Transformation-Invariant Metamaterials

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The fundamental semiconductor concept of doping has recently been transplanted to photonics in the platform of epsilon-near-zero (ENZ) media. By doping nonmagnetic impurities, ENZ media can exhibit almost arbitrary magnetism. However, this original photonic doping approach results only in isotropic media and thus cannot achieve impedance matching for all incident angles. We extend the photonic doping approach of ENZ media by adding anisotropy, which enables full transparency with omnidirectional impedance matching. More importantly, such anisotropically doped ENZ media preserve their material parameters under arbitrary coordinate transformations, thereby providing a powerful platform to construct various ideal transformation optical devices. As an example, a full-parameter omnidirectional invisibility cloak is demonstrated to hide objects from a wide range of incident angles. The transformation-invariant material proposed not only supplements and extends the rising technologies of ENZ media but also constitutes a significant step towards the practical implementation of ideal transformation optical devices.

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The connection between geometry and material in optics has been known for centuries, since the age of Fermat's principle [1], which governs the behavior of optical rays at a scale much larger than the optical wavelength. With the advent of metamaterials [2–4] this century, the geometry of light has been reestablished at the level of Maxwell's equations via the invention of transformation optics (TO) [5–7]; TO is applicable to all scales from macroscopic to deep subwavelength, controlling not only optical rays but also electromagnetic fields [5]. However, ideal TO devices are in general extremely inhomogeneous and anisotropic, and they normally require both electric and magnetic responses along three orthogonal directions [8]. As a result, a general strategy to realize ideal TO devices is still lacking. Most previous TO demonstrations [9–16] relied on the so-called eikonal approximation, which is valid only for optical rays, whereas it forfeits the power of TO in controlling electromagnetic fields.

The original transformation theory [5,9] adopts by default a standard design procedure that starts with vacuum, or an isotropic background medium (see Refs. [17–20] for general TO approaches applicable for anisotropic and arbitrary initial media), followed by a coordinate transformation. It subsequently leads to complex constitutive parameters that are difficult to realize. To tackle this problem, we ask a question. Can one start with a special metamaterial rather than vacuum and let it satisfy the following two conditions: (1) the constitutive parameters (e.g., permittivity and permeability) of the metamaterial are

invariant under an arbitrary coordinate transformation, and (2) the metamaterial is fully transparent to light illuminated from all angles? Apparently, such a metamaterial, once found, will offer a general platform to design and implement many ideal TO devices, paving the way to practically exploiting geometry of light at all scales.

No conventional optical material is eligible for the two demanding conditions. However, there might exist an exception, the epsilon-near-zero (ENZ; i.e., permittivity $\epsilon \simeq 0$) media [21,22]. From the perspective of TO, since any mathematical product involving zero is still zero, the ENZ property, i.e., $\epsilon \simeq 0$, is invariant under any coordinate transformation. However, conventional ENZ media have no control over the magnetic permeability μ , so the value of μ still changes during transformation. This situation has been changed by the recent breakthrough in ENZ technologies that transplanted the concept of “doping” from semiconductor physics [23]. Surprisingly, ENZ media doped with nonmagnetic impurities can exhibit arbitrary magnetism, including a zero value of permeability, i.e., $\mu \simeq 0$, giving rise to epsilon-mu-near-zero (EMNZ) media. Apparently, both the permittivity and the permeability of the EMNZ media are unchanged under arbitrary coordinate transformations, hence satisfying our condition (1). Moreover, within such EMNZ media, the efficient phase-invariant wave tunneling effect in ENZ media [21] can occur in arbitrarily shaped two-dimensional (2D) areas; namely, EMNZ media enable phase-invariant transparency for an arbitrary 2D geometry [23–25]. Unfortunately, such

transparency occurs for normal illumination only, being unable to satisfy the required condition (2).

The reason why the doped ENZ media cannot be fully transparent for all angles is because the original doping approach [23] produces only isotropic media, which cannot achieve impedance matching to the ambient environment for all angles. Here, we extend the photonic doping approach of ENZ media by including anisotropy. Such anisotropically doped ENZ media satisfy both of our requirements of omnidirectional impedance matching and constitutive parameter invariance under arbitrary coordinate transformations (note that previously reported anisotropic ENZ media [26–29] do not possess these two properties), thereby providing a general platform to construct various ideal TO devices.

Let us start with the original photonic doping approach [23]. We consider a 2D scenario and TM polarized

electromagnetic waves (electric fields in the x - y plane and magnetic fields along the z direction). To achieve an EMNZ medium, we follow Ref. [23] to dope an ENZ medium [as shown in Fig. 1(a)] with perfect electric conductor (PEC) particles (the simplest dopants [23]). See the details in Sec. I of the Supplemental Material [30]. According to Ref. [23], when PEC particles occupy almost the whole area of the ENZ medium (i.e., the “superdense doping case” [23]), as shown in Fig. 1(b), the effective permeability is nearly zero, i.e., $\mu_z \approx 0$. Apparently, a slab of undoped ENZ medium with $\varepsilon \approx 0$ and $\mu = 1$ has an infinitely large impedance $\sqrt{\mu/\varepsilon}$, which will reflect all waves emitted from a point source [simulated in Fig. 1(d)]. The impedance of the doped ENZ medium with $\varepsilon_{x,y} \approx 0$ and $\mu_z \approx 0$, instead, can be tuned to match the external environment and thus achieve transparency [23], but only for normal incidence. Consequently,

FIG. 1. Photonic doping of ENZ media. (a) An ENZ medium. (b) Isotropic doping of an ENZ medium. (c) Anisotropic doping of an ENZ medium. (d)–(f) Field patterns when electromagnetic waves radiated from a point source are incident on the slabs of media shown in (a)–(c).

FIG. 2. Coordinate transformation and device applications of anisotropically doped ENZ media. (a) Virtual space. The yellow arrow denotes an optical ray. (b) Coordinate transformation corresponding to an anisotropically doped ENZ slab. (c) Principal value invariance under coordinate transformation. (d) A flat superlens, (e) an omnidirectional retroreflector, and (f) an omnidirectional invisibility cloak constructed by the anisotropically doped ENZ media.

the electromagnetic waves radiating from a point source are still almost fully reflected [Fig. 1(e)]. Only negligible transmission occurs for the tiny amount of waves whose wave vectors are almost normal to the slab surface (i.e., propagating along the $+x$ direction).

Nevertheless, if we connect the PEC particles along the x direction and let them periodically separate rather than immerse in the ENZ medium [Fig. 1(c)], the electromagnetic waves radiating from the point source can perfectly propagate through the slab without any reflection [Fig. 1(f)]. Since the waves from the point source contain all wave vectors, the slab is fully transparent for all incident angles. This anisotropically doped ENZ medium in Fig. 1(c) has unusual effective electromagnetic parameters. Since the electric field in the anisotropically doped ENZ medium along the x direction (parallel to the PEC interfaces) is almost zero, to produce a finite electric displacement, the effective permittivity shall be infinite in this direction, i.e., $\epsilon_x \simeq \infty$; on the other hand, the electric displacement in the y direction is still zero (i.e., $D_y \simeq 0$), which maintains $\epsilon_y \simeq 0$. Meanwhile, since the connection of PEC particles has a negligible influence on the magnetic flux in the z direction, $\mu_z \simeq 0$ is preserved. We have thus constructed a new medium with $\epsilon_x \simeq \infty$, $\epsilon_y \simeq 0$, and $\mu_z \simeq 0$. (See Sec. I of the Supplemental Material [30] for the full transparency property of this medium.) Note that similar multilayer structures have been used to construct previously reported anisotropic ENZ media without control of the permeability [26–29]; their wave response is similar to that in Fig. 1(e), but not to the all-angle full transparency in Fig. 1(f).

The full transparency for all incident angles in Fig. 1(f) can be explained with TO. If we take an imaginary vertical line [marked in green in Fig. 2(a)] in vacuum and stretch it into a finite slab in Fig. 2(b), the constitutive parameters of the transformed slab will take the form of $\epsilon_x \simeq \infty$, $\epsilon_y \simeq 0$, $\mu_z \simeq 0$, identical to those of our proposed metamaterial in Fig. 1(c). This explains the full transparency resulting from the anisotropic doping approach. Moreover, since the newly introduced $\epsilon_x \simeq \infty$ by anisotropic doping is also unchanged under arbitrary coordinate transformations, arbitrary coordinate transformations do not change the principal values of the constitutive parameters of the anisotropically doped ENZ media but only rotate their optical axes (details are provided in Sec. II of the Supplemental Material [30]), as illustrated in Fig. 2(c). Such a transformation invariance property applies only to the proposed anisotropically doped ENZ media; it does not hold for the previously reported anisotropic ENZ media [20].

Here, we take three examples to show potential applications of the media in constructing ideal TO devices (corresponding coordinate transformations are provided in Sec. III of the Supplemental Material [30]). The first example in Fig. 2(d) shows an impedance-matched flat superlens constructed by the newly developed anisotropically doped ENZ media. Previously, superresolution was achieved either with flat negative index materials without magnification [31,32], or with hyperbolic materials which must have a curved interface to maintain the phase continuity of images [33,34]. Yet in our example shown in Fig. 2(d), the image of two point sources with distance of

FIG. 3. Demonstration of an invisibility cloak based on anisotropic photonic doping of ENZ media. (a) Schematic of the experimental setup. One horn antenna launches electromagnetic waves on one side of the parallel-plate waveguide, while the other measures transmission on the other side. Both antennas can sweep angles azimuthally. The height of the parallel plane waveguide is 35 mm. (b)–(d) Photos of the cloak details. $p = 6$ mm, $d_1 = 0.5$ mm, $d_2 = 0.5$ mm.

half a wavelength can be amplified by the flat superlens to a distance of eight wavelengths without phase variance, and perfectly coupled to free space without reflection. The second example in Fig. 2(e) shows a wide-angle retroreflector which can perfectly reflect a Gaussian beam back along the opposite direction. Previously, retroreflectors were demonstrated by either complex metamaterial or metasurface designs [11,35]. In our second example, we can construct a wide-angle retroreflector merely with our

proposed anisotropically doped ENZ media, as shown in Fig. 2(e). Finally, in Fig. 2(f), we consider an impedance-matched wide-angle invisibility cloak without parameter reduction, which has never been achieved before due to the singular constitutive parameters required [5,9,10,12–16,36,37]. Here, with the anisotropically doped ENZ media, we can construct such an ideal cloak that can perfectly conceal the central rhombic region from external detection. The wave displacements can be removed by

FIG. 4. Performance of the invisibility cloak based on anisotropic photonic doping of ENZ media. (a)–(c) Simulated field patterns at 4.29 GHz for cases of bare Teflon background, PEC obstacle, and cloak when the waves are normally incident. (d)–(f) Measured transmissions at 4.27 GHz corresponding to (a)–(c). (g)–(i) Simulated field patterns at 4.29 GHz for 10-, 20-, and 30-deg incidences. (j)–(l) Measured transmissions at 4.27 GHz for 10-, 20-, and 30-deg incidences.

extra anisotropic media (see Sec. IV of the Supplemental Material [30] for more mathematical details).

We use the third example, i.e., the invisibility cloak, to demonstrate experimentally the effectiveness of our method. ENZ media have been demonstrated in several previous works both in microwave [38,39] and optical regimes [40,41]. Here, we adopt the method of structural dispersion [38] for microwave demonstration. For convenience, the cloak is designed with only four pieces of anisotropically doped ENZ metamaterials. Figure 3(a) shows the schematic of experimental setup in a parallel-plate metallic (aluminum) waveguide environment. The two parallel plates of the waveguide are separated vertically by 35 mm, filled with Teflon (relative permittivity, 2.1) as the background material. The cloak [the red region in Fig. 3(a)] is implemented with thin aluminum plates that are stacked with periodicity $p = 6$ mm [indicated in Fig. 3(c); Figs. 3(b)–3(d) are the photos of details of the cloak]. Note that there is a thin empty channel with thickness $d_1 = 0.5$ mm between adjacent aluminum plates. The cutoff frequency of the channels is 4.29 GHz, at which the thin channels can be effectively treated as ENZ media [38]. Two extra-thin coupling regions [with thickness $d_2 = 0.5$ mm, indicated in Fig. 3(d)] on two sides of the cloak structure are designed to enhance coupling from the Teflon background to the thin air channels. We excite the TE₁ mode of the parallel-plate waveguide at 4.29 GHz (near the cutoff frequency of the thin channels) with a homemade lens antenna and measure the transmission on the other side of the setup (see measurement details in Sect. V of the Supplemental Material [30]). To facilitate the measurement, we fabricate the background Teflon to be two semicircular disks so that we can sweep the incident and receiving angles [as illustrated in Fig. 3(a)].

Figures 4(a)–4(c) show the simulated field patterns at 4.29 GHz for cases of the bare Teflon background, a bare rhombic obstacle, and the same obstacle in the presence of designed cloak, respectively. The results clearly show significant scattering from the obstacle [see Fig. 4(b)], while the cloak can smoothly guide incident waves around and dramatically suppress scattering [see Fig. 4(c)]. Figures 4(d)–4(f) show the measured transmission for varying receiving angles at 4.27 GHz of the above three cases in Figs. 3(a)–3(c), respectively. The results are normalized to the maximum transmission through the background Teflon.

Then we alter the incident angle. We only take 10, 20, and 30 deg as the oblique incident angles; larger incident angles require a larger construction beyond the size of the current setup. The simulated field patterns at 4.29 GHz and the measured transmission at 4.27 GHz on the receiving side for different incident angles are shown in Figs. 4(g)–4(i) and 4(f)–4(l), respectively. For relatively small incident angles (e.g., <20 deg), the measured transmission can be above 94%. The slightly lower transmission

(above 80%) of 30 deg is because of the larger leakage and dissipation along the longer propagation path in the cloak.

The anisotropically doped ENZ metamaterial demonstrated in this Letter is transparent to any incident illumination. Its constitutive parameters are invariant under arbitrary coordinate transformations. These remarkable properties make it a powerful platform to construct ideal TO devices. Owing to ENZ properties, this scheme is limited to a narrow bandwidth. Although our experimental demonstration is performed at microwave frequencies, the principle of anisotropic photonic doping is applicable to both metals and dielectrics, and it can be generalized to higher frequencies. Possible optical realizations can be achieved by combining optical ENZ media [25,40,41] and high refractive index metamaterials [42,43], where large material dissipations must be carefully considered. Our photonic doping approach has thus far been limited to 2D and currently cannot be translated to 3D systems [23]. However, the concept of transformation-invariant metamaterial is valid for both 2D and 3D. Our study can be readily extended to acoustics, thermal dynamics, mechanics, etc., paving the road to the full control of impedance and geometry of other physical fields.

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