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Teo, Edmund K. H.; Garg, Hari K.; Zheng, Yuanjin; Cao, Mingzheng

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An Adaptive Filtering Algorithm for Direct-Conversion Receivers: Architecture and Performance Analysis

Yuanjin Zheng, *Member, IEEE*, Mingzheng Cao, *Student Member, IEEE*, Edmund K. H. Teo, and Hari K. Garg, *Senior Member, IEEE*

Abstract—An adaptive filtering algorithm is proposed in this paper to remove I/Q mismatch, dc offsets, flicker noise, and intersymbol interference (ISI) simultaneously in a direct-conversion receiver. I/Q mismatch is cancelled by a real valued adaptive mismatch canceller, and dc offsets are removed with one complex tap. In addition, flicker noise is modeled as a complex autoregressive (AR) random process so the system to be identified transforms to an ARX model. After estimating the coefficients in the model during the training period, the desired signal can be estimated using the decision feedback method. To accelerate the convergence of the algorithm and to reduce the estimation variance, an internal iterative algorithm is introduced. The convergence analysis of the proposed algorithm is also given, and the closed form of the minimum mean square error of the proposed algorithm is derived. Simulation results are provided to verify the superior performance of the proposed algorithm.

Index Terms—ARX model, convergence, dc offsets, direct-conversion receivers, flicker noise, I/Q mismatch, minimum mean-square error (MMSE).

I. INTRODUCTION

IN THE PAST, direct-conversion receivers (DCRs), illustrated in Fig. 1, are rarely used due to issues such as I/Q mismatch, dc offsets, even-order distortion, and flicker noise [1]–[4]. Among these issues, dc offsets and flicker noise are more serious and challenging [2], [5].

Several methods have been proposed to solve I/Q mismatch, dc offsets, and flicker noise [2]–[4], [7]. In [2], the dc offset is removed by employing an ac coupling which may corrupt the signal's dc components due to its high corner frequency [1], [7]; in addition, the flicker noise is regarded as equivalent in its effect on intersymbol interference (ISI) and is mitigated only by employing a finite-impulse response (FIR) minimum mean-square-error linear equalizer (MMSE-LE); it is also difficult to design a suitable ac coupling because the exact $1/f$ roll-off frequency is unknown in practice.

In this paper, a novel adaptive method to estimate the transmitted signal corrupted by ISI, I/Q mismatch, dc offsets, and

Fig. 1. Physical model of a DCR with a system block diagram of our baseband filter.

flicker noise is proposed. A real valued adaptive mismatch canceller is employed to cancel I/Q mismatch, and the varying dc offsets are removed by one complex tap [3], [8]. In addition, the flicker noise is modeled as a complex AR random process (Fig. 2), so the system transforms to an ARX model [9], [10]. By estimating the coefficients in the model during the training period, the desired signal can then be estimated by the decision feedback method. To accelerate the convergence of the algorithm and to reduce the estimation variance, an internal iterative algorithm is introduced [8]. On top of that, the convergence analysis of the proposed algorithm is given, and the closed form of the MMSE of the algorithm is derived as the lower bound of its performance.

The received signal model, architecture of the proposed adaptive filters, detailed algorithm, convergence analysis, and simulation results are illustrated and explained in Sections II–VI, respectively. Finally, the conclusions are drawn in Section VII.

II. SIGNAL MODEL

Let $t(n)$, $\bar{d}(n)$, $\tilde{d}(n)$ and $f(n)$ denote the transmitted signal, constant dc offset, varying dc offset, and flicker noise, respectively. In this paper, the bold lower case letters represent complex variables, the upper case letters represent vectors, and bold

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Y. Zheng is with the ICS Laboratory, Institute of Microelectronics, Singapore 117685 (e-mail: yuanjin@ime.a-star.edu.sg).

M. Cao, E. K. H. Teo, and H. K. Garg are with the Department of Electrical and Computer Engineering, Faculty of Engineering, National University of Singapore, Singapore 117576 (e-mail: eleghk@nus.edu.sg).

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Fig. 2. PSD of flicker noise.

upper case letters represent matrices. Here, n denotes the discrete time index. Then, the received signal $\bar{\mathbf{x}}(n)$ corrupted by ISI, dc offsets, and flicker noise can be written as

$$\bar{\mathbf{x}}(n) = \sum_{i=0}^{N_h-1} \mathbf{h}_i \mathbf{t}(n-i) + \tilde{\mathbf{d}}(n) + \bar{\mathbf{d}}(n) + \mathbf{f}(n). \quad (1)$$

Here, $(\mathbf{h}_0 = 1)$, $\mathbf{h}_i (i = 0, 1, \dots, N_h - 1)$ are the coefficients of the channel discrete-time impulse response (assuming minimum phase channel in this paper) and N_h is the order of channel response. Since $\mathbf{f}(n)$ is a highly colored noise, it can be modeled as an AR random process as [9], [11]

$$\mathbf{f}(n) = \sum_{i=1}^{N_a-1} (-\mathbf{a}_i) \mathbf{f}(n-i) + \mathbf{v}(n). \quad (2)$$

Here, $N_a - 1$ is the order, $\{\mathbf{a}_i\} (i = 1, 2, \dots, N_a - 1)$ are the coefficients, and $\mathbf{v}(n)$ is the innovation. $\mathbf{a}_i$ is chosen to make the power spectral density (PSD) of the generated $\mathbf{f}(n)$ close to the PSD of the flicker noise. Denote $A(z)$ as

$$A(z) = 1 + \sum_{i=1}^{N_a-1} \mathbf{a}_i z^{-i}. \quad (3)$$

Then, the z -transform of $\mathbf{f}(n)$ can be represented as

$$F(z) = \frac{V(z)}{A(z)} \quad (4)$$

where $V(z)$ is the z -transform of $\mathbf{v}(n)$ in (2).

Next, we take the transceiver I/Q mismatch into consideration. Assume that α and β denote the amplitude gains of the I and Q channels, respectively; ϕ denotes the phase splitter mismatch and is split equally between the I and Q channels for symmetry; $\tilde{\mathbf{m}}(n)$ denotes the equivalent transceiver I/Q mismatch induced signal, where $\tilde{m}_I(n)$ and $\tilde{m}_Q(n)$ are its real and

imaginary parts, respectively. Then, $\tilde{m}_I(n)$ and $\tilde{m}_Q(n)$ can be modeled by using a matrix notation according to [4] as

$$\begin{pmatrix} \tilde{m}_I(n) \\ \tilde{m}_Q(n) \end{pmatrix} = \begin{pmatrix} \alpha \cos(\phi/2) & \beta \sin(\phi/2) \\ \alpha \sin(\phi/2) & \beta \cos(\phi/2) \end{pmatrix} \begin{pmatrix} \bar{x}_I(n) \\ \bar{x}_Q(n) \end{pmatrix} = \tilde{\mathbf{M}} \begin{pmatrix} \bar{x}_I(n) \\ \bar{x}_Q(n) \end{pmatrix}. \quad (5)$$

Here, $\tilde{\mathbf{M}}$ represents the mismatch matrix. The mismatch model in (5) is applied to an AR model in (1) through signal $\bar{\mathbf{x}}(n)$. Since all of the delay effect due to the channel is modeled in (1), the mismatch delay (if any) is also modeled in (5).

When $\alpha = \beta$ and $\phi = 0^\circ$, (5) becomes $\mathbf{x}(n) = \alpha \bar{x}_I(n) + \beta \bar{x}_Q(n)$, and there is no I/Q mismatch. When $\alpha \neq \beta$, amplitude mismatch occurs. When $\phi \neq 0^\circ$, phase mismatch occurs. If mismatch exists, it can be mitigated by inverse modeling (e.g., refer to [11, Sec. 3.6.4]). Considering that there are four elements in the matrix $\tilde{\mathbf{M}}$, a four-real-tap equalizer is employed to model the inverse of it. According to the model in (5), a four-real-tap equalizer can remove both the amplitude mismatch and the phase mismatch simultaneously.

Furthermore, considering the effect of additive white Gaussian noise (AWGN) $\tilde{\mathbf{n}}(n)$, the digitalized baseband signal after the ADC can be represented as

$$\mathbf{x}(n) = x_I(n) + jx_Q(n) = \tilde{\mathbf{m}}(n) + \tilde{\mathbf{n}}(n). \quad (6)$$

Note that this paper deals with the baseband equivalent model. It can be verified that the noise generated in the RF front-end can be modeled equivalently in (6), which is a digital baseband equivalent model [13].

III. ARCHITECTURE OF THE ADAPTIVE FILTER

Based on the system model in Section II, an integrated adaptive filter and equalizer is proposed in Fig. 3. It is composed of, in sequence, a mismatch canceller, a dc offsets canceller, a complex feed-forward equalizer (CFE), and a complex decision-feedback equalizer (CDFE). In this sequence, the mismatch matrix will also be applied to the dc offset and flicker noise. However, the model is still valid, as typically the dc offset is generated due to the self-mixing of the mixer and transistor mismatch in either mixer in the I or Q channel [1]. Thus, the equivalent dc offset in the I and Q channels can be different, and this can be modeled by the I/Q mismatch model. Similarly, flicker noise is mainly generated due to the usage of CMOS transistors in building the RF front-end (LNA and mixer). Since the transistors built into the I channel have some mismatch with those in the Q channel during fabrication, the equivalent total flicker noise generated in the I channel is different from that in the Q channel, and thus the I/Q mismatch model is also applied to flicker noise to account for this effect.

In the mismatch canceller, which is a four-real-tap equalizer, the taps m_1 and m_4 are used to cancel the amplitude imbalance while the taps m_2 and m_3 are used to cancel the component of $\bar{x}_Q(n)$ in $x_I(n)$ and the component of $\bar{x}_I(n)$ in $x_Q(n)$, respectively. The output of the mismatch canceller $\mathbf{y}(n)$ is treated as a complex valued signal with the signal in the I channel as its real part and the signal in the Q channel as its imaginary part. At

Fig. 3. Signal flow and the proposed adaptive filter.

$\mathbf{y}(n)$, a complex valued tap cancels off the dc offsets [3]. After which, CFFE mitigates ISI followed by CDFE, which mitigates flicker noise and provides data estimation.

IV. ALGORITHM

Using the model in Section II, the output of the mismatch canceller, as in Fig. 3, is

$$\mathbf{y}(n) = y_I(n) + jy_Q(n) \quad (7)$$

$$y_I(n) = m_1(n) \times x_I(n) + m_2(n) \times x_Q(n) \quad (8)$$

$$y_Q(n) = m_3(n) \times x_I(n) + m_4(n) \times x_Q(n). \quad (9)$$

Here, $\{m_i\} (i = 1, \dots, 4)$ denotes the taps of the mismatch canceller, $\tilde{\mathbf{w}}^*(n)$ denotes the tap of the dc offsets canceller, and then the output $\mathbf{y}'(n)$ can be represented as

$$\mathbf{y}'(n) = \mathbf{y}(n) + \tilde{\mathbf{w}}^*(n). \quad (10)$$

The superscript “*” denotes complex conjugation, and all of the taps of the filters are given in their complex conjugation forms in Fig. 3. During the derivation of the algorithm, AWGN is dropped. Despite this, simulated results (with AWGN) using the derived algorithm are still very good. In addition, in practical situations, the input SNR for receiver is normally high (around 20 dB), so neglecting AWGN here is reasonable. Referring to Fig. 3, $\mathbf{y}'(n)$ is $\mathbf{x}(n)$ with mismatch and dc offsets cancelled.

Drop $\tilde{\mathbf{d}}(n)$, $\bar{\mathbf{d}}(n)$ and replace $\bar{\mathbf{x}}(n)$ in (1) with $\mathbf{y}'(n)$. Take the z transform and, according to (2)–(4), we obtain

$$A(z)Y'(z) = B(z)T(z) + V(z). \quad (11)$$

Here, $B(z) = A(z)H(z)$. $Y'(z)$, $T(z)$ and $H(z)$ are the z -transforms of $\mathbf{y}'(n)$, $\mathbf{t}(n)$ and $\mathbf{h}(n)$, respectively. $B(z)$ can also be represented as

$$B(z) = 1 - \sum_{i=1}^{N_b-1} b_i z^{-i} \quad (N_b = N_a + N_h - 1). \quad (12)$$

In this paper, we assume that the channel is minimum phase.

Since (11) is an ARX model, given $\mathbf{y}'(n)$ and $\mathbf{t}(n)$, the coefficients of $A(z)$ and $B(z)$ can be estimated as $\{\hat{\mathbf{a}}_i\} (i = 1, 2, \dots, N_a - 1)$ and $\{\hat{\mathbf{b}}_i\} (i = 1, 2, \dots, N_b - 1)$, respectively, during training period. Then, the desired signal can be estimated during decision period by [8], [10]

$$\hat{\mathbf{t}}(n) = \left(\mathbf{y}'(n) + \sum_{i=1}^{N_a-1} \hat{\mathbf{a}}_i \mathbf{y}'(n-i) \right) + \sum_{i=1}^{N_b-1} \hat{\mathbf{b}}_i \hat{\mathbf{t}}(n-i). \quad (13)$$

Here, $\hat{\mathbf{t}}(n)$ denotes the estimated transmitted sequence $\mathbf{t}(n)$. The output $\mathbf{z}(n)$ and the filtering error signal $\mathbf{e}(n)$ during training period are

$$\mathbf{z}(n) = \sum_{j=0}^{k_1-1} \tilde{\mathbf{w}}_j^*(n) \mathbf{y}'(n-j) + \sum_{k=1}^{k_2-1} \tilde{\mathbf{w}}_k^*(n) \mathbf{t}(n-k) \quad (14)$$

$$\mathbf{e}(n) = \mathbf{t}(n) - \mathbf{z}(n) \quad (15)$$

where k_1 and k_2 denote the orders of the CFFE and CDFE taps, respectively. $\tilde{\mathbf{w}}_j^*(n) (j = 0, 1, \dots, k_1 - 1)$, $\tilde{\mathbf{w}}_k^*(n) (k = 0, 1, \dots, k_2 - 1)$ and $\mathbf{t}(n)$ denote the CFFE tap values, CDFE tap values, and the training signal, respectively. The coefficients of $A(z)$ and $B(z)$ in (11) are estimated by the taps $\{\tilde{\mathbf{w}}_j^*\}$ and $\{\tilde{\mathbf{w}}_k^*\}$ in (14), respectively, by using the least-mean-square (LMS) algorithm [11], [12]. Without loss of generality, assume $k_1 = N_a$ and $k_2 = N_b$. In this case, the estimated coefficients $\{\hat{\mathbf{a}}_i\}$ and $\{\hat{\mathbf{b}}_i\}$ will be obtained by the adaptations of $\{\tilde{\mathbf{w}}_j^*\}$ and $\{\tilde{\mathbf{w}}_k^*\}$, respectively.

Denote μ_m, μ_d, μ_f and μ_b as the step sizes for adjusting $\{m_i\} (i = 1, \dots, 4)$, $\tilde{\mathbf{w}}^*$, $\{\tilde{\mathbf{w}}_j^*\} (j = 0, 1, \dots, k_1 - 1)$ and $\{\tilde{\mathbf{w}}_k^*\} (k = 2, \dots, k_2 - 1)$, respectively. According to the LMS algorithm, the closed forms of the taps adaptations are given as

$$m_i(n+1) = m_i(n) - \mu_m \times \nabla m_i(n), \quad i = 1, \dots, 4 \quad (16)$$

$$\tilde{\mathbf{w}}^*(n+1) = \tilde{\mathbf{w}}^*(n) + 2 \mu_d \sum_{j=0}^{k_1-1} \hat{\mathbf{w}}_j(n) \mathbf{e}(n) \quad (17)$$

$$\tilde{\mathbf{w}}_j^*(n+1) = \tilde{\mathbf{w}}_j^*(n) + 2 \mu_f \mathbf{y}^{t*}(n-j) \mathbf{e}(n), \quad j = 0, 1, \dots, k_1 - 1 \quad (18)$$

$$\tilde{\mathbf{w}}_k^*(n+1) = \tilde{\mathbf{w}}_k^*(n) + 2 \mu_b \mathbf{t}^*(n-k) \mathbf{e}(n), \quad k = 1, 2, \dots, k_2 - 1 \quad (19)$$

where $\{\nabla m_i\}$ in (16) can be derived by taking partial derivatives of $|e(n)|^2$ with respect to $\{m_i\}$.

It is obvious that the adaptations of $\{m_i\}$, $\tilde{\mathbf{w}}^*$, $\{\tilde{\mathbf{w}}_j^*\}$ and $\{\tilde{\mathbf{w}}_k^*\}$ influence mutually. Hence, we propose an internal iterative algorithm below to accelerate the convergence of the algorithm and to reduce the estimation variance [8].

Let $\tilde{N}$ denote the total number of internal iterations. $m_i^{(l)}(n)$, $\tilde{\mathbf{w}}^{(l)}(n)$, $\tilde{W}^{(l)}(n)$, $\tilde{W}^{(l)}(n)$ and $e^{(l)}(n)$ denote the updated values of $m_i(n)$, $\tilde{\mathbf{w}}^*(n)$, $\tilde{W}^{(l)}(n)$, $\tilde{W}^{(l)}(n)$ and $e(n)$, respectively, after $l(l=0, 1, \dots, \tilde{N}-1)$ internal iterations at time n . To reduce the complexity of the algorithm, $m_i(n-j)$ and $\tilde{\mathbf{w}}^*(n-j)(j=1, 2, \dots, k_1-1)$ are only updated and do not take part in the internal iterations at time n . Simulation results show that whether or not $m_i(n-j)$ and $\tilde{\mathbf{w}}^*(n-j)(j=1, 2, \dots, k_1-1)$ take part in the internal iterations makes little difference to the performance.

Hence, the internal iterations for $l=0, 1, \dots, \tilde{N}-1$ are as follows:

$$m_i^{(l+1)}(n) = m_i^{(l)}(n) - \mu_m \times H_i \left(X(n), e^{(l)}(n), \tilde{W}^{(l)}(n) \right) \quad (20)$$

$$\tilde{\mathbf{w}}^{*(l+1)}(n) = \tilde{\mathbf{w}}^{*(l)}(n) - \mu_d \times I \left(\tilde{W}^{(l)}(n), e^{(l)}(n) \right) \quad (21)$$

$$\begin{aligned} \tilde{W}^{(l+1)}(n) &= \tilde{W}^{(l)}(n) \\ &\quad - \mu_f \times J \left(X(n), m_i^{(l)}(n), \tilde{\mathbf{w}}^{*(l)}(n), e^{(l)}(n) \right) \end{aligned} \quad (22)$$

$$\tilde{W}^{(l+1)}(n) = \tilde{W}^{(l)}(n) - \mu_b \times K \left(T'(n), e^{(l)}(n) \right) \quad (23)$$

$$e^{(l+1)}(n) = G \left(X(n), m_i^{(l+1)}(n), \tilde{\mathbf{w}}^{*(l+1)}(n), \tilde{W}^{(l+1)}(n), \tilde{W}^{(l+1)}(n) \right) \quad (24)$$

where $G(\bullet)$, $H(\bullet)$, $I(\bullet)$, $J(\bullet)$ and $K(\bullet)$ above are general symbolic functions.

At time $n+1$, the initial tap values are the latest updated tap values at time n . The next internal iteration goes so forth. After all of the taps converged to their optimum values, the training sequence $\mathbf{t}(n)$ is replaced by the estimated sequence $\hat{\mathbf{t}}(n)$.

V. PERFORMANCE ANALYSIS

A. Convergence Analysis

Denote

$$A = [\mathbf{a}_0 \quad \mathbf{a}_1 \quad \dots \quad \mathbf{a}_{N_a-1}]^H (\mathbf{a}_0 = 1) \quad (25)$$

$$B' = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \dots \quad \mathbf{b}_{N_b-1}]^H \quad (26)$$

$$Y'(n) = [\mathbf{y}^*(n) \quad \mathbf{y}^*(n-1) \quad \dots \quad \mathbf{y}^*(n-N_a+1)]^H \quad (27)$$

$$\begin{bmatrix} \mathbf{R}_Y \\ \mathbf{R}_T \end{bmatrix} = E \begin{bmatrix} Y'(n)Y'^H(n) \\ T'(n)T'^H(n) \end{bmatrix}. \quad (28)$$

During training period, according to (14), $\mathbf{z}(n)$ can be written as

$$\mathbf{z}(n) = \tilde{W}^H(n)Y'(n) + \tilde{W}^H(n)T'(n). \quad (29)$$

It is obvious that the algorithm will converge only when the taps of all of the equalizers converge and the convergence of CFFE and CDFE plays a dominant role. Denote $\lambda_{Y'_{\max}}$ and $\lambda_{T'_{\max}}$ as the maximum eigenvalues of $\mathbf{R}_{Y'}$ and $\mathbf{R}_{T'}$, respectively. When $\{m_i\}(i=1, \dots, 4)$ and $\tilde{\mathbf{w}}^*$ are kept fixed, $\mathbf{y}'(n)$ can be regarded as a channel response of input $\mathbf{x}(n)$ and 1. Therefore, when only considering CFFE and CDFE, they will converge when μ_f and μ_b meet the following requirements (see [11, Sec. 6.2]):

$$0 < \mu_f < 1/\lambda_{Y'_{\max}} \quad (30)$$

$$0 < \mu_b < 1/\lambda_{T'_{\max}}. \quad (31)$$

Equations (30) and (31) only guarantee the convergence of CFFE and CDFE. When $\tilde{W}^H(n)$ and $\tilde{W}^H(n)$ are adapted to two sets of values, e.g., $\tilde{W}_c^H$ and $\tilde{W}_c$ (not necessarily the optimum convergence values), the algorithm will converge when $\{m_i\}(i=1, \dots, 4)$ and $\tilde{\mathbf{w}}^*$ converge as well. In the following convergence analysis of $\{m_i\}(i=1, \dots, 4)$ and $\tilde{\mathbf{w}}^*$, $\tilde{W}^H(n)$ and $\tilde{W}^H(n)$ are regarded as constant values $\tilde{W}_c^H$ and $\tilde{W}_c$.

Denote

$$X_I(n) = [x_I(n) \quad x_I(n-1) \quad \dots \quad x_I(n-N_a+1)]^T \quad (32)$$

$$X_Q(n) = [x_Q(n) \quad x_Q(n-1) \quad \dots \quad x_Q(n-N_a+1)]^T \quad (33)$$

$$P = \begin{bmatrix} E[Y'(n)\mathbf{t}^*(n)] \\ E[T'(n)\mathbf{t}^*(n)] \end{bmatrix} \quad (34)$$

$$\mathbf{R} = E \left(\begin{bmatrix} Y'(n) \\ T'(n) \end{bmatrix} [Y'^H(n) \quad T'^H(n)] \right). \quad (35)$$

Denote ξ as the mean square error. Then

$$\begin{aligned} \xi &= E[|e(n)|^2] \\ &= E[|\mathbf{t}^2(n)|] - \left(\frac{\tilde{W}_c}{\tilde{W}_c} \right)^H \\ &\quad \times P - P^H \left(\frac{\tilde{W}_c}{\tilde{W}_c} \right) + \left(\frac{\tilde{W}_c}{\tilde{W}_c} \right)^H \mathbf{R} \left(\frac{\tilde{W}_c}{\tilde{W}_c} \right). \end{aligned} \quad (36)$$

Denote

$$\begin{bmatrix} \mathbf{R}_{XI} \\ \mathbf{R}_{XQ} \\ \mathbf{R}_{IQ} \end{bmatrix} = E \begin{bmatrix} X_I(n)X_I^T(n) \\ X_Q(n)X_Q^T(n) \\ X_I(n)X_Q^T(n) \end{bmatrix}. \quad (37)$$

According to the LMS algorithm

$$m_1(n+1) = m_1(n) - \mu_m \frac{\partial |e(n)|^2}{\partial m_1}. \quad (38)$$

Denote $v_{m_1}(n) = m_1(n) - m_{1o}$, where the subscript “o” represents the optimum value. By taking the expectation of (38) and setting $(\partial \xi / \partial m_1)$ to zero, it can be obtained that

$$E[v_{m_1}(n+1)] = (1 - 2\mu_m \tilde{W}_c^H \mathbf{R}_{XI} \tilde{W}_c) E[v_{m_1}(n)]. \quad (39)$$

Therefore, the convergence condition for m_1 is

$$0 < \mu_m < (\widehat{W}_c^H \mathbf{R}_{XI} \widehat{W}_c)^{-1}. \quad (40)$$

By similar derivations, it can be obtained that (40) is also the convergence condition for m_3 . The convergence condition for m_2 and m_4 is

$$0 < \mu_m < \left(\widehat{W}_c^H \mathbf{R}_{XQ} \widehat{W}_c \right)^{-1}. \quad (41)$$

Combining (40) and (41), the I/Q mismatch canceller (m_i) will converge when

$$0 < \mu_m < \min \left(\left(\widehat{W}_c^H \mathbf{R}_{XI} \widehat{W}_c \right)^{-1}, \left(\widehat{W}_c^H \mathbf{R}_{XQ} \widehat{W}_c \right)^{-1} \right). \quad (42)$$

Denote

$$\widehat{W}_c = [\widehat{\mathbf{w}}_{c,0}^*, \widehat{\mathbf{w}}_{c,1}^*, \dots, \widehat{\mathbf{w}}_{c,N_a-1}^*]^H \quad (43)$$

$$\widetilde{W}_c = [\widetilde{\mathbf{w}}_{c,0}^*, \widetilde{\mathbf{w}}_{c,1}^*, \dots, \widetilde{\mathbf{w}}_{c,N_a-1}^*]^H. \quad (44)$$

It can be obtained that

$$\begin{aligned} E[\mathbf{v}_{\bar{w}}^*(n+1)] &= E[\mathbf{v}_{\bar{w}}^*(n)] - 2\mu_d E[\mathbf{v}_{\bar{w}}^*(n)] \widehat{W}_c^H \mathbf{I}_{N_a \times N_a} \widehat{W}_c \\ &\quad \times E[\mathbf{v}_{\bar{w}}^*(n)] \left\{ 1 - 2\mu_d \left[\sum_{i=0}^{N_a-1} \widehat{\mathbf{w}}_{c,i}^* \sum_{i=0}^{N_a-1} \widehat{\mathbf{w}}_{c,i} \right] \right\}. \end{aligned} \quad (45)$$

Here, $\mathbf{v}_{\bar{w}}(n) = \bar{\mathbf{w}}(n) - \bar{\mathbf{w}}_o$, $\bar{\mathbf{w}}_o$ is obtained by setting $(\partial \xi / \partial \bar{\mathbf{w}}^*)$ to zero. Then, the convergence condition of the dc offset canceller ($\bar{\mathbf{w}}^*$) is

$$0 < \mu_d < \left[\sum_{i=0}^{N_a-1} \widehat{\mathbf{w}}_{c,i} \sum_{i=0}^{N_a-1} \widehat{\mathbf{w}}_{c,i}^* \right]. \quad (46)$$

Because we can always find μ_f, μ_b, μ_m and μ_d to satisfy (30), (31), (42), and (46) simultaneously, the mean convergence of the algorithm is guaranteed.

B. MMSE Analysis

According to (11), $\mathbf{t}(n)$ in time domain can be written as

$$\mathbf{t}(n) = [A^H \quad B'^H] \begin{bmatrix} Y_o'(n) \\ T'(n) \end{bmatrix} - \mathbf{v}(n). \quad (47)$$

Here, $Y_o'(n)$ represents the received signal vector with mismatch and dc offsets fully cancelled, i.e., $\{m_i\}$ and $\bar{\mathbf{w}}$ are at their optimum values $\{m_{io}\}$ and $\bar{\mathbf{w}}_o$, respectively. Let $\widehat{W}(n)$ and $\widetilde{W}(n)$ denote the estimated vectors of A and B' , respectively, $\Delta \widehat{W}(n)$ and $\Delta \widetilde{W}(n)$ represent the weight-error vectors, and $\Delta Y'(n)$ represents the error vector caused by the estimation error of $\{m_i\}$ and $\bar{\mathbf{w}}$. Then, after all of the taps of the

mismatch canceller, dc canceller, CFFE, and CDFE are close to their optimum, according to (14), the output $\mathbf{z}(n)$ can be represented as

$$\begin{aligned} \mathbf{z}(n) &= \widehat{W}^H(n) Y'(n) + \widetilde{W}^H(n) T'(n) \\ &= [A^H \quad B'^H] \begin{bmatrix} Y_o'(n) \\ T'(n) \end{bmatrix} \\ &\quad + [\Delta \widehat{W}^H(n) \quad \Delta \widetilde{W}^H(n)] \begin{bmatrix} Y_o'(n) \\ T'(n) \end{bmatrix} \\ &\quad + A^H \Delta Y'(n) + \Delta \widehat{W}^H(n) \Delta Y'(n). \end{aligned} \quad (48)$$

From (15), (46), and (48), it can be obtained that

$$\begin{aligned} \mathbf{e}(n) &= [-\Delta \widehat{W}^H(n) \quad -\Delta \widetilde{W}^H(n)] \begin{bmatrix} Y_o'(n) \\ T'(n) \end{bmatrix} \\ &\quad - A^H \Delta Y'(n) - \Delta \widehat{W}^H(n) \Delta Y'(n) - \mathbf{v}(n). \end{aligned} \quad (49)$$

Recall that $\xi = E[|e(n)|^2]$. Assume that $\mathbf{v}(n)$ is white, zero mean with covariance σ_v^2 ; $\mathbf{v}(n)$, $\Delta \widehat{W}(n)$, $\Delta Y'(n)$, $Y_o'(n)$, and $\Delta \widetilde{W}(n)$ are mutually independent; $\Delta \widehat{W}(n)$, $\Delta Y'(n)$ and $\Delta \widetilde{W}(n)$ are zero mean; and $\Delta \widehat{W}(n)$ and $\Delta \widetilde{W}(n)$ are independent of $Y_o'(n)$ and $T'(n)$, respectively. Then

$$\begin{aligned} E \left\{ \mathbf{v}(n) \left[T'^H(n) \Delta \widetilde{W}(n) \right. \right. \\ \left. \left. - \Delta Y'^H(n) A - Y_o'^H(n) \Delta \widehat{W}(n) \right] \right\} = 0 \end{aligned} \quad (50)$$

$$\begin{aligned} E \left\{ \left[\Delta \widetilde{W}^H(n) T'(n) - A^H \Delta Y'(n) \right. \right. \\ \left. \left. - \Delta \widehat{W}^H(n) Y_o'(n) \right] \mathbf{v}^*(n) \right\} = 0. \end{aligned} \quad (51)$$

Denote

$$\begin{bmatrix} \mathbf{R}_{\Delta Y'}(n) \\ \mathbf{R}_{Y_o'}(n) \\ \mathbf{K}_{\widehat{W}}(n) \\ \mathbf{K}_{\widetilde{W}}(n) \end{bmatrix} = E \begin{bmatrix} \Delta Y'(n) \Delta Y'^H(n) \\ Y_o'(n) Y_o'^H(n) \\ \Delta \widehat{W}(n) \Delta \widehat{W}^H(n) \\ \Delta \widetilde{W}(n) \Delta \widetilde{W}^H(n) \end{bmatrix}. \quad (52)$$

Denote $\xi_1(n)$ as the error caused by the weight-error vectors $\Delta \widehat{W}(n)$ and $\Delta \widetilde{W}(n)$, $\xi_2(n)$ and $\xi_3(n)$ as the errors caused by the $A^H \Delta Y'(n)$ and $\Delta \widehat{W}^H(n) \Delta Y'(n)$, respectively. Then, according to the derivations in [11, Sec. 6.3.1], we have

$$\xi_1(n) = \text{tr} \left\{ [\mathbf{K}_{\widehat{W}}(n) \quad \mathbf{K}_{\widetilde{W}}(n)] \begin{bmatrix} \mathbf{R}_{Y_o'} \\ \mathbf{R}_{T'} \end{bmatrix} \right\} \quad (53)$$

$$\xi_2(n) = \text{tr} \left\{ E[A^H \Delta Y'(n) \Delta Y'^H(n) A] \right\} \quad (54)$$

$$\xi_3(n) = \text{tr} [\mathbf{K}_{\widehat{W}}(n) \mathbf{R}_{\Delta Y'}(n)] \quad (55)$$

$$\begin{aligned} \xi &= \xi(n) |_{n \rightarrow \infty} \\ &= (\xi_1(n) + \xi_2(n) + \xi_3(n) + \sigma_v^2) |_{n \rightarrow \infty} \end{aligned} \quad (56)$$

According to the misadjustment analysis of the LMS algorithm in [11, Sec. 6.3.3], it can be approximated that

$$\xi_1(\infty) = \sigma_v^2 [\mu_f \text{tr}(\mathbf{R}_{Y_o'}) + \mu_b \text{tr}(\mathbf{R}_{T'})] \quad (57)$$

$$\text{tr}(\mathbf{R}_{Y_o'}) = [m_{1o}^2 + m_{3o}^2 \quad m_{2o}^2 + m_{4o}^2 \quad 2(m_{1o}^2 m_{2o}^2 + m_{3o}^2 m_{4o}^2)] \\ \cdot \left[\text{tr} \begin{pmatrix} \mathbf{R}_{XI} \\ \mathbf{R}_{XQ} \\ \mathbf{R}_{IQ} \end{pmatrix} \right] + N_a C \quad (58)$$

where C is a constant that is determined by the optimum values of $\{m_i\} (i = 1, \dots, 4)$ and $\bar{\mathbf{w}}$.

Similarly, it can be obtained that

$$\xi_2(\infty) \geq \sigma_v^2 \left[\mu_m |\mathbf{a}_{\min}|^2 \text{tr}(\mathbf{R}_{XI} + \mathbf{R}_{XQ}) + \mu_d \sum_{i=0}^{N_a-1} |\mathbf{a}_i|^2 \right] \quad (59)$$

Here, $\mathbf{a}_{\min}$ denotes the minimum element of A in (25).

The weight-error vector $\Delta \hat{\mathbf{W}}(n)$ is much smaller than the true vector A when the taps $\hat{\mathbf{w}}_j(n)$ are near convergence. Assuming that the value of $\Delta \hat{\mathbf{W}}(n)$ is 10% that of A , then $\xi_3(n)$ is only 1% of $\xi_2(n)$. Therefore, $\xi_3(n)$ can be ignored in (56). Thus, when $n \rightarrow \infty$, (56) can be written as

$$\xi(\infty) \approx \xi_1(\infty) + \xi_2(\infty) + \sigma_v^2. \quad (60)$$

When there is no mismatch canceller, dc offsets canceller, and CDFE, $m_{1o} = m_{3o} = 1, m_{2o} = m_{4o} = 0, \bar{\mathbf{w}}_o = 0, \mu_b = \mu_m = \mu_d = 0$, then (60) becomes

$$\xi(\infty) = \sigma_v^2 \mu_f \text{tr}(\mathbf{R}_{Y_o'}) + \sigma_v^2. \quad (61)$$

If there is no I/Q mismatch, dc offsets, or flicker noise, then $\{\mathbf{a}_i\} = 0 (i = 1, 2, \dots, N_a - 1)$. The received signal to be processed by the adaptive filter becomes the transmitted signal that is only corrupted by ISI and a white noise with variance of σ_v^2 . The autocorrelation matrix of the transmitted signal corrupted by ISI in this case becomes $\mathbf{R}_{Y_o'}$. Then, (61) becomes the MMSE of a standard LMS algorithm.

VI. SIMULATION RESULTS

In Fig. 4, five curves are illustrated to show the efficiency of the mismatch canceller when the dc offset canceller is absent. In the simulations, 16 quaternary amplitude modulation (QAM) signals are passed through a given channel and then corrupted by I/Q mismatch and AWGN. In condition 1, $\alpha = 1.5, \beta = 0.5, \phi = 10^\circ, \tilde{N} = 6$, and, in condition 2, $\alpha = 1.5, \beta = 0.5, \phi = 20^\circ, \tilde{N} = 6$. Without the mismatch canceller, the bit error rates (BERs) of the two curves are around 10^{-1} , but with the canceller they decreased dramatically.

The method to cancel dc offsets by one tap is discussed in many papers [3], [8]. Fig. 5 is obtained when quaternary phase-shift keying (QPSK) signals are passed through a given channel. When only flicker noise and ISI exist, and only CFFE is employed, the BER is around 10^{-1} . However, after employing the CDFE, the BER reaches 10^{-4} at SNR = 20 dB. When ISI, flicker noise, mismatch ($\alpha = 1.2, \beta = 0.8, \phi = 10^\circ$), and varying dc offsets with the similar energy of the transmitted

Fig. 4. BER performance of the mismatch canceller.

Fig. 5. BER performance of the proposed algorithm.

signal exist simultaneously and $\tilde{N} = 6$, the BER reaches 10^{-4} at SNR = 25 dB.

Fig. 6 is obtained when the QPSK signal is passed through a given channel and corrupted by I/Q mismatch, dc offsets, flicker noise, and AWGN as modeled in Section II. In the simulations, the internal iterative number $\tilde{N} = 6$. The learning curve of the algorithm is obtained by an ensemble average of the sequence $|e(n)|^2$ over 100 independent runs. As shown in Fig. 6, the learning curve with internal iterations converges within 1000 iterations, which is much faster than that without internal iterations. The MSE of the algorithm with internal iterations is also smaller than that of the algorithm without internal iterations. Generally, the proposed algorithm is applied to a slow fading channel. For a fast fading channel, the proposed algorithm should improve convergence speed by incorporating some dedicated speeding algorithm.

To verify the proposed algorithm, comprehensive simulation results are given when binary phase-shift keying (BPSK), QPSK, and 16-QAM signals are passed through a given

Fig. 6. Learning curve of the proposed algorithm without and with internal iterations.

Fig. 7. Comprehensive BER performance of BPSK.

channel and are corrupted by all of the distortions such as I/Q mismatch, dc offsets, flicker noise, and AWGN, respectively. As shown in Figs. 7–9, the lower bound is obtained when the transmitted signal is only corrupted by ISI and AWGN. In the simulations, $N = 6$, and $\text{SDR} = 10$ dB where SDR represents the power ratio of the desired signal to the dc offset. The flicker noise is generated as modeled in Section II, but its power is changed accordingly. In condition 1, $\alpha = 1.05$, $\beta = 0.95$, $\phi = 5^\circ$, and $\text{SFR} = 20$ dB. Here, SFR represents the power ratio of the desired signal to the flicker noise. In condition 2, $\alpha = 1.05$, $\beta = 0.95$, $\phi = 5^\circ$, and $\text{SFR} = 10$ dB. In condition 3, $\alpha = 1.1$, $\beta = 0.9$, $\phi = 10^\circ$, and $\text{SFR} = 10$ dB. As shown in Figs. 7–9, the BER curves of the three conditions are close to one another and are robust to mismatch and flicker noise.

Fig. 10 shows the learning curve of MSE and the derived MMSE. It is obtained when a QPSK signal is passed through a given channel and corrupted by I/Q mismatch, dc offsets, and flicker noise. The learning curve of the algorithm is obtained by an ensemble average of the sequence $|e(n)|^2$ over 100 indepen-

Fig. 8. Comprehensive BER performance of QPSK.

Fig. 9. Comprehensive BER performance of 16 QAM.

Fig. 10. Learning curve of MSE and the derived MMSE.

dent runs. As shown in Fig. 10, the derived MMSE provides a close lower bound for the steady-state MSE.

VII. CONCLUSION

In this paper, an adaptive algorithm has been proposed to cancel I/Q mismatch, dc offsets, flicker noise, and ISI simultaneously in both the I and Q channels in DCRs by using adaptive filters. To accelerate the convergence of the algorithm and to reduce the estimation variance, an internal iterative algorithm is proposed. In addition, the convergence analysis of the proposed algorithm is given, and the closed form of the MMSE of the proposed algorithm is derived. Simulation results are provided to verify the superior performance of the proposed algorithm.

REFERENCES

- [1] B. Razavi, "Design considerations for direct-conversion receivers," *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, vol. 44, no. 6, pp. 428–435, Jun. 1997.
- [2] W. Namgoong and T. H. Meng, "Direct-conversion RF receiver design," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 518–529, Mar. 2001.
- [3] M. Faulkner, "DC offset and IM2 removal in direct conversion receivers," *IEE Proc. Commun.*, vol. 149, no. 3, pp. 179–184, 2002.
- [4] J. K. Cavers and M. W. Liao, "Adaptive compensation for imbalance and offset losses in direct conversion transceivers," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 581–588, Oct. 1993.
- [5] B. Razavi, "RF IC design challenges," in *Proc. Design Automation Conf.*, San Francisco, CA, 1998, pp. 408–418.
- [6] A. A. Abidi, "Direct-conversion radio transceivers for digital communications," *IEEE J. Solid-State Circuits*, vol. 30, pp. 1399–1410, 1995.
- [7] H. Yoshida, H. Tsurumi, and Y. Suzuki, "DC offset canceller in a direct conversion receiver for QPSK signal reception," in *Proc. 9th IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, 1998, vol. 3, pp. 1314–1318.
- [8] C. Mingzheng, Z. Yuanjin, and H. K. Garg, "A novel adaptive equalizer and compensator for direct conversion receivers," in *Proc. 9th IEEE Singapore Int. Conf. Commun. Syst.*, 2004, pp. 436–440.
- [9] C. Mingzheng, Z. Yuanjin, and H. K. Garg, "A novel algorithm for DC offsets and flicker noise cancellation in direct conversion receivers," in *Proc. 9th IEEE Singapore Int. Conf. Commun. Syst.*, 2004, pp. 441–445.
- [10] L. Ljung, *System Identification: Theory for the User*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1999.
- [11] B. F. Boroujeny, *Adaptive Filters: Theory and Applications*. New York: Wiley, 1998.
- [12] S. Haykin, *Adaptive Filter Theory*, 4th ed. Upper Saddle River, NJ: Prentice-Hall, 2002.
- [13] J. G. Proakis, *Digital Communication*, 4th ed. New York: McGraw-Hill, 2001.

Yuanjin Zheng (M'03) received the B.S. and M.Eng. degrees from Xi'an Jiaotong University, China, in 1993 and 1996, respectively, and the Ph.D. degree from Nanyang Technological University, Singapore, in 2001.

From July 1996 to April 1998, he was with the National Key Lab of Optical Communication Technology, University of Electronic Science and Technology of China, as a Research Scientist. In March 2001, he joined the Institute of Microelectronics, A*STAR, Singapore, where he has helped to develop various wireless such as WLAN, WCDMA, and UWB radio transceiver ICs. His research interests are RF transceiver and communication systems, analog and digital ICs, and DSP algorithm design and implementations. He has published over 60 international journal and conference papers and one book chapter and has four U.S. patents granted and two U.S. patents pending.

Mingzheng Cao (S'07) received the B.Sc. degree in electrical engineering and automation from Tianjin University, Tianjin, China, in 1996, and the M.Sc. degree in electrical engineering from the National University of Singapore (NUS), Singapore, in 2005. He is currently working toward the Ph.D. degree in electrical engineering at the New Jersey Institute of Technology (NJIT), Newark.

From 1996 to 2001, he was with the China Petroleum and Chemical Corporation (Sinopec), and from 2001 to 2003, he was with Automation and Drive Group, Siemens Ltd., China. During his graduate studies at NUS, he was with the Institute of Microelectronics (IME) of Singapore, where he was involved with the distortion mitigation for direct-conversion receiver design. His research focuses on signal processing for wireless communications.

E. K. H. Teo received the B.Eng. (Hons.) degree in electrical engineering from the National University of Singapore (NUS), Singapore, in 2007.

He is now with the Defence Science Organisation (DSO) National Laboratories, Singapore. His research interests are digital signal processing and system implementations on FPGAs.

Hari K. Garg (SM'07) received the B.Tech. degree in electrical engineering from the Indian Institute of Technology, Delhi, in 1981, the M.Eng. and Ph.D. degrees from Concordia University, Montreal, QC, Canada, in 1983 and 1985, respectively, and the M.B.A. degree from Syracuse University, Syracuse, NY, in 1995.

He was a faculty member with Syracuse University from 1985 until 1995. He has been with the National University of Singapore since 1995, with the exception of 1998–1999, when he was with Philips. His research interests are in the areas of digital signal/image processing, wireless communications, coding theory, and digital watermarking. He has published extensively on these and related topics. He is also the founder of several companies in the space of mobile telephony.