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Time-averaged in-line digital holography is applied for vibration analysis. In particular, by use of a double-exposure approach, simultaneous determination of vibration mode shape and mean static state deformation during a vibration cycle are obtained. The subtraction of two numerically reconstructed digital holograms recorded at the same resonant frequency but with a small difference in amplitude shows the mixing of Bessel-type time-averaged fringes owing to vibration and of the double-exposure fringes owing to differences in the mean deformation of the object. It is shown that separation of these fringe patterns can be readily accomplished numerically. An experimental demonstration of this effect by use of in-line digital holography for relatively small membranes is demonstrated. © 2006 Optical Society of America

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1. Introduction

Digital holography,¹ with its ability to reconstruct numerically both the phase and the amplitude² of the object wave, has found applications in imaging, microscopy, and static and dynamic metrology.^{3–6} In-line digital holography, which makes efficient use of the full sensor area for image recording, is becoming the preferred choice for digital holography. Furthermore, the need for a small recording distance compared with that for an off-axis setup⁷ reduces noise caused by speckles. The main problem for in-line digital holography is the overlapping of a zero-order wave and a twin image wave with a real image wave. This overlap can be suppressed by digital methods^{8,9}; one approach is simply to subtract the individual reference and object beams from the hologram. Digital holography is now being explored for dynamic applications by use of time-averaged recording. Picart *et al.* showed that time-averaged off-axis digital holography when it is numerically reconstructed provides the familiar J_0 fringes seen in conventional time-averaged holography.¹⁰ Time-averaged digital holography was further extended to the detection of

so-called hidden stationary deformations of vibrating surfaces¹¹ and to the visualization of high-contrast fringes in low-resolution digital holography by use of the concept of the zero-crossing phase.¹² More recently, time-averaged digital holography was demonstrated as a low-cost device for dynamic model characterization of musical instruments.¹³ These methods use either off-axis geometry or a quasi-Fourier configuration that requires large distances of the object from the CCD during hologram recording and thus has been applied for relatively large objects (greater than 30 mm in diameter).

In this paper time-averaged in-line digital holography is demonstrated, for the first time to our knowledge, for vibration analysis of membranes 10 mm in diameter. It is also shown that using double-exposed time-averaged in-line holography suppresses the effects of a zero-order wave and a twin image wave simultaneously without any additional preprocessing of the holograms. However, this double exposure results in the mixing of time-averaged and mean deformation fringes. A method for the numerical separation of vibration and mean deformation fringes is also proposed and experimentally verified. We envisage that with this approach the technique can be extended to dynamic testing of microelectromechanical systems (MEMS) diaphragms.

2. Time-Averaged In-Line Digital Holography

Let the displacement of a sinusoidally vibrating object be given as

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$$\mathbf{z}(x, y, t) = \mathbf{z}(x, y) \cos \omega t, \quad (1)$$

where ω is the angular frequency of vibration. The instantaneous object wave, $O(x, y, t)$, scattered from the vibrating object is

$$O(x, y, t) = O_0(x, y) \exp[i\phi_0(x, y)] \times \exp\{i[\mathbf{K} \cdot \mathbf{z}(x, y, t)]\}, \quad (2)$$

where $O_0(x, y)$ is the complex amplitude of the scattered light when the object is stationary and $\mathbf{K}$ is the sensitivity vector. $\phi_0(x, y)$ is the phase that represents the mean deformation state of the vibrating object and is called the mean static state.

An in-line Fresnel digital hologram $H(\xi, \eta)$ formed by the interference of this object wave with reference wave $R(\xi, \eta)$ at CCD plane (ξ, η) can be written as

$$H(\xi, \eta) = |O(\xi, \eta)|^2 + |R(\xi, \eta)|^2 + O^*(\xi, \eta)R(\xi, \eta) + O(\xi, \eta)R^*(\xi, \eta). \quad (3)$$

For time-averaged recording of holograms, the exposure time is much larger than the period of object vibration.

For a CCD with $M \times N$ pixels of size $\Delta\xi \times \Delta\eta$, digitally sampled hologram $H(m, n)$ becomes¹⁴

$$H(m, n) = \left[H(\xi, \eta) * \text{rect}\left(\frac{\xi}{\alpha\Delta\xi}, \frac{\eta}{\beta\Delta\eta}\right) \right] \times \text{rect}\left(\frac{\xi}{M\Delta\xi}, \frac{\eta}{N\Delta\eta}\right) \text{comb}\left(\frac{\xi}{\Delta\xi}, \frac{\eta}{\Delta\eta}\right), \quad (4)$$

where $*$ represents two-dimensional convolution and (α, β) are the fill factors of the CCD pixels; rect and comb are the rectangle and comb functions, respectively.

Reconstructed wave field $U(x', y')$ a distance d' from the hologram plane is obtained by discrete Fresnel transformation of the digital hologram multiplied by a numerical reference beam $R(m, n)$ and an impulse response $g(m, n)$ of the coherent optical system. It can be expressed as

$$U(x', y') = \mathfrak{F}[H(m, n)R(m, n)g(m, n)]. \quad (5)$$

For in-line digital holography, the zero-order wave and the twin image overlap this real image wave. Apart from a magnification term, the reconstructed wave can be written as

$$U(x', y') = O_0(x', y') \exp[i\phi(x', y')] J_0[\mathbf{K} \cdot \mathbf{z}(x', y')] + U', \quad (6)$$

where the first term on the right-hand side is the reconstructed real image wave and U' is the sum of the background reference wave (zero-order wave) and the twin image wave. The background noise in the final reconstructed real image of the object is due to

the zero-order wave and an out-of-focus twin image wave and does not change with different deformation states of the same object. However, because the speckle pattern changes stochastically between exposures, the background noise in both exposures may not be exactly equal.

To eliminate the background reference beam and the twin image in time-averaged digital holography, we propose using a double-exposure method. Here two digital holograms of the object vibrating at the same frequency but at slightly different amplitudes are recorded and subtracted.

3. Double-Exposure Time-Averaged In-Line Digital Holographic Interferometry

Consider an object vibrating at angular frequency ω at two amplitudes, $\mathbf{z}_1$ and $\mathbf{z}_2$. The phases of the mean deformation states are ϕ_1 and ϕ_2 , respectively. Then the object waves that correspond to these two states can be written as

$$O_1(x, y, t) = O_0(x, y) \exp[i\phi_1(x, y)] \times \exp\{i[\mathbf{K} \cdot \mathbf{z}_1(x, y, t)]\}, \quad (7a)$$

$$O_2(x, y, t) = O_0(x, y) \exp[i\phi_2(x, y)] \times \exp\{i[\mathbf{K} \cdot \mathbf{z}_2(x, y, t)]\}. \quad (7b)$$

Time-averaged digital holograms are recorded for these two states of an object. On numerical reconstruction we get the wave fields

$$U_1(x', y') = O_0(x', y') \exp[i\phi_1(x', y')] J_0[\mathbf{K} \cdot \mathbf{z}_1(x', y')] + U_1', \quad (8a)$$

$$U_2(x', y') = O_0(x', y') \exp[i\phi_2(x', y')] J_0[\mathbf{K} \cdot \mathbf{z}_2(x', y')] + U_2'. \quad (8b)$$

As mentioned above, U_1' and U_2' are not exactly the same because of variations in the stochastic speckle pattern between the two holograms. This difference also adds to the random noise in the amplitude and the phase terms of the reconstructed real image wave. The subtraction of wave fields (we ignore this background noise) can thus be written as

$$(U_1 - U_2) = O_0(x', y') \{ \exp[i\phi_1(x', y')] \times J_0[\mathbf{K} \cdot \mathbf{z}_1(x', y')] - \exp[i\phi_2(x', y')] \times J_0[\mathbf{K} \cdot \mathbf{z}_2(x', y')] \}, \quad (9)$$

or

$$I = |U_1 - U_2|^2 = |O_0(x', y') \{ \exp[i\phi_1(x', y')] \times J_0[\mathbf{K} \cdot \mathbf{z}_1(x', y')] - \exp[i\phi_2(x', y')] \times J_0[\mathbf{K} \cdot \mathbf{z}_2(x', y')] \}|^2. \quad (10)$$

This wave field difference and the resultant intensity show the mixing of the mean static deformation

Fig. 1. (a) In-line digital holographic setup for vibration measurement. (b) Schematic of the vibrating membrane.

fringes and the zero-order Bessel fringes that is due to time-averaged recording. Because this mixing is complex and not easy to isolate, the following procedure is used to separate the Bessel fringes from the deformation fringes: The amplitudes and the phases of the individual wave fields are extracted first. The difference in amplitudes then provides the Bessel-type vibration fringes, whereas the difference in phase gives the mean deformation as $\cos^2$ fringes. These can be represented as

$$\Delta I = |U_1|^2 - |U_2|^2$$

$$= O_0^2(x', y') \{ J_0^2[\mathbf{K} \cdot \mathbf{z}_1(x', y')] - J_0^2[\mathbf{K} \cdot \mathbf{z}_2(x', y')] \}, \quad (11a)$$

$$\Delta \phi = \phi_1(x', y') - \phi_2(x', y'). \quad (11b)$$

4. Experimental Results

The experimental setup for in-line digital holography used for vibration study is shown in Fig. 1(a). A frequency-doubled Nd-YAG laser operating at 532 nm is split into two beams by a variable beam splitter. The object beam is expanded and illuminates the object, and the other beam forms a collimated reference beam.

The object, a thin aluminum membrane bonded to a circular metal ring of outer and inner diameters 13 and 10 mm, respectively [Fig. 1(b)], is attached to an earphone. The earphone is excited by a frequency generator that can be tuned to vibrate at different sinusoidal frequencies and amplitudes. One controls

Fig. 2. Subtracted wave fields of membranes vibrating at the same frequency but different amplitudes, showing mixing of the vibration and mean static deformation fringes. Vibration frequencies are (a) 2.0, (b) 5.25, (c) 6.0, and (d) 7.75 kHz.

the static deformation of the membrane by varying the amplitude of the excitation signal using the frequency generator. An 8 bit digital CCD with 2029×2044 square pixels $9 \mu\text{m}$ in size is used to record the holograms at 30 frames/s. The object is placed 500 mm from the CCD, which satisfies the sampling theorem requirement for efficient use of the full sensor area.⁷ In-line time-averaged digital holograms are recorded at different amplitudes of vibration and at frequencies that range from 1 to 10 kHz. Thus the exposure time is greater than the period of vibration, giving rise to time-averaged recording. Numerical reconstruction is performed with MATLAB software.

For time-averaged recording, the amplitude is modulated by the J_0 function and the phase represents information about the mean static state of the object. If there is any change in the mean static state of the vibrating object at different amplitudes, then, as discussed above, the subtracted wave fields (or intensity) of two time-averaged holograms would show mixing of time-averaged and static deformation fringes [Eqs. (9) and (10)]. Double-exposure holograms of the membrane are recorded at the same frequency but at different amplitudes. Subtraction of the reconstructed wave fields that correspond to resonant frequencies 2.0, 5.25, 6.0, and 7.75 kHz are shown in Fig. 2.

Two sets of fringes that correspond to Bessel fringes and mean deformation fringes are clearly shown in Fig. 2. The deformation fringes extend to the area where the diaphragm is bonded to the frame, indicating poor and nonuniform bonding. Furthermore, whereas the fringe pattern appears to indicate that the two sets of fringes are simply superposed, it

Fig. 3. Vibration mode patterns at frequencies (a) 2.0, (b) 5.25, (c) 6.0, and (d) 7.75 kHz.

can be seen from Eq. (9) that this is not so. Indeed, if the two patterns were simply superposed, one could readily separate them by considering the amplitude and the phase of the wave field subtraction.

To separate the Bessel fringes from the mean deformation fringes it is thus necessary first to determine the amplitude and the phase of the two time-averaged digital holograms and then to subtract them as given in Eqs. (11). The mode patterns of the vibrating membrane that correspond to the same frequencies discussed above are shown in Fig. 3. These patterns are obtained by subtraction of reconstructed intensities [Eq. 11(a)] from two time-averaged holograms recorded at the same vibration frequencies but with slightly different amplitudes. One can obtain similar patterns by displaying the intensity of a single time-averaged hologram. These patterns are similar to those observed in conventional time-averaged holographic interferometry. For a single time-averaged hologram, the overlapping zero-order wave and twin image effect needs to be eliminated by preprocessing of the hologram.^{8,9} For two-exposure subtraction, no such preprocessing is required.

The mean static state of the membrane during vibration changes the phase of the wave field. Hence, to display only the difference in mean static state between two wave fields, one extracts the phase from each wave field and then subtracts them as in Eq. 11(b). Figure 4 shows the resultant phase difference, which represents only the difference in mean static states for a membrane vibrating at different frequencies. The difference in amplitudes between the two subtracted images for four frequencies is similar, and hence the difference in mean static state fringes is identical. Closer inspection of the fringe

Fig. 4. Subtraction of phase gives the fringe pattern that corresponds to mean static deformation during a change in vibration amplitudes.

pattern indicates what appear to be additional phase effects that are due to the fact that the membrane is vibrating at resonance. These additional phase changes mask the mean deformation fringes in these regions. Also, the amplitudes at resonant frequencies are much higher than for static deformation if the plate were not vibrating, as is to be expected. This is clear from Fig. 5, which shows the same membrane but subject to static deformation at zero vibration frequency. Only the fringes that are due to differences in static states are clearly seen. Indeed, even if the plate were vibrating, as long as the amplitude of vibration is small, only fringes caused by differences in the static state are observed. Also note that, in this case [Fig. 5], the difference in amplitude between the two exposures is two to three times larger than the amplitude difference for Fig. 4. The deformations in the region where the membrane is attached to the

Fig. 5. Mean static deformation at two amplitudes of a stationary plate.

ring are also distinct, indicating improper bonding of the diaphragm.

5. Conclusions

In this paper, in-line time-averaged digital holography for the study of the vibration characteristics of a thin circular aluminum membrane 10 mm in diameter has been demonstrated. A double-exposure approach to simultaneous determination of vibration mode shape and mean-static state deformation that uses a vibration cycle has been presented. This approach is also suitable for simultaneous suppression of background noise, caused by in-line geometry, without any preprocessing of holograms. However, the double-exposure method results in the superposition of the Bessel fringes owing to vibration and of the mean deformation fringes owing to double exposure. Separation of these fringes requires that the amplitudes and phases of the individual holograms be determined first and then subtracted. For amplitude subtraction, enhanced and multiplied Bessel fringes are observed, whereas phase subtraction gives the mean deformation fringes. It can also be observed that, in addition to mean deformation fringes, the phase difference shows the phase jump that is due to the resonating membrane that masks the fringes in certain regions. Numerical reconstruction of phase information is an additional advantage of digital holography compared with electronic speckle pattern interferometry¹⁵ for vibration analysis. This method can be readily extended to dynamic testing of MEMS diaphragms of ~1 mm size. The main problem for small objects is speckle noise. This noise can be controlled by reduction of the distance of the object from the CCD during hologram recording, which for in-line geometry can be done easily.

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