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Charge Collection From Within a Collecting Junction Well

Oka Kurniawan and Vincent K. S. Ong

Abstract—This paper provides the analytical equation for the charge collection from a collecting region with a finite dimension. Electron-beam-induced current has widely been used for semiconductor characterization. The availability of analytical expressions would further enhance the study and development of various measurement techniques. Nevertheless, most devices are fabricated with junctions that have finite dimensions, which are usually either L shaped or U shaped. For these cases, the analytical expressions are lacking. This paper provides the derivation of the electric current profile when an electron beam scans from within the collecting region for these two cases. The computation was verified with a semiconductor device simulation program on a computer and was found to be in good agreement. This paper then gives a discussion on the effects of certain parameters such as the junction depth and junction width, the diffusion length, and the depth of the generation volume.

Index Terms—Charge-carrier processes, electron beam application, semiconductor material measurements, simulation.

I. INTRODUCTION

ANALYTICAL expressions for the charge collection profile due to the bombardment of electron beam were derived for the case of the L-shaped and the U-shaped collectors. The electron-beam-induced current (EBIC) technique of the scanning electron microscope has widely been used in semiconductor material characterizations [1]. It has found wide acceptance, particularly in the characterization of minority-carrier transport properties in bipolar transistors and photodiodes [2].

In the EBIC technique, the sample is bombarded with an electron beam to generate the electron-hole pairs (ehps) within the material. These ehps tend to diffuse away from the generation volume and recombine. If a built-in electric field is in the neighborhood of the generation source, the minority carriers can be separated from the majority carriers. This separation is sometimes termed as a collection in nuclear instrumentation [3]. This charge collection results in an induced current in the external circuit.

The many applications of EBIC have arisen from the availability of the analytical equations of the EBIC profile. The two most used collector configurations for EBIC measurements are: 1) the normal-collector configuration and 2) the planar-collector

Fig. 1. (a) Normal-collector configuration. (b) Planar-collector configuration.

configuration (see Fig. 1) [1]. For these two configurations, analytical expressions for the EBIC profile have been derived in the literature.

Berz and Kuiken, utilizing the method of images, derived the EBIC expression for the normal-collector configuration as a function of Bessel functions [4]. In 1982, Donolato proposed a simpler expression that contains only elementary functions [5]. In the following year, von Roos and Luke derived the EBIC expression that includes the finite surface recombination velocity of the back contact into consideration [6], [7]. So far, Donolato's expression for the normal-collector configuration has been widely used for analysis in minority-carrier properties [8]–[13].

For the planar-collector configurations, Ioannou and Dimitriadis derived the EBIC expression for the case of large surface recombination velocity, resulting in an expression with Bessel functions [14]. The expression was used for diffusion length measurement in the asymptotic region. Donolato, on the other hand, derived the expression for the case of zero surface recombination velocity [15]. The expression for any value of surface recombination velocity was derived by Boersma *et al.* [16]. This expression was used in the analysis

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Fig. 2. L-shaped geometry of a junction well.

Fig. 3. U-shaped geometry of a junction well.

and development of measurement techniques for the planar-collector configuration [17], [18].

In planar technology, however, many devices are constructed from p-n junctions with a finite junction depth, as shown in Figs. 2 and 3. This means that, when one uses the expression available for the normal-collector configuration, the junction must be assumed to be infinitely deep, and the generation of ephs is located near the surface. Similarly, when one uses the expression for the planar-collector configuration, the junction must be assumed to be very narrow.

Several works have been done to study the collection with a finite junction depth. Holloway studied the charge collection when a light spot impinges upon the semiconductor outside the junction well [19]. It was shown that the sample thickness, as well as the depth of the junction, affects the current profile. A thorough analysis of the EBIC profile from a sharp L-shaped geometry was done by Soukup and Ekstrand [20]. The analytical expressions were derived for collections from both inside and outside the junction well. The expressions, however, involve a nonelementary function such as the Bessel function. It will be shown later that an EBIC expression that contains only elementary functions can be found for the case when the beam is inside the junction well.

The EBIC expression provided by Soukup and Ekstrand is valid for the L-shaped junction shown in Fig. 2. For this case,

the width of the collecting region was assumed to be infinitely wide. In other words, the charge collection by the horizontal junction is from zero to infinity. In bipolar devices, junction wells have a finite width. Moreover, the shape of the junction may not be L shaped but rather U shaped, as shown in Fig. 3. Currently, the authors have not found any EBIC expression for this case with finite width and finite junction depth collection yet. Boudjani *et al.* measured diffusion lengths from finite dimensions of bipolar transistors by using EBIC [21]. In their paper, they used the assumption that the depth of the junction well is much larger than the depth of the generation volume. They also used an asymptotic expression in order to get the value of the diffusion length. The availability of an analytical expression will help enhance the study of charge collection in devices where the dimension of the junction can no longer be assumed to be infinite.

This paper provides analytical EBIC expressions when the generation source is within the well region. Two cases are considered: 1) the L-shaped junction and 2) the U-shaped junction. The effects of certain parameters, such as the junction depth and the junction width, diffusion lengths, and the depth of the generation source, are then analyzed.

II. DERIVATIONS

The analytical expression for the case of the L-shaped geometry is first derived. The Green's function method was used for this purpose. The expression for the case of the U-shaped geometry will similarly be derived using the Green's function. For the latter case, we choose to start the derivation in a slightly different manner, i.e., by applying the reciprocity theorem [22]–[24].

The expressions were derived for a 2-D case (i.e., the x - z plane), which is similar to [5]. In most EBIC linescan measurements, such as diffusion length measurement, the beam is scanned along one direction [25]. It is commonly assumed that the dimension in the other direction, i.e., the y -axis, is large. The current along the y -direction due to a point source can be solved by calculating the current at an infinitely small dimension δy at the junction due to a line source that extends in the y -direction [19], [26]. This reduces the problem from a 3-D one to a 2-D one. The loss in accuracy will not be very significant.

A. Expression for the L-Shaped Junction

First, the mathematical problem for a point source was formulated. The expression for the extended generation volume is simply the convolution of this point source expression with the distribution of the generation source. This is only true when the generation volume is completely inside the junction well. For the L-shaped geometry, the point source current collected from the two sides of the junction can be written as

$$Q(x', z') = D \left\{ \int_0^h \frac{\partial G(x, z|x', z')}{\partial x} \Big|_{x=0} dz - \int_0^d \frac{\partial G(x, z|x', z')}{\partial z} \Big|_{z=h} dx \right\} \quad (1)$$

where D is the diffusion constant, G is the Green's function of the problem, h is the junction depth, and d is the junction width. This expression simply means that the current due to a point source located at (x', z') is the sum of the gradient of the concentration at the two sides of the junction. The concentration profile in 2-D satisfies the continuity equation

$$\frac{\partial^2 q(x, z)}{\partial x^2} + \frac{\partial^2 q(x, z)}{\partial z^2} - \lambda^2 q(x, z) = -\frac{h(x - x', z - z')}{D} \quad (2)$$

where q is the concentration of the minority charge carrier, which, in this case, is assumed to be a hole. The term $\lambda = 1/L = 1/\sqrt{D\tau}$, where L is the diffusion length and is related to the lifetime τ as shown. The right-hand-side term $h(x, z)$ is the projection of the generation volume onto the x - z plane. The Green's function, therefore, also satisfies the continuity equation as follows [27]:

$$\frac{\partial^2 G(x, z|x', z')}{\partial x^2} + \frac{\partial^2 G(x, z|x', z')}{\partial z^2} - \lambda^2 G(x, z|x', z') = -\frac{\delta(x - x')\delta(z - z')}{D} \quad (3)$$

which is the same as the continuity equation when the source is a point source.

The Green's function must also satisfy the same boundary condition as that of the concentration. For simplicity, we assume that the boundary at $x = d$ satisfies the Neumann boundary condition. In other words, the boundary acts to make the concentration gradient at that boundary equal to zero. These same boundary conditions apply for the case of zero surface recombination velocity. Furthermore, we assume that the ohmic contact at the top surface spans throughout the well without short-circuiting the p-n junction. This simplification allows us to use a homogeneous boundary condition at the top surface, and the fringing effects can be neglected. At the junction, the concentration is zero. This is simply because all the minority carriers that arrive at the junction are assumed to be collected and transported to the other side. Therefore, we can write the boundary conditions for the Green's function as follows:

$$\begin{aligned} G &= 0, & \text{for } x = 0 \text{ and } 0 \leq z \leq h \\ \frac{\partial G}{\partial x} &= 0, & \text{for } x = d \text{ and } 0 \leq z \leq h \\ G &= 0, & \text{for } z = 0 \text{ and } 0 \leq x \leq d \\ G &= 0, & \text{for } z = h \text{ and } 0 \leq x \leq d. \end{aligned} \quad (4)$$

The details of the calculation of the Green's function are given in the Appendix. Here, we will only show the final result. The solution is given in (A16), i.e.,

$$\begin{aligned} G_I(x, z|x', z') &= \sum_n \frac{2 \sin(p_n x')}{d D \mu_n h \sinh(\mu_n h)} \\ &\quad \times \sinh(\mu_n (h - z')) \sin(p_n x) \sinh(\mu_n z) \\ G_{II}(x, z|x', z') &= \sum_n \frac{2 \sin(p_n x')}{d D \mu_n h \sinh(\mu_n h)} \sinh(\mu_n z') \\ &\quad \times \sin(p_n x) \sinh(\mu_n (h - z)) \end{aligned} \quad (5)$$

where region I is for $0 \leq z \leq z'$, region II is for $z' \leq z \leq h$, and $n = 1, 3, 5, \dots$ to infinity. In Eq. (5)

$$p_n = \frac{n\pi}{2d}, \quad \mu_n = (p_n^2 + \lambda^2)^{1/2}. \quad (6)$$

Equation (5) can then be substituted into (1) to give the EBIC profile

$$\begin{aligned} Q(x', z') &= \sum_n \frac{2 \sin(p_n x')}{d \sinh(\mu_n h)} \\ &\quad \times \left\{ \frac{p_n}{\mu_n^2} [\sinh(\mu_n (h - z')) (\cosh(\mu_n z') - 1) \right. \\ &\quad \left. + \sinh(\mu_n z') (\cosh(\mu_n (h - z')) - 1)] \right. \\ &\quad \left. - \frac{\sinh(\mu_n z')}{p_n} \right\}. \end{aligned} \quad (7)$$

It can be seen that the expression contains only elementary functions. The above equation can numerically be computed to give the charge collection profile for the case where the beam is inside the junction well.

B. Expression for the U-Shaped Junction

To derive the EBIC profile for this U-shaped geometry, we will begin by utilizing the reciprocity theorem [22]–[24]. This theorem states that the charge collection current satisfies the homogeneous continuity equation

$$\frac{\partial^2 Q(x', z')}{\partial x^2} + \frac{\partial^2 Q(x', z')}{\partial z^2} - \lambda^2 Q(x', z') = 0 \quad (8)$$

and for the case of the U-shaped geometry, it also satisfies the boundary conditions

$$\begin{aligned} Q &= 1, & \text{for } x = 0 \text{ and } 0 \leq z \leq h \\ Q &= 1, & \text{for } x = d \text{ and } 0 \leq z \leq h \\ Q &= 0, & \text{for } z = 0 \text{ and } 0 \leq x \leq d \\ Q &= 1, & \text{for } z = h \text{ and } 0 \leq x \leq d. \end{aligned} \quad (9)$$

These conditions are obvious from its physical configuration. When the beam is located at the ohmic contact, the collection is zero. On the other hand, when the beam is located at the three junction sides, the collection is unity. Solving (8) gives the expression of the charge collection for the case of the U-shaped geometry. Therefore, the expression for the minority carrier need not be computed beforehand. This theorem simplifies the derivation of the charge collection profile for certain geometries.

However, in this paper, we use the Green's function method and, thus, indirectly prove that the solution is the same when we derived the expression starting from the continuity equation for the minority-carrier concentration, which was the case for the L-shaped junction. It was shown in [27] that the solution of the homogeneous problem with the inhomogeneous boundary conditions, such as what we have, can be expressed as

$$Q(x', z') = \oint Q_s \frac{\partial G}{\partial \mathbf{n}} \cdot d\mathbf{A} \quad (10)$$

where Q_s is the value at the surface, and the gradient is normal outward at the surface. This expression means that the collected current is simply obtained by multiplying the charge collection at the boundary with the gradient of the Green's function in the direction normal to that boundary. In this case, the Green's function satisfies the homogeneous continuity equation

$$\frac{\partial^2 G(x, z|x', z')}{\partial x^2} + \frac{\partial^2 G(x, z|x', z')}{\partial z^2} - \lambda^2 G(x, z|x', z') = 0 \quad (11)$$

and the homogenous boundary conditions

$$\begin{aligned} G &= 0, & \text{for } x = 0 \text{ and } 0 \leq z \leq h \\ G &= 0, & \text{for } x = d \text{ and } 0 \leq z \leq h \\ G &= 0, & \text{for } z = 0 \text{ and } 0 \leq x \leq d \\ G &= 0, & \text{for } z = h \text{ and } 0 \leq x \leq d. \end{aligned} \quad (12)$$

A Green's function similar to the one for the L-shaped geometry can be obtained for the case of the U-shaped geometry. It can be seen that only one boundary condition changes from the previous case. Furthermore, to satisfy this new condition, the same function $\sin(p_n x)$ [refer to (A4) in the Appendix] can be used to satisfy the two boundaries at $x = 0$ and $x = d$. However, now, the eigenvalues must change to satisfy $G = 0$ at $x = d$, or in other words, $\sin(p_n x) = 0$. Hence, we have

$$p_n = \frac{n\pi}{d} \quad (13)$$

where $n = 1, 2, 3, \dots$ to infinity. Therefore, the Green's function that satisfies (11) and (12) is

$$\begin{aligned} G_I(x, z|x', z') &= \sum_n \frac{2 \sin(p_n x')}{d \mu_n h \sinh(\mu_n h)} \sinh(\mu_n (h - z')) \\ &\quad \times \sin(p_n x) \sinh(\mu_n z) \\ G_{II}(x, z|x', z') &= \sum_n \frac{2 \sin(p_n x')}{d \mu_n h \sinh(\mu_n h)} \sinh(\mu_n z') \\ &\quad \times \sin(p_n x) \sinh(\mu_n (h - z)). \end{aligned} \quad (14)$$

By using the values in (9), (10) can be expanded to

$$\begin{aligned} Q(x', z') &= \int_0^h \frac{\partial G(x, z|x', z')}{\partial x} \Big|_{x=0} dz \\ &\quad - \int_0^d \frac{\partial G(x, z|x', z')}{\partial z} \Big|_{z=h} dx - \int_0^h \frac{\partial G(x, z|x', z')}{\partial x} \Big|_{x=d} dz \end{aligned} \quad (15)$$

which is similar to (1), but with an additional collecting junction term. Note that the diffusivity D is missing from the denominator in (14). This is because the Green's function satisfies the homogenous equation (11). The solution, however, still gives a correct answer, because the term D does not appear in (15) as well. In the previous solution, the D term is canceled out. Thus,

the reciprocity theorem is shown to be valid. Substituting the Green's function into (14) gives

$$\begin{aligned} Q(x', z') &= \sum_{n=1}^{\infty} \frac{2 \sin(p_n x') (1 - (-1)^n)}{d \sinh(\mu_n h)} \\ &\quad \times \left\{ \frac{p_n}{\mu_n^2} [\sinh(\mu_n (h - z')) (\cosh(\mu_n z') - 1) \right. \\ &\quad \left. + \sinh(\mu_n z') (\cosh(\mu_n (h - z')) - 1)] \right. \\ &\quad \left. + \frac{\sinh(\mu_n z')}{p_n} \right\}. \end{aligned} \quad (16)$$

This expression can numerically be evaluated to give the charge collection due to a point source in the U-shaped geometry.

Equations (7) and (15) give the profile for a point generation source. In reality, the generation of ephs usually occurs in a finite volume. In this case, the expression for the point source must be convoluted with the distribution of the generation volume as follows:

$$I_N(x', z') = \iint Q(x, z) g(x - x', z/R) dx dz \quad (17)$$

where $g(x - x', z/R)$ is the distribution of the generation volume in 2-D. The depth position is now a function of the electron penetration range R , which is determined by the beam energy. The common models for this generation volume usually involve Gaussian-like distributions [5], [28], [29].

III. COMPUTATION AND SIMULATION

In order to see the effects of certain parameters on the EBIC profile, (7) and (16) were numerically evaluated using Matlab on a standard personal computer. The infinite summation in (16) was approximated by using 400 terms. For the L-shaped geometry, the investigated parameters were the junction depth h , the diffusion length L , and the depth of the generation source z . The profile was computed from $x = 0$ to $x = d$. Since the boundary at $x = d$ is the Neumann boundary condition, the effect of the width of the well d is not significant in this case. For the case of the U-shaped geometry, however, this parameter d was also varied to see its effect on the profile.

The computation results from the analytical equation derived in Section II were verified using the MEDICI 2-D device simulator. The reason for using a simulator is that, on a simulator, the parameters being investigated can be varied in a precise manner. The structures shown in Figs. 2 and 3 were constructed with a fine grid of $0.1 \mu\text{m}$ at the junction, the generation source, and regions near the contacts. Structures with three different junction depths were constructed. The constructed junction depths were 1, 3, and $5 \mu\text{m}$. For the case of the U-shaped junction well, the widths were similarly varied to 5, 7, and $9 \mu\text{m}$. These dimensions are slightly larger compared to the ones in [21]. The purpose of this is to exaggerate the effect of the parameters contained in the derived equations.

The doping was uniform in both regions and was set to $1 \times 10^{18} \text{ cm}^{-3}$. The lifetime was set to give the appropriate values of diffusion lengths. The diffusion lengths used were 2,

Fig. 4. Effect of junction depth on the EBIC profile from within the L-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $L = 5 \mu\text{m}$, $z = 0.3 \mu\text{m}$, and $d = 5 \mu\text{m}$.

Fig. 5. Effect of diffusion length on the EBIC profile from within the L-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $h = 5 \mu\text{m}$, $z = 0.3 \mu\text{m}$, and $d = 5 \mu\text{m}$.

3, 4, 5, and $7 \mu\text{m}$. In order to approximate a point generation source, the simulator used a square generation source, with $0.2 \mu\text{m}$ on each side. It has been shown in [18] that this geometry can be used to approximate a round generation source with a radius of $0.1 \mu\text{m}$. The center of the generation volume was set at five different values. They are 0.3, 0.6, 0.9, 1.2, and $1.5 \mu\text{m}$.

IV. RESULTS

Figs. 4–6 show the results for the L-shaped geometry, whereas Figs. 7–10 show the results for the U-shaped geometry. In these figures, I_N is the normalized EBIC. It can be seen that the results computed from the analytical equation agree with the results from the MEDICI simulator. The accuracy seems to be affected by the number of terms that are summed up in (7) and (16). In Figs. 5 and 8, where the plot is in natural logarithm, the plots from the analytical equation for the case of small diffusion lengths look slightly oscillatory. The cause seems to be the denominator term $\sinh(\mu_n h)$, which tends to go to infinity very quickly. This causes the series to converge prematurely. As the value of the diffusion length is reduced, the term λ and, therefore, μ_n , is increased. For large values of h and

Fig. 6. Effect of the depth of the generation source on the EBIC profile from within the L-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $L = 5 \mu\text{m}$, $h = 5 \mu\text{m}$, and $d = 5 \mu\text{m}$.

Fig. 7. Effect of the junction depth on the EBIC profile from within the U-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $L = 5 \mu\text{m}$, $z = 0.3 \mu\text{m}$, and $d = 5 \mu\text{m}$.

Fig. 8. Effect of the diffusion length on the EBIC profile from within the U-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $h = 5 \mu\text{m}$, $z = 0.3 \mu\text{m}$, and $d = 5 \mu\text{m}$.

μ_n , the hyperbolic sine term will go to infinity very quickly, and therefore, for a given precision, the number of n terms will be very limited. This accuracy can be improved by using a higher precision number in the computation.

Fig. 9. Effect of the depth of the generation source on the EBIC profile from within the U-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $L = 5 \mu\text{m}$, $h = 5 \mu\text{m}$, and $d = 5 \mu\text{m}$.

Fig. 10. Effect of the width of the junction on the EBIC profile from within the U-shaped geometry junction. The lines are computed from the analytical equation, whereas the points are from the MEDICI simulation. The parameters are $L = 5 \mu\text{m}$, $z = 0.3 \mu\text{m}$, and $h = 5 \mu\text{m}$.

Another observation is that, in Figs. 4 and 7, for the case of $h = 1 \mu\text{m}$, the computed results from the analytical equation is rather low compared to the results from the MEDICI simulator. This could be due to the effect of the finite generation source in MEDICI. This effect is more pronounced when the generation source is near the collecting junction. The generation source in MEDICI has a dimension of $0.2 \mu\text{m}$ and is therefore comparable to the ratio of $h/L = 1/5 = 0.2$.

The finite generation volume in MEDICI causes a shift at the center of mass of the generation source. In order to explain this, let us consider the current value at x distance away from the vertical collectors. With this consideration, it is assumed that the contributions due to the vertical junctions are negligible. According to [5], the collection can be written as

$$Q(z') = \frac{\sinh(\lambda z')}{\sinh[\lambda(h - z')]} \quad (18)$$

It can be seen that at $z' = 0$, the collection probability is zero, whereas at $z' = h$, the collection probability is unity. The finite dimension of the generation volume causes a different contribution between the top of the generation volume and the

bottom of the generation volume. At the top, the collection probability can be written as

$$Q_t = \frac{\sinh(\lambda(z' - \Delta z))}{\sinh[\lambda(h - (z' - \Delta z))]} \quad (19)$$

where Δz is half the dimension of the generation volume, i.e., $0.1 \mu\text{m}$. At the bottom, the collection probability is

$$Q_b = \frac{\sinh(\lambda(z' + \Delta z))}{\sinh[\lambda(h - (z' + \Delta z))]} \quad (20)$$

It can be seen that $Q_b > Q_t$ for all values of z . Therefore, the effective center of mass of the point source is, in fact, below the middle point between the top and the bottom edges of the generation volume used in the simulation. The new center of mass can be calculated from

$$z'_m = \frac{\int_{z' - \Delta z}^{z' + \Delta z} zQ(z)dz}{\int_{z' - \Delta z}^{z' + \Delta z} Q(z)dz} \quad (21)$$

where z'_m is the new center of mass for the MEDICI generation source, and $Q(z)$ can be approximated from (18) for the case where the beam is far away from the vertical junctions. For the case of $z' = 0.3 \mu\text{m}$, $L = 5 \mu\text{m}$, and $h = 5 \mu\text{m}$, the new center of mass was computed to be $z'_m = 0.3122 \mu\text{m}$. The amount of shift is affected mainly by the location of z' . The effect of this shift is not significant for large values of h . However, for small values of h , it is more pronounced.

Since the center of mass shifts down below z' , the curves from the MEDICI simulations would produce current values that are higher than the results from the analytical equations, with the point source located at z' . The computational results from the analytical equations, then, should use the values of the shifted center of mass in order to get the same curves as those from MEDICI. Increasing the z' values used in the computed analytical equations would give a higher charge collection probability, and this would coincide with the curve from the MEDICI simulations in Figs. 4 and 7.

In Figs. 5 and 8, the results from the analytical equations are found to be lower than the results from the MEDICI simulations for small values of diffusion length. This can, again, be explained from the finite dimension of the MEDICI generation source. Let us, again, consider the collection due to a source far away from the vertical junctions. The collection is affected mainly by the distance from the source to the bottom junction, i.e., $(h - z')/L$. The charge collection is exponentially related to this distance. The shift of the center of mass shortens this distance to $(h - z' - \Delta z)/L$, where $\Delta z' = z'_m - z'$. The actual shift is therefore $\Delta z'/L$. It can be seen that small values of L causes the effect of this shift to be more pronounced.

Similar observations can also be seen in almost all figures in regions that are very close to the vertical collectors. However, for this case, the effect is due to the finite lateral dimension of the source in MEDICI. The computed results are slightly lower than the one obtained from the MEDICI simulations for the regions that are very close to the vertical collectors, i.e.,

for regions near $x = 0$ for the L-shaped configuration and for regions near $x = 0$ and $x = d$ for the U-shaped configuration. This explains all discrepancies between the analytical equations and the MEDICI simulations.

Though the dimensions used in the simulations are larger than in most current devices, the equations are still valid as long as the point source assumption is satisfied. This assumption is satisfied when the dimensions are about one order of magnitude larger than the size of the generation volume. When the point source assumption cannot be satisfied, the current must be computed using (17) instead. This applies to smaller and shallower junctions.

The advantage of having an analytical expression is that the effects of the parameters can be seen more clearly in the equation. This is not very obvious when computer simulation is used. The effects of the parameters on the charge collection are discussed in Section V.

V. EFFECTS OF THE PARAMETERS

Figs. 4 and 7 show the effect of changing the junction depth on the EBIC profile. It can be seen that shallower junction depths give higher EBIC profiles. This result agrees with theory since shallower junctions have a higher probability for the charge to be collected. The difference becomes imperceptible as the junction depth becomes comparable to the diffusion length. This is because the changes in the z -direction are approximately exponential. In order to see these changes better, the plot should be in the logarithmic scale.

Another interesting observation is that the collection is dominated by the bottom junction side as the beam moves farther away from the vertical junction sides. If the width is large enough, we could expect the profile to approach a constant value. The difference is quite considerable when the junction depth is smaller than the diffusion length.

Figs. 5 and 8, on the other hand, give the plots for the effect of changing the diffusion length. Larger diffusion lengths result in higher EBIC profiles. This diffusion length value affects the slope of the profile in the logarithmic scale. For the case of the L-shaped geometry, if the bottom junction side is not present, the profile becomes that of the normal-collector configuration, with infinite surface recombination velocity.

The effect is mostly significant for beam distances that are farther away from the vertical collecting junction. However, the changes become negligible as the diffusion length becomes comparable to the distance between the source and the collecting junction. Therefore, in order to extract diffusion lengths from these geometries, the diffusion length must be smaller than the dimension of the junction well.

In Figs. 6 and 9, the results show the effect of the depth of the generation source. It can physically be explained that deeper generation sources result in higher EBIC profiles. This is simply because the distance between the generation source and the bottom collecting junction decreases, which results in a higher collection probability. In practice, it is impossible to generate a point source beyond a certain depth inside the material by using electron beam bombardment. The electron beam generates ephs in a volume similar in shape to

a teardrop [1]. Thus, a convolution, as given in (17), must be used.

Fig. 10 shows the effect of the width of the junction well for the case of the U-shaped geometry. In addition to extending the profile wider, the width also slightly affects the minimum EBIC current value. Wider junctions give lower values of the EBIC minimum. This can be explained by considering the effect of the diffusion length on the recombination of the minority carriers as the width is increased.

VI. CONCLUSION

This paper has provided the analytical equations for the charge collection profile when the beam scans from within the junction well. Two geometrical cases were considered: 1) the L-shaped junction and 2) the U-shaped junction. For these two cases, the analytical solution for a point generation source was derived using the Green's function. The case for extended generation volume can simply be obtained by convolution.

The computation results were verified with a semiconductor device simulation program on a computer to check its accuracy. The results show that the computation is in good agreement with the simulations. A discussion on the accuracy of the computation was also given.

The effects of the parameters were then investigated. The results were explained from a physical point of view by taking into account the diffusion lengths and the geometry of the structure.

APPENDIX

In this section, the details for the derivation of the EBIC profile are given. The problem is to solve (3) with the boundary conditions given in (4). In order to solve this, the eigenfunction expansion is used. The first step is to divide the region into two for simplification, where region I lies in $0 \leq z \leq z'$, and region II lies in $z' \leq z \leq h$. Within each region, the Green's function satisfies the homogenous equation

$$\frac{\partial^2 G(x, z|x', z')}{\partial x^2} + \frac{\partial^2 G(x, z|x', z')}{\partial z^2} - \lambda^2 G(x, z|x', z') = 0. \quad (\text{A1})$$

To solve this differential equation, a separation-of-variables technique is used. Let $G = X(x)Z(z)$. Substituting this into (A1) and dividing by XZ gives

$$\frac{X''}{X} + \frac{Z''}{Z} - \lambda^2 = 0 \quad (\text{A2})$$

where we denote the second derivatives of X and Z as X'' and Z'' , respectively. It can be seen that this equation can be separated into two ordinary differential equations (ODEs). The first ODE is

$$\frac{X''}{X} = -p^2 \quad (\text{A3})$$

where we have chosen the constant to be negative. In this case, the solution is in the form of sine and cosine. In order to satisfy

the boundary condition $G = 0$ at $x = 0$, we choose

$$X(x) = \sin(px) \quad (\text{A4})$$

and to satisfy the second condition at $x = d$, we need to have $\cos(px) = 0$ or

$$p_n = \frac{n\pi}{2d} \quad (\text{A5})$$

where n is an odd number, starting from 1, i.e., $n = 1, 3, 5, \dots$

The second ODE can be written in the following form:

$$Z'' - \mu_n^2 Z = 0 \quad (\text{A6})$$

where $\mu_n = (\lambda^2 + p_n^2)^{1/2}$. The solution is in the form of an exponential, and to satisfy $G = 0$ at $z = 0$, we can choose for region I

$$Z_{nI} = \sinh(\mu_n z) \quad (\text{A7})$$

and to satisfy $G = 0$ at $z = h$, we can choose for region II

$$Z_{nII} = \sinh(\mu_n(h - z)). \quad (\text{A8})$$

Therefore, the solution can be written as a linear combination of the individual solutions. That is,

$$\begin{aligned} G_I(x, z|x', z') &= \sum_{n=1,3,5,\dots}^{\infty} C_n \sin(p_n x) \sinh(\mu_n z) \\ G_{II}(x, z|x', z') &= \sum_{n=1,3,5,\dots}^{\infty} D_n \sin(p_n x) \sinh(\mu_n(h - z)) \end{aligned} \quad (\text{A9})$$

where C_n and D_n are constants with respect to x and z .

Since the solution must be continuous at $z = z'$, the solution must satisfy $G_I = G_{II}$, and if we utilize the orthogonality of $\sin(p_n x)$, we can obtain

$$C_n \sinh(\mu_n z') = D_n \sinh(\mu_n(h - z')) \quad (\text{A10})$$

from which we can define a new constant as follows:

$$E_n \equiv \frac{C_n}{\sinh(\mu_n(h - z'))} = \frac{D_n}{\sinh(\mu_n z')}. \quad (\text{A11})$$

Therefore, we can write the Green's function in (A9) in terms of E_n as follows:

$$\begin{aligned} G_I(x, z|x', z') &= \sum_{n=1,3,5,\dots}^{\infty} E_n \sinh(\mu_n(h - z')) \sin(p_n x) \sinh(\mu_n z) \\ G_{II}(x, z|x', z') &= \sum_{n=1,3,5,\dots}^{\infty} E_n \sinh(\mu_n z') \sin(p_n x) \sinh(\mu_n(h - z)). \end{aligned} \quad (\text{A12})$$

Now, we need to obtain the expression for E_n . For simplicity, let us write the Green's function in the form $G = \sum g_n(z, x', z') \sin(p_n x)$. Substituting this into (3) by multiplying it with $\sin(p_n x)$ and then integrating from 0 to d gives

$$\left\{ -(\mu_n)^2 g_n + \frac{d^2 g_n}{dz^2} \right\} \frac{d}{2} = -\frac{\sin(p_n x') \delta(z - z')}{D}. \quad (\text{A13})$$

The summation results in only one term due to the orthogonal property of $\sin(p_n x)$. Integrating this at a very small interval around $z = z'$ gives

$$\frac{d}{2} \left[\frac{dg_n}{dz} \right]_{z'-\varepsilon}^{z'+\varepsilon} = -\frac{\sin(p_n x)}{D} \quad (\text{A14})$$

where $\varepsilon \rightarrow 0$. Solving this equation results in the following expression for E_n :

$$E_n = \frac{2 \sin(p_n x)}{d D \mu_n h \sinh(\mu_n h)}. \quad (\text{A15})$$

Therefore, the final solution of the Green's function is given by

$$\begin{aligned} G_I(x, z|x', z') &= \sum_{n=1,3,5,\dots}^{\infty} \frac{2 \sin(p_n x)}{d D \mu_n h \sinh(\mu_n h)} \\ &\quad \times \sinh(\mu_n(h - z')) \sin(p_n x) \sinh(\mu_n z) \\ G_{II}(x, z|x', z') &= \sum_{n=1,3,5,\dots}^{\infty} \frac{2 \sin(p_n x)}{d D \mu_n h \sinh(\mu_n h)} \\ &\quad \times \sinh(\mu_n z') \sin(p_n x) \sinh(\mu_n(h - z)). \end{aligned} \quad (\text{A16})$$

The collected current for the L-shaped junction is obtained by substituting this expression into (1). The result is given in (7).

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