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Imperfect Predictive Maintenance Model for Multi-State Systems with Multiple Failure Modes and Element Failure Dependency

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Abstract - **The objective of this study is to develop a practical statistical model for imperfect predictive maintenance based scheduling of multi-state systems (MSS) with reliability dependent elements and multiple failure modes. The system is modeled using a Markov state diagram and reliability analysis is performed using the Universal Generating Function (UGF) technique. The model is simulated for a case study of a power generation - transmission system. The various factors influencing the predictive maintenance (PdM) policy such as maintenance quality and user threshold demand are examined and the impact of the variation of these factors on system performance is quantitatively studied. The model is found to be useful in determining downtime schedules and estimating times to replacement of an MSS under the PdM policy. The maintenance schedules are devised based on a "system-perspective" where failure times are estimated by analyzing the overall performance distribution of the system. Simulation results of the model reveal that a slight improvement in the "maintenance quality" can postpone the system replacement time by manifold. The consistency in the quality of maintenance work with minimal variance is also identified as a very important factor that enhances the system's future operational and downtime event predictability. Moreover, the studies reveal that in order to reduce the frequency of maintenance actions, it is necessary to lower the minimum user expectations from the system, ensuring at the same time that the system still performs its intended function effectively. The model proposed can be utilized to implement a PdM program in the industry with a few modifications to suit the individual industry's needs.**

I. INTRODUCTION

Maintenance has evolved from the age-old ad hoc corrective (or reactive) maintenance [**1**] (CM) to preventive maintenance (PM) [**2**] and then to the presently popular predictive maintenance (PdM) [**3, 4**]. However, it is well recognized that both the CM and PM are ineffective. In the case of CM, the "completely failed" system is highly degraded, making maintenance very difficult, time-consuming and expensive. Also, CM is associated with large and unpredictable downtimes resulting in low mean availability, increased delays, larger inventory storage requirements and increased forgone production losses. As for PM, the fixed downtime intervals imply more-than-necessary repair frequency during the initial periods of the system operation that could increase the probability of maintenance-induced failures. On the other hand, as the system ages and enters into its wear-out period, PM results in less-than-necessary repair frequency, thereby increasing the probability of unanticipated catastrophic failures and making PM similar to CM.

In PdM, which is also referred to as condition-based PM [**5**], the maintenance schedule and frequency match the age or health of the system at all times, making the schedule nearly optimum, prolonging the time to replacement (TTR) as a consequence. The expected times to future failure of a system are estimated during each operational period based on the variation pattern of its physical properties (condition monitoring) that are indicative of its state of degradation using implanted sensors, and the downtime schedule for each operation cycle is determined based on the estimated future failure times. Past research studies show that the average system reliability (and yield), availability and mean system performance are the highest for PdM and the incurred maintenance operation costs are the lowest [**6**]. The spare part requirements and delay times are also reduced due to reliable predictions of future downtime events.

However, there are currently two main obstacles to the practical implementation of the PdM policy. Firstly, there is no simple concrete statistical model that PdM can be based upon. The past models developed are theoretical in their approach with idealistic assumptions and fitting parameters, rendering them unfit for practical real-world implementation. For example, in [**7**], it was proposed that the system being repaired could be restored to either the "as-good-as-new" condition or the "as-bad-as-old" condition with complementary probabilities, failing to account for the possibility that the system's restoration could be somewhere in between these two possible extreme cases. Although the *virtual age* model proposed by Kijima et. al. [**8**] to account for the imperfect restoration helped overcome the above-mentioned problem, the determination of the effective age parameter '*a*' in the proposed model is not given, making its implementation vague.

Secondly, the implementation of PdM requires advanced monitoring technologies, real-time data acquisition systems with sophisticated data storage and speed requirements and signal processing and filtering techniques [**9**], making the implementation of PdM complex and expensive. However, with the advances in sensor technologies today, this difficulty is gradually overcome [**10**].

In this work, we will focus on the first obstacle which is to develop a comprehensive and practical statistical model for PdM. *Imperfect maintenance* will be considered in this work for practical applications. This imperfect maintenance is a term frequently used to refer to maintenance activities in which the future reliability and degradation trend of the system depends on the skill and quality of the current and previous repair works performed. In other words, imperfect maintenance accounts for the impact of maintenance quality on the future reliability of repairable systems. In reality, maintenance

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strategies for most systems can be categorized under imperfect maintenance because repair / replacement is typically performed only for some of the components in the system; while there are other components which are degraded but not to the extent of needing a repair / replacement. Therefore, from a "system perspective", repair does not rejuvenate a system to its original zero degradation state, calling for the need to use imperfect maintenance models.

The structure of this paper is as follows. Section II gives a brief review on the various existing models for imperfect maintenance. Section III introduces the methodology for the multi-state system PdM modeling and the description of the system case study. Section IV describes the various results from the model simulation and finally, a short summary of the work done and results achieved is presented in Section V.

II. IMPERFECT MAINTENANCE

Various models have been proposed for imperfect maintenance in the past from different perspectives as reviewed in complete detail in **[11]**. Basically, there are four classes of models developed so far.

The first class of models was based on a *probabilistic approach* **[12]** – **[14]** where it was assumed that the system undergoes "perfect renewal" to "as-good-as-new" condition with a constant probability of *p* and "minimal repair" to "as-bad-as-old" condition with a probability of *(1-p)*. Further enhancement to this probabilistic approach was to consider the probabilities as time-varying functions, *p(t)* and *[1-p(t)]*, to account for the change in these values with the aging system's degradation **[15, 16]**. Makis and Jardine **[17]** further account for the probability that the repair is unsuccessful and causes a catastrophic complete system failure and the *p(t)* function was modified to *p(n ,t)* to describe the probability accounting for the number of previous failures, *n*, undergone by the system prior to the current one.

The second class of models was based on the *improvement factor* method where the system was analyzed by looking at the failure rate. Certain models were proposed to reflect the reduction in failure rate after repair **[18, 19]**. The degree of improvement in the failure rate was called improvement factor and "failure rate" was used as the threshold reliability index.

The third class of models was based on the age of the system. The most popular model in this class is known as the *virtual age model* proposed by Kijima et al, **[8, 20]**. The virtual age of the system after the n^{th} repair (V_n) is expressed as: $V_n = V_{n-1} + a \cdot X_n$ where X_n is n^{th} failure time, V_{n-1} is the virtual age after $(n-1)^{th}$ repair and "*a*" is the virtual age parameter ($0 \leq a \leq 1$). However, the method of estimating the parameter "*a*" is not mentioned in the literatures. Another age-based model called the *proportional age setback model* was proposed in **[21]** which is very similar to the virtual age model, except that the effects of equipment working conditions and surveillance effectiveness on imperfect maintenance and corresponding age reduction are accounted for in addition to the maintenance work quality.

The fourth class of models was based on the *system degradation* where the system is considered to suffer random shocks at variable intervals of time causing it to undergo progressive increments of damage **[22, 23]**. When a threshold cumulative damage level is reached, the system is interpreted to have failed. The effect of the imperfect maintenance actions is described by the degree of reduction of the cumulative system damage after repair as compared to that before repair. Wang et al. **[24]** – **[27]** treated imperfect maintenance by modeling the decrease in system lifetime with the increase in the number of repairs. They also modeled the time between maintenance actions using a quasi-renewal process **[26]**.

There are other models to account for imperfect maintenance, and readers may refer to references **[14]** and **[28]** for detailed information. However, all the above-mentioned models are focused on a binary system where the system operates in only two discrete performance states, viz. "functional" or "non-functional". Also, most of the models assume a "single unit" system with negligible maintenance duration and the impact of maintenance quality on the system reliability is not included. For practical applications, multi-state systems should be considered **[29, 30]**, and the study of the impact of maintenance quality is especially useful as it directly impacts the time to replacement of the system. It is with the motivation to enable PdM model to be applicable in a practical environment that this work is produced.

In this work, we analyze the system from a multi-state perspective for a generic complex n-component system with any system structure (series, parallel, k-out-of-n structures etc…). Imperfect *PdM* is modeled by considering the effect of the mean and variance in the quality of maintenance work separately. The impact of the skill of maintenance work and the spare part product quality on the future reliability of a system is modeled by a parameter called the *Restoration Factor* (RF) which describes the percentage recovery in the system performance for the new operation cycle, after maintenance, relative to the previous operation cycle of the system. Being a quality index, *RF* is assumed to have a *normal distribution* with its mean (μ_{RF}) and standard deviation (σ_{RF}) giving a clear indication of the skill and consistency of maintenance work performed respectively. The model developed here is widely applicable to most industrial systems and its application to a dependent multi-state system (MSS) with multiple failure modes is illustrated in this work.

Although an imperfect maintenance policy for multi-state systems (MSS) has been proposed earlier in **[31]** based on the *proportional age setback model* **[21]**, the maintenance duration is assumed to be negligible, and the *user threshold demand* is assumed to be constant. Furthermore, no method for the determination of the time to replacement (TTR) of the system is discussed. For practical applications, the model proposed in this work considers the finite maintenance duration and its variability, and the system replacement time is determined using a simple approach.

The novelty of the work lies in characterizing the system performance variation in dependent and multiple failure mode MSS for different maintenance work quality standards represented by the restoration factor (RF) distribution and different user threshold demands (W). In other words, the system's performance capability is being examined from the user's perspective. The system is modeled using a Markov State Diagram in this work, which is found to be a good choice for modeling complex systems **[32]**. Readers may refer to **[33]** for the earlier study of PdM applied to the simplest case of an independent MSS with no element failure dependency and single failure mode.

III. METHODOLOGY & SYSTEM CASE STUDY

There are two types of *multi-state systems (MSS)*. One is the *flow transmission system* **[29]** in which the performance (degradation) of the system is characterized and measured in terms of its *productivity* or *capacity*. Typical examples are (a) hydraulic systems where performance is measured in terms of *volume* and *mass flow rates (tons/min)*, (b) power systems with its *power generating capacity* and (c) continuous production systems with its *rate of production*.

The other type is the *task processing system* **[30]** in which performance is described in terms of *processing speed* or *response time*. Typical examples include server, control and other software systems where the performance index of interest is the speed of processing data and instructions, expressed in mega bits per second (mbps). The analysis of these two types of MSS mentioned above are different owing to the different properties being examined in them and the different nature of their basic functions.

The system examined in this work is a power generator system which is a flow transmission MSS. The topology of the system consists of 3 generator elements as shown in Fig 1. Elements 1 and 2 are in parallel with each other and they are collectively in series with element 3. The performance (degradation) index of the system is the power processing capacity of each generator element expressed in the unit of megawatts (MW). In this study, element 2 is considered to be dependent on element 1 and element 3 degrades under the influence of two failure modes which may be dependent on each other.

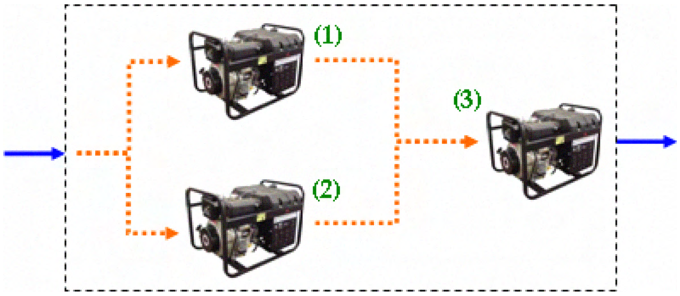

Fig. 1. Power generator system topology examined in this case study.

In this work, the power generator system is analyzed using the *Markov* process **[29, 30]**. Each element has its own *markov state diagram* with different states of performance and corresponding degradation rates (conventionally known as "failure rate") as shown in Fig 2. The parameters g_{ij} represent the j^{th} discrete degraded state of performance of the i^{th} element in the system. The symbol $\lambda_{i,j}^{(m)}$ is the degradation rate of element *m* where its performance degrades from the i^{th} state to the j^{th} state. The operational lifespan of the system may be classified into different operation cycles. The k^{th} *operation cycle* is defined as the operating time interval between the $(k\text{-}1)^{th}$ and k^{th} maintenance actions. Referring to the Markov State diagram shown in Fig 2, the numerical values for the various states of performance of each element and the degradation rate (λ) are given in Table I.

The system in this study is modeled such that Element 2 is dependent on Element 1 and Element 3 is influenced by two failure modes (FM). The symbol *p* in Table I denotes the conditional probability of occurrence of FM B given that FM A has occurred. In other words, FM A is assumed to be the failure mode which will definitely occur while FM B is triggered depending on FM A.

Each performance rate in Table I corresponds to a discrete amount of power processing capacity that every element of the generator system can process. For example, *Element 3* has *3* discrete states of performance. It could be processing power at its maximum capacity (performance) of g_{31} = 10 MW or intermediate capacity of g_{32} = 5 MW or at g_{33} = 0 MW implying complete non-functional failure. Elements 1, 2 and 3 each have 3 discrete states of performance, thus making up a total of 3 × 3 × 3 = 27 discrete *system performance rates*.

Under some reasonably general conditions, the failures of a complex system can be shown to follow the exponential distribution even though the individual components in the system may follow other failure distributions **[32]**. Thus, if the power generator system in Fig 2 is assumed to be complex, it will follow the exponential failure distribution pattern, thereby justifying the use of a *Markov State Diagram*, in which the degradation rates are all time-independent constants.

During the operation of the system, the elements transit from one state of performance to another in a period of time. Using the concept of *hazard rate*, the inter-state transitions can be described using the *degradation rate*, which is expressed as the number of such state transitions per year (unit time). The hypothetical values assumed for the degradation rates of each element are shown in Table I. The values of these parameters in Table I for various state transitions may be extracted from past maintenance data records and condition monitoring data of previously operated similar systems using the standard *Maximum Likelihood Estimate (MLE)* procedure for the exponential distribution **[34]**.

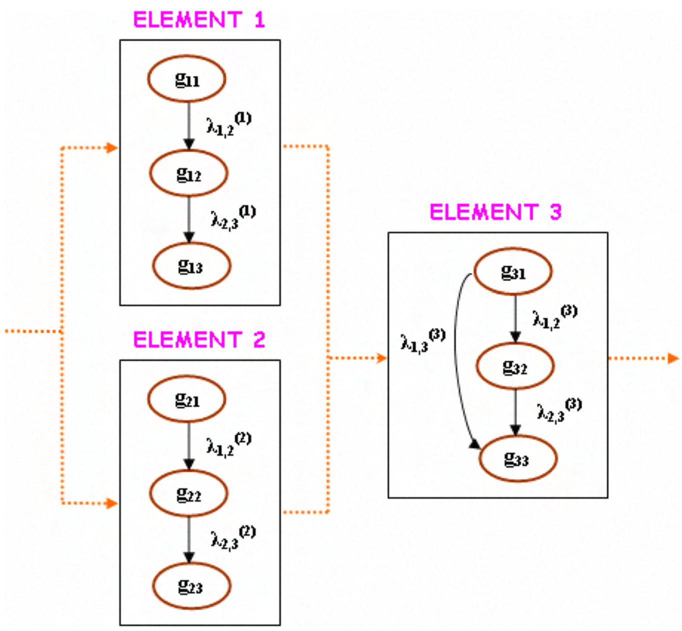

Fig. 2. Markov State Diagram for the 3-element power generator system.

For example, if *n* observations on *n* different identical systems are made from their historical maintenance records and the duration (t_i) for transition between consecutive states of performance $g_{i,j}$ and $g_{i,j+1}$ is measured, then the degradation rate $\lambda_{j,j+1}$ could be estimated using the exponential MLE as in (1) where t_i is the time to degradation (TTD) of the i^{th} observation from state *j* to state *j+1*.

$$\hat{\lambda}_{j,j+1} = \left(\frac{n}{\sum_{i=1}^{n} t_i} \right) \tag{1}$$

As mentioned in Table I, the values of $\lambda_{1,2}^{(3)}$ and $\lambda_{1,3}^{(3)}$ are assumed to depend on the failure modes A and B and the conditional probability, *p*. If FM A causes FM B, then element 3 is under the influence of two failure modes and it is most likely to undergo a catastrophic breakdown from state g_{31} to state g_{33}. The probability for such a *catastrophic* failure is the conditional probability (*p*) of FM B occurring given that FM A is present. In contrast, when FM B is not triggered by FM A with a corresponding probability of (*1 - p*), then element 3 *degrades gradually* from state g_{31} to g_{32} and eventually to g_{33}, since it is under the influence of only one failure mode. Based on the above proposition, the expressions for $\lambda_{1,2}^{(3)}$ and $\lambda_{1,3}^{(3)}$ in terms of λ_A, λ_B and *p* may be expressed as in (2) and (3). The terms λ_A and λ_B refer to the degradation rates of element 3 with respect to failure modes A and B respectively.

$$\lambda_{1,2}^{(3)} = (1-p)\cdot\lambda_A \tag{2}$$

$$\lambda_{1,3}^{(3)} = p\cdot(\lambda_A + \lambda_B) \tag{3}$$

TABLE I
NUMERICAL VALUES OF PERFORMANCE STATES AND TRANSITION DEGRADATION RATES IN THE MARKOV MODEL

Element (#)	Performance State (MW)	Degradation Rates (yr^{-1})	Remarks
1	$g_{11} = 6.0$ $g_{12} = 3.5$ $g_{13} = 0.0$	$\lambda_{1,2}^{(1)} = 0.020$ $\lambda_{2,3}^{(1)} = 0.030$	Element 1 has 3 distinct states of performance. It has only one failure mode and it is independent of other elements.
2	$g_{21} = 4.0$ $g_{22} = 2.5$ $g_{23} = 0.0$	$\lambda_{1,2}^{(2)} = 0.035$ $\lambda_{2,3}^{(2)} = 0.045$	Element 2 performance distribution dependent on Element 1 performance state.
3	$g_{31} = 10.0$ $g_{32} = 5.0$ $g_{33} = 0.0$	$\lambda_A = 0.050$ $\lambda_B = 0.080$ $\lambda_{2,3}^{(3)} = 0.040$	Element 3 has two failure modes {A, B} where failure mode A always exist and the conditional probability of occurrence of FM B given that FM A exists is *p*. The values of $\lambda_{1,2}^{(3)}$ and $\lambda_{1,3}^{(3)}$ depend on λ_A, λ_B and *p*.

It is to be noted that the model above is based on the observed system dynamics of the generator being examined. Different systems have different natures and modes of failure and in each case, the model in equations (2) and (3) will change according to the particular system. Therefore, equations (2) and (3) are not standard expressions. Rather, they are models used to describe the nature of failure of the generator system.

There are two important factors which affect the predictive maintenance (PdM) policy. They are the restoration factor (RF) and the user threshold demand (W). Since the impact of these two factors on PdM will be investigated in this work, let us now discuss these two factors in detail.

A. PdM Model Parameters

1) Restoration Factor (RF)

To study the impact of the quality of a maintenance work on system performance quantitatively under the PdM policy, a new term called the *Restoration Factor* (RF) is introduced. It represents the percentage recovery of the system's mean performance in the k^{th} operation cycle (after the *(k-1)*th maintenance action) relative to its mean performance during the previous *(k-1)*th operation cycle. The k^{th} *maintenance action (cycle)* refers to the downtime duration between the successive k^{th} and *(k+1)*th operation cycles. The better the maintaining quality is, the higher the RF.

Based on the definition of RF, the system mean performance during the k^{th} operation cycle, denoted by $G_k(t)$, may be expressed in terms of the corresponding mean performance in the *(k-1)*th operation cycle, $G_{k-1}(t)$ using the restoration factor of the preceding *(k-1)*th maintenance cycle, *RF[k-1]* as follows:

$$G_k(t) = G_{k-1}(t)\cdot RF[k-1] \tag{4}$$

An RF value of 100% represents the system being maintained to an *as-good-as-new (renewal process)* condition. However, this is practically unachievable unless the system is *replaced* (expensive) instead of being *maintained* (repaired) upon failure.

The RF value is not a constant throughout the system life cycle. It is a random variable that can vary during every maintenance action because it is influenced by various factors such as the concentration and attentiveness of the maintenance personnel (state of mind), the ability to accurately locate the point of defect due to the complexity of the failure, availability of appropriate spare parts and maintenance tools etc. Also, the same system might be maintained by different personnel during different downtimes having different capabilities in performing the same maintenance work. Moreover, at the system-level, not all the components undergo a repair / replacement during the maintenance task. This variability in the quality of maintenance work and confinement of repair to certain critical components of the system renders RF to be considered as a *random variable*.

The RF parameter is modeled as a random variable following a *Normal Distribution* as given by (5) in this work. As RF is always positive, it should be modeled by a statistical distribution with a positive valued random variable. However, the ensemble of the many RF values over a period of operation time and a large number of equipment maintenance actions justify its representation by a normal distribution with large mean value and moderately low standard deviation, such that the probability of having negative value is negligible, as the tail of the normal p.d.f narrows down around RF = 0, making the probability, $Pr(RF < 0)$ very small (rare event). In (5), μ_{RF} is the *mean* value of the RF distribution and σ_{RF} is the corresponding *standard deviation*.

$$RF \equiv N(\mu_{RF}, \sigma_{RF}^2) \tag{5}$$

As most industries are adopting the conventional CM policy, past maintenance history of downtime events for previously failed systems could be used to estimate RF values and then use them to predict the parameters of the RF distribution. The RF estimate for a particular k^{th} maintenance action, denoted by RF_k, can be calculated using (6) where T_{k+1} and T_k represent the durations of the $(k+1)^{th}$ and k^{th} operation cycles respectively. Quantities t_{k+1} and t_k refer to the absolute time instance of the $(k+1)^{th}$ and k^{th} failures respectively, relative to the initial time of system operation $(t = 0)$; τ_k and τ_{k-1} are the downtime durations for the k^{th} and $(k-1)^{th}$ repair actions respectively. Similar values of RF can be estimated for all possible k values and for many such identical systems. The obtained RF data set could then be used to compute μ_{RF} and σ_{RF} parameters for the RF distribution using the standard statistical mean and variance expressions. Based on the estimated values of μ_{RF} and σ_{RF} from past maintenance records, the performance trends for future operation cycles of new similar systems can be modeled based on Eq. (4) where $RF[k-1]$ is a random number representing the expected maintenance quality for the $(k-1)^{th}$ maintenance action, which is generated from the normal distribution with parameters μ_{RF} and σ_{RF} using the random number generator function.

$$RF_k = \frac{T_{k+1}}{T_k} = \frac{t_{k+1} - (t_k + \tau_k)}{t_k - (t_{k-1} + \tau_{k-1})} \tag{6}$$

This approach of computing the RF distribution parameters from past CM maintenance records is based on the assumption that the quality of maintenance in both CM and PdM policies is similar because the nature of the maintenance work is essentially the same. Note that only hands-on field repair on the system is termed as "maintenance" in this work. Minor control signal adjustments to the automated system's parameters is not considered as a "maintenance" activity as it does not involve or require any hands-on maintenance personnel skills.

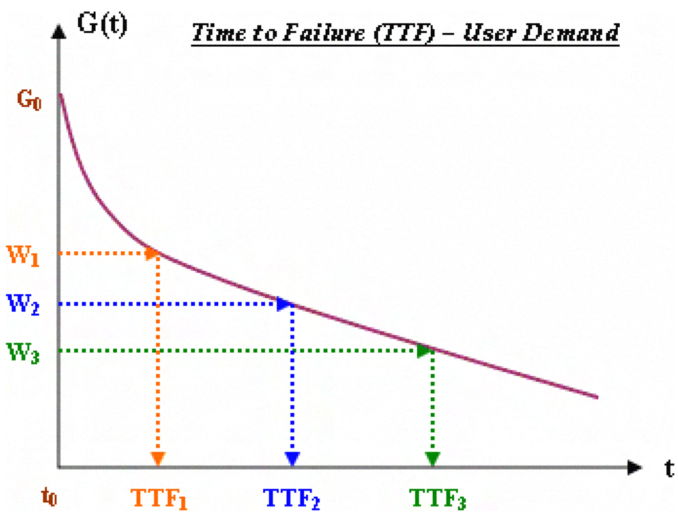

Fig. 3. Schematic illustration of the relationship between time to failure (TTF) and user demand (W).

2) User Threshold Demand (W)

In this study, the system's performance is analyzed from the *user's perspective*. The user of the system sets a minimum expectation from the system, called the *user threshold demand*, represented as *W*. During each operation cycle, as the system's mean performance $G(t)$ drops below the user set demand of W, the system is "interpreted" to have "*failed*" from the *user's perspective* even though the system might not have physically failed in reality, i.e. the user's "dissatisfaction" is considered as "system failure" in this case. The higher the user threshold demand (*W*), the sooner will be the time to failure (*TTF*) and replacement (*TTR*) of the system as seen in Fig 3. The *time to failure* for a system during the k^{th} operation cycle is estimated by solving (7).

$$G_k(t) - W = 0 \tag{7}$$

B. Markov Chain Diagram

Having described the parameters RF and W, we now develop and analyze the Markov model for the system. From the Markov State diagram for each element shown in Fig 2, the set of simultaneous differential equations describing the state probability expressions may be extracted. They are listed in equations (8) – (16) below. Every state is described by its corresponding differential equation. In Fig 2, since each element has 3 states, there are a total of 3 + 3 + 3 = 9 dependent differential equations to be solved simultaneously, with the following initial conditions: $p_{11}(0) = p_{21}(0) = p_{31}(0) = 1$, $p_{12}(0) = p_{13}(0) = p_{22}(0) = p_{23}(0) = p_{32}(0) = p_{33}(0) = 0$. These initial conditions correspond to the probability of the system's elements in their most efficient states of performance $\{g_{11}, g_{21}, g_{31}\}$ being equal 100% at time t = 0.

$$\dot{p}_{11}(t) = -\lambda_{1,2}^{(1)} \cdot p_{11}(t) \tag{8}$$

$$\dot{p}_{12}(t) = +\lambda_{1,2}^{(1)} \cdot p_{11}(t) - \lambda_{2,3}^{(1)} \cdot p_{12}(t) \tag{9}$$

$$\dot{p}_{13}(t) = +\lambda_{2,3}^{(1)} \cdot p_{12}(t) \tag{10}$$

$$\dot{p}_{21}(t) = -\lambda_{1,2}^{(2)} \cdot p_{21}(t) \tag{11}$$

$$\dot{p}_{22}(t) = +\lambda_{1,2}^{(2)} \cdot p_{21}(t) - \lambda_{2,3}^{(2)} \cdot p_{22}(t) \tag{12}$$

$$\dot{p}_{23}(t) = +\lambda_{2,3}^{(2)} \cdot p_{22}(t) \tag{13}$$

$$\dot{p}_{31}(t) = -\left(\lambda_{1,2}^{(3)} + \lambda_{1,3}^{(3)}\right) \cdot p_{31}(t) \tag{14}$$

$$\dot{p}_{32}(t) = +\lambda_{1,2}^{(3)} \cdot p_{31}(t) - \lambda_{2,3}^{(3)} \cdot p_{32}(t) \tag{15}$$

$$\dot{p}_{33}(t) = +\lambda_{2,3}^{(3)} \cdot p_{32}(t) + \lambda_{1,3}^{(3)} \cdot p_{31}(t) \tag{16}$$

The terms $p_{i,j}(t)$ in the above equations denote the probability that element i is in state j at any arbitrary time $t \geq 0$.

C. Universal Generating Function (UGF)

The *UGF* methodology **[30, 35]** is an essential tool to obtain the *performance distribution* of the *overall system* from the performance distribution of the individual elements of the system. A *performance distribution* is a probability distribution table listing the various states of performance of the element/system and their corresponding time-varying state probability expressions. The *element performance distributions* for all the 3 elements of the generator system are described in Table II based on the Markov analysis results in the previous section.

The *system performance distribution* needs to be obtained from the individual *element performance distributions* in order to characterize the system's reliability and performance variation denoted by R(t) and G(t) respectively, which is our final objective. This is made possible using the *Universal Generating Function* (UGF) represented as *U(z)*, which is a *z-transform* based approach first proposed by *Ushakov* (1987) **[36]**. The UGF is an efficient tool for complex multi-state systems (*MSS*) reliability assessment as it greatly reduces the problem complexity and computational intensity by modularizing a system into its components and analyzing each component of the system individually, thereby enabling a complex problem to be broken into sub-problems each of which can be solved separately, with ease.

For the generator system in this work, the UGF method reduces the total number of differential equations to only 3 + 3 + 3 = *9* in (8) – (16) as compared to using a single "overall-system" markov analysis which would have required a maximum of 3 × 3 × 3 = *27* differential equations, corresponding to the 27 discrete and distinct system states.

As mentioned earlier, element 2 of the generator system is dependent on element 1. This dependency can be modeled in such a way that the *performance distribution* of element 2 depends on the *performance state* of element 1.

TABLE II
PERFORMANCE DISTRIBUTION OF THE THREE ELEMENTS OF THE GENERATOR SYSTEM

Element	*Performance Distribution*		
1	$g_{11} = 6.0$	$g_{12} = 3.5$	$g_{13} = 0.0$
	$p_{11}(t)$	$p_{12}(t)$	$p_{13}(t)$
2	$g_{21} = 4.0$	$g_{22} = 2.5$	$g_{23} = 0.0$
	$p_{21}(t)$	$p_{22}(t)$	$p_{23}(t)$
3	$g_{31} = 10.0$	$g_{32} = 5.0$	$g_{33} = 0.0$
	$p_{31}(t)$	$p_{32}(t)$	$p_{33}(t)$

Representing the discrete random variable for performance of elements 1 and 2 as G_1 and G_2, if we assume the following dependency condition given in Table III, we have the following equations:

TABLE III
CONDITIONAL PERFORMANCE DISTRIBUTION FOR ELEMENT 2 OF THE GENERATOR SYSTEM

Element 1 state	***Element 2 Performance Distribution***
$G_1 = \{g_{11}\}$ →	$\mathbf{G_{2A}} = \{g_{21}, g_{22}\}$ & $\mathbf{P_{2A}} = \{p_{21A}(t), p_{22A}(t)\}$
$G_1 = \{g_{12}, g_{13}\}$ →	$\mathbf{G_{2B}} = \{g_{22}, g_{23}\}$ & $\mathbf{P_{2B}} = \{p_{22B}(t), p_{23B}(t)\}$

$$P(G_2 = g_{21}) = p_{21}(t) = P(G_{2A} = g_{21} | G_1 = g_{11}) = p_{11}(t) \cdot p_{21A}(t) \quad (17)$$

$$\begin{aligned} P(G_2 = g_{22}) &= p_{22}(t) = P(G_{2A} = g_{22} | G_1 = g_{11}) + P(G_{2B} = g_{22} | G_1 = \{g_{12}, g_{13}\}) \\ &= p_{11}(t) \cdot p_{22A}(t) + [p_{12}(t) + p_{13}(t)] \cdot p_{22B}(t) \end{aligned} \quad (18)$$

$$\begin{aligned} P(G_2 = g_{23}) &= p_{23}(t) = P(G_{2B} = g_{23} | G_1 = \{g_{12}, g_{13}\}) \\ &= [p_{12}(t) + p_{13}(t)] \cdot p_{23B}(t) \end{aligned} \quad (19)$$

Equations (17) – (19) describe the *conditional probability theory* used to determine the resultant state probability expressions $p_{21}(t)$, $p_{22}(t)$ and $p_{23}(t)$ for dependent element 2.

Using the *UGF* approach, the z-polynomial *u-functions*, *u(z)* for individual elements 1, 2 and 3 of the system may now be expressed as follows,

$$u_1(z) = p_{11}(t) z^{g_{11}} + p_{12}(t) z^{g_{12}} + p_{13}(t) z^{g_{13}} \quad (20)$$

$$u_2(z) = p_{21}(t) z^{g_{21}} + p_{22}(t) z^{g_{22}} + p_{23}(t) z^{g_{23}} \quad (21)$$

$$u_3(z) = p_{31}(t) z^{g_{31}} + p_{32}(t) z^{g_{32}} + p_{33}(t) z^{g_{33}} \quad (22)$$

To formulate the *system's overall performance distribution* in terms of the individual *element performances*, a *system structure function* **[30]**, φ, is constructed. This φ function depends on the *system topology* (series – parallel architecture) and also the type of MSS being analyzed (*flow transmission* or *task processing*). The system topology of the generator system in Fig 2 consists of elements 1 and 2 in parallel to each other and the parallel combination in turn in series with element 3.

The net useful power output of the system (performance), G_s, will be the minimum of the total amount of power processed by the parallel combination of elements {1,2} given by (G_1 + G_2) and the serially connected element 3 having a power processing capacity represented by the random variable, G_3. Based on this configuration, the *system structure function*, φ, for the *flow transmission power generator system* in Fig 2 is given by (23), where G_s denotes the overall system performance (output power) random variable.

$$G_S = \varphi(G_1, G_2, G_3) = \min\{(G_1 + G_2), G_3\} \quad (23)$$

Equations (20) – (22) only describe the *u-functions:* $u_1(z)$, $u_2(z)$ and $u_3(z)$ for each individual element of the system. It is however necessary to obtain the *system u-function*, $U_s(z)$, for the entire system in order to extract the *system performance distribution* of interest. $U_s(z)$ can be obtained using the *composition operator* approach **[30]**, Ω_φ, making use of the individual element *u-functions* in (20) – (22) and the *system structure function* , $\boldsymbol{\varphi}$ in (23).

Using the element state probabilities in Table II and the expressions in (20) - (23), the *system UGF* represented by $U_s(z)$ is obtained as:

$$\begin{aligned} U_S(z) &= \Omega_\varphi\{u_1(z), u_2(z), u_3(z)\} \\ &= \sum_{c=1}^{c=3} \sum_{b=1}^{b=3} \sum_{a=1}^{a=3} p_{1a}(t) p_{2b}(t) p_{3c}(t) * z^{\varphi(G_1, G_2, G_3)} \quad (24) \\ &= \sum_{c=1}^{c=3} \sum_{b=1}^{b=3} \sum_{a=1}^{a=3} p_{1a}(t) p_{2b}(t) p_{3c}(t) * z^{\min\{g_{1a} + g_{2b}, g_{3c}\}} \end{aligned}$$

The number of terms embedded in the summation of (24) is equal to the product of the number of performance states of elements 1, 2 and 3 which is equal to 3 × 3 × 3 = *27*. The powers of the z-polynomial in (24) are the various *system performance states* and the corresponding polynomial coefficients are the respective *state probability* expressions. The pair of these data forms the *system performance distribution* as shown in Table IV below. Note that not all the 27 states may be distinct. Some of them may end up with the same system performance rate values, in which case, the associated state probabilities are summed up.

Representing $\{g_{s1}, g_{s2},, ..., g_{s27}\}$ and $\{p_{s1}t), p_{s2}(t), ..., p_{s27}(t)\}$ as the set of system performance states and their probabilities, a simplified form of (24) is:

$$U_S(z) = \sum_{i=1}^{27} p_{si}(t) \cdot z^{g_{si}} \tag{25}$$

The relationship between the state probabilities for each element is: $p_{11}(t) + p_{12}(t) + p_{13}(t) = 1$; $p_{21}(t) + p_{22}(t) + p_{23}(t) = 1$; $p_{31}(t) + p_{32}(t) + p_{33}(t) = 1 \ \forall\, t$.

For the system, $\sum_{i=1}^{27} p_{si}(t) = 1 \ \forall\, t$.

From Table IV, the power capacity of g_1 = 10 MW corresponds to the maximum performance rate of the "*fully functional*" system. On the other hand, the power capacity of g_{27} = 0 MW represents the total failure event where the power system is "*completely non-functional*" i.e. not able to process any power at all. The power processing capacities of 8.5 MW, 7.5 MW, 6 MW etc… correspond to the intermediate degraded states where the system is only "*partially functional and efficient*" in its performance.

TABLE IV
SYSTEM PERFORMANCE DISTRIBUTION TABLE OBTAINED USING THE UGF APPROACH

System performance state - G_S / MW	10	8.5	7.5	…….	0
State probability – $P_S(t)$	$p_{s1}(t)$	$p_{s2}(t)$	$p_{s3}(t)$	…….	$p_{s27}(t)$

D. System Reliability & Performance

From the system performance distribution, the *Reliability (Survival) Function* of the system, $R_1(t)$ for the 1st operation cycle, can be defined as the probability that the system's performance (G_S) is above the minimum user-set threshold demand value, W. This is consistent with our earlier definition of *failure* in *Section III.A.2* where the system is considered to have failed from the *user's perspective* once its mean performance, $G(t)$, drops below the threshold user demand (W). Therefore, $R_1(t)$ is expressed as follows:

$$R_1(t) = P(G_S \geq W) = \left\{ \sum_{i=1}^{27} p_{si}(t) \;\middle|\; g_{si} \geq W \right\} \tag{26}$$

where W is the *minimum threshold demand* setting representing the *minimum user expectation* from the system.

The mean performance of the system for the 1st operation cycle, $G_1(t)$, can be modeled from the *system performance distribution.* Since Table IV is a *probability distribution function* (p.d.f) of a discrete statistical random variable of the system performance, G_S, the mean or expectation of G_S, denoted by $E(G_S)$, can therefore be expressed as follows:

$$G_1(t) = E(G_S) = [\sum_{i=1}^{27} p_{si}(t) \cdot g_{si}] \tag{27}$$

In (27), $G_1(t)$ is the system mean performance for the *1st operation cycle*; the summation term is the usual statistical definition for "*expectation of a random variable*".

The performance variation of the system for a general k^{th} operation cycle, $G_k(t)$, may now be described in terms of $G_1(t)$ by (28) based on the earlier expressions in (4) and (27). The *estimated time to failure* (TTF) for every operation cycle, k, is found by solving (7) numerically, where $G_k(t)$ is described by (28).

$$G_k(t) = G_1(t) \cdot \prod_{r=1}^{k-1} RF[r] \tag{28}$$

E. Modeling of Maintenance Cycle

The duration for different maintenance actions (*downtime duration*) in any maintenance policy is always a variable due to many factors. For example, the root cause of each failure could be different; the degree of the damages caused by the failures can be different on different occasions too. Some of the failure sites might be externally accessible and maintenance could be performed without dismantling the system, thus requiring less repair time; whereas some others could be situated deep inside the system that requires the system to be opened out for failure analysis and restoration work which could end up to be very time-consuming. As a result of all these variations, the maintenance duration needs to be modeled by a random variable with a stochastic distribution. The *Weibull* and *Gamma* distributions are commonly used for *downtime* or *repair* distributions **[37]**. Here, we use the *Weibull distribution* for downtime event modeling. The shape factor β is assumed to be 1 to reflect the age-independent randomness in the maintenance durations. The value for the scale factor, η is set to 0.02 years according to maintenance duration records for power generators, as revealed in **[38]**.

As the system continues to age, the degree of failure and extent of damage of the to-be-maintained system becomes more pronounced and severe even under the PdM policy, due to the effect of irreparable wear-and-tear effects. Thus, the later stages of system failures are more difficult and time-consuming to maintain and restore as compared to the initial failures. Therefore, it would be appropriate to model the scale factor, η, of the downtime distribution as an arbitrary increasing function of the operation cycle, k, to represent the increased downtime periods during subsequent repair actions, as the system's rate of degradation increases with time for *imperfect maintenance*.

With the mathematical model developed, we can now simulate the model using *Matlab* and study the effect of the various factors described, on the system reliability and performance characteristics for the MSS *PdM* policy. Due to the unavailability of real industrial data, the numerical values of the Markov degradation rates in this power system case study are hypothetically assumed for the sake of illustration; hence, the magnitudes of the computation results may not reflect the reality. In other words, the results described in the following sections serve only to show the practical usefulness and applicability of the model.

IV. RESULTS AND DISCUSSION

A. Impact of Threshold Demand (W)

Figures 4(a) – 4(c) show the system mean performance curves for three threshold demand (***W***) values of 9 MW, 8 MW and 7 MW respectively at a given RF distribution with parameters μ_{RF} = 95% and σ_{RF} = 0%. One can see from these figures that the higher the threshold demand (W), the sooner the time to failure *(TTF*) and the higher the mean frequency of maintenance actions to be performed. This is because setting a higher threshold demand (W) implies that the system's mean performance would degrade below the threshold level in a shorter span of time as illustrated earlier in Fig 3.

Time to Replacement, represented as *TTR*, is defined as the instant when the degrading system's mean performance can never be restored to above the minimum required threshold (W) anymore in spite of any further maintenance work. In such an event, further repair work is not beneficial because the user's minimum expectations can no longer be satisfied, and *replacement* of the system is therefore the only alternative option. Fig 5 clearly illustrates the *replacement criteria*.

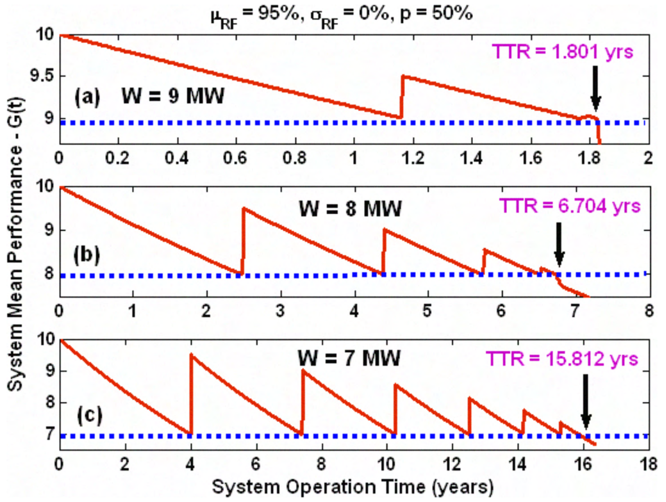

Fig. 4. System mean performance variation for (a) W = 9MW, (b) W = 8 MW and (c) W = 7 MW.

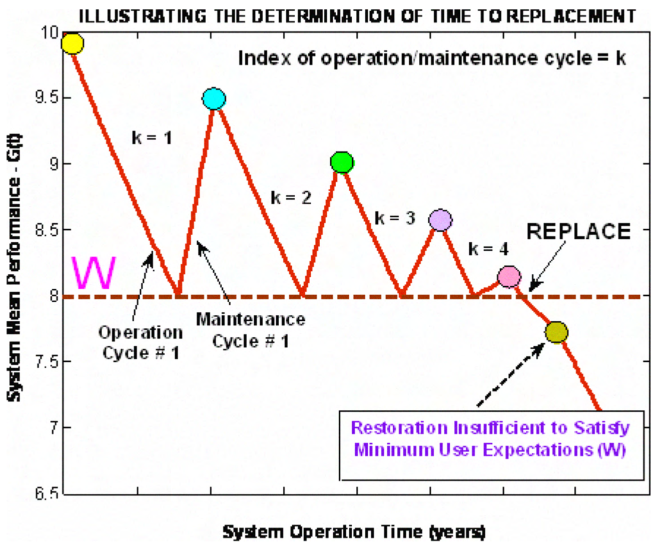

Fig. 5. Determination of time to replacement (TTR) from system mean performance curve. Symbol 'k' represents the k^{th} operation cycle.

TABLE V
TIME TO REPLACEMENT (TTR) VERSUS THRESHOLD DEMAND (W) TRENDS

Threshold Demand W (MW)	Time to Replacement (TTR) / yrs
μ_{RF} = 95%; σ_{RF} = 0%; p = 50%.	
9.0	1.801
8.0	6.704
7.0	15.81

Table V shows the computed TTR for different threshold demand values when the RF distribution is fixed at μ_{RF} *= 95%* and σ_{RF} *= 0%*. From Table V, it can be seen that if the threshold demand W is increased from 7 MW to 8 MW, *TTR* drops by approximately *57.6%*. Similarly, further increase in the threshold demand from 8 MW to 9 MW again causes the replacement time to drop further by around *73.1%*. Therefore, it is important for the user *not* to choose a very high W value close to the maximum performance capability of the system (10 MW in this case study). Instead, a moderate threshold demand under which the system can still function effectively should be chosen.

B. *Impact of Mean Restoration Factor (μ_{RF})*

Fig 6 shows the typical variation of system mean performance curves respectively for various μ_{RF} values of 90%, 95% and 97.5% respectively keeping the parameters σ_{RF} = 0%, p = 50% and W = *7 MW* fixed. The figure shows that the higher the $\mu_{RF,}$ the higher the average performance at any point in time, as expected.

Large values of μ_{RF} coupled with low σ_{RF} indicate very high quality of maintenance work and imply large restorations in the system performance during every maintenance. As a result, the system's initial performance during the start of every operation cycle is relatively high and it takes longer time for the system's performance to degrade to below the minimum threshold demand (W) in that operation cycle. This implies extended *times to failure (TTF)* and hence prolonged *time to replacement (TTR)*.

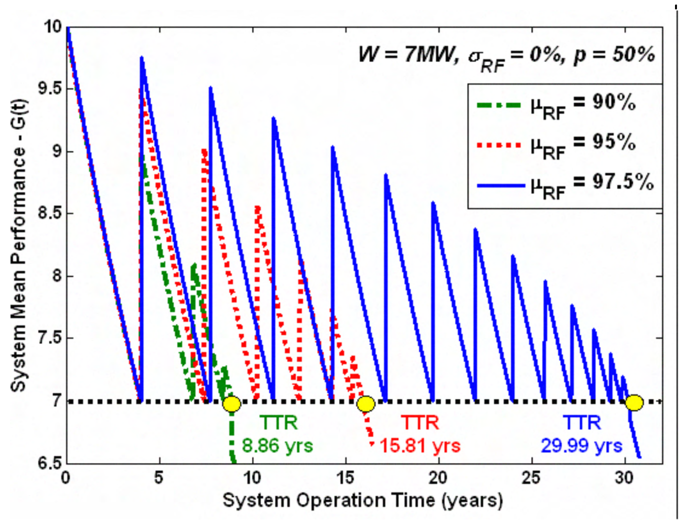

Fig. 6. System mean performance variation for various mean restoration factors of μ_{RF} = 90%, 95% and 97.5%, given W = 7MW, σ_{RF} = 0% and p = 50%.

TABLE VI
TIME TO REPLACEMENT (TTR) VERSUS THRESHOLD DEMAND (W) TRENDS

Mean RF (μ_{RF})	Time to Replacement (TTR) / yr
W = 7 MW; σ_{RF} = 0%; p = 50%.	
97.5%	29.99
95.0%	15.81
92.5%	11.19
90.0%	8.856
85.0%	6.508

Table VI shows that the TTR of the system increases largely by 36.1% as the μ_{RF} is increased from 85% to 90%. Further increase in μ_{RF} from 90% to 95% prolongs the TTR value further by as large as 78.5%. It is therefore necessary for an industry to strive to improve its μ_{RF} to as much as possible. The cost incurred in improving the μ_{RF} must be justified in comparison to the cost savings in maintenance and prolonged TTR achieved as a result of the improvement.

TABLE VII
RESTORATION FACTOR FOR DIFFERENT MAINTENANCE ACTIONS OBTAINED USING RANDOM NUMBER GENERATION FOR TWO DIFFERENT SIMULATIONS, SIM A AND SIM B WHERE MEAN RESTORATION FACTOR = 85%.

Maintenance #	1	2	3	4	5
Simulation A (SIM A)	0.984	0.879	0.998	0.964	0.782
Simulation B (SIM B)	0.807	0.842	1.004	-----	-----

C. *Impact of Variation in Restoration Factor (σ_{RF})*

Figure 7 illustrates the large deviation between a possible *optimistic-case* (SIM A) and *pessimistic-case* (SIM B) *scenario* of a system's performance variation pattern when the σ_{RF} is as high as 10%. The curves were generated by keeping all the parameters fixed at μ_{RF} = 85%, σ_{RF} = 10%, p = 50% and W = 6 MW. The random number generator in *Matlab* provides two entirely different set of values of *RF[k]* from the same *RF distribution* during the two separate simulations. The various RF values at each maintenance cycle for SIM A and SIM B are shown in Table VII.

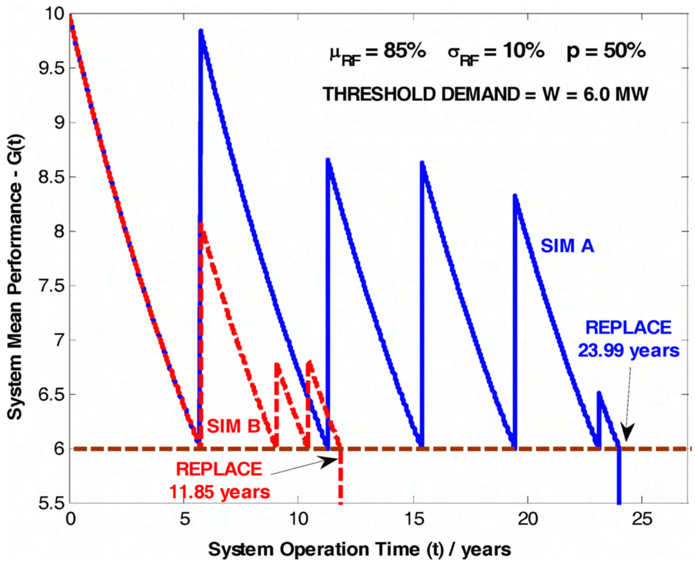

Fig. 7. Impact of inconsistency in maintenance work quality on variation in system performance trends.

Observations from Fig 7 and Table VII reveal that if the initial maintenance goes bad by chance, then future maintenance actions will not be effective in restoring the system to a satisfactory level of performance regardless of how good these future maintenance actions are. A bad repair work during the initial stages of system operation causes irreparable damage to its reliability and this damage cannot be compensated for by trying to improve the quality of future repair works on the system. The ultimate effect is a drastic reduction in the time to replacement (TTR). In Fig 7, the RF value for the first maintenance in the case of SIM A and SIM B are 98.4% and 80.7% respectively. Due to the low initial RF for the case of SIM B, the TTR for SIM B is as low as 11.85 years in comparison to the longer (almost double) TTR value of 23.99 years for SIM A despite the subsequent higher RF for SIM B as shown in Table VII. Therefore, it is crucial to maintain consistency in maintenance quality for a longer life cycle of the equipment. Note that maintenance quality includes the technical skill and competency of the maintenance personnel as well as the quality of spare parts used during repair.

It is therefore clearly evident that keeping the maintenance actions as consistent as possible is of paramount importance in order to enhance the predictability of future downtime schedules, facilitate efficient pre-planning of inventory stocks, reduce delay times and downtime durations, thereby increasing the mean availability and production output of the system.

D. *Effect of Multiple Failure Modes*

In the beginning of Section III, we mentioned that element 3 of the power generator system is under the influence of two failure modes where FM B is dependent on FM A with a conditional probability, *p*. Fig 8 shows the system performance variation trends for the extreme values of *p = 0%* and *p = 100%* keeping the other parameters μ_{RF} = 95%, σ_{RF} = 0% and *W* = 8 MW fixed.

When p = 0%, the element is under the influence of only FM A and therefore, it has a lower degradation rate as it gradually degrades with a stepwise state transition from $g_{31} \rightarrow g_{32} \rightarrow g_{33}$. Therefore, when p = 0%, the system has a prolonged lifespan and the maintenance intervals are more spread out. In contrast, when p = 100%, the FM A is certain to cause FM B and therefore the element which is now under the influence of both the failure modes has a very high effective degradation rate and hence, it is most likely to undergo a catastrophic breakdown directly from the best state of g_{31} to the worst state of g_{33}. As a result, the lifespan of the system is very short and the maintenance frequency is very high. From Fig 8, while the TTR value for p = 100% is only 4.226 years, the corresponding value for p = 0% is 15.81 years which is about four times larger. This shows the significant effect of the presence of multiple failure modes.

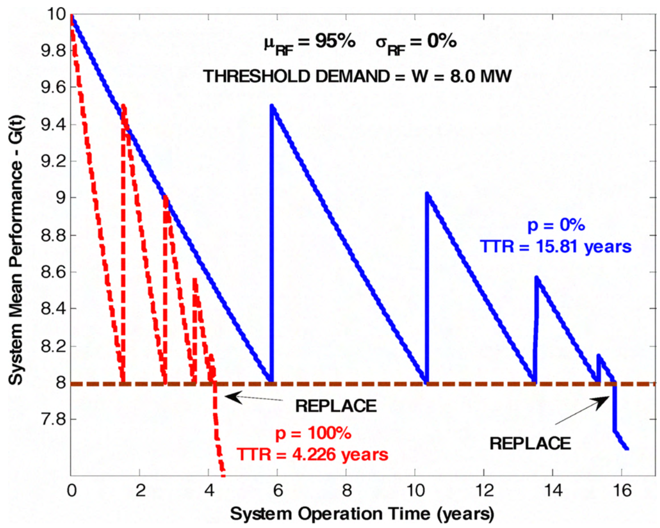

Fig. 8. Impact of the presence of multiple failure modes on the system performance trends.

E. *Determination of Maintenance Schedule*

Based on the simulations of the proposed model, the *times to failure* (TTF) data can be obtained from the numerical solution of (7) and an estimate of the *maintenance (downtime)* *schedules* can be constructed. Table VIII shows a typical example of a maintenance schedule derived from the model for the following four different cases:

TABLE VIII
DETERMINATION OF MAINTENANCE SCHEDULE FROM PDM MODEL SIMULATION FOR FOUR DIFFERENT CASES.

DOWNTIME (MAINTENANCE) SCHEDULE (all units in YEARS)				
Downtime #	(i) W = 8 MW μ_{RF} = 95% σ_{RF} = 0% p = 50%	(ii) W = 7 MW μ_{RF} = 95% σ_{RF} = 0% p = 50%	(iii) W = 7 MW μ_{RF} = 90% σ_{RF} = 0% p = 50%	(iv) W = 8 MW μ_{RF} = 95% σ_{RF} = 0% p = 100%
REPLACE	*6.704*	*15.81*	*8.856*	*4.226*
1	2.479	3.982	3.982	1.556
2	4.384	7.385	6.777	2.762
3	5.729	10.21	8.400	3.607
4	6.498	12.48	REPLACE	4.095
5	REPLACE	14.16	-----	REPLACE
6	-----	15.27	-----	-----
7	-----	REPLACE	-----	-----

(i) μ_{RF} = 95%, σ_{RF} = 0%, W =8 MW, p = 50%.
(ii) μ_{RF} = 95%, σ_{RF} = 0%, W =7 MW, p = 50%.
(iii) μ_{RF} = 90%, σ_{RF} = 0%, W =7 MW, p = 50%.
(iv) μ_{RF} = 95%, σ_{RF} = 0%, W =8 MW, p = 100%.

F. Comparison of Dependent & Independent Systems

In this study, Element 2 is modeled to be dependent on Element 1. In order to quantitatively examine the impact of this dependency on the system lifespan, two simulations are performed. One of them models the dependency of element 2 according to Table III and equations (17) – (19), using the conditional probability theory. The other simulation assumes element 2 to be independent of element 1. In both these cases, the values of the degradation rates, $\lambda_{1,2}^{(2)}$ and $\lambda_{2,3}^{(2)}$ are the same and the values of the other PdM parameters μ_{RF} = 95%, σ_{RF} = 0%, W = 8 MW and p = *50%* are all kept fixed. Fig 9 illustrates the performance degradation trends for these two different cases of dependent and independent systems.

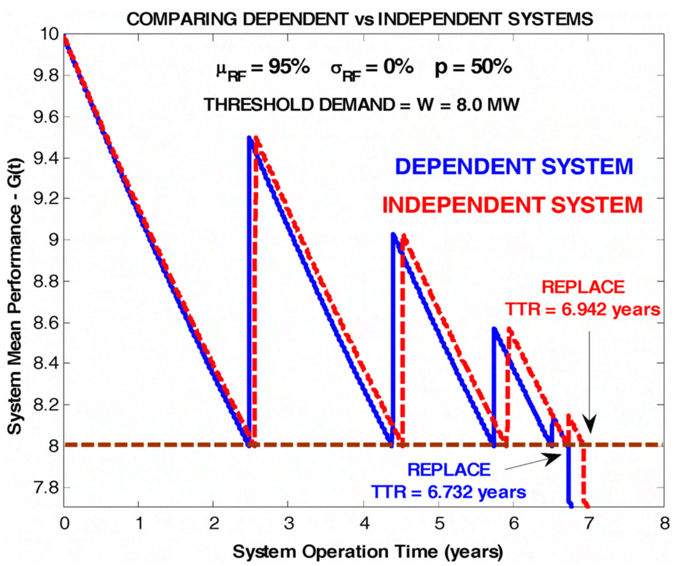

Fig. 9. Comparing the performance variation of a system with dependent and independent elements.

It may be noticed that in the case of a dependent system, since element 2 degrades along with element 1 because of the dependency, the effective degradation rate of the system is higher and hence, it is to be replaced in a shorter period of time. In contrast, when the elements are independent of each other, degradation of element 1 does not affect the performance of element 2 and as a result, the effective degradation rate of the system is lower resulting in prolonged operation times and longer time to replacement. In Fig 9, the TTR value for dependent system was 6.732 years while for the independent system, it is 3% larger at 6.942 years.

V. CONCLUSION

A statistical model for the *PdM policy* of *MSS* based systems has been developed by combining the *Universal Generating Function (UGF)* and *Markov Chain analysis* theories. *Restoration factor (RF)*, which is indicative of maintenance work quality and *threshold demand (W)*, which represents the *minimum user expectations* are identified as important *PdM* parameters, and their impacts on the system performance, downtime schedule and replacement time were quantitatively examined.

Using the stochastic model for the *restoration factor (RF)*, system performance variation for various μ_{RF}, σ_{RF} and W values were simulated and presented graphically. The results clearly indicate the significant impact of μ_{RF}, σ_{RF} and W on system reliability. A highly skilled maintenance crew (high μ_{RF}) can help improve the system reliability and maintainability to a large extent, thus saving costs and reducing wear and tear of the system and in turn prolonging its useful lifespan. Consistent performance of maintenance (low σ_{RF}) is also very essential for more accurate predictability of future downtime schedules and times to system replacement (*TTR*) which in turn assist the management to precisely pre-plan the production activities so as to meet the timely customer market demands.

Throughout this study, the model developed and the results shown were all based on the case study of a simple 3-element power generator system MSS. However, it is important to take note that the exact same procedure described in this work could be applied to any n-element MSS of any type (flow transmission or task processing) with any arbitrary topology. The only feature to take note of is the system structure function, $G_S = \varphi(G_1, G_2, \ldots, G_n)$ which will vary for different systems depending on its MSS classification and its topology **[30]**.

A company's long term financial position hinges largely on its ability to reduce plant operational and maintenance costs, which currently accounts for as much as around 15 – 70% of its overall production expenses **[1]**. The new *PdM* policy proposed in this study can definitely lower the maintenance cost, and hence increase the company's profitability.

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BIOGRAPHY

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