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CNN-Based Distributed Adaptive Control for Vehicle-Following Platoon with Input Saturation

Xianggui Guo, Jianliang Wang, Fang Liao, and Rodney Swee Huat Teo

Abstract—A neural network-based distributed adaptive approach combined with sliding mode technique is proposed for vehicle-following platoons in the presence of input saturation, unknown unmodeled nonlinear dynamics and external disturbances. A simple and straightforward strategy by adjusting only a single parameter is proposed to compensate for the effect of input saturation. Two spacing policies (i.e., traditional constant time headway (TCTH) policy and modified constant time headway (MCTH) policy) are used to guarantee string stability and maintain the desired spacing. Chebyshev neural networks (CNN) are used to approximate the unknown nonlinear functions in the followers on-line, and the implementation of the basis functions of CNN depends only on the leader's velocity and acceleration. Furthermore, unlike existing approaches, the nonlinearities of consecutive vehicles need not satisfy the matching condition. Finally, simulations are carried out to illustrate the effectiveness and advantage of the proposed methods, first using a numerical example, followed by a practical example of a high speed train platoon.

Index Terms—String stability, Actuator saturation, Constant time headway (CTH) policy, Chebyshev Neural Network (CNN), Sliding mode

I. INTRODUCTION

Intelligent vehicle highway systems and intelligent transportation systems have recently attracted considerable attention among researchers for addressing awfully crowded urban traffic [1]. The vehicle-following platoon architecture, which requires vehicles within each lane to be organized into platoons and capable of maintaining a small inter-vehicle spacing, may help to improve lane capacity and reduce energy consumption [2]. It is desirable, from the point of preventing collisions, that the real distances between adjacent vehicles maintain a desired safety spacing [2]–[4] and inter-vehicle spacing errors

do not amplify as they propagate along the platoon [4], i.e., guaranteeing *string stability* [5]. Control methods for ensuring platoon string stability exist in [3], [4], [6]–[9], however, under the unrealistic assumption that initial spacing/velocity errors are zero. Since the nonzero initial spacing/velocity errors may result in *string instability*, our previous result [10] removes this assumption and further avoids the low traffic density of the traditional constant-time headway (TCTH) policy. However, the nonlinear vehicle dynamics considered in [10] have two assumptions that the nonlinearities of the consecutive vehicles (including the leader) should satisfy some Lipschitz constraints and should exactly match each other. These may restrict the application domain of the proposed approach in [10] in real applications. On the other hand, although the unreliable vehicular networking with packet loss, transmission error and communication time delay has recently been investigated based on graph theory [2], [11]–[13], input saturation unavoidable in almost all real applications due to the limited capability of any physical actuator and/or safety constraints is not considered. If its adverse effect is neglected, input saturation will usually result in performance degradation and even instability [14]–[16]. For this reason, many effectiveness methods have been proposed to handle input saturation, such as adaptive compensation [14], [15] and neural network control [16] methods, etc. However, the control problem for vehicle-following platoon with input saturation becomes much more complex and still remains an open problem. How to deal with this problem has been a task of major practical interest as well as theoretical significance.

Unmodeled nonlinear dynamics often exist in vehicle-following systems because of the measurement noise, modeling errors, and environment disturbances such as wind gusts and rough road surface. Platoon control with unmodeled nonlinear dynamics is challenging since they can severely degrade the performance of the platoon and even result in string instability of the whole vehicle-platoon. Therefore, how to mitigate the effects of unmodeled nonlinear dynamics is very interesting. It is known that neural networks (NN) have the capability to approximate any continuous functions over a compact set to arbitrary accuracy [17]. In particular, Chebyshev neural network (CNN) is a functional link network whose input is generated by using a subset of Chebyshev polynomials (CP), and it has been shown that CNN has powerful approximation capabilities [19]. Although radial basis function neural network (RBFNN) has been successfully employed to solve the control problem for vehicle-following platoon in [20], [21], the key issue of string stability of platoon control is not considered. In this paper, we are interested in investigating

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the string stability issue with CNN (instead of RBFNN) due to the fact that only one parameter (i.e., the order of the CP basis) is required to determine the CP basis [17], which will significantly simplify our control design.

Motivated by the above discussion, the problem of distributed adaptive NN control for vehicle-following platoon with unmodeled nonlinear dynamics, unknown external disturbances and input saturation is investigated. The proposed control schemes can guarantee the string stability of the whole vehicle-following platoon and the uniform ultimate boundedness of all signals. The spacing errors will converge to a small neighborhood of the origin. The main contributions of the proposed schemes are highlighted as follows.

- 1) As shown in Remark 4 of Section III.A, unlike the existing methods in [14]–[16], by exploring a simple and straightforward method of adjusting only one parameter, the effect of input saturation can be attenuated.
- 2) Unlike our previous result [10], vehicles are governed by a more general model where the nonlinearities of consecutive vehicles are not required to satisfy matching conditions. In addition, two control schemes respectively based on TCTH policy and modify CTH (MCTH) policy are proposed. The MCTH policy is employed to increase the traffic density and simultaneously removes the unrealistic assumption in [3], [4], [6]–[9] that all initial spacing/velocity errors must be zero.
- 3) Integral sliding mode technique [22] combined with adaptive CNN is developed for the control of vehicle-following platoon with unmodeled nonlinear dynamics, unknown external disturbances and input saturation. The implementation of the basis functions of CNN depends only on the leader's velocity and acceleration. By adopting the CNN technique, the requirement of matching conditions for the nonlinearities of consecutive vehicles as in [10] is removed. It is worth mentioning that a common assumption in [16], [17] that the reference signal and its derivatives are assumed to be bounded will not be required.

This paper is organized as follows. Section II provides the vehicle-following platoon model and preliminaries. In Section III, novel distributed adaptive CNN control schemes based on TCTH and MTCH policies are presented for the vehicle-following platoons with input saturation and uncertain nonlinear effects. String stability is also proved in this section. Simulation results are provided to show the effectiveness and advantages in Section IV. Concluding remarks are presented in Section V.

Throughout this paper, the following notations are used: 1) Let $\mathcal{R}$ denote real numbers, $\mathcal{R}^n$ denote the real n vector, and $\mathcal{R}^{n \times m}$ denote the real $n \times m$ matrices; 2) $\|\cdot\|$ stands for the Euclidean norm of a vector; 3) The symbol of $|\cdot|$ represents the absolute value of real numbers; 4) $\text{tr}\{X\}$ denotes the trace of the matrix X ; 5) The notation $\text{sgn}(\cdot)$ is the sign function, i.e., $\text{sgn}(x) = 1$ if $x > 0$; $\text{sgn}(x) = 0$ if $x = 0$; and $\text{sgn}(x) = -1$ if $x < 0$.

II. VEHICLE-FOLLOWING PLATOON MODEL AND PRELIMINARIES

A. Vehicle-Following Platoon Description

Suppose a vehicle platoon with N follower vehicles and one leader vehicle (labeled as 0) runs in a straight line as shown in Fig. 1. Let $(x_i(t), v_i(t)) \in \mathcal{R}^2$ denote the position and velocity of vehicle $i \in \mathcal{V}_N$, $\mathcal{V}_N = \{1, 2, \dots, N\}$, and $(x_0(t), v_0(t), a_0(t)) \in \mathcal{R}^3$ denote the position, velocity and acceleration of the leader, respectively. Each follower vehicle i in the platoon is characterized by the following second-order nonlinear time-variant dynamics

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), i \in \mathcal{V}_N \\ \dot{v}_i(t) &= \text{sat}(u_i(t)) + f_i(x_i(t), v_i(t), t) + w_i(t) \end{aligned} \quad (1)$$

where $u_i(t)$ denotes the control input, $f_i(x_i(t), v_i(t), t)$ is the unknown nonlinear effect, which is assumed to be smooth continuous bounded on a compact set Ω , and models vehicle acceleration disturbances, wind gust, parameters uncertainties and intermediate uncertainties induced by networks. This unknown nonlinear function $f_i(x_i(t), v_i(t), t)$ can be considered as a bounded unmodeled dynamic, which is more general than the one considered in [10]. The disturbance input $w_i(t)$ is assumed bounded by $\|w_i(t)\| \leq \bar{w}$. The control input $u_i(t)$ is subject to saturation type nonlinearity described by

$$\text{sat}(u_i(t)) = \begin{cases} u_{Mi} \text{sgn}(u_i(t)), & \text{if } |u_i(t)| \geq u_{Mi} \\ u_i(t), & \text{if } |u_i(t)| \leq u_{Mi} \end{cases} \quad (2)$$

where u_{Mi} is a known bound of $\text{sat}(u_i(t))$. Obviously, the relationship between the applied control $\text{sat}(u_i(t))$ and the input control $u_i(t)$ has a sharp corner when $|u_i(t)| = u_{Mi}$. Similar to [15], [16], $\text{sat}(u_i(t))$ is approximated by a smooth function defined as

$$g(u_i(t)) = u_{Mi} \tanh(u_i(t)/u_{Mi}). \quad (3)$$

The approximation of the saturation function and the approximation error $\Delta u_i(t) = \text{sat}(u_i(t)) - g(u_i(t))$ are shown in Fig. 2(a). From Fig. 2(a), one can find the approximation error $\Delta u_i(t)$ is a bounded function in time and its bound can be obtained as

$$\begin{aligned} |\Delta u_i(t)| &= |\text{sat}(u_i(t)) - g(u_i(t))| \\ &\leq u_{Mi}(1 - \tanh(1)) := \bar{D}_i. \end{aligned}$$

Note that, according to the mean-value theorem, $g(u_i(t))$ can be rewritten as

$$g(u_i(t)) = g(u_0) + g_{u_i}(t)(u_i(t) - u_0) \quad (4)$$

where

$$\begin{aligned} g_{u_i}(t) &= \frac{\partial g(u_i(t))}{\partial u_i(t)} \Big|_{u_i(t)=u_{\mu i}} \\ &= \frac{4}{(e^{u_i(t)/u_{Mi}} + e^{-u_i(t)/u_{Mi}})^2} \Big|_{u_i(t)=u_{\mu i}} \leq 1 \end{aligned}$$

with $u_{\mu i} = \mu u_i(t) + (1 - \mu)u_0$ ($0 < \mu < 1$). The above relation (4) has been used extensively in the literature, such as [15], [16]. Since $g(0) = 0$, by choosing $u_0 = 0$, (4) can be further rewritten as

$$g(u_i(t)) = g_{u_i}(t)u_i(t). \quad (5)$$

Fig. 1. Configuration of Vehicle-Following Platoon under TCTH Policy

Fig. 2. (a) Saturation function $\text{sat}(u_i(t))$; Approximation function $g(u_i(t))$; Approximation error $\text{sat}(u_i(t)) - g(u_i(t))$; (b) $\frac{\partial g(u_i(t))}{\partial u_i(t)}$.

Then, the follower dynamics (1) can be transformed as follows

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), i \in \mathcal{V}_N \\ \dot{v}_i(t) &= g_{u_i}(t)u_i(t) + f_i(x_i(t), v_i(t), t) \\ &\quad + \Delta u_i(t) + w_i(t) \end{aligned} \quad (6)$$

where $\Delta u_i(t) = \text{sat}(u_i(t)) - g(u_i(t))$. In addition, from Fig. 2(b), one can find $0 < \frac{\partial g(u_i(t))}{\partial u_i(t)} \leq 1$ for any $u_i(t)$, therefore, there exists a sufficiently small parameter $g_m > 0$ such that $0 < g_m \leq g_{u_i}(t) \leq 1$ as in [15].

Remark 1. In fact, the parameter $g_{u_i}(t) \in (0, 1]$ in (5) can be viewed as an indicator for the degree of saturation at the actuator. When $g_{u_i}(t)$ approaches zero, there is almost no feedback from input $u_i(t)$, while $g_{u_i}(t) = 1$ means that $u_i(t)$ does not saturate, and $\Delta u_i(t) = 0$. When $u_i(t) \rightarrow \infty$, $g_{u_i}(t)$ will approach zero, which will cause deep saturation, and there is no miracle control method for this case. Therefore, the parameter g_m can be viewed as a true reflection of the extent of saturation to be tolerated by control design.

Suppose that the dynamics of the leader is governed by

$$\dot{x}_0(t) = v_0(t); \quad \dot{v}_0(t) = a_0(t) \quad (7)$$

where the acceleration $a_0(t)$ is a known function of time. According to practical experience, it is reasonable to assume that $v_0(t)$ and the acceleration $a_0(t)$ acting on the leader are bounded but the position $x_0(t)$ may be unbounded. For example, the velocity and acceleration of a bus on the road should be limited, where the velocity constraint is due to safety considerations and acceleration constraint is due to hardware limitations and comfortableness of drivers and passengers [23]. It is worth mentioning that all exiting results about tracking control problems by using neural network approach require that the reference signals (i.e., the leader's signals ($x_0(t), v_0(t), a_0(t)$) here) are assumed to be bounded (see, e.g., [16], [17], [23] and the references therein). Obviously, this assumption is not reasonable to the problem considered in this paper since the position $x_0(t)$ is unbounded here.

Remark 2. In the follower vehicle dynamics (1), each vehicle's nonlinear dynamic function $f_i(x_i, v_i, t)$ are supposed to be unknown, hence it can meet the requirement of a lot of

practical engineering. However, most of the existing non-linear control schemes are limited to many strict assumptions, for example, in [18], the nonlinearities $f_i(x_i, v_i, t)$ of the agents (including the leader) should exactly match each other.

B. Chebyshev Neural Network (CNN)

The neural network (NN) structure employed in this paper is a single layer Chebyshev neural network (CNN). The CNN has been shown to be capable of universally approximating non-linear systems with any degree of nonlinearity to any degree of accuracy due to their inherent approximation capabilities [17]. CNN is a functional link network based on Chebyshev polynomials (CP). Thus, according to the universal approximation property of CNN, a nonlinear function $f(Z) : \mathcal{R}^m \rightarrow \mathcal{R}$ can be approximated by CNN as

$$f(Z) = W^* \xi(Z) + \varepsilon(Z) \quad (8)$$

where $Z \in \mathcal{R}^m$ is the input vector of CNN, $W^* \in \mathcal{R}^{1 \times (mN_1+1)}$ is the optimal weight vector of the CNN with N_1 denoting the order of the CP and $\xi(Z) \in \mathcal{R}^{(mN_1+1) \times 1}$ being the CP basis function, and $\varepsilon(Z)$ is the function reconstruction error. For a given vector $Z \in \mathcal{R}^m$, $\xi(Z)$ is given by

$$\xi(Z) = \begin{bmatrix} 1 & T_1(z_1) & \cdots & T_{N_1}(z_1) & \cdots \\ T_1(z_m) & \cdots & T_{N_1}(z_m) \end{bmatrix}^T$$

where $T_k(z_j)$ ($k = 1, \dots, N_1; j = 1, \dots, m$) can be generated by the following two-term recursive formula [17]

$$\begin{aligned} T_{k+1}(z_j) &= 2z_j T_k(z_j) - T_{k-1}(z_j) \\ T_0(z_j) &= 1, T_1(z_j) = z_j, z_j \in \mathcal{R}. \end{aligned}$$

C. Useful Lemma

Before proceeding to the control design, the following lemma to be used for control design and system stability analysis is introduced.

Lemma 1. [24] Let function $V(t) \geq 0$ be a continuous function defined $\forall t \geq 0$ and $\dot{V}(t) \leq -\gamma V(t) + \varepsilon$, where $\gamma > 0$ and ε are constants, then,

$$V(t) \leq (V(0) - \frac{\varepsilon}{\gamma})e^{-\gamma t} + \frac{\varepsilon}{\gamma}.$$

III. DISTRIBUTED ADAPTIVE NN CONTROL DESIGN AND STABILITY ANALYSIS

In this section, the sliding mode control and CNN techniques are used to construct the distributed adaptive NN control schemes based on traditional constant time headway (TCTH) and modify constant time headway (MCTH) policies to achieve the bounded stability of individual vehicle and string stability of the whole vehicle platoon. In addition, by adopting CNN technique, the requirement of matching conditions for the nonlinearities of the consecutive vehicles as in [10] is removed.

A. Control Scheme I: TCTH Policy

The aim of the platoon control is to track the speed of the leader while maintaining a desired safety spacing between consecutive vehicles. One of the major spacing policies for vehicle-following platoons is constant time headway (CTH) policy, which is shown in Fig. 1. For the CTH policy, the desired inter-vehicle spacing varies with vehicle velocity, which accords with driver behaviors to some extent but limits the achievable traffic capacity [8]. The traditional CTH (TCTH) policy [3], [4] is given as follows:

$$e_i^x(t) = x_{i-1}(t) - x_i(t) - \delta_i - h_i v_i(t) \quad (9)$$

where $\delta_i > 0$ and $h_i > 0$ are the required i th standstill spacing and constant time headway, respectively. Defining the spacing between consecutive vehicles as

$$d_i(t) = x_{i-1}(t) - x_i(t),$$

the control objective here is to design a distributed adaptive neural network sliding mode algorithm for (1) with a leader (7) such that $v_i(t) \rightarrow v_0(t)$ and $d_i(t) \rightarrow \delta_i + h_i v_i(t)$. Simultaneously, for the vehicle-following platoon control problem, strong string stability requires that

$$|e_N^x(t)| \leq |e_{N-1}^x(t)| \leq \dots \leq |e_1^x(t)|, \quad (10)$$

i.e., the error propagation transfer function $G_i(s) := \frac{E_{i+1}^x(s)}{E_i^x(s)}$ satisfies $|G_i(s)| \leq 1$ for all $i \in \mathcal{V}_N$, where $E_i^x(s)$ denotes the Laplace transform of $e_i^x(t)$. As in exiting results, the case of zero initial spacing/velocity errors is considered first in this subsection, while the case of nonzero initial spacing/velocity errors is considered in subsection II.B. Since the initial spacing errors $e_i^x(0)$ are assumed to be zero, define an integral sliding surface to achieve the above objectives as:

$$s_i(t) = e_i^x(t) + \int_0^t \lambda e_i^x(\tau) d\tau \quad (11)$$

where λ is a positive constant. In order to guarantee the string stability, we also construct a coupled sliding surface as follows

$$S_i(t) = \begin{cases} q s_i(t) - s_{i+1}(t), & i \in \mathcal{V}_N \setminus \{N\} \\ q s_i(t), & i = N \end{cases} \quad (12)$$

where q is a positive constant which couples sliding surfaces $s_i(t)$ and $s_{i+1}(t)$, and the fact $s_{N+1}(t) = 0$ is used since $s_{N+1}(t)$ does not exist in the case of the last vehicle (i.e., $i = N$). Then, the following relationship between $S_i(t)$ and $s_i(t)$ can be obtained

$$S(t) = Q s(t) \quad (13)$$

where

$$Q = \begin{bmatrix} q & -1 & \dots & 0 & 0 \\ 0 & q & -1 & \dots & 0 \\ & & \ddots & & \\ 0 & 0 & \dots & q & -1 \\ 0 & 0 & \dots & 0 & q \end{bmatrix}^T$$

$$s(t) = \begin{bmatrix} s_1(t) & s_2(t) & \dots & s_N(t) \end{bmatrix}^T$$

$$S(t) = \begin{bmatrix} S_1(t) & S_2(t) & \dots & S_N(t) \end{bmatrix}^T.$$

Since $q > 0$, we can obtain the inverse of Q as

$$Q^{-1} = \begin{bmatrix} \frac{1}{q} & \frac{1}{q^2} & \dots & \frac{1}{q^{N-1}} & \frac{1}{q^N} \\ 0 & \frac{1}{q} & \frac{1}{q^2} & \dots & \frac{1}{q^{N-1}} \\ & & \ddots & & \\ 0 & 0 & \dots & \frac{1}{q} & \frac{1}{q^2} \\ 0 & 0 & \dots & 0 & \frac{1}{q} \end{bmatrix}.$$

Consequently, it follows the equivalence of $S_i(t)$ and $s_i(t)$ for any fixed platoon size N .

Remark 3. It should be pointed out that the transformation (13) is of course true for any fixed platoon size N . However, the element $\frac{1}{q^N}$ in Q^{-1} approaches 0 as N grows towards infinity. In particular choosing a smaller value of q even increases the rate with which $\frac{1}{q^N}$ approaches 0. Fortunately, the number of vehicles in every platoon will be finite. For a long platoon, a relative large value of q (the extreme case $N = \infty$, let $q = 1$) is required to guarantee the mapping between $s_i(t)$ and $S_i(t)$.

It is to be noted that if all follower vehicles' velocities $v_i(t)$ ($i \in \mathcal{V}_N$) approach the leader's velocity $v_0(t)$, then the nonlinear function $f_i(x_i(t), v_i(t), t)$ converges to the desired nonlinear function $F_i(x_0(t), v_0(t), t)$ defined by

$$F_i(x_0(t), v_0(t), t) = f_i(\int_0^t v_0(\tau) d\tau - \sum_{j=1}^i \delta_j - \sum_{j=1}^i h_j v_0(t), v_0(t), t).$$

Using the approximation property of the CNN as (8), the unknown function $F_i(x_0(t), v_0(t), t)$ ($i \in \mathcal{V}_N$) can be approximated over a compact set Ω_0 by

$$F_i(x_0(t), v_0(t), t) = W_i^* \xi_i(v_0(t), a_0(t)) + \varepsilon_i(v_0(t), a_0(t)) \quad (14)$$

where $W_i^* \in \mathcal{R}^{1 \times (2N_1+1)}$ with N_1 being the order of the CP is the optimal weight matrix of CNN, $\varepsilon_i(v_0(t), a_0(t)) \in \mathcal{R}$ is the CNN approximation error, and $\xi_i(v_0(t), a_0(t)) \in \mathcal{R}^{(2N_1+1) \times 1}$ is called the CP basis function. To simplify writing, $\xi_i(v_0(t), a_0(t))$ and $\varepsilon_i(v_0(t), a_0(t))$ are abbreviated to $\xi_i(t)$ and $\varepsilon_i(t)$ respectively in some subsequent formulae. In addition, there exist optimal constant weights W_i^* such that the absolute value of $\varepsilon_i(t)$ is assumed to be less than a small positive constant $\bar{\varepsilon}_i$ (i.e., $|\varepsilon_i(t)| \leq \bar{\varepsilon}_i$). Moreover, W_i^* is bounded by $\text{tr}\{W_i^{*T} W_i^*\} \leq \bar{W}_i$. However, it is difficult to determine the bounds $\bar{W}_i$ and $\bar{\varepsilon}_i$, therefore, we further assume that $\bar{W}_i$ and $\bar{\varepsilon}_i$ are unknown positive parameters.

To facilitate the development of the main result, we denote

$$\Phi_i(t) = f_i(x_i(t), v_i(t), t) - F_i(x_0(t), v_0(t), t). \quad (15)$$

Since $f_i(x_i(t), v_i(t), t)$ is assumed to be smooth continuous bounded on the compact set Ω , the variable $\Phi_i(t)$ has a maximum $\bar{\Phi}_i$ on the compact set Ω . Then, recalling $|\Delta u_i(t)| \leq \bar{D}_i$, $|w_i(t)| \leq \bar{w}_i$ and $|\varepsilon_i(t)| \leq \bar{\varepsilon}_i$, one can obtain that

$$-qh_i[\Phi_i(t) + \Delta u_i(t) + w_i(t) + \varepsilon_i(t)]S_i(t) \leq qh_i\gamma_i^*|S_i(t)| \quad (16)$$

where $\gamma_i^* = \bar{\Phi}_i + \bar{D}_i + \bar{w}_i + \bar{\varepsilon}_i$.

Next, define $\tilde{W}_i(t) = W_i^* - W_i(t)$ and $\tilde{\gamma}_i(t) = \gamma_i^* - \gamma_i(t)$ ($i \in \mathcal{V}_N$), where $W_i(t)$ is the estimation of W_i^* denoted in (14), and $\gamma_i(t)$ is used to estimate γ_i^* denoted in (16). Then, the control law for the i th vehicle in the formation is defined as

$$u_i(t) = \frac{1}{h_i g_m} \frac{(v_{i-1}(t) - v_i(t))^2 S_i(t)}{|(v_{i-1}(t) - v_i(t)) S_i(t)| + \sigma_{1i}} + \frac{K}{2q h_i g_m} S_i(t) + \frac{1}{q h_i g_m} \frac{A_i^2(t) S_i(t)}{|A_i(t) S_i(t)| + \sigma_{1i}} + \frac{1}{g_m} \gamma_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) + \frac{1}{g_m} \frac{|W_i(t) \xi_i(t)|^2 S_i(t)}{|W_i(t) \xi_i(t) S_i(t)| + \sigma_{1i}} \quad (17)$$

where

$$A_i(t) = \begin{cases} q \lambda e_i^x(t) - \dot{e}_{i+1}^x(t) - \lambda e_{i+1}^x(t) & \text{for } i \in \mathcal{V}_N \setminus N \\ q \lambda e_i^x(t), & \text{for } i = N, \end{cases} \quad (18)$$

and the adaptive laws for $\gamma_i(t)$ and $W_i(t)$ are given by

$$\begin{aligned} \dot{\gamma}_i(t) &= \iota_{1i} q h_i [S_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) - \sigma_{2i} \gamma_i(t)] \\ \dot{W}_i(t) &= -\iota_{2i} q h_i \xi_i^T(t) S_i(t) - \iota_{2i} \sigma_{3i} W_i(t) \end{aligned} \quad (19)$$

with $\gamma_i(0) \geq 0$. Here, K , ι_{1i} and ι_{2i} are any positive constants, while σ_{1i} , σ_{2i} , σ_{3i} and ϵ_i are small positive constants.

Then, the following theorem, which guarantees the bounded stability of individual vehicle and the string stability of the whole vehicle-following platoon, can be derived.

Theorem 1. Consider the vehicle-following platoon (1) with a leader (7) subject to input saturation constraint (2) under the assumption that the initial spacing errors $e_i^x(0)$ ($i \in \mathcal{V}_N$) are zero. For a sufficiently large positive constant V_{max} , if the initial condition satisfies

$$\sum_{i=1}^N \text{tr} \left\{ \tilde{W}_i^T(0) \tilde{W}_i(0) \right\} + \|\tilde{S}(0)\|^2 \leq \frac{2V_{max}}{\kappa} \quad (20)$$

where

$$\begin{aligned} \tilde{S}(t) &= \begin{bmatrix} S^T(t) & \tilde{\gamma}^T(t) \end{bmatrix}^T \\ S(t) &= \begin{bmatrix} S_1(t) & \cdots & S_N(t) \end{bmatrix}^T \\ \tilde{\gamma}(t) &= \begin{bmatrix} \tilde{\gamma}_1(t) & \cdots & \tilde{\gamma}_N(t) \end{bmatrix}^T \\ \kappa &= \max\{1, \min_{1 \leq i \leq N} \{\iota_{1i}\}, \min_{1 \leq i \leq N} \{\iota_{2i}\}\}, \end{aligned}$$

and $\tilde{W}_i(t)$ and $\tilde{\gamma}_i(t)$ are as defined just above (17), then, the distributed adaptive NN control law (17)-(19) guarantees that the spacing errors $e_i^x(t)$ in (9) converge to a small neighborhood of the origin by appropriately choosing design parameters, while string stability of the vehicle-following platoon also can be guaranteed for $i \in \mathcal{V}_N$ when q satisfies $0 < q \leq 1$.

Proof. Construct the following Lyapunov function candidate for vehicle i :

$$V_i(t) = \frac{1}{2} S_i^2(t) + \frac{1}{2\iota_{1i}} \tilde{\gamma}_i(t)^2 + \frac{1}{2\iota_{2i}} \text{tr} \left\{ \tilde{W}_i^T(t) \tilde{W}_i(t) \right\}$$

where $\tilde{\gamma}_i(t)$ and $\tilde{W}_i(t)$ are the parameter estimation errors.

Taking the time derivative of $V_i(t)$ for $t > 0$ results in

$$\begin{aligned} \dot{V}_i(t) &= S_i(t) \dot{S}_i(t) - \frac{1}{\iota_{1i}} \tilde{\gamma}_i(t) \dot{\gamma}_i(t) \\ &\quad - \frac{1}{\iota_{2i}} \text{tr} \left\{ \tilde{W}_i^T(t) \dot{W}_i(t) \right\}. \end{aligned} \quad (21)$$

Recalling the definitions of (9)-(12) and invoking $s_{N+1}(t) = 0$, one can obtain that

$$\begin{aligned} \dot{S}_i(t) &= q[\dot{e}_i^x(t) + \lambda e_i^x(t)] - [\dot{e}_{i+1}^x(t) + \lambda e_{i+1}^x(t)] \\ &= q\{v_{i-1}(t) - v_i(t) - h_i[\text{sat}(u_i(t)) + w_i(t)] \\ &\quad + f_i(x_i(t), v_i(t), t)] + \lambda e_i^x(t)\} \\ &\quad - [\dot{e}_{i+1}^x(t) + \lambda e_{i+1}^x(t)] \\ &= q\{v_{i-1}(t) - v_i(t) - h_i[g_{u_i}(t)u_i(t) \\ &\quad + \Delta u_i(t) + \Phi_i(t) + W_i^* \xi_i(t) + \varepsilon_i(t) \\ &\quad + w_i(t)]\} + A_i(t), \text{ for } i \in \mathcal{V}_N \setminus N \\ \dot{S}_i(t) &= q\{v_{i-1}(t) - v_i(t) - h_i[\text{sat}(u_i(t)) \\ &\quad + f_i(x_i(t), v_i(t), t)]\} + q \lambda e_i^x(t) \\ &= q\{v_{i-1}(t) - v_i(t) - h_i[g_{u_i}(t)u_i(t) \\ &\quad + \Delta u_i(t) + \Phi_i(t) + W_i^* \xi_i(t) + \varepsilon_i(t) \\ &\quad + w_i(t)]\} + A_i(t), \text{ for } i = N, \end{aligned} \quad (22)$$

where $\Phi_i(t)$ and $A_i(t)$ are defined as in (15) and (18), respectively.

The parameter γ_i^* is estimated by adaptive law $\gamma_i(t)$ in (19), and the initial value of $\gamma_i(t)$ has to be selected as $\gamma_i(0) \geq 0$ to guarantee that $\gamma_i(t) \geq 0$ for all $t \in [0, \infty)$. Then, using the control law (17), we have

$$\begin{aligned} &- q h_i g_{u_i}(t) u_i(t) S_i(t) \\ &= - \frac{q g_{u_i}(t)}{g_m} \frac{(v_{i-1}(t) - v_i(t))^2 S_i^2(t)}{|(v_{i-1}(t) - v_i(t)) S_i(t)| + \sigma_{1i}} - \frac{g_{u_i}(t) K}{2g_m} S_i^2(t) \\ &\quad - \frac{q h_i g_{u_i}(t)}{g_m} \frac{|W_i(t) \xi_i(t)|^2 S_i^2(t)}{|W_i(t) \xi_i(t) S_i(t)| + \sigma_{1i}} - \frac{g_{u_i}(t)}{g_m} \frac{A_i^2(t) S_i^2(t)}{|A_i(t) S_i(t)| + \sigma_{1i}} \\ &\quad - \frac{q h_i g_{u_i}(t)}{g_m} \gamma_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) S_i(t) \\ &\leq - \frac{q(v_{i-1}(t) - v_i(t))^2 S_i^2(t)}{|(v_{i-1}(t) - v_i(t)) S_i(t)| + \sigma_{1i}} - q h_i \gamma_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) S_i(t) \\ &\quad - \frac{q h_i |W_i(t) \xi_i(t)|^2 S_i^2(t)}{|W_i(t) \xi_i(t) S_i(t)| + \sigma_{1i}} - \frac{A_i^2(t) S_i^2(t)}{|A_i(t) S_i(t)| + \sigma_{1i}} - \frac{K}{2} S_i^2(t) \end{aligned} \quad (23)$$

where we have used the facts that $0 < g_m \leq g_{u_i}(t) < 1$ and $\gamma_i(t) \geq 0$.

Applying the following inequalities

$$\begin{aligned} &- \frac{q(v_{i-1}(t) - v_i(t))^2 S_i^2(t)}{|(v_{i-1}(t) - v_i(t)) S_i(t)| + \sigma_{1i}} \\ &\leq -q|v_{i-1}(t) - v_i(t)| |S_i(t)| + q \sigma_{1i} \\ &\quad - \frac{A_i^2(t) S_i^2(t)}{|A_i(t) S_i(t)| + \sigma_{1i}} \\ &\leq -|A_i(t) S_i(t)| + \sigma_{1i} \\ &\quad - \frac{q h_i |W_i(t) \xi_i(t)|^2 S_i^2(t)}{|W_i(t) \xi_i(t) S_i(t)| + \sigma_{1i}} \\ &\leq -q h_i |W_i(t) \xi_i(t)| |S_i(t)| + q h_i \sigma_{1i} \\ &\leq q h_i W_i(t) \xi_i(t) S_i(t) + q h_i \sigma_{1i} \end{aligned}$$

into (23), one can obtain that

$$\begin{aligned} &- q h_i g_{u_i}(t) u_i(t) S_i(t) \\ &\leq -q|v_{i-1}(t) - v_i(t)| |S_i(t)| - |A_i(t) S_i(t)| \\ &\quad + q h_i W_i(t) \xi_i(t) S_i(t) - \frac{K}{2} S_i^2(t) \\ &\quad - q h_i \gamma_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) S_i(t) + \Sigma_i \end{aligned} \quad (24)$$

where $\Sigma_i = (q + 1 + q h_i) \sigma_{1i}$.

By using the property $0 \leq |\eta| - \eta \tanh(\frac{\eta}{\epsilon}) \leq 0.2785\epsilon$ for $\forall \epsilon > 0$ and $\eta \in \mathcal{R}$ [10], one obtains

$$\gamma_i^* |S_i(t)| \leq \gamma_i^* S_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) + 0.2785 \gamma_i^* \epsilon_i. \quad (25)$$

It then follows from (16), (22), (24) and (25) that

$$\begin{aligned}
& \dot{S}_i(t)S_i(t) \\
& \leq -qh_i\gamma_i(t)\tanh\left(\frac{S_i(t)}{\epsilon_i}\right)S_i(t) - \frac{K}{2}S_i^2(t) \\
& \quad -qh_i\tilde{W}_i(t)\xi_i(t)S_i(t) + qh_i\gamma_i^*S_i(t)\tanh\left(\frac{S_i(t)}{\epsilon_i}\right) \\
& \quad + 0.2785qh_i\gamma_i^*\epsilon_i + \Sigma_i \\
& = qh_i\tilde{\gamma}_i(t)\tanh\left(\frac{S_i(t)}{\epsilon_i}\right)S_i(t) - qh_i\sigma_{2i}\tilde{\gamma}_i(t)\gamma_i(t) \\
& \quad + qh_i\sigma_{2i}\tilde{\gamma}_i(t)\gamma_i(t) - \frac{K}{2}S_i^2(t) - qh_i\tilde{W}_i(t)\xi_i(t)S_i(t) \\
& \quad - \sigma_{3i}\text{tr}\left\{\tilde{W}_i^T(t)W_i(t)\right\} + \sigma_{3i}\text{tr}\left\{\tilde{W}_i^T(t)W_i(t)\right\} \\
& \quad + 0.2785qh_i\gamma_i^*\epsilon_i + \Sigma_i
\end{aligned} \tag{26}$$

where the following relationships have been used

$$\begin{aligned}
(v_{i-1}(t) - v_i(t))S_i(t) - |v_{i-1}(t) - v_i(t)||S_i(t)| & \leq 0 \\
A_i(t)S_i(t) - |A_i(t)S_i(t)| & \leq 0.
\end{aligned}$$

Noting that $W_i(t) \in \mathcal{R}^{1 \times (2N_1+1)}$, $\tilde{W}_i(t) \in \mathcal{R}^{1 \times (2N_1+1)}$, $\xi_i(t) \in \mathcal{R}^{(2N_1+1) \times 1}$ and $S_i(t) \in \mathcal{R}$, we can obtain that

$$\begin{aligned}
& -qh_i\tilde{W}_i(t)\xi_i(t)S_i(t) - \sigma_{3i}\text{tr}\left\{\tilde{W}_i^T(t)W_i(t)\right\} \\
& = -qh_i\text{tr}\left\{\xi_i^T(t)\tilde{W}_i^T(t)S_i(t)\right\} - \sigma_{3i}\text{tr}\left\{W_i^T(t)\tilde{W}_i(t)\right\}.
\end{aligned}$$

Then, combining the adaptive laws (19) with (26), (21) can be rewritten as

$$\begin{aligned}
\dot{V}_i(t) & \leq -\frac{K}{2}S_i^2(t) + qh_i\sigma_{2i}\tilde{\gamma}_i(t)\gamma_i(t) + \sigma_{3i}\text{tr}\left\{\tilde{W}_i^T(t)W_i(t)\right\} \\
& \quad + 0.2785qh_i\gamma_i^*\epsilon_i + \Sigma_i.
\end{aligned} \tag{27}$$

By completion of squares and the Young's inequality, we have

$$\begin{aligned}
\tilde{\gamma}_i(t)\gamma_i(t) & = -\frac{1}{2}\gamma_i^2(t) - \frac{1}{2}\tilde{\gamma}_i^2(t) + \frac{1}{2}(\gamma_i^*)^2 \\
& \leq -\frac{1}{2}\tilde{\gamma}_i^2(t) + \frac{1}{2}(\gamma_i^*)^2
\end{aligned} \tag{28a}$$

$$\begin{aligned}
& \text{tr}\left\{\tilde{W}_i^T(t)W_i(t)\right\} \\
& = \text{tr}\left\{\tilde{W}_i^T(t)(W_i^* - \tilde{W}_i(t))\right\} \\
& = -\text{tr}\left\{\tilde{W}_i^T(t)\tilde{W}_i(t)\right\} + \text{tr}\left\{\tilde{W}_i^T(t)W_i^*\right\} \\
& \leq -\frac{1}{2}\text{tr}\left\{\tilde{W}_i^T(t)\tilde{W}_i(t)\right\} + \frac{1}{2}\bar{W}_i
\end{aligned} \tag{28b}$$

where the bound $\text{tr}\left\{W_i^{*T}W_i^*\right\} \leq \bar{W}_i$ has been used.

Applying (28a) and (28b) to (27), one can obtain that

$$\begin{aligned}
\dot{V}_i(t) & \leq -\frac{K}{2}S_i^2(t) - \frac{qh_i\sigma_{2i}}{2}\tilde{\gamma}_i^2(t) - \frac{\sigma_{3i}}{2}\text{tr}\left\{\tilde{W}_i^T(t)\tilde{W}_i(t)\right\} \\
& \quad + \frac{qh_i\sigma_{2i}}{2}(\gamma_i^*)^2 + \frac{\sigma_{3i}}{2}\bar{W}_i + 0.2785qh_i\gamma_i^*\epsilon_i + \Sigma_i.
\end{aligned} \tag{29}$$

Next, a global Lyapunov function $V(t)$ is constructed as follows:

$$V(t) = \sum_{i=1}^N V_i(t).$$

In light of (29), it then follows that

$$\dot{V}(t) \leq -\zeta_1 V(t) + \zeta_2 \tag{30}$$

where the positive parameters ζ_1 and ζ_2 are given as follows

$$\begin{aligned}
\zeta_1 & = \min\{K, \min_{1 \leq i \leq N} \iota_{1i}qh_i\sigma_{2i}, \min_{1 \leq i \leq N} \iota_{2i}\sigma_{3i}\} \\
\zeta_2 & = \sum_{i=1}^N [qh_i(\frac{\sigma_{2i}}{2}\gamma_i^{*2} + 0.2785\gamma_i^*\epsilon_i) + \frac{\sigma_{3i}}{2}\bar{W}_i + \Sigma_i].
\end{aligned}$$

Thus, by using Lemma 1, the following inequality holds:

$$V(t) \leq (V(0) - \frac{\zeta_2}{\zeta_1})e^{-\zeta_1 t} + \frac{\zeta_2}{\zeta_1} \leq V(0) + \frac{\zeta_2}{\zeta_1}, \tag{31}$$

where the fact $V(0) = \sum_{i=1}^N [\frac{1}{2}S_i^2(0) + \frac{1}{2\iota_{1i}}\tilde{\gamma}_i(0)^2 + \frac{1}{2\iota_{2i}}\text{tr}\{\tilde{W}_i^T(0)\tilde{W}_i(0)\}] \geq 0$ is used for the last inequality. Therefore, it is straightforward to show that $S_i(t)$, $\gamma_i(t)$ and $W_i(t)$ are uniformly ultimately bounded if the initial condition is bounded as in (20). It is clear that reducing ζ_2 , meanwhile increasing ζ_1 will lead to smaller bounds of $S_i(t)$, i.e., $S_i(t)$ can converge to an arbitrarily small neighborhood of zero by choosing the design parameters appropriately. Then, according (11) and (12), $s_i(t)$ and the spacing error $e_i^x(t)$ also converges to a small neighborhood of zero.

In what follows, the string stability of the whole vehicle-following platoon is proved following the approach in the Laplace domain presented in [7]. Since $S_i(t) = qs_i(t) - s_{i+1}(t)$, when $S_i(t)$ converges to an arbitrary small neighborhood of zero by choosing the design parameters appropriately, we have

$$q(e_i^x(t) + \int_0^t \lambda e_i^x(\tau)d\tau) = e_{i+1}^x(t) + \int_0^t \lambda e_{i+1}^x(\tau)d\tau. \tag{32}$$

Taking Laplace transform of (32), we can get

$$q(E_i^x(s) + \frac{\lambda}{s}E_i^x(s)) = E_{i+1}^x(s) + \frac{\lambda}{s}E_{i+1}^x(s),$$

i.e., $G_i(s) = \frac{E_{i+1}^x(s)}{E_i^x(s)} = q$. Then, when $0 < q \leq 1$, $|G_i(s)| \leq 1$ is guaranteed and string stability of the whole vehicle platoon is also guaranteed. This completes the proof. $\square$

Remark 4. Since actuator physical constraints can severely degrade the whole vehicle-following platoon, control design for vehicle-following platoon subject to input saturation and nonlinear unmodeled dynamics presents a tremendous challenge. To overcome the effect of input saturation, only one parameter g_m needs be adjusted here, which is much simpler than most existing results such as [14]–[16]. It is clear in (17) that the vehicle-following platoon system performance is affected by g_m . Therefore, an appropriate value of g_m is important for eliminating the effect of input saturation. It should be pointed out that (23) will fail if $g_{u_i}(t) < g_m$.

Remark 5. In the proof of Theorem 1, the strong string stability of the whole vehicle-following platoon is proved following the approach in the Laplace domain under the condition that $S_i(t) = qs_i(t) - s_{i+1}(t)$ converges to zero. However, according to the proof of Theorem 1, the sliding mode motion converges to a neighborhood of $S_i(0) = 0$. This is due to actuator saturation nonlinearities and/or un-modeled dynamics, etc. By adjusting the design parameters, this neighborhood of $S_i(0)$ can be made sufficiently small so that the subsequent analysis can be done on the sliding manifold $S_i(t) = 0$. It is further highlighted that the incorporation of zero initial conditions (the MCTH policy (34) also has zero initial conditions), hence guarantees that $s_i(0) = S_i(0) = 0$ and the system starts on the sliding manifold, eliminating completely the reaching phase that normally exists in most sliding model control schemes. Therefore, even though strong string stability before reaching the sliding manifold is not proven in theory in Theorem 1, the proposed control law (17) effectively guarantees the string

stability not only in the transient but also in the steady-state phase of the system response, as is demonstrated by the numerical examples of Section IV. Nevertheless, a more efficient and effective notion of and approach to string stability when the system is knocked off the sliding surface is a very interesting topic for future research.

Remark 6. Two observations can be made of the control scheme in Theorem 1. First, although the TCTH policy as a variable spacing policy has the advantage of improving string stability and can ensure string stability using on-board information only [4], [9], one obvious drawback of this policy is low traffic density at high speed. In practice, with the same speed, the higher the traffic density is, the higher the traffic flow is, which can be provided for avoiding traffic jam. Second, a common assumption, like most of the existing results such as [6]–[9], that the initial spacing errors $e_i^x(0)$ are all zero is necessary for achieving string stability in Theorem 1. However, in practice, the initial spacing errors are not zero in most cases, which makes the assumption restrictive. Consequently, in order to overcome the above two drawbacks, a modified CTH (MCTH) policy will be provided in the next subsection.

When the input saturation is not considered in the vehicle-following platoon, the following corollary can be obtained.

Corollary 1. Consider the vehicle-following platoon (1) with a leader (7) under assumption that the initial spacing errors $e_i^x(0) (i \in \mathcal{V}_N)$ are zero. For a sufficiently large positive constant V_{max} , if the initial condition satisfies (20), the distributed adaptive NN control laws (17)–(19) with $g_m = 1$ guarantee that the spacing errors $e_i^x(t)$ in (9) converge to a small neighborhood around origin by appropriately choosing design parameters, while string stability of the vehicle-following platoon also can be guaranteed for $i \in \mathcal{V}_N$ when q satisfies $0 < q \leq 1$.

B. Control Strategy II: MCTH Control Laws

From a practical point of view, with increasing traffic density (more vehicles on the way), the traffic flow also increases. Therefore, with the aim of overcoming the aforementioned drawback in Remark 6 arising from the use of TCTH policy (low traffic density) and inspired by [4], [25], [26], a MCTH policy is given out as follows:

$$\tilde{e}_i^x(t) = x_{i-1}(t) - x_i(t) - \delta_i - h_i(v_i(t) - v_0(t)). \quad (33)$$

Compared with the TCTH policy in (9), the advantage of the MCTH policy (33) is that the desired spacing between consecutive vehicles is a constant spacing δ_i , which will increase the traffic density at high speed, and will improve the traffic flow on highways or urban roads that have heavy traffic. Furthermore, with the aim of removing the aforementioned zero initial spacing error assumption and similar to our recent work [10], (33) is further modified as follows:

$$\begin{aligned} e_i^x(t) &= \tilde{e}_i^x(t) - \Xi_i(t) \\ \Xi_i(t) &= [\tilde{e}_i^x(0) + (\zeta_i \tilde{e}_i^x(0) + \dot{\tilde{e}}_i^x(0))t]e^{-\zeta_i t} \end{aligned} \quad (34)$$

where $\tilde{e}_i^x(0) = \tilde{e}_i^x(t)|_{t=0}$, $\dot{\tilde{e}}_i^x(0) = \dot{\tilde{e}}_i^x(t)|_{t=0}$ and ζ_i is a strictly positive constant. In addition, we can obtain that

$$e_i^x(t)|_{t=0} = 0, \quad \dot{e}_i^x(t)|_{t=0} = 0$$

which shows that the modified spacing errors $e_i^x(t)$ in (34) are initially zero for arbitrary initial spacing errors. Furthermore, adopt the same sliding mode surfaces $s_i(t)$ and $S_i(t)$ in (11) and (12), respectively.

Then, based on the new spacing policy (34), the adaptive NN control law is designed as follows:

$$\begin{aligned} u_i(t) &= \frac{1}{h_i g_m} \frac{(v_{i-1}(t) - v_i(t) - \dot{\Xi}(t))^2 S_i(t)}{|(v_{i-1}(t) - v_i(t) - \dot{\Xi}(t)) S_i(t)| + \sigma_{1i}} + \frac{K}{2q h_i g_m} S_i(t) \\ &\quad + \frac{1}{q h_i g_m} \frac{A_i^2(t) S_i(t)}{|A_i(t) S_i(t)| + \sigma_{1i}} + \frac{1}{g_m} \gamma_i(t) \tanh\left(\frac{S_i(t)}{\epsilon_i}\right) \\ &\quad + \frac{1}{g_m} \frac{|W_i(t) \xi_i(t)|^2 S_i(t)}{|W_i(t) \xi_i(t) S_i(t)| + \sigma_{1i}} \end{aligned} \quad (35)$$

where

$$\begin{aligned} A_i(t) &= \begin{cases} q h_i a_0(t) + q \lambda e_i^x(t) - \dot{e}_{i+1}^x(t) - \lambda e_{i+1}^x(t) & \text{for } i \in \mathcal{V}_N \setminus N \\ q h_i a_0(t) + q \lambda e_i^x(t), & \text{for } i = N. \end{cases} \\ \dot{\Xi}_i(t) &= -\zeta_i [\tilde{e}_i^x(0) + (\zeta_i \tilde{e}_i^x(0) + \dot{\tilde{e}}_i^x(0))t]e^{-\zeta_i t} \\ &\quad + (\zeta_i \tilde{e}_i^x(0) + \dot{\tilde{e}}_i^x(0))e^{-\zeta_i t}. \end{aligned} \quad (36)$$

The adaptive laws for $\gamma_i(t)$ and $W_i(t)$ are the same as (19).

Note that if all vehicles' velocities $v_i(t)$ ($i \in \mathcal{V}_N$) approach the leader's velocity $v_0(t)$ under the MCTH policy (34), the nonlinear function $f_i(x_i(t), v_i(t), t)$ will converge to a desired nonlinear function $F_i(x_0(t), v_0(t), t)$ defined by

$$F_i(x_0(t), v_0(t), t) = f_i\left(\int_0^t v_0(\tau) d\tau - \sum_{j=1}^i \delta_j, v_0(t), t\right).$$

The following theorem states that the control law (35) guarantees the bounded stability of individual vehicle and string stability of whole vehicle platoon under the MCTH policy (34).

Theorem 2. Consider the vehicle-following platoon (1) with a leader (7) subject to input saturation constraint (2). For a sufficiently large positive constant V_{max} , if the initial condition satisfies (20), the distributed adaptive NN control law in (35), (36), and (19) guarantees that the spacing errors $e_i^x(t)$ in (34) converge to a small neighborhood around origin by appropriately choosing design parameters, while string stability of the vehicle-following platoon also can be guaranteed for $i \in \mathcal{V}_N$ when q satisfies $0 < q \leq 1$.

Proof. The proof is immediate and follows along the same lines that are developed for the proof of Theorem 1. $\square$

Remark 7. Two points should be noticed about Theorem 2. First, the main idea of the MCTH policy (34) used in Theorem 2 is to transform a nonzero initial spacing error problem to a zero initial spacing error problem. Similar to [10], the MCTH policy (34) effectively handles the effect of non-zero initial spacing errors, as is shown in Section IV. Second, the steady-state traffic density of (33) or (34) will be the same as the one of the constant spacing policy used in [6]–[9] and be much higher than the one for the TCTH policy (9) [9]. This improvement in traffic density is the outcome of MCTH policy (33) (and hence of MCTH policy (34)) where the leader

velocity $v_0(t)$ is used so that the time headway h_i is taken with respect to the relative velocity $v_i(t) - v_0(t)$, instead of the standard absolute velocity $v_i(t)$ as in existing literature. This requirement of leader velocity $v_0(t)$ can be achieved via vehicle-to-vehicle (V2V) communication.

Remark 8. In comparison with the existing results that also consider adaptive NN control for vehicle-following platoons [20], [21], the main differences in our result are summarized as follows: (a) Actuator saturation nonlinearity is not considered in [20], [21]; (b) As compared with RBFNN used in [20], [21], the key advantage of CP basis function lies in the fact that only one parameter (i.e., N_1 , the order of CP) is required to determine the CP basis function [17]; (c) The string stability issue as a key issue of platoon control is not considered in [20], [21] which means the vehicles in the platoon may come into collision with each other; (d) Zero initial spacing errors are assumed in [20], [21].

IV. SIMULATION STUDY AND PERFORMANCE RESULTS

In this section, the feasibility of the proposed methods is illustrated via a numerical example and a practical example. All numerical simulations were performed with Matlab version 2007a and a notebook computer with 1.8GHz Intel (R) Core (TM) i5-3337U CPU and 8.00GB RAM using Windows 10.

A. Example 1 (Numerical Example)

In this subsection, we will verify the effectiveness of the proposed distributed adaptive neural control based on TCTH policy (9) and MCTH policy (34) for a vehicle platoon of six follower vehicles with input saturation as in (1) and a leader as in (7). In the simulation, the unmodeled dynamic nonlinear function $f_i(x_i(t), v_i(t), t)$, external disturbances $w_i(t)$ and the leader's acceleration are given as

$$\begin{aligned} f_i(x_i(t), v_i(t), t) &= \frac{0.5x_i^2(t)}{1+x_i^2(t)} + \varphi_i \sin\left(\frac{\pi}{4} + \varphi_i v_i(t)\right) \\ w_i(t) &= 0.1 \sin(t), i \in \mathcal{V}_N \\ a_0(t) &= \begin{cases} 1.5 \text{ m/s}^2, & 2s \leq t < 5s \\ -0.5 \text{ m/s}^2, & 6s \leq t < 10s \\ 0 \text{ m/s}^2, & \text{otherwise.} \end{cases} \end{aligned}$$

Tables I and II show the values of the simulation parameters in this section. Furthermore, the input vector of the CNN is $(v_0(t), a_0(t))^T$.

TABLE I
NUMERICAL SIMULATION PARAMETERS' VALUES FOR $i \in \mathcal{V}_N$

Parameter Name	Simulation Values
Standstill Distance	$\delta_i = 0.8m$
Time Headway	$h_i = 1s$
Leader Initial	$x_0(0) = 20m; v_0(0) = 1m/s$
The Order of CP	$N_1 = 2$
$\iota_{1i}, \iota_{2i}, \epsilon_i$	$\iota_{1i} = 20, \iota_{2i} = 0.2, \epsilon_i = 0.1$
$\sigma_{ji}, j = 1, 2, 3$	$\sigma_{1i} = \sigma_{2i} = 0.3, \sigma_{3i} = 1$
$\gamma_i(0), W_i(0)$	$\gamma_i(0) = 1, W_i(0) = 0_{1 \times 5}$

Case I: Zero Initial Spacing Errors with TCTH Policy

According to the definition of TCTH policy (9), the initial values of $e_i^x(0)$ are zero. For this case, the controller parameters are taken as $K = 5$, $q = 0.85$ and $\lambda = 0.1$. In the

TABLE II
THE PARAMETERS $x_i(0)$, $v_i(0)$ AND φ_i FOR $i \in \mathcal{V}_N$

i	1	2	3	4	5	6
$x_i(0)$	18.5m	17m	15.5m	14m	12.5m	11m
$v_i(0)$	1m/s	1m/s	1m/s	1m/s	1m/s	1m/s
φ_i	1	0.1	-0.8	0.1	-1.2	0.2

Fig. 3. (a) Spacing error $e_i(t)$; (b) Distance $d_i(t)$; (c) Velocity $v_i(t)$; (d) Control input $u_i(t)$ under Corollary 1 without input saturation.

following, the comparison between Corollary 1 and Theorem 1 is studied so as to demonstrate the effectiveness of the proposed single parameter g_m in handling control saturation. First of all, when there does not exist input saturation, the simulation results obtained by using Corollary 1 are shown in Fig. 3. It is found from Fig. 3(a) that the amplitude of the spacing error decreases through the string of vehicles (i.e., $\|e_6(t)\| \leq \dots \leq \|e_1(t)\|$), which implies that the string stability of the whole vehicle platoon is achieved despite the nonlinear unmodeled dynamics and external disturbances. Fig. 3(b) shows that inter-vehicle collision can be avoided since the distances between consecutive vehicles are always positive. Figs. 3(b) and 3(c) show that inter-vehicle spacings of all consecutive vehicles converge to 4.3m and the velocities of the vehicles follow the trajectory of the leader and maintain the desired velocity 3.5m/s finally. The control input $u_i(t)$ is shown in Fig. 3(d), where we can find the maximum value of the input is 3.779. Due to the space constraint, the responses of the input $u_i(t)$ will be omitted in the sequel.

Next, given the maximum control effort of 3.7799 in Fig. 3(d), we consider an input saturation limit of $u_{Mi} = 1.5$. The simulation results under Corollary 1 and Theorem 1 with

$g_m = 0.6$ are shown in Fig. 4 and Fig. 5, respectively. With the controller in Corollary 1, it can be found from Figs. 4(a) and 4(b) that the input saturation causes significant performance degradation and string instability, which may cause inter-vehicle collision. However, with the controller in Theorem 1, Figs. 5(a) and (b) show that collision is avoided and string stability is achieved despite the existence of input saturation, demonstrating the effectiveness and advantage of the proposed method. Referring to Figs. 5(b) and 5(c), it is found that the inter-vehicle spacings $d_i(t)$ are proportional to vehicle speed $v_i(t)$, and can become very large at high speed, which decreases the capacity of traffic flow.

Case II: Nonzero Initial Spacing/Velocity Errors with MTH Policy

According to the definition of MTH policy (33), the initial values of $\tilde{e}_i^x(0)$ are nonzero as $\tilde{e}_i^x(0) = 1m, i \in \mathcal{V}_N$ while $e_i^x(0) = 0m, i \in \mathcal{V}_N$ with the MTH policy (34). For this case, the controller parameters are taken as $K = 5, q = 0.96, \lambda = 0.5$ and $\zeta_i = 20$. In addition, the parameter g_m is assumed to be 0.5. Since the input saturation limit $u_{M,i}$ determines the control capacity and saturation degree of the actuator, the nonzero initial spacing error condition considered here requires higher input saturation limit, i.e., the input saturation limit $u_{M,i}$ here is chosen as 2 (instead of 1.5 in Fig. 5 in Case I above). The simulation results are shown in Fig. 6. Referring to Fig. 5(a), it is found that Fig. 6(a) has a faster convergence rate and higher accuracy control performance than Fig. 5(a) since larger control capacity ($u_{M,i} = 2$ instead of $u_{M,i} = 1.5$) has been used. From Figs. 6(b) and 6(c), by using MTH policy (34), the inter-vehicle distance ($0.8m$) is much smaller than the one ($4.3m$) in Figs. 5(b) and 5(c), which will increase the traffic density significantly. For the transient phase, it is seen from Fig. 5(c) and Fig. 6(c) that the controller (35) of Theorem 2 has a faster transient response and a higher accuracy control performance than the controller (17) of Theorem 1.

B. Example 2 (Practical Example)

Consider a platoon of high-speed trains movement on a railway line. The platoon consists of six follower trains and a leader train. The dynamic motion for each follower train is the same as in [27]. After minor transformations, the follower train with actuator saturation is given by (1) with

$$\begin{aligned} f_i(x_i(t), v_i(t), t) &= -c_{i0} - c_{i1}v_i(t) - c_{i2}v_i^2(t) \\ u_i(t) &= \frac{\tau_i(t)}{m_i} \\ w_i(t) &= 0, i \in \mathcal{V}_N \end{aligned}$$

where $x_i(t)$, $v_i(t)$, m_i and $\tau_i(t)$ are the position, speed, mass and control force of the i th train, respectively. The leader high-speed train's behaviour is independent of its followers, and the acceleration $a_0(t)$ in (7) varies during the motion process as

$$a_0(t) = \begin{cases} 0.3t \text{ km/h}^2, & 5h \leq t < 10h \\ -0.3t \text{ km/h}^2, & 16h \leq t < 20h \\ 0 \text{ km/h}^2, & \text{otherwise.} \end{cases}$$

The system parameters of each high-speed train and the initial positions and speeds of each high-speed train are as in [27] and listed in Table III and Table IV, respectively. The desired spacing between consecutive trains is $\delta_i = 20km$, while the

Fig. 4. (a) Spacing error $e_i(t)$; (b) Distance $d_i(t)$; (c) Velocity $v_i(t)$ under Corollary 1 with input saturation.

Fig. 5. (a) Spacing error $e_i(t)$; (b) Distance $d_i(t)$; (c) Velocity $v_i(t)$ under Theorem 1 with input saturation.

time headway is $h_i = 1h$. In addition, inspired by [17], in order to obtain good control performance, the input vector of the CNN is normalized as $(v_0(t)/220, a_0(t)/220)^T$. The input saturation limit is taken as $u_{M,i} = 500$, and the control parameters are chosen as $k = 10, q = 0.95, \lambda = 5, \zeta_i = 15, g_m = 0.5$. All other design parameters are the same as those in Example 1. According to the definition of $\tilde{e}_i^x(t)$ in (33), the initial spacing errors $\tilde{e}_i^x(0)$ are nonzero as $\tilde{e}^x(0) = [20, 34, 82.5, 103, 64.5, 213]km$. Under the same initial conditions of the adaptive estimations, the proposed controller (35) of Theorem 2 performs as shown in Fig.

Fig. 6. (a) Spacing error $e_i(t)$; (b) Distance $d_i(t)$; (c) Velocity $v_i(t)$ under Theorem 2 with input saturation.

7. It can be observed that the control method can achieve string stability in presence of input saturations, nonlinear dynamics and nonzero initial spacing errors. It is clear that the distances between two neighboring trains converge to the desired spacing 20km and the velocities of the followers converges to that of the leader. It should be highlighted that although the maximum spacing error $e_i^x(t)$ is 150km , which is much larger than the desired spacing 20km , there still do not exist collisions in Fig. 7(b). The main reason for this phenomenon is owed to the use of CTH policy, which also demonstrates the superiority in improving string stability by using the MCTH policy (34). All these simulations show the effectiveness and advantage of the proposed method.

TABLE III
PARAMETERS OF HIGH-SPEED TRAINS FOR $i \in \mathcal{V}_N (N = 6)$

Symbol	Value
m_i	$80 \times 10^4 \text{kg}$
c_{i0}	0.01176N/kg
c_{i1}	0.00077616Ns/mkg
c_{i2}	$1.6 \times 10^{-5} \text{Ns}^2/\text{m}^2 \text{kg}$
δ_i, h_i	$20\text{km}, 1\text{h}$

TABLE IV
THE PARAMETERS $x_i(0)$ AND $v_i(0)$ FOR $i \in \mathcal{V}_N \cup \{0\} (N = 6)$

i	0	1	2	3	4	5	6
$x_i(0)(\text{km})$	148	118	94	71.5	48.5	24	1
$v_i(0)(\text{km/h})$	220	210	190	140	120	160	10

V. CONCLUSION

Based on TCTH and MCTH policies, two distributed adaptive NN control schemes are proposed for vehicle-following platoons. The proposed controller can force the followers to track the leader's trajectory while maintaining a desired

Fig. 7. (a) Spacing error $e_i(t)$; (b) Distance $d_i(t)$; (c) Velocity $v_i(t)$ under Theorem 2 with input saturation.

safety spacing simultaneously, even in the presence of input saturation, unknown unmodeled dynamics and external disturbances. A simple and straightforward strategy by adjusting only a single parameter is proposed to attenuate the effect of input saturation. Chebyshev neural networks (CNN) is used to approximate the unknown nonlinear function in the followers on-line, and the implementation of CP basis function depends only on the leader's velocity and acceleration. In addition, by adopting the CNN technique, the matching conditions as in the existing methods are removed. Future research will consider the effect of time-delay, measurement noise and so on.

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