core fibres and D-shaped cladding fibres. The results reveal that the temperature built in a fibre is not solely responsible for the TMI, but the pump saturation effect, coupling strength and thermal lensing effect also play a significant role in governing the TMI behaviour.
- (c) Parametric studies for a TMI suppressed step-index fibre was numerically investigated with an emphasis on thermal load and its dependence on key fibre design parameters encompassing fibre length, cladding (pumping) radius, core NA, core radius and ion concentration in PD and non-PD fibres. In a PD fibre, the main contribution to the TMI is found QD-induced thermal load despite presence of the PD induced thermal load. It is also found that the PD induced thermal load is shaped by signal-induced thermal load rather than PD-pump-induced thermal load. Additionally, I proposed fibre parameters within practically achievable range for suppressing the TMI as well as the nonlinearities in PD fibres. The simulation result suggests that 1.7 kW output power is achievable in the TMI-suppressed PD fibre, and simultaneously it does not worsen the nonlinear threshold power, within practical design parameters.
- (d) I numerically analysed the TMI behaviour, TMI threshold power and output power in Yb-doped ring-core and multicore fibres. It is found that the annular cores greatly suppress TMI as compared to the circular core fibre. Reduced heat generation in the ring-core and multicore structures was suggested as a main reason for the TMI suppression. A ring-shaped fundamental mode in the annular cores can be converted to a Gaussian-like mode in the far-field with 88% of conversion efficiency.
- (e) The ring-core fibre with outside circular diameter of 16 μm and core thickness of 2 μm was fabricated. I made the experimental lasing setup for the ring-core fibre, and tested that the slope efficiency is about 47.2% for the FM output power with respect to the LD. Such low efficiency was caused by the relatively small overlap between FM and doping area, which worsens the laser efficiency. New fibre fabrication is suggested to investigate the TMI suppression in a ring core fibre.

6.3 Recommendations for future research

In the following, I list the possible further work below:

- (a) For the TMI is related to the similar pattern between signal intensity and changed refractive index, it may work to break such pattern by introducing the artificial grating in fibre to suppress the TMI and increase threshold power.
- (b) Experimental test of the TMI suppression in the annular-core (ring-core and multicore) fibres compared to the circular core fibre in high power fibre amplifier system.
- (c) Experimental comparison of the TMI between TMI-suppressed PD fibre and the circular core PD fibre in high power fibre laser and amplifier systems.
- (d) Experimental test of the total thermal load and averaged thermal load in the circular core fibres with same CCAR and V parameter, but different ion concentration. This is to verify the simulation result that the total thermal load at the TMI threshold remains consistent when PD fibres hold the same CCAR and V parameter. However, the averaged thermal load varies with the ion concentration even in the PD fibres with same CCAR and V parameter, which is misleading in predicting the TMI behaviour at some conditions (for instance, different ion concentration).

Appendix

In this section, I check the accuracy of proposed numerical 4D split-step FFT based BPM in comparison to the prior simulation and experimental results reported by other researchers. The numerical 4D model used in this thesis is compared with experimental test shown in [116] and semi-analytical model presented in [78]. Table A-1 shows the fibre parameters in experimental setup [116] and numerical 4D model. Figures A-1 to A-5 illustrate the evolution of modal powers along fibre length, calculated by the numerical 4D model, at the input pump power of 2000 W, 2050 W, 2100 W, 2150 W and 2200 W, respectively. Figure A-6 presents the HOM power content versus input pump power, which is obtained from Figures A-1 to A-5. It is seen that the TMI threshold power (HOM takes 5% of total output power) is at about 2120 W close to 2200 W in experiment.

Table A-1 Fibre parameters and TMI threshold power in experiment and 4D model

	Experiment [116]	Numerical 4D Model
Core diameter	20 μm	20 μm
Cladding radius	400 μm	400 μm
Core NA	0.06	0.06
Ion concentration	$6 \times 10^{25}/\text{m}^3$	$6 \times 10^{25}/\text{m}^3$
Fibre length	9.7 m	9.7 m
Signal wavelength	1064 nm	1064 nm
Pump wavelength	976 nm	976 nm
Pump absorption	Above 95%	Above 95%
Input FM power	10 W	10 W
Input HOM power	1 μW	1 μW
TMI threshold power	2200 W	2120 W
Average thermal load	23.1 W/m	22.7 W/m
Maximum thermal load	74.0 W/m	78.2 W/m

Figure A-1: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 2000 W calculated by the numerical 4D model.

Figure A-2: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 2050 W calculated by the numerical 4D model.

Figure A-3: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 2100 W calculated by the numerical 4D model.

Figure A-4: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 2150 W calculated by the numerical 4D model.

Figure A-5: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 2200 W calculated by the numerical 4D model.

Figure A-6: HOM power content versus input pump power calculated by the numerical 4D model.

Table A-2 shows the fibre parameters in semi-analytical model [78] and numerical 4D model used in this thesis. I selected the same parameters as in [78]. Figures A-7 to A-11 illustrate the evolution of modal powers along fibre length, calculated by the numerical 4D model, at the input pump power of 760 W, 780 W, 820 W, 840 W and 860 W, respectively. Figure A-12 presents the HOM power content versus input pump power in semi-analytical model [78] and numerical 4D model. The results agree well within 5% discrepancy at largest. The disparity grows with higher HOM power content due to the incorporation of thermal lensing effect in the numerical 4D model.

Table A-2 Fibre parameters in semi-analytical model and numerical 4D model

	Semi-analytical Model [78]	Numerical 4D Model
Core diameter	50 μm	50 μm
Cladding radius	250 μm	250 μm
Core NA	0.06	0.06
Ion concentration	$6 \times 10^{25}/\text{m}^3$	$6 \times 10^{25}/\text{m}^3$
Fibre length	1.6 m	1.6 m
Signal wavelength	1064 nm	1064 nm
Pump wavelength	976 nm	976 nm
Input FM power	30 W	30 W
Input HOM power	3 μW	3 μW

Figure A-7: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 760 W calculated by the numerical 4D model.

Figure A-8: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 780 W calculated by the numerical 4D model.

Figure A-9: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 820 W calculated by the numerical 4D model.

Figure A-10: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 840 W calculated by the numerical 4D model.

Figure A-11: Evolution of modal powers along fibre length in (a) linear scale and (b) log scale at input pump power of 860 W calculated by the numerical 4D model.

Figure A-12: HOM power content versus input pump power calculated by the semi-analytical model [78] and numerical 4D model.

In the proposed numerical 4D model, I select 128 points along the fibre transverse (x and y directions), 64 time grid points for one time cycle (defined as $1/\Delta\omega$) and step size of $10\text{ }\mu\text{m}$ along fibre length. If larger grid points are selected, for instance, 256 points along fibre transverse, 128 points for one time cycle or step size smaller than $10\text{ }\mu\text{m}$, the running time of 4D model doubles while the accuracy remains nearly same. Additionally, when the number of grid points become smaller, such as 64 points along fibre transverse, 32 points for one time cycle or step size much larger than $10\text{ }\mu\text{m}$, the model is inaccurate to simulate the TMI behaviour in fibre.

Author's Publications

Journal Paper:

1. N. Xia and S. Yoo, "Mode instability in ytterbium-doped non-circular fibres," *Opt. Express* 25(12), 13230-13251 (2017).
2. N. Xia and S. Yoo, "Power Scaling in Photodarkening Fibre by Transverse Mode Instability Suppression," submitted to the *Journal of Lightwave Technology* and is under peer review now.
3. N. Xia and S. Yoo, "Suppression of transverse mode instability in an annular core fibre," being under revision and will be submitted soon.

Conference Paper:

1. N. Xia and S. Yoo, "Mode instability in a Yb-doped stretched core fibre," in *Proceedings of Photonics West Conference, Fibre Lasers XIV: Technology and Systems*, 10083 (2017).
2. S. Yoo, N. Xia, R. Sidharthan, D. Ho, Y. Zhou, J. Ji and D. Yue "Asymmetric large mode area fibres," in *Proceedings of CLEO-PR Conference (IEEE, 2017)*.
3. S. Yoo, J. Ji, X. Wu, S. Raghavan, D. Ho, N. Xia, J. Sahu and J. Nilsson, "Mode area scalability in rectangular core fibre," in *Proceedings of IEEE Conference on Photonics*, 323–324 (IEEE, 2015).

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