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3- D Formulation of Formation Flight Based on Model Predictive Control with Collision Avoidance Scheme

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This paper addresses the application of Model Predictive Control (MPC) approach for formation flight control and coordination of autonomous unmanned aerial vehicles. The Nominal Decentralized Model Predictive Control (NDMPC) is presented and its enhancement to deal with the model uncertainty and disturbances, called the Robust Decentralized Model Predictive Control (RDMPC), is also proposed and analyzed. Obstacle avoidance control scheme for any shape and size of obstacles is addressed under the unified MPC framework. The simulation results show that this MPC scheme can successfully achieve collision-free formation flights.

I. Introduction

Research activities in the formation flight of unmanned aerial vehicles (UAVs) have increased substantially in the last few years. UAVs formation flight can potentially be useful for increased surveillance coverage, better target acquisition and increased security measures¹⁻⁵.

Different approaches have been developed for formation flight control. Formation control based on linear dynamic models of aircraft has been discussed^{4, 6-10}. The advantage of the linear control is that it is intuitive and easy to synthesize, but it can not handle the constraints directly and may not valid for large initial conditions since it is designed around a fixed operating point. Some researchers address the nonlinear formation flight control problem by using feedback linearization¹¹⁻¹³ and adaptive control^{15, 14-17}. Although these nonlinear control methods can deal with the unmodeled dynamics, they cannot handle the constraints directly and the implementation of such controller may result in ill-defined control inputs. In this paper, we try to address those challenges by developing formation flight control scheme with the following properties: guaranteed stability, intuitive, and easy to implement under constraints.

Model Predictive Control (MPC) is one of the frequently applied advanced control methods in industry. It is traditionally applied to plants where the dynamics are slow to permit a sampling rate amenable to optimal input computations between samples. Recently, with the advent of faster modern computers, it has become possible to extend the MPC method to systems exhibiting relatively fast dynamics. MPC can also handle constraints in relative ease¹⁸⁻¹⁹. The application of MPC to formation control of ground robots has been reported in Ref. 20, where a dual mode controller is used. To ensure the stability, the controller has to switch from an MPC type of controller to a terminal state controller, i.e. input-output feedback linearization controller. MPC has also been used in the aerial vehicle formation control, for example see Ref. 21,22. In Ref. 21, the formation commands (heading angle, speed, etc) are given to each agent and the control algorithm is used to reduce the pilots' stress during the formation flight. In that work, the MPC is used for maintaining the formation in the presence of gusts or other disturbances, but not for trajectory following or formation reconfiguration.

In the reported works of formation flight control using MPC, although centralized optimal or suboptimal approaches have been used in different studies, for instance in Ref. 23, it is clear that as the number of vehicles increases, the solution of such large-scale, centralized, nonconvex optimization problems become prohibitive. This is true even when the most advanced optimization solvers and much simplified linear dynamics are used²⁴. The main challenge is to formulate simpler decentralized systems which will result in a formation behavior similar to what is obtainable by a centralized approach²⁵. This paper is an attempt towards addressing such challenge. In this paper, the dynamical coupling between the individual aircraft is neglected. We are interested in decentralized control for

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dynamically decoupled and independently actuated systems. Although several papers have addressed the decentralized MPC for the formation flight control problem²⁶⁻²⁹, none has discussed the robustness of the controller when there are both disturbances and model uncertainties in the system. Also, there are two drawbacks in the existing collision avoidance scheme by using MPC: 1) the shape of the obstacles is limited to rectangle; 2) the small obstacles have not been taken into account in the obstacle avoidance constraint formulation. This paper also tries to address such drawbacks in the collision avoidance scheme.

In summary, the contribution of this paper is in the development of a robust collision-free decentralized formation flight control system using the unified MPC idea. Specifically, the paper addresses the followings:

1. In the robustness under input disturbances (e.g. crosswind) and model uncertainty: predict the effects of uncertainty and then compensate for the uncertainties as constraints to nominal states based on the range of the disturbances and model uncertainty.
2. In the obstacle avoidance scheme: extend the current collision avoidance algorithm to include various shapes of obstacles and prevent the collision with small obstacles by using the spatial horizon to detect them, which is then translated into an additional position constraint.

The paper is organized as follows. Robust decentralized model predictive formation flight control is presented in section II, including its problem formulation and the details of the MPC controller design. In section III, the obstacle avoidance control scheme is addressed. Some simulation results are presented in section IV. Finally, conclusions and future work are presented in section V.

II. Formation Control Strategy

The overall formation flight control scheme using a Robust Decentralized Model Predictive Control (RDMPC) approach is illustrated in Fig. 1. The main concept of the approach is to use the model of the plant to predict the future evolution of the system³⁶. Based on this prediction, at each time step k a certain performance index is optimized under operating constraints with respect to a sequence of future input moves. The first of such optimal moves is the control action applied to the plant at time k . At time $k+1$, a new optimization is solved over a shifted prediction horizon.

Figure 1. Decentralized UAV formation flight control

The RDMPC scheme is formulated here for dynamically decoupled systems. The coupling in the formulation, if any, come only from the cost function and constraints of an optimization problem. The main idea of the proposed framework is to break a centralized MPC problem into problems of smaller sizes^{25, 37-40}. Each MPC controller is associated with a different vehicle and computes the local control inputs based only on its states and that of its neighbors. On each vehicle, the current state and the model of its neighbors are used to predict their possible trajectories and move accordingly (similar to what we do while driving cars). The information exchange topology and inter-vehicle constraints are described by a graph structure in the problem formulation or depends on the task descriptions. The cost function will depend on the formation's mission and include terms that minimize relative

distances and/or velocities between neighboring vehicles. The coupling constraints arise from collision avoidance. The interaction graph is full (each vehicle has interactions with all the other vehicles) but it is approximated with a time-varying graph based on a “closest spatial neighbors” model.

II.1 Problem Formulation and Decentralized Control Scheme

In this paper, the inputs, outputs, and states constraints are all regarded as polytopes. Some pertinent definitions are given below. For more details, readers may refer to Ref. 4.

Definition 1 (polyhedron): A convex set $\mathcal{Q} \in \mathbb{R}^n$ given as an intersection of a finite number of closed half-spaces

$$\mathcal{Q} = \left\{ \mathbf{x} \in \mathbb{R}^n \mid \mathbf{Q}^x \mathbf{x} \leq \mathbf{Q}^c \right\}$$

is called polyhedron.

Definition 2 (polytope): A bounded polyhedron $\mathcal{P} \in \mathbb{R}^n$

$$\mathcal{P} = \left\{ \mathbf{x} \in \mathbb{R}^n \mid \mathbf{P}^x \mathbf{x} \leq \mathbf{P}^c \right\}$$

is called polytope.

One of the fundamental properties of polytope is that it can also be described by its vertices

$$\mathcal{P} = \left\{ \mathbf{x} \in \mathbb{R}^n \mid \mathbf{x} = \sum_{i=1}^{vp} \alpha_i V_p^{(i)}, 0 \leq \alpha_i \leq 1, \sum_{i=1}^{vp} \alpha_i = 1 \right\},$$

where $V_p^{(i)}$ denotes the i -th vertex of $\mathcal{P}$, and vp is the total number of vertices of $\mathcal{P}$. Therefore, in the constraints or obstacles representations, we can either use the half-space form or vertex form to denote the polytopes.

Consider a set of N_v linear decoupled UAV dynamical systems, where the i -th vehicle dynamics is described by the discrete-time time-invariant state equation

$$\begin{aligned} \mathbf{x}_{k+1}^i &= \mathbf{f}^i(\mathbf{x}_k^i, \mathbf{u}_k^i) = (\mathbf{A}^i + \hat{\mathbf{A}}^i) \mathbf{x}_k^i + \mathbf{B} \mathbf{u}_k^i + \mathbf{D} \mathbf{w}_k^i \\ \mathbf{y}_k^i &= \mathbf{h}^i(\mathbf{x}_k^i, \mathbf{u}_k^i) = \mathbf{C}^i \mathbf{x}_k^i \end{aligned} \quad (1)$$

where the state update function $\mathbf{f} : \mathbb{R}^6 \times \mathbb{R}^3 \rightarrow \mathbb{R}^6$ and output map $\mathbf{h} : \mathbb{R}^6 \rightarrow \mathbb{R}^6$ are linear or piecewise linear functions of its inputs. The states and inputs of the vehicle at time k are denoted by $\mathbf{x}_k \in \mathbb{R}^6$ and $\mathbf{u}_k \in \mathbb{R}^3$, respectively. In particular

$$\mathbf{u}_k = \begin{bmatrix} N_{cmd} \\ E_{cmd} \\ h_{cmd} \end{bmatrix} \quad \mathbf{y}_k = \begin{bmatrix} N \\ E \\ h \\ \dot{N} \\ \dot{E} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{k,pos} \\ \mathbf{y}_{k,vel} \end{bmatrix} \quad \mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{k,pos} \\ \mathbf{x}_{k,vel} \end{bmatrix} \quad (2)$$

and $\mathbf{x}_{k,pos} \in \mathbb{R}^3$ is the vector of N , E , h coordinates and $\mathbf{x}_{k,vel} \in \mathbb{R}^3$ denotes a vector of states corresponding to N -, E - and h -axis velocity at time k .

In general, Eq. (1) subjects to the disturbance input

$$\mathbf{w}_k^i \in \mathcal{W} \subset \mathbb{R}^q \quad (3)$$

Hereafter, a disturbance sequence $\mathbf{w}_k^i$ satisfying Eq. (3) will be called admissible.

Additionally, as indicated in Eq. (1), there may also be model uncertainty in the system, indicated by the uncertainty set $\mathcal{A}^i = \left\{ \hat{\mathbf{A}}^i \mid \frac{\|\hat{\mathbf{A}}^i\|}{\|\mathbf{A}^i\|} = \varepsilon \right\}$, where the constant matrix $\mathbf{A}^i$ gives the average value of the state matrix, and

$\hat{A}^i$ describes its statistical variation. For example, if we assume that the uncertainty in the state matrix is 10% of its nominal value, then $\frac{\|\hat{A}^i\|}{\|A^i\|} = 0.1$.

Let $\mathcal{U}_k^i \in \mathbb{R}^{m^i}$ and $\mathcal{Y}_k^i \in \mathbb{R}^{p^i}$ denote the set of feasible inputs and outputs of the i -th vehicle

$$\mathbf{y}_k^i \in \mathcal{Y}_k^i \quad \mathbf{u}_k^i \in \mathcal{U}_k^i \quad k \geq 0 \quad (4)$$

where $\mathcal{Y}_k^i$ and $\mathcal{U}_k^i$ are given polytopes.

In the case of UAV, Eq. (4) often indicates the constraints on the speed and acceleration of the vehicle as follows:

$$\begin{aligned} \mathbf{y}_{vel} \in \mathcal{Y}_v &= \{ \mathbf{z} \in \mathbb{R}^3 \mid \underline{\mathbf{y}}_{vel} \leq \mathbf{z}_i \leq \bar{\mathbf{y}}_{vel}, i=1,2,3 \} \\ \dot{\mathbf{y}}_{vel} \in \mathcal{Y}_a &= \{ \mathbf{z} \in \mathbb{R}^3 \mid \underline{\dot{\mathbf{y}}}_{vel} \leq \mathbf{z}_i \leq \bar{\dot{\mathbf{y}}}_{vel}, i=1,2,3 \} \end{aligned}$$

The set of N_v constrained systems will be referred to as the multi-vehicle system. Let $\tilde{\mathbf{x}}_k \in \mathbb{R}^{\tilde{n}}$ with $\tilde{n} = \sum_i n^i$ and $\tilde{\mathbf{u}}_k \in \mathbb{R}^{\tilde{m}}$ with $\tilde{m} = \sum_i m^i$ be the vectors which collect the states and inputs of the multi-vehicle system at time k , i.e., $\tilde{\mathbf{x}}_k = [\mathbf{x}_k^1; \dots; \mathbf{x}_k^{N_v}]^T$, $\tilde{\mathbf{u}}_k = [\mathbf{u}_k^1; \dots; \mathbf{u}_k^{N_v}]^T$, with $\tilde{\mathbf{x}}_{k+1} = \mathbf{f}(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k)$. The equilibrium pair of the i -th vehicle is denoted by $(\mathbf{x}_e^i, \mathbf{u}_e^i)$, while $(\tilde{\mathbf{x}}_e^i, \tilde{\mathbf{u}}_e^i)$ denotes the corresponding equilibrium for the multi-vehicle system.

So far the individual vehicles belonging to the multi-vehicle system are completely decoupled. In this research, the effects of coupling in dynamics are not investigated and are not taken into account in the problem formulation. We consider an optimal control problem for the multi-vehicle system where cost function and constraints couple the dynamic behavior of individual systems. A graph topology⁴² is used in this paper to present the connections between the UAVs in the formation group.

Graph theory for constraint formulation

A directed graph $\mathcal{G}$ consists of a set of vertices, denoted $\mathcal{V}$, and a set of edges $\mathcal{A} \subset \mathcal{V}^2$, where $a = (\alpha, \beta) \in \mathcal{A}$ and $\alpha, \beta \in \mathcal{V}$. The first element of a is denoted $tail(a)$, and the second is denoted $head(a)$. We assume that $tail(a) \neq head(a)$ for all a , meaning that the graph has no self-loops. We also assume that each element of $\mathcal{A}$ is unique. A graph with the property that for any $(\alpha, \beta) \in \mathcal{A}$, the arc $(\beta, \alpha) \in \mathcal{A}$ as well is said to be undirected. The in (out)-degree of a vertex, denoted $d_i(\alpha)$ ($d_o(\alpha)$), is the number of edges with α as its head (tail). If every possible arc exists, the graph is said to be complete.

In this research, a graph topology is used to represent the coupling in the following way. The i -th system is associated with the i -th node of the graph. If an edge $(i, j) \in \mathcal{A}$ connecting the i -th and j -th node is present, then the cost and the constraints of the optimal control problem will have a component which is a function of both $\mathbf{x}^i$ and $\mathbf{x}^j$. The edges representing coupling change with time. Therefore, before defining the optimal control problem, we need to define a graph (which can be time-varying) $\mathcal{G}(t) = (\mathcal{V}, \mathcal{A}(t))$ associated with vehicle formation group.

Using the graph structure defined previously, the optimization problem is formulated as follows. Denote with $\hat{\mathbf{x}}_k^i$ the states of all neighbors of the i -th vehicle plus that of itself at time k , i.e., $\hat{\mathbf{x}}_k^i = \{ \mathbf{x}_k^j \in \mathbb{R}^{n^j} \mid (i, j) \in \mathcal{A}(k) \text{ or } j = i \}$, $\hat{\mathbf{x}}_k^i \in \mathbb{R}^{\hat{n}_k^i}$ with $\hat{n}_k^i = n^i + \sum_{j|(i,j) \in \mathcal{A}(k)} n^j$. Analogously, $\hat{\mathbf{u}}_k^i \in \mathbb{R}^{\hat{m}_k^i}$ and $\hat{\mathbf{y}}_k^i \in \mathbb{R}^{\hat{p}_k^i}$ denote the inputs and outputs of all the neighbors of the i -th vehicle and itself at time k , respectively. Let

$$q^{i,j}(\mathbf{x}^i, \mathbf{x}^j) \leq 0 \quad (5)$$

define the interconnection constraints between the i -th and the j -th vehicle, with $q^{i,j} : \mathbb{R}^{n^i} \times \mathbb{R}^{n^j} \rightarrow \mathbb{R}^{nc^{i,j}}$.

Remarks:

Depending on the group tasks, the interconnection function $q^{i,j}(\mathbf{x}^i, \mathbf{x}^j)$ can assume several variant forms:

- For inter-agent collision avoidance, these constraints define non-convex requirements in the following way:

$$q^{i,j}(\mathbf{x}_k^i, \mathbf{u}_k^i, \mathbf{x}_k^j, \mathbf{u}_k^j) = d_{safe} - \|\mathbf{y}_{k,pos}^i - \mathbf{y}_{k,pos}^j\|_p \leq 0 \quad (6)$$

where the parameter d_{safe} in Eq. (6) represents a lower bound on the norm of relative position between neighboring vehicles. The subscript p denotes the type of norm which will be used in the optimization problem. $p = 2$ leads to a circle protection zones around vehicle and the optimization problem is quadratic. If $p = 1$ or ∞ , then the protection zones around vehicle are square and the optimization problem is linear.

– Since the team of vehicles is equipped with a communication network, and that the connectivity of the network depends upon the relative distance between neighboring vehicles. Therefore, maintaining network connectivity constrains the maximum allowable distance between vehicles. These constraints define convex requirements in the following way:

$$q^{i,j}(\mathbf{x}_k^i, \mathbf{u}_k^i, \mathbf{x}_k^j, \mathbf{u}_k^j) = \|\mathbf{y}_{k,pos}^i - \mathbf{y}_{k,pos}^j\|_p - d_{com} \leq 0 \quad (7)$$

where d_{com} represents the diameter of the detection cylinder, i.e., the maximum allowable distance between vehicles.

– For formation maintaining or formation reconfiguration, these constraints define convex requirements in the following way:

$$q^{i,j}(\mathbf{x}_k^i, \mathbf{u}_k^i, \mathbf{x}_k^j, \mathbf{u}_k^j) = \mathbf{y}_{k,pos}^i - \mathbf{y}_{k,pos}^j = l \quad (8)$$

where l specify the formation pattern between neighboring vehicles.

In general, guaranteed collision avoidance and network connectivity between any vehicle pair in a formation flight problem would necessitate the use of a full graph for describing inter-vehicle constraints (vehicle protection zones should not intersect for every vehicle pair). This would prevent the use of any approach but a centralized one. For practical decentralization purposes, it is usually sufficient for each vehicle to consider only a neighboring subset of all vehicles to accomplish formation flight. Due to possible changes of the required formation, this subset is likely to change leading to a time-varying interconnection graph. Furthermore, the allowed number of vehicles in these neighboring subsets might be limited. In fact, the maximum number of agents whose detection cylinder is in contact with the detection cylinder of the computing agent is 6 (see Fig. 2).

Consider the following overall cost function:

$$J(\tilde{\mathbf{x}}, \tilde{\mathbf{u}}) = \sum_{i=1}^{N_V} J_k^i(\tilde{\mathbf{x}}_k^i, \tilde{\mathbf{u}}_k^i) \quad (9)$$

where $J^i : \mathbb{R}^{\tilde{n}_k^i} \times \mathbb{R}^{\tilde{m}_k^i} \rightarrow \mathbb{R}$ is the cost associated with the i -th vehicle and is a function of its states and the states of its neighbor vehicles. Assume that J is a positive convex function with $J(\tilde{\mathbf{x}}_e, \tilde{\mathbf{u}}_e) = 0$. The cost function in Eq. (9) is assumed to have the following form:

$$J_k^i(\tilde{\mathbf{x}}_k^i, \tilde{\mathbf{u}}_k^i) = \left\| \begin{bmatrix} \mathbf{y}^i - \mathbf{y}_f^i \\ \hat{\mathbf{y}}_k^i - \hat{\mathbf{y}}_f^i \\ \Delta \mathbf{y}_k^i - \Delta \mathbf{y}_f^i \\ \Delta \hat{\mathbf{y}}_k^i - \Delta \hat{\mathbf{y}}_f^i \end{bmatrix} \right\|_p + \left\| R \begin{bmatrix} \mathbf{u}^i \\ \hat{\mathbf{u}}_k^i \end{bmatrix} \right\|_p \quad (10)$$

Figure 2 Agent in the center of the cluster exchange information with most 6 neighbors

where $\Delta \mathbf{y}_k^i$ is a stacked vector of relative outputs $(\mathbf{y}^i - \mathbf{y}^j)$ for all j such that $(i, j) \in \mathcal{A}(k)$. It represents the difference between the i -th vehicle outputs and its neighbors' outputs. The variable $\Delta \hat{\mathbf{y}}$ denotes a similar collection of relative outputs between the neighbors of the i -th vehicle, i.e., it is comprised of $(\mathbf{y}^q - \mathbf{y}^r)$ output differences for all q, r such that $(i, q) \in \mathcal{A}(k)$ and $(i, r) \in \mathcal{A}(k)$. Subscript f denotes reference values for all variables. The previous general cost function involves weights on the outputs and control inputs of the i -th vehicle and its neighbors, as well as weighting the relative outputs between them.

Centralized solutions to the vehicles formation control problem are characterized by the presence of a large number of states, and can be solved using techniques in Ref. 43. However, for large-scale systems the computational requirements grow exponentially with problem size, making such an approach intractable and impractical. We tackle the complexity associated with the design of controllers for such class of large-scale systems by using decentralized optimal control schemes.

Given a certain graph interconnection structure $\mathcal{G}(t)$, let the following linear, discrete-time system be associated with the i -th vehicle at time k :

$$\begin{aligned}\hat{\mathbf{x}}_{k+1}^i &= \mathbf{f}^i(\hat{\mathbf{x}}_k^i, \hat{\mathbf{u}}_k^i) = (\hat{\mathbf{A}}^i + \hat{\hat{\mathbf{A}}}^i)\hat{\mathbf{x}}_k^i + \hat{\mathbf{B}}^i\hat{\mathbf{u}}_k^i + \hat{\mathbf{D}}^i\hat{\mathbf{w}}_k^i \\ \hat{\mathbf{y}}_k^i &= \mathbf{h}^i(\hat{\mathbf{x}}_k^i, \hat{\mathbf{u}}_k^i) = \hat{\mathbf{C}}^i\hat{\mathbf{x}}_k^i\end{aligned}\quad (11)$$

subject to the disturbance input, model uncertainty

$$\hat{\mathbf{w}}_k^i \in \hat{\mathcal{W}}^i \subset \mathbb{R}^{\hat{q}}, \hat{\mathbf{A}}^i \in \hat{\mathcal{A}}^i = \left\{ \hat{\mathbf{A}}^i \mid \frac{\|\hat{\hat{\mathbf{A}}}^i\|}{\|\hat{\mathbf{A}}^i\|} = 0.1 \right\} \quad (12)$$

and to the state and control constraints

$$\hat{\mathbf{x}}_k^i \in \hat{\mathcal{X}}^i \subset \mathbb{R}^{\hat{n}_k^i}, \hat{\mathbf{u}}_k^i \in \hat{\mathcal{U}}^i \subset \mathbb{R}^{\hat{m}_k^i} \quad (13)$$

where $(\hat{\mathbf{A}}^i + \hat{\hat{\mathbf{A}}}^i) = \text{diag}(\dots, (\mathbf{A}^j + \hat{\mathbf{A}}^j), \dots) \in \mathbb{R}^{\hat{n}_k^i} \times \mathbb{R}^{\hat{n}_k^i}$ for all $(i, j) \in \mathcal{A}(k)$ or $j = i$, i.e., since the vehicles in the decentralized formation are dynamically decoupled, the state matrix for each decoupled system is the concatenation of the its own state matrix and those of its neighbor. Analogously, the matrices $\hat{\mathbf{B}}$, $\hat{\mathbf{D}}$ and $\hat{\mathbf{C}}^i$ can also be defined.

The equilibrium pair of each decoupled system is denoted by $(\hat{\mathbf{x}}_e^i, \hat{\mathbf{u}}_e^i)$, and it will be assumed in this paper that:

- The pair $(\hat{\mathbf{A}}^i, \hat{\mathbf{B}}^i)$ is stabilizable
- The sets $\hat{\mathcal{W}}^i, \hat{\mathcal{X}}^i, \hat{\mathcal{U}}^i$ contain the origin as an interior point
- $\hat{\mathcal{W}}^i$ and $\hat{\mathcal{U}}^i$ are compact sets

II.2 MPC-Based Control Development

The goal here is to obtain, via model predictive control, a nonlinear state feedback

$$\hat{\mathbf{u}}_k^i = \mathbf{g}(\hat{\mathbf{x}}_k^i) \quad (14)$$

which drives the state of system Eq. (11) “as close as possible” to the equilibrium of the multi-vehicle system $(\hat{\mathbf{x}}_e^i, \hat{\mathbf{u}}_e^i)$ while satisfying the states and input constraints Eq. (13) for all admissible disturbance Eq. (12) and model uncertainty. A further objective is that the nonlinear feedback Eq. (14) reduces to a linear well-tuned feedback whenever this is compatible with the constraints Eq. (13).

Clearly, the presence of a persistent disturbance and model uncertainty Eq. (12) acting on the system Eq. (11) means it is not possible to guarantee asymptotical convergence, i.e. that $\lim_{k \rightarrow \infty} \hat{\mathbf{x}}_k^i = \hat{\mathbf{x}}_e^i$. Then an achievable goal, the best that can be hoped for, is to steer the initial state $\hat{\mathbf{x}}_0^i$ to a neighborhood of the equilibrium point, $\boldsymbol{\Omega}$. Of course the requirement for optimal regulation performance then translates to the target set $\boldsymbol{\Omega}$ being as small as possible

since this amounts to minimizing the state sensitivity to disturbances and model uncertainty. To meet the desire for linear control when $\hat{\mathbf{x}}_k^i \in \boldsymbol{\Omega}$, consider a stabilizing LTI state feedback

$$\hat{\mathbf{u}}_k^i = F^i \hat{\mathbf{x}}_k^i \quad (15)$$

designed so as to provide a satisfactory, or optimal in some sense (e.g. LQ, $\mathcal{H}_\infty, l_1$), control performance to decoupled system Eq. (11) in the absence of constraints. The control law Eq. (15) will be regarded as a-priori fixed throughout this paper and referred to as the nominal feedback. The corresponding asymptotically stable nominal close-loop system is

$$\hat{\mathbf{x}}_{k+1}^i = \Phi \hat{\mathbf{x}}_k^i + \hat{A}^i \hat{\mathbf{x}}_k^i + \hat{D}^i \hat{\mathbf{w}}_k^i, \hat{\mathbf{w}}_k^i \in \mathcal{W}^i, \hat{A}^i \in \hat{\mathcal{A}}^i \quad (16)$$

where $\Phi = \hat{A}^i + \hat{B}^i F^i$, and $\Phi + \hat{A}^i$ is the closed-loop state transition matrix for which it is assumed that $\Phi + \hat{A}^i$ has all its eigenvalues strictly inside the unit circle.

In order to characterize the target set $\boldsymbol{\Omega}$, it is convenient to introduce the notion of the invariant set⁴⁴:

Definition

A set $\Sigma \subset \mathbb{R}^{\hat{n}_k^i}$ is disturbance and model uncertainty invariant for the uncontrolled system Eq. (16) if for any $\hat{\mathbf{x}} \in \Sigma$, $\Phi \hat{\mathbf{x}} + \hat{A}^i \hat{\mathbf{x}} + \hat{D}^i \hat{\mathbf{w}}_k^i \in \Sigma$ for all $\hat{\mathbf{w}}_k^i \in \mathcal{W}^i$ and $\hat{A}^i \in \hat{\mathcal{A}}^i$.

In particular, if in the above definition $\mathcal{W}^i = \{0\}$ and $\hat{\mathcal{A}}^i = \{0\}$, the set Σ is simply invariant. Let $\hat{\mathcal{X}}_c^i = \hat{\mathcal{X}}^i \cap (F^i)^{-1}(\mathcal{U}^i)$ be the set of the states which satisfy both the state and the control constraints in Eq. (13) under the nominal feedback Eq. (15). The control objectives can therefore be accomplished by taking the target set $\boldsymbol{\Omega}$ as the minimal invariant subset of $\hat{\mathcal{X}}_c^i$ for the closed-loop system Eq. (16). Such an $\boldsymbol{\Omega}$ can be identified using some results presented in Ref. 45. Let

$$\mathcal{R}_j^i = \sum_{n=0}^{j-1} \left[\left(\Phi + \hat{A}^i \right)^n \hat{D}^i \mathcal{W}^i + \left(\hat{\mathcal{A}}^i \right)^{n+1} \hat{\mathbf{x}}_0^i \right] \quad (17)$$

denote the set of states of the nominal linear closed-loop system Eq. (16) which are reachable in j steps from the equilibrium point. Then:

The sequence of sets $\mathcal{R}_j^i$ has a limit $\mathcal{R}_\infty^i$ as $j \rightarrow \infty$, and $\mathcal{R}_\infty^i$ is a compact invariant set.

$\mathcal{R}_\infty^i$ is the minimal invariant subset, i.e. if Σ is invariant then $\mathcal{R}_\infty^i \subset \Sigma$. Therefore, there exists a invariant subset of $\hat{\mathcal{X}}_c^i$ if and only if the following assumption holds: $\mathcal{R}_\infty^i \subset \hat{\mathcal{X}}_c^i$

Restated the decoupled control objective

Motivated by the above discussion, the control objective is restated in a more convenient form as follows.

Design a nonlinear state feedback Eq. (14) such that, for all initial states $\hat{\mathbf{x}}_0^i$ in a suitable domain of attraction, the following two requirements are met:

(R1) The closed-loop system

$$\begin{aligned} \hat{\mathbf{x}}_{k+1}^i &= (\hat{A}^i + \hat{A}^i) \hat{\mathbf{x}}_k^i + \hat{B}^i \mathbf{g}(\hat{\mathbf{x}}_k^i) + \hat{D}^i \hat{\mathbf{w}}_k^i, \hat{\mathbf{w}}_k^i \in \mathcal{W}^i, \hat{A}^i \in \hat{\mathcal{A}}^i, \\ \hat{\mathbf{y}}_k^i &= \hat{C}^i \hat{\mathbf{x}}_k^i \end{aligned} \quad (18)$$

satisfies the constraints Eq. (13) for all $k \geq 0$;

(R2) the nonlinear feedback Eq. (14) asymptotically approaches the nominal feedback Eq. (15), i.e.

$$\lim_{k \rightarrow \infty} [\mathbf{g}(\hat{\mathbf{x}}_k^i) - F^i \hat{\mathbf{x}}_k^i] = 0 \quad (19)$$

Notice that Eq. (15) in turn implies that

$$\hat{\mathbf{x}}_k^i \rightarrow \mathcal{R}_\infty^i \text{ as } t \rightarrow \infty \quad (20)$$

To see this, note that Eq. (18) can be rewritten as

$$\begin{aligned} \hat{\mathbf{x}}_{k+1}^i &= (\Phi + \hat{A}^i) \hat{\mathbf{x}}_k^i + \hat{B}^i [\mathbf{g}(\hat{\mathbf{x}}_k^i) - F^i \hat{\mathbf{x}}_k^i] + \hat{D}^i \hat{\mathbf{w}}_k^i \\ \hat{\mathbf{y}}_k^i &= \hat{C}^i \hat{\mathbf{x}}_k^i \end{aligned} \quad (21)$$

By Eq. (19), the bracketed term vanishes for $t \rightarrow \infty$. Hence Eq. (20) can be thought of as the state equation of a linear asymptotically stable system subject to a vanishing input $\mathbf{c}_k^i = \mathbf{g}(\hat{\mathbf{x}}_k^i) - F^i \hat{\mathbf{x}}_k^i$ and to a persistent input $\hat{\mathbf{w}}_k^i$. For such a system $\lim_{k \rightarrow \infty} \left[\hat{\mathbf{x}}_k^i - \sum_{t=0}^{k-1} (\Phi + \hat{A}^i)^t \hat{D}^i \hat{\mathbf{w}}_{k-t-1}^i \right] = 0$, from which Eq. (20) follows.

It is convenient to introduce the control variable $\mathbf{c}_k^i = \mathbf{g}(\hat{\mathbf{x}}_k^i) - F^i \hat{\mathbf{x}}_k^i$ as the difference between the control input $\hat{\mathbf{u}}_k^i$ and the nominal feedback $F^i \hat{\mathbf{x}}_k^i$. System Eq. (11) is accordingly rewritten as

$$\begin{aligned} \hat{\mathbf{x}}_{k+1}^i &= (\Phi + \hat{A}^i) \hat{\mathbf{x}}_k^i + \hat{B}^i \mathbf{c}_k^i + \hat{D}^i \hat{\mathbf{w}}_k^i \\ \hat{\mathbf{y}}_k^i &= \hat{C}^i \hat{\mathbf{x}}_k^i \\ \hat{\mathbf{u}}_k^i &= \mathbf{c}_k^i + F^i \hat{\mathbf{x}}_k^i \end{aligned} \quad (22)$$

and requirement given in Eq. (19) becomes

$$(R2') \quad \lim_{k \rightarrow \infty} \mathbf{c}_k^i = 0. \quad (23)$$

Robust decentralized predictive control

Base on the above definitions, we first formulate a Nominal Decentralized Model Predictive Control (NDMPC) scheme which synthesizes the control law neglecting disturbances and model uncertainty based in the cost function, system models, and constraints previously defined. Next we shall introduce a modified controller, referred to as Robust Decentralized Model Predictive Control (RDMPC), to account for disturbances and model uncertainty. In order to do that, some more notations are introduced:

- $\mathbf{c}_{k+t|k}^i, t \geq 0$ denotes the future control moves, with respect to the nominal feedback, planned at time k;
- $\hat{\mathbf{x}}_{k+t|k}^i$ and $\hat{\mathbf{u}}_{k+t|k}^i$ denote the disturbance- and model uncertainty-free predictions of $\hat{\mathbf{x}}_{k+t}^i$ and $\hat{\mathbf{u}}_{k+t}^i$ respectively, given the state $\hat{\mathbf{x}}_k^i$ and $\mathbf{c}_{k+j|k}^i, 0 \leq j \leq t$.

NDMPC

Given a certain graph interconnection structure $\mathcal{G}(t)$, the NDMPC algorithm can be summarized as follows:

At time k, given the states $\hat{\mathbf{x}}_k^i$

$$\min_{\mathbf{C}_k^i} \sum_{t=0}^{N-1} J_k^i(\hat{\mathbf{x}}_{k+t|k}^i, \hat{\mathbf{u}}_{k+t|k}^i) + J_N^i(\hat{\mathbf{x}}_{k+N|k}^i, \hat{\mathbf{u}}_{k+N|k}^i) \quad (24)$$

subject to the nominal system dynamics (with out disturbance and model uncertainty)

$$\begin{aligned} \hat{\mathbf{x}}_{k+t+1|k}^i &= \Phi^i \hat{\mathbf{x}}_{k+t|k}^i + \hat{B}^i \mathbf{c}_{k+t|k}^i, t \geq 0 \\ \hat{\mathbf{y}}_{k+t|k}^i &= \hat{C}^i \hat{\mathbf{x}}_{k+t|k}^i \\ \hat{\mathbf{u}}_{k+t|k}^i &= \mathbf{c}_{k+t|k}^i + F^i \hat{\mathbf{x}}_{k+t|k}^i, t \geq 0 \\ \hat{\mathbf{x}}_{k|k}^i &= \hat{\mathbf{x}}_k^i \\ \mathbf{c}_{k+t|k}^i &= 0, t \geq N \end{aligned} \quad (25)$$

and state and control constraints

$$\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}^i, \hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}^i, t \geq 0 \quad (26)$$

interconnection constraints for vehicle i and its neighboring vehicles j , i.e. $(i, j) \in \mathcal{A}(k)$

$$q^{i,j}(\mathbf{x}_{k+t+1|k}^i, \mathbf{u}_{k+t+1|k}^i, \mathbf{x}_{k+t+1|k}^j, \mathbf{u}_{k+t+1|k}^j) \leq 0, t \geq 0, (i, j) \in \mathcal{A}(k) \quad (27)$$

where $\mathbf{C}_k = [\mathbf{c}_{k|k}^i; \mathbf{c}_{k+1|k}^i; \dots; \mathbf{c}_{k+N-1|k}^i]^T$ is the control sequence and let $\mathbf{C}_k^* = [\mathbf{c}_{k|k}^{i*}; \mathbf{c}_{k+1|k}^{i*}; \dots; \mathbf{c}_{k+N-1|k}^{i*}]^T$ be the optimal solution;

(1) set $\mathbf{c}_k^i = \mathbf{c}_{k|k}^{i*}$ and apply $\hat{\mathbf{u}}_k^i = \mathbf{c}_k^i + F^i \hat{\mathbf{x}}_k^i$

Remark:

- The infinitely many constraints Eq. (26) can actually be reduced to a finite number. In fact, a finite integer $i^* \geq 0$ can be determined off-line⁴⁵ such that Eq. (26) needs only to be imposed for $t = 0, 1, \dots, N + i^*$.
- As stated before, the interconnection constraints Eq. (27) can represent all kinds of constraints between vehicles in the graph, depending on the task, network connectivity constraints and configuration constraints can be added to the optimization problem. For the collision & obstacle avoidance, depending on the size of the obstacle, the position constraint may be added to account for the small obstacles.

RDMPC

The constraints Eq. (26) involve only the nominal (disturbance-, model uncertainty-free) predictions. Therefore, they do not provide any guarantee that the true system state and input will satisfy the constraints Eq. (13) due to the presence of the disturbance and model uncertainty. Hence NDMPC, as well as many standard predictive control algorithms, can lose feasibility and hence their guarantee of stability.

The RDMPC replaces the original constraints Eq. (26) with more stringent ones which preserve feasibility despite the presence of disturbance and model uncertainty. The true state and input response are given by

$$\begin{aligned} \hat{\mathbf{x}}_{k+t}^i &= \hat{\mathbf{x}}_{k+t|k}^i + \sum_{j=1}^t \left[\left(\Phi + \hat{A}^i \right)^{j-1} \hat{D}^i \hat{\mathbf{w}}_{k+t-j}^i + \left(\hat{A}^i \right)^j \hat{\mathbf{x}}_{k|k}^i \right] \\ \hat{\mathbf{u}}_{k+t}^i &= \hat{\mathbf{u}}_{k+t|k}^i + \sum_{j=1}^t F^i \left[\left(\Phi + \hat{A}^i \right)^{j-1} \hat{D}^i \hat{\mathbf{w}}_{k+t-j}^i + \left(\hat{A}^i \right)^j \hat{\mathbf{x}}_{k|k}^i \right] \end{aligned} \quad (28)$$

where the first terms represent the disturbance-, model uncertainty-free predictions and the latter terms denote the forced responses due to the disturbance and model uncertainty. Hence with $\hat{\mathcal{R}}_t^i$ defined as in Eq. (17), a sufficient condition for the constraints $\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}^i$ and $\hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}^i$ to be satisfied, is to impose restricted constraints on the nominal predictions, i.e.

$$\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}_t^i, \hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}_t^i, t \geq 0 \quad (29)$$

with $\hat{\mathcal{X}}_t^i = \hat{\mathcal{X}}^i \sim \mathcal{R}_t^i$ and $\hat{\mathcal{U}}_t^i = \hat{\mathcal{U}}^i \sim F^i \mathcal{R}_t^i$. (30)

In fact, $\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}^i \sim \mathcal{R}_t^i \Rightarrow \hat{\mathbf{x}}_{k+t|k}^i + \mathcal{R}_t^i \subset \hat{\mathcal{X}}^i$ and consequently $\hat{\mathbf{x}}_{k+t}^i \subset \hat{\mathcal{X}}^i$ for all admissible disturbances and model uncertainty. The same arguments can be used to show that $\hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}_t^i$ implies $\hat{\mathbf{u}}_{k+t}^i \in \hat{\mathcal{U}}^i$. Notice that $\hat{\mathcal{X}}_t^i$ and $\hat{\mathcal{U}}_t^i$ are non-empty due to assumption (A5).

Based on above arguments, the following RDMPC algorithm is proposed. To make the RDMPC practically implementable, it is necessary to reduce the constraints Eq. (29) to a finite number. Since $\mathbf{c}_{k+t|k}^i = 0$ for $t \geq N$, in fact⁴⁵, the constraints $\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}_t^i$ and $\hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}_t^i \forall t \geq N$ are equivalent to the terminal constraint $\hat{\mathbf{x}}_{k+N|k}^i \in \hat{\mathcal{X}}^i \sim \mathcal{R}_N^i = \hat{\mathcal{X}}_N^i$. Hence, constraints Eq. (29) can be expressed as

$$\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}_t^i, \hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}_t^i, t = 0, 1, \dots, N \quad (31)$$

RDMPC algorithm:

At time k , given the states $\hat{\mathbf{x}}_k^i$, compute the control $\hat{\mathbf{u}}_k^i$ as in the NDMPC algorithm by only replacing the constraints Eq. (26) with the constraints Eqs. (30), (31).

Feasibility and stability

In this part, the properties of RDMPC will be analyzed.

Definition

A control sequence $\mathbf{C}_k = [\mathbf{c}_{k|k}^i; \mathbf{c}_{k+1|k}^i; \dots; \mathbf{c}_{k+N-1|k}^i]^T$ is said to be admissible for state $\hat{\mathbf{x}}_k^i$ if the constraints Eqs. (25), (27), (30) and (31) are satisfied. A state $\hat{\mathbf{x}}_k^i$ is said to be feasible if there exists a sequence $\mathbf{C}_k$ admissible for $\hat{\mathbf{x}}_k^i$.

Lemma

For system Eq. (11) under RDMPC feedback $\hat{\mathbf{u}}_k^i = \mathbf{c}_k^i + F^i \hat{\mathbf{x}}_k^i$ with $\mathbf{c}_k^i = \mathbf{c}_{k|k}^{i*}$, the following implication holds

$$\mathbf{C}_k^* \text{ is admissible for } \hat{\mathbf{x}}_k^i \Rightarrow \tilde{\mathbf{C}}_{k+1} = [\mathbf{c}_{k+1|k}^{i*}; \mathbf{c}_{k+2|k}^{i*}; \dots; \mathbf{c}_{k+N-1|k}^{i*}; 0]^T \text{ is admissible for } \hat{\mathbf{x}}_{k+1}^i. \quad (32)$$

Proof.

Denote by $(\hat{\mathbf{x}}_{k+t|k}^i, \hat{\mathbf{u}}_{k+t|k}^i)$ the predictions associated to the control sequence $\mathbf{C}_k^*$ and state $\hat{\mathbf{x}}_k^i$, and by $(\tilde{\mathbf{x}}_{k+1+t-1|k+1}^i, \tilde{\mathbf{u}}_{k+1+t-1|k+1}^i)$ the predictions for control $\tilde{\mathbf{C}}_{k+1}$ state $\hat{\mathbf{x}}_{k+1}^i$. It can be checked that

$$\begin{aligned} \tilde{\mathbf{x}}_{k+1+t-1|k+1}^i &= \hat{\mathbf{x}}_{k+t|k}^i + \left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}_k^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i, \quad k \geq 1, \\ \tilde{\mathbf{u}}_{k+1+t-1|k+1}^i &= \hat{\mathbf{u}}_{k+t|k}^i + F^i \left[\left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}_k^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \right], \quad k \geq 1. \end{aligned}$$

Since by assumption, $\mathbf{C}_k^*$ is admissible for $\hat{\mathbf{x}}_k^i$, we have $\hat{\mathbf{x}}_{k+t|k}^i \in \hat{\mathcal{X}}_t^i$ and $\hat{\mathbf{u}}_{k+t|k}^i \in \hat{\mathcal{U}}_t^i$, hence

$$\begin{aligned} \tilde{\mathbf{x}}_{k+1+t-1|k+1}^i &\in \hat{\mathcal{X}}_t^i + \left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i, \\ \tilde{\mathbf{u}}_{k+1+t-1|k+1}^i &\in \hat{\mathcal{U}}_t^i + F^i \left[\left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \right]. \end{aligned}$$

Using Eq. (30) and the properties of the set difference $\sim$, and by noticing that due to the optimization process we can assume that the initial state $\hat{\mathbf{x}}_0^i$ is closer to the equilibrium point than $\hat{\mathbf{x}}_k^i$ for $k \geq 1$:

$$\begin{aligned} &\hat{\mathcal{X}}_t^i + \left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \\ &= \left(\hat{\mathcal{X}}^i - \hat{\mathcal{R}}_t^i \right) + \left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \\ &= \left[\left(\hat{\mathcal{X}}^i - \hat{\mathcal{R}}_{t-1}^i \right) \sim \left(\left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_0^i \right) \right] + \left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \\ &= \left[\hat{\mathcal{X}}_{t-1}^i \sim \left(\left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_0^i \right) \right] + \left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \\ &\subset \hat{\mathcal{X}}_{t-1}^i \end{aligned}$$

and similarly $\hat{\mathcal{U}}_t^i + F^i \left[\left(\Phi + \hat{A}^i \right)^{t-1} \hat{D}^i \hat{\mathcal{W}}^i + \left(\hat{A}^i \right)^t \hat{\mathbf{x}}_k^i \right] \subset \hat{\mathcal{U}}_{t-1}^i$. Therefore, $\tilde{\mathbf{x}}_{k+1+t-1|k+1}^i \in \hat{\mathcal{X}}_{t-1}^i$, $\tilde{\mathbf{u}}_{k+1+t-1|k+1}^i \in \hat{\mathcal{U}}_{t-1}^i$ from which $\tilde{\mathbf{C}}_{k+1}$ is admissible for $\hat{\mathbf{x}}_{k+1}^i$.

Theorem:

Provided that the initial state $\hat{\mathbf{x}}_0^i$ is feasible, system Eq. (11) under RDMPC feedback $\hat{\mathbf{u}}_k^i = \mathbf{c}_k^i + F^i \hat{\mathbf{x}}_k^i$ with $\mathbf{c}_k^i = \mathbf{c}_{k|k}^{i*}$, satisfies the following properties:

- (i) $\hat{\mathbf{x}}_k^i \in \hat{\mathcal{X}}^i$ and $\hat{\mathbf{u}}_k^i \in \hat{\mathcal{U}}^i$ for all $k \geq 0$;
- (ii) $\lim_{k \rightarrow \infty} \mathbf{c}_k^i = 0$;
- (iii) $\hat{\mathbf{x}}_k^i \rightarrow \mathcal{R}_\infty^i$ as $k \rightarrow \infty$

Proof:

The hypothesis that $\hat{\mathbf{x}}_0^i$ is feasible along with Eq. (32) imply that $\hat{\mathbf{x}}_k^i$ is feasible for all $k \geq 0$ and hence (i) holds. To show convergence of $\mathbf{c}_k^i$, introduce the Bellman function $V_k = V(\hat{\mathbf{x}}_k^i) = \sum_{t=0}^{N-1} (\mathbf{c}_{k+t|k}^{i*})^T \Psi(\mathbf{c}_{k+t|k}^{i*})$, $\Psi = \Psi^T \succ 0$. Define the cost $\tilde{V}_{k+1}$ associated to the control sequence $\tilde{\mathbf{C}}_{k+1}$, i.e. $\tilde{V}_{k+1} = \sum_{t=1}^{N-1} (\mathbf{c}_{k+t|k}^{i*})^T \Psi(\mathbf{c}_{k+t|k}^{i*}) = V_k - (\mathbf{c}_k^{i*})^T \Psi(\mathbf{c}_k^{i*})$. However, at sample instant $k+1$, an admissible $\mathbf{C}_{k+1}^*$ is selected after the optimization and therefore can be different from $\tilde{\mathbf{C}}_{k+1}$. Hence it is clear that

$$V_{k+1} \leq \tilde{V}_{k+1} \Rightarrow V_k - V_{k+1} \geq (\mathbf{c}_k^{i*})^T \Psi(\mathbf{c}_k^{i*}) \geq 0 \quad (33)$$

So $\{V_k\}_{k \geq 0}$ is a nonnegative monotonic nonincreasing scalar sequence and as $k \rightarrow \infty$, must converge to $V_\infty < \infty$. Summing the $V_k - V_{k+1}$ of Eq. (33), for k from 0 to ∞ , we have

$$\infty > V_0 - V_\infty \geq \sum_{k=0}^{\infty} (\mathbf{c}_k^{i*})^T \Psi(\mathbf{c}_k^{i*}) \geq 0 \Rightarrow \lim_{k \rightarrow \infty} (\mathbf{c}_k^{i*})^T \Psi(\mathbf{c}_k^{i*}) = 0$$

which as $\Psi \succ 0$, proves (ii). Finally, due to the assumption (A4) and (ii),

$$\begin{aligned} \lim_{k \rightarrow \infty} \hat{\mathbf{x}}_k^i &= \lim_{k \rightarrow \infty} \left[(\Phi + \hat{A}^i)^k \hat{\mathbf{x}}_0^i + \sum_{t=1}^k \left((\Phi + \hat{A}^i)^{t-1} \hat{B}^i \mathbf{c}_{k-t}^i \right) + \sum_{t=1}^k \left((\Phi + \hat{A}^i)^{t-1} \hat{D}^i \hat{\mathbf{w}}_k^i \right) \right] \\ &= \lim_{k \rightarrow \infty} \left[\sum_{t=1}^k \left((\Phi + \hat{A}^i)^{t-1} \hat{D}^i \hat{\mathbf{w}}_k^i \right) \right] \in \mathcal{R}_\infty^i \end{aligned}$$

which in turn proves (iii).

Hence, by virtue of the Theorem, under initial feasibility RDMPC guarantees convergence of the state to the terminal region $\Omega = \mathcal{R}_\infty^i$. Notice that the lemma and theorem given above only give sufficient conditions under which RDMPC guarantees the required objectives (R1) and (R2').

III. Collision Avoidance Integration

Based on the environmental information, obstacles can be divided into two groups: the known obstacles and the unknown (pop-up) obstacles. If there are only some known obstacles and no pop-up obstacles in the environment, several approaches to this kind of

Figure 3 UAVs penetrating the small size obstacle

collision avoidance problem have been reported in literature. A comprehensive survey of such methods is given in Ref. 46.

Recently, more researches are focused on the pop-up obstacle avoidance control schemes or the inter-agents avoidance control. For example, obstacle avoidance schemes in an unknown environment have been studied in Ref. 29 and 47-57. Some of the design techniques for the formation control and pop-up obstacle avoidance have been successfully developed without explicitly taking constraints into account^{48, 52-54}. Even where constraints are considered, this is done only implicitly. In such case, the collision avoidance constraints are normally translated into the minimization of barrier/potential functions whose value becomes very large when a direction leading to collision is chosen. Such an approach does not account for real-life issues like limited speed, acceleration, and actuation authority of the vehicle. Other researchers⁴⁹⁻⁵¹ have applied MPC idea to the problem of collision avoidance. The predictive feature of MPC makes it inherently more robust for use in unknown environments, and the inputs/states constraints can be modeled naturally in the problem formulation. Mixed Integer Linear Programming (MILP) combined with MPC^{29, 55-57} has been widely used in the collision avoidance of multiple agents' coordination. But there are some drawbacks in this collision avoidance method. Firstly, the obstacle's shape is restricted as rectangular in the problem formulation, while in the real-life environment, the shape of the obstacles can be of any form and formulating it as a box may waste the space that maybe useful for the vehicle to perform the necessary maneuver. Secondly, since the predicted position is used in the collision avoidance constraint formulation, if the obstacle's width is smaller than the vehicle's one step predictive travel distance (this can happen if the time step is long and the vehicle velocity is high), such algorithm may fail to prevent the collisions. Figure 3 illustrates such scenario, where the obstacle's (red) width is the same as the vehicle's longest travel distance in one predictive step, and since the MPC does not see the obstacle in its predicted step, it will lead the vehicles to the trajectories penetrating the obstacle. To deal with this problem and the variety of the obstacle shapes, the following schemes are proposed.

Receding horizons can be divided into two families, i.e., temporal and spatial ones. The temporal horizons manage time frames while the spatial horizons manage physical areas. A temporal horizon, as the same in most of the MPC literatures¹⁹, defined as the prediction horizon and unique for each vehicle ($h_i, i = 1, 2, \dots, n$), sets the temporal windows in the future over which the optimizer of the cost function and the collision avoidance constraints will be applied, starting from the present time. The temporal horizon is used to deal with the obstacles whose size is bigger than one-step predictive distance.

A number of spatial horizons can also be defined. One example can be the sensor detection horizon⁵⁰ that allows for detection of near-by pop-up small size obstacles. The sensor detection horizon is used to deal with the small size obstacles.

1. For large obstacles:

For safety concern, each vehicle is modeled as large round moving obstacle. In this paper, a polytope is introduced to present various shapes of the obstacles. To avoid collisions, the following hard constraints must be satisfied by each trajectory point

$$\mathbf{x}_{k+i} \notin \bigcup \mathbf{P}_j$$

where $\bigcup \mathbf{P}_j$ is the union of the polytopes in the neighborhood of the aircraft j . The presentation of such constraints is a logic form. Interested reader should refer to Refs. 58 or 59 for the details of the polytope obstacle modeling.

The whole formation flight control optimization problem becomes non-convex after adding the collision avoidance constraints. The conversion from the collision avoidance constraints to mixed integer constraints is done in YALMIP⁵⁹ or MPT⁵⁸ using a standard big-M formulation. In order to solve the non-convex optimization problem more efficiently, bounds on the variables are added explicitly

$$\bar{\mathbf{x}} \leq \mathbf{x}_{k+i} \leq \underline{\mathbf{x}}$$

where $\bar{\mathbf{x}}$ and $\underline{\mathbf{x}}$ denote the upper and lower bounds for the aircraft states.

2. For small obstacles

In this section, the small obstacle's shape is specified as a circle (note that if the shape of the obstacle is not a circle, the collision avoidance algorithm still works after some modifications which will not be detailed in this paper). This shape is similar to the vehicle spatial horizon coverage (sensor's detection area are represented by a circle). In the proposed approach, position constraints are computed using the intersection points between the vehicle spatial horizon H_i and the obstacle.

Suppose the obstacles detection sensor's range H_i is little bit larger than the distance that a vehicle can travel in one predictive step. When the obstacle is small enough that it lies in the vehicle spatial horizon H_i as in Fig. 4, a temporary and smaller vehicle spatial horizon H_{i^*} can be set, based on the distance between the vehicle and the obstacle center and the intersection points with the obstacle.

A position constraint can be added to deal with such scenario

$$\text{norm}\left(\begin{bmatrix} \hat{x}_{i,k+1} \\ \hat{y}_{i,k+1} \end{bmatrix} - \begin{bmatrix} x_{i,k} \\ y_{i,k} \end{bmatrix}\right) \leq H_{i^*}$$

Note that this position constraint prevents the next vehicle predictive position to penetrate the small obstacle, and this constraint adds no difficulty to the optimization problem since it is a convex inequality constraints on the predictive states. The total collision avoidance control scheme is shown below in Fig. 5.

Figure 4 Collision avoidance with a small obstacle

Figure 5 Collision Avoidance Flow chart

IV. Simulation Results

This section presents some simulation results of the collision-free decentralized control scheme and due to the difficulty of plotting high-dimensional uncertainty polytopes, the RDMPC formation flight is not simulated in this paper. The vehicle model used in the simulations is a common kind of small-size helicopter with autopilot installed. Most of the simulations are considered in a 2D space representation, which can be extended to a 3D space in a straightforward manner. The 2D representation is selected since it facilitates the visualization of the vehicle trajectories in a clearer manner. Four cases are simulated to illustrate the methodology. Throughout this section, the interconnection will be described by a directed graph, which is determined by the two closest visible neighbors to each vehicle.

It is important to note that the method proposed in this paper can easily accommodate any other particular UAV dynamics described by higher fidelity, heterogeneous, higher order, more complex linear or piecewise-linear models. Within the decentralized MPC framework, each vehicle is modeled as a low-order linear system that represents the controlled dynamics of the vehicle. The coupling between vehicles stems only from the common objective of the team (formation) and its constraints.

The simplified vehicle dynamics can be obtained using identification techniques based on position step responses of the closed-loop controlled nonlinear vehicles model. In this simulation, a third-order position command to position output discrete-time LTI model has been identified based on the simulated nonlinear model response. The details of the simulation setup are elaborated below.

1. The predictive control horizon N_u and the predictive horizon N_2 are set as $N_u = 3$ and $N_2 = 5$. The sample time is 0.4s.
2. Given the system dynamics and a “two-closest-neighbors” interconnection policy, each vehicle solves the decentralized optimization problem Eq. (24) with the quadratic cost function:

$$J_k^i(\hat{\mathbf{x}}_{k+tlk}^i, \hat{\mathbf{u}}_{k+tlk}^i) = (\Delta \hat{\mathbf{u}}_{k+tlk}^i)^T \mathcal{Q}_{\Delta u} (\Delta \hat{\mathbf{u}}_{k+tlk}^i) +$$

$$(\hat{\mathbf{x}}_{k+tlk, pos}^i - \hat{\mathbf{x}}_{f, pos}^i)^T \mathcal{Q}_{pos} (\hat{\mathbf{x}}_{k+tlk, pos}^i - \hat{\mathbf{x}}_{f, pos}^i) + (\hat{\mathbf{x}}_{k+tlk, vel}^i - \hat{\mathbf{x}}_{f, vel}^i)^T \mathcal{Q}_{vel} (\hat{\mathbf{x}}_{k+tlk, vel}^i - \hat{\mathbf{x}}_{f, vel}^i)$$

3. No terminal cost and constraint $J_N^i(\hat{\mathbf{x}}_{k+Nlk}^i, \hat{\mathbf{u}}_{k+Nlk}^i) = 0$
4. The vehicles in the formation have identical dynamics.
5. Linear constraints on single vehicle's inputs and velocity are:
 $|\Delta u| \leq [3 \ 3 \ 3]^T m, |v| \leq [5 \ 5 \ 5]^T m$
6. The non-convex interconnection constraints (collision avoidance) are represented by:

$$q^{i,j}(\mathbf{x}_k^i, \mathbf{u}_k^i, \mathbf{x}_k^j, \mathbf{u}_k^j) = d_{safe} - \|\mathbf{y}_{k, pos}^i - \mathbf{y}_{k, pos}^j\|_{\infty} \leq 0, \quad d_{safe} = 0.5m$$

For pop-up constraint, the position constraint $norm\left(\begin{bmatrix} \hat{x}_{i,k+1} \\ \hat{y}_{i,k+1} \end{bmatrix} - \begin{bmatrix} x_{i,k} \\ y_{i,k} \end{bmatrix}\right) \leq H_i^*$ is added.

7. Network connectivity constraints :
 $q^{i,j}(\mathbf{x}_k^i, \mathbf{u}_k^i, \mathbf{x}_k^j, \mathbf{u}_k^j) = \|\mathbf{y}_{k, pos}^i - \mathbf{y}_{k, pos}^j\|_{\infty} - d_{com} \leq 0, \quad d_{com} = 3m$
8. The weights in the cost function are chosen to be $\mathcal{Q}_{\Delta u} = I_n$ for all vehicles, where I_n denotes the identity matrix. Other weights are chosen as $\mathcal{Q}_{pos} = \mathcal{Q}_{vel} = 5I_n$

Case 1: Formation flight with any shape and size obstacle avoidance

In Fig. 6, the leader (black), wingman1 (blue), and wingman2 (green) are initially aligned along the y axis when the leader is commanded to fly to the target position (25,0) with the two wingman maintaining the formation, i.e. the commanded separation between each other is 1 m in the y axis. Fig. 6 shows that the formation is maintained and the fleet successfully avoid the triangular obstacle shown in red.

Figure 6 Triangle Obstacle Collision Avoidance

Figure 7 Small Size Obstacles Avoidance

In Fig. 7, the scenario is the similar to that in Fig. 6 except that the obstacles are round and small and the target position for leader is (33,0). The radiuses of the obstacles are 0.8 m and 1.5 m respectively. Since the first obstacle's size is smaller than the vehicle longest travel distance (2 m) in one sample time, in the MPC controller setup, the position constraints have been added to the MPC formulation. In after the vehicles successfully bypassed the obstacles, because of the short predictive horizon and relatively more weight on the target acquisition there is overshoot in the leader's trajectory.

Case 2: Formation flight with moving agent and moving obstacle collision avoidance

In Fig. 8, initially, the leader (black) and wingman1 (blue) are located on the y axis at x=-3 m, while wingman2 (green) is at the coordinate of (30, 0). The leader is commanded to fly to the its target position at (30,0), and the wingman2 is commanded to fly to the target position at (-3,0). The wingman1 is trying to maintain the formation

with leader during the task. As we can see from the Fig. 8 that the formation is maintained while the vehicles are avoiding the inter-agent collision.

In Fig. 9, the scenario is the same as in Fig. 8 except that a moving obstacle replaces the wingman2. Since the

Figure 8 Inter-agents Collision Avoidance

Figure 9 Collision Avoidance with Moving Obstacle

moving obstacle can not be controlled to change its trajectory, then the vehicles in the formation make a larger adjustment to avoid collision as compared to Fig. 8.

Case 3: Formation splitting

The scenario in Fig. 10 is more complicated than previous two cases. Initially, the leader (black), wingman1 (blue), wingman2 (green), wingman3 (cyan), wingman4 (magenta) are aligned on the y axis at $x=0$. Firstly the leader is commanded to fly to the target position at (15, 0), and the formation is commanded to maintain the V shape as indicated by the red dotted line in the figure. Secondly, the 5-vehicle formation is commanded split into two formation groups, i.e., group 1 (leader, wingman1 and wingman2) and group 2 (wingman3 and wingman4). The leader and wingman 3 are commanded to fly to the targets (50,0) and (50,-15) respectively. Fig. 10 indicates that the vehicles can execute the commands while successfully maintaining safety. Notice that there is overshoot on the trajectories; this is mainly due to the short predictive horizons. If the computational power is enough, better performance can be attained by setting the predictive horizon longer.

Figure 10 Formation Split and Obstacle avoidance

Case 4: Formation flight in 3D

This simulation is use to demonstrate that 2D scenarios can be also extended to 3D case as can be seen from Fig. 11. In the 3-D collision avoidance constraint formulation, several options can be chosen, for example, the vehicle can bypass the obstacles either in the X-Y, X-Z or Y-Z plane depending on the current situation and the group mission requirements. In Fig. 11, we make the collision avoidance in the X-Z plane and let the leader fly below of the obstacle since this kind of maneuver can lead to fuel saving compare to flying up to make the collision avoidance.

Figure 11 3D Formation Flight

V. Conclusions and Future Work

The 3-D decentralized vehicle formation flight control problem has been formulated using decomposition in a hierarchical fashion. The collision-free control scheme and the robustness analysis have been proposed under the unified MPC framework. The overall control scheme is formulated as mixed binary convex optimization problems of small sizes in order to reduce the computational power. Simulation results demonstrate that the proposed control method is able to maintain the desired formation geometry within the constraints posted and guarantees collision-free flight.

As part of the future work, we will concentrate on simulation of the formation flight with uncertainty as well as the controller implementation.

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