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Relations among stability parameters in the stable surface layer: Golder curves revisited

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Abstract

A nomogram was prepared by [Golder, 1972. Boundary Layer Meteorology 3, 47–58] to compute the surface layer parameters in stable conditions. This note revisits the Golder's curves and examines the methodology underlying their derivation in stable conditions. The inherent limitation in the methodology used for construction of Golder's curves was also noticed by Trombetti et al. (1986). Surface layer fluxes computed using the parameters derived from modified curves are found to be closer to the turbulence measurements from CASES-99 experiment for stable conditions than those calculated from the [Golder, 1972. Boundary Layer Meteorology 3, 47–58] curves.

Keywords: Nomogram; Bulk Richardson number; Surface layer; Golder's curves

1. Introduction

Golder (1972) proposed a methodology for constructing a nomogram between $\ln(z/z_0)$ (z is the height above the ground and z_0 is the roughness length) and gradient Richardson number (Ri) for a fixed value of bulk Richardson number (Rb). This nomogram was widely used for describing the fluxes in the atmospheric models. However, Golder used the derivative of potential temperature in the definition of Rb rather than the temperature difference in the bulk layer. These two methods give rise to different expressions of Rb in terms of z/L (L is Monin–Obukhov length). Trombetti et al. (1986) also pointed out the discrepancy in Golder's curves and indicated the conditions for their validity. Golder (1972) suggested taking $z = \bar{z}$ as the mean geometrical height between the two levels where temperatures are measured and wind speed u at $\bar{z}$. Wang (1981) proposed z as the upper temperature level whereas Sedefian and Bennet (1980) used $z = \bar{z}$ and u at the upper level.

In this note, we revisit Golder's curves by introducing them first, and then using a systematic approach to construct the modified curves in stable conditions. The surface fluxes computed using the parameters drawn from these curves are compared with those based on turbulence measurements in stable conditions from CASES-99 experiment.

2. Methodology

From similarity theory, under horizontally homogeneous and steady state conditions:

$$\frac{kz}{u_*} \frac{\partial u}{\partial z} = \Phi_m\left(\frac{z}{L}\right), \quad (1)$$

$$\frac{kz}{\theta_*} \frac{\partial \theta}{\partial z} = \Phi_h\left(\frac{z}{L}\right). \quad (2)$$

Here $k = 0.4$ is von Karman constant, z is height above the ground, u_* is friction velocity, θ_* is friction temperature, u is mean wind speed, θ is potential temperature, Φ_m and Φ_h are universal functions of the stability parameter z/L , where L is defined as

$$L = \frac{u_*^2 T_0}{kg\theta_*}, \quad (3)$$

g is the acceleration due to gravity and T_0 is the air temperature.

Integration of Eqs. (1) and (2) gives (Panofsky and Dutton, 1984, p. 134):

$$u = \frac{u_*}{k} \left[\ln \frac{z}{z_0} - \Psi_m\left(\frac{z}{L}\right) \right], \quad (4)$$

$$\theta - \theta_0 = \frac{\theta_*}{k} \left[\ln\left(\frac{z}{z_0}\right) - \Psi_h\left(\frac{z}{L}\right) \right], \quad (5)$$

where θ_0 is the temperature at $z = z_0$ and

$$\Psi_m\left(\frac{z}{L}\right) = \int_{z_0/L}^{z/L} (1 - \Phi_m(\zeta)) \frac{d\zeta}{\zeta}, \quad (6)$$

in which $\zeta = z/L$.

The gradient Richardson number (Ri) is defined as

$$Ri = \frac{g}{T_0} \frac{\frac{\partial \theta}{\partial z}}{\left[\frac{\partial u}{\partial z} \right]^2}. \quad (7)$$

This can be expressed as a function of z/L using Eqs. (1)–(3) as

$$Ri = \frac{z}{L} \frac{\Phi_h(z/L)}{\Phi_m^2(z/L)}. \quad (8)$$

In stable conditions, Φ_m and Φ_h are expressed as linear functions of z/L (Businger et al., 1971). Golder's curves are based on the relation

$$\Phi_m = \Phi_h = 1 + \beta \frac{z}{L}, \quad (9)$$

where β is a constant and $\Psi_m = \Psi_h$. From Eqs. (8) and (9),

$$\frac{z}{L} = \frac{Ri}{1 - \beta Ri}. \quad (10)$$

2.1. Golder's method

Golder (1972) defined the bulk Richardson number (Rb) as

$$Rb = \frac{g}{T_0 u^2} \frac{\partial \theta}{\partial z} z^2 \quad (11)$$

From Eqs. (7) and (11), Rb is related to Ri (Panofsky and Dutton, 1984, p. 143) as

$$Ri = \frac{Rb}{p^2}, \quad (12)$$

where

$$p = \frac{\partial(\ln u)}{\partial(\ln z)} = \frac{z}{u} \frac{\partial u}{\partial z} \quad (13)$$

Golder expressed u and $\frac{\partial u}{\partial z}$ as a function of Ri in Eq. (13). Using Eqs. (1), (4) and (10) in (13), p can be expressed (Panofsky and Dutton, 1984, p. 138) as

$$p = \frac{\Phi_m(Ri)}{\left[\ln \left(\frac{z}{z_0} \right) - \Psi_m(Ri) \right]}. \quad (14)$$

In stable conditions, Φ_m and Ψ_m in terms of Ri can be given as

$$\Phi_m(Ri) = \frac{1}{1 - \beta Ri}, \quad (15)$$

$$\Psi_m(Ri) = -\frac{\beta Ri}{1 - \beta Ri}. \quad (16)$$

Eqs. (15) and (16) are valid only if K_h/K_m is assumed to be 1. Here K_h and K_m are eddy diffusivities for heat and momentum respectively. The value of β was taken as 7 by Golder (1972).

Notice that Eq. (12) is a quadratic equation in Ri . This can be solved for the lowest value of Ri for given values of $\ln(z/z_0)$ and Rb .

2.2. Alternative procedure

Golder (1972) used the gradient of the potential temperature in the definition of Rb as shown in Eq. (11). On the other hand, the gradient of the potential temperature should be approximated as the potential temperature difference divided by the layer thickness and accordingly Rb is defined as

$$Rb = \frac{g}{T_0} \frac{(\theta - \theta_0)}{u^2} z. \quad (17)$$

Using Eqs. (4) and (5) along with Eqs. (3) and (9) in Eq. (17), we get

$$Rb = \frac{z}{L} \left[\ln\left(\frac{z}{z_0}\right) + \beta \frac{z}{L} \right]^{-1} \quad (18)$$

This gives

$$\frac{z}{L} = \frac{Rb \ln\left(\frac{z}{z_0}\right)}{1 - \beta Rb}. \quad (19)$$

For given values of $\ln(z/z_0)$ and Rb , Eq. (19) is used to compute z/L which in turn is utilized to calculate Ri from Eqs. (8) and (9) as

$$Ri = \frac{z/L}{1 + \beta z/L}. \quad (20)$$

Notice that in the expressions of Rb and Ψ_m , we assumed $z \gg z_0$, and thus, z_0 compared to z has been ignored in order to minimize the degrees of freedom for constructing nomogram for known values of $\ln(z/z_0)$ and Rb . Since the value of Rb is assumed to be known from the observations, we have taken z as the upper level at which the measurements of temperature and wind speed are available, in both Golder's and modified methodology. The same approach has also been adopted by Trombetti et al. (1986).

3. Results and discussion

In Golder's formulation, for given values of Rb and $\ln(z/z_0)$, the lowest value of Ri was computed from the quadratic equation deduced from Eq. (12). These values of Ri were used to construct a nomogram between $\ln(z/z_0)$ and Ri for a fixed value of Rb . This reproduces the nomogram given by Golder (1972) in Fig. 1a.

For constructing a nomogram based on Eq. (14) by expressing p explicitly as a function of z/L , we express Eq. (12) in terms of z/L by using Eqs. (1), (4), (9) and (20), as

$$Rb = \frac{z}{L} \left(1 + \beta \frac{z}{L} \right) \left[\ln \left(\frac{z}{z_0} \right) + \beta \frac{z}{L} \right]^{-2} \quad (21)$$

This is a second-degree polynomial in z/L . Trombetti et al. (1986) have also expressed Rb as a function of z/L , which reduces to a second degree polynomial in z/L for the linear relation (9) for Φ_m . The Eq. (21) will have two values of z/L for a given value of Rb and $\ln(z/z_0)$. The lowest value of z/L computed from Eq. (21) is utilized for calculating Ri from Eq. (20). This method leads to the nomogram as given by Golder (1972). However, Golder's expression for Rb in terms of z/L , Eq. (21), is different from that derived in our approach, Eq. (18). The discrepancy, which arises due to the incorrect definition of Rb used by Golder (1972) in constructing the nomogram, was also noticed by Trombetti et al. (1986).

Figs. 1a and b shows the variation of $\ln(z/z_0)$ with Ri for a fixed Rb . Golder's (1972) nomogram is shown in Fig. 1a and the nomogram constructed on the basis of the modified procedure is given in Fig. 1b. The curves are steeper with the modified method than those obtained from Golder's method.

In Fig. 1, $\beta = 7$ is taken following Golder (1972). However, the well-established value of $\beta = 5$ as proposed by Dyer (1974) is adopted for computation of results in the rest of this paper. Turbulence measurements from CASES-99 (Mahrt et al., 2001) in stable conditions for $Rb < 0.15$ (Sharan et al., 2003) are used for validating the fluxes computed using the parameters obtained from the nomogram. Wind and temperature measurements at 10 m level are used. The temperature at z_0 is obtained by extrapolating temperatures measured at 10 and 1.5 m. Friction velocity (Fig. 2a) and heat flux (Fig. 2b) computed from the modified nomogram are found to be closer to the turbulence observations than those obtained from Golder's (1972) nomogram. This is confirmed quantitatively by computing the Normalised mean square error (NMSE). The NMSE values are 0.096 and 0.05 for u_* computed from Golder and modified nomogram respectively. The corresponding value of NMSE for heat flux reduces from 0.498 to 0.112 with the modified curves. In addition, Figs. 3 and 4 reveal that the surface layer parameters computed from modified curves are found to be closer to the observations than those obtained from Golder's curve. The fluxes computed using the procedure outlined in Trombetti et al. (1986) are found to be almost same to those obtained here based on the assumption that z is taken as the upper level, z_0 as the lower level and $u = u(z)$.

For comparison with turbulence data, the commonly used value of $\beta = 5$ is taken. However, by using the value of $\beta = 7$ following Golder (1972), the NMSE values are 0.191 and 0.069 for u_* computed from Golder and modified nomograms respectively. The corresponding values for the surface heat flux are 1.017 and 0.139.

Both nomograms are based on the use of linear functions of z/L for non-dimensional wind and temperature profiles. However, the applicability of the linear function is limited

to $Rb < 1/\beta$ (Sharan et al., 2003). Much larger values of Rb are often observed in weak wind stable conditions.

4. Conclusions

This note revisits the Golder's (1972) curves relating Ri and $\ln(z/z_0)$ for fixed values of Rb and examines the methodology underlying their derivation. New curves derived from the modified method are shown to differ somewhat from those given by Golder. Both qualitative and quantitative analysis of the surface layer fluxes derived from the two methods in relation to the observed turbulent fluxes in stable conditions from CASES-99 experiment show that the modified method performs better.

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Fig. 3. Evaluation of (a) friction velocity, (b) heat flux and (c) stability parameter z/L computed from Golder's nomogram (-----) and modified nomogram (——) with CASES-99 data ($\circ$) for the night of October 10–11, 1999.

Fig. 4. Evaluation of (a) friction velocity (b) heat flux (c) stability parameter z/L computed from Golder's nomogram (-----) and modified nomogram (——) with CASES-99 data ($\circ$) for the night of October 11–12, 1999.

Fig. 1

Fig. 2

Fig. 3

Fig. 4