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Amplitude and phase analysis in digital dynamic holography

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Lensless in-line digital holographic interferometry has the potential for vibration analysis of objects smaller than 5 mm in diameter. This is particularly useful for dynamic characterization of microelectromechanical systems devices. To achieve this, there is a need to magnify the object wave, which is done using a diverging beam. It is observed that an increase in the object-to-CCD distance increases the sensitivity of the amplitude-modulated time-average fringes. At the same time the effect on phase information that represents the mean static deformation of a vibrating object is studied. It is also observed that a reduction in the object-to-CCD distance increases the phase sensitivity as evidenced by the double-exposure time-average fringes. The experimental observation and a theoretical explanation for this contradictory phenomenon are presented. © 2006 Optical Society of America

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Digital holography¹ is increasingly being used for quantitative measurement of both the amplitude and the phase of object waves, and applications in particle imaging,² microscopy,³ static and dynamic metrology,^{4,5} and vibration analysis have been demonstrated.⁶ The in-line geometry proposed by Gabor⁷ is a preferred choice in digital holography, because it utilizes the full sensor area of a CCD array for hologram recording. The disadvantage of overlapping of a zero-order wave and a twin image can be suppressed digitally.^{8,9} While microscopic objectives have been used to magnify the object wave, for a lensless system a diverging beam provides the geometrically magnified object wave.¹⁰ Time-average digital holography was recently exploited for vibration analysis using an off-axis¹¹ setup and a quasi-Fourier setup.¹² These setups use a large recording distance between object and CCD and have thus been applied only for objects larger than 30 mm. Recently, in-line digital holographic interferometry has been applied for vibration analysis of objects 10 mm in diameter.¹³ A double-exposure time-average method was also proposed for recording and separation of vibration amplitude as well as mean deformation analysis, which simultaneously suppresses the zero-order and twin image waves. To further explore the use of lensless digital dynamic holography for microelectromechanical systems diaphragm characterization, the use of a diverging object wave was explored. Interesting features become apparent when both the time-average amplitude fringes and the mean deformation phase images are observed for the diverging object wave. The vibration amplitude and the mean deformation of the object are represented by the amplitude and the phase of the reconstructed real image wave, respectively. It is observed that for diverging object waves the sensitivity of the vibration amplitude fringes increases with increased distance, while the sensitivity of phase information is reduced, and vice versa. This effect is demonstrated here, and an explanation is proposed.

Consider a harmonically excited object placed in the (x, y) plane with an out-of-plane displacement $\vec{z} = \vec{z}(x, y)\cos\omega t$, where ω is the frequency of vibration. The instantaneous wave scattered by the object is

$$O(x, y, t) = O_0(x, y)\exp[i\phi_0(x, y)]\exp[i(\vec{K} \cdot \vec{z})], \quad (1)$$

where $O_0(x, y)$ is the complex amplitude, $\phi_0(x, y)$ is the phase and $\vec{K}$ is the sensitivity vector that depends on the angles of illumination and observation.

A time-average hologram $H(\xi, \eta)$, where the exposure time is much larger than the period of vibration, τ , can be written as

$$H(\xi, \eta) = \int_0^\tau I(\xi, \eta) d\tau, \quad (2)$$

where $I(\xi, \eta)$ represent the interference between the object and the reference waves at the hologram plane (ξ, η) , which is placed at a distance D from the object plane. The reconstructed wave field at plane (x', y') placed at a distance D' from the hologram plane is obtained by either the Fourier transform method or the convolution method. The convolution method is preferred for shorter recording distances, and the image size is independent of reconstruction distance. For $D'=D$, the reconstructed real image wave from the time-average hologram remains fundamentally the same as the recorded object wave, except for the magnification term, i.e.,

$$U(x', y') = O_0(x', y')\exp[i\phi_0(x', y')]J_0[\vec{K} \cdot \vec{z}(x', y')]. \quad (3)$$

Here J_0 is the zero-order Bessel function of the first kind. The numerically reconstructed amplitude and phase can be separately calculated from the real image wave as

Fig. 1. (a) In-line digital holographic recording mechanism. (b) Object illumination by the diverging beam.

$$|U(x', y')| = |O_0(x', y')| |J_0[\vec{K} \cdot \vec{z}(x', y')]| + \text{background noise}, \quad (4a)$$

$$\phi_0(x', y') = \tan^{-1} \frac{\text{Im}[U(x', y')]}{\text{Re}[U(x', y')]} \quad (4b)$$

For traditional time-average holographic interferometry, Eq. (4a) is adequate, as it gives the mode shapes and amplitudes of vibration. The phase term does not contain useful information and is a source of speckle noise.

Now, consider two time-averaged holograms recorded at the same vibration frequency but with different amplitudes of vibration. Subtraction of the individually reconstructed amplitudes provides J_0 -type fringes with different visibility. However, in this case the subtraction of phases gives the difference in mean deformation of the object during its vibration cycle.

An in-line digital hologram is the interference of the object wave with the in-line plane reference wave and is recorded by the CCD sensors placed at the plane (ξ, η) as shown in Fig. 1(a). For good-quality amplitude reconstruction, the sampling theorem should be fulfilled across the entire CCD sensor area. This means that the interference fringe spacing must be larger than two pixels of the CCD sensor. If L is the object size, then for a CCD sensor containing N pixels, $\Delta\xi$ in size, the shortest distance ($D_{\min}$) between the object and CCD sensor is given by

$$D_{\min} = \frac{\Delta\xi}{\lambda} (N\Delta\xi + L), \quad (5)$$

where λ is the wavelength of light. It is clear from this equation that better sampling is achieved at larger recording distances. Illumination by a diverging beam will magnify the object wave by a factor $(D+d)/d$, where d is the distance of the object from the diverging source. At the same time, the objective speckle size at the hologram plane at a distance D with an object size L is $\lambda D/L$, as shown in Fig. 1(b). Hence, the smaller the object-to-CCD distance, the smaller the speckle size and the larger the phase sensitivity.

The experimental setup for in-line digital holographic interferometry is shown in Fig. 2. A thin aluminum membrane 5 mm in diameter is excited sinusoidally at different frequencies and amplitudes by an earphone connected to a frequency generator.

Light from a frequency-doubled Nd:YAG laser operating at a 532 nm wavelength is coupled into a single-mode fiber coupler that is further split equally into two beams. One of the diverging beams illuminates the vibrating object, while the other is collimated to form the in-line reference beam. An 8-bit digital CCD sensor with 2029×2044 square pixels $9 \mu\text{m}$ in size is used to record the time-average holograms at 30 frames/s. The numerical reconstruction process is performed using a fast Fourier transform algorithm,¹⁵ which is simulated in MATLAB.

The minimum object distance from the CCD to satisfy the sampling theorem requirement is 388 mm. In this experiment, three distances are selected, 270, 350, and 410 mm, and both the vibration (J_0) and the mean deformation fringes are analyzed at these distances. As discussed previously, the vibration mode shape and its amplitude information can be reconstructed from the amplitude part of the reconstructed real image wave [Eq. (4a)], while the reconstructed phase information gives the mean static deformation of the object during vibrations [Eq. (4b)].

The amplitudes reconstructed from the holograms at the three distances at a frequency of 2.6 kHz and an excitation voltage of 9.5 V are shown in Fig. 3. The reconstructed amplitude fringes show the vibration mode of the membrane. It can be clearly seen that the sensitivity of the vibration fringes increases as the distance increases.

A mean static deformation in the vibrating membrane can be introduced by an offset voltage or can occur naturally due to nonlinear mechanical effects. These deformation fringes can be analyzed using the reconstructed phase information. The subtraction of reconstructed phases during double exposure represents only the mean static deformation fringes at

Fig. 2. Experimental setup of in-line digital holographic interferometry.

Fig. 3. Reconstructed intensity from time average holograms at (a) 270, (b) 350, (c) 410 mm.

Fig. 4. Phase subtraction showing the deformation fringes reconstructed at (a) 270 mm, (b) 350 mm, (c) 410 mm.

relatively lower amplitudes of vibrations, and as vibration amplitude increases it mixes with the mean offset deformation caused by vibrations. Figure 4 shows the subtraction of reconstructed phases from two time-averaged holograms recorded at the frequency 2.6 kHz and at different amplitudes of vibration (excitation voltages are 5 and 8.5 V) corresponding to three distances. The mean deformation fringes can be clearly observed at the minimum recording distance of 270 mm, but as the distance increases the fringes are not seen. The effect of the zero-crossing phase,⁶ which was used to determine the dark fringes for pure sinusoidal excitation, also appears in the phase-subtracted image, especially at larger reconstruction distances.

In conclusion, a lensless in-line digital holographic interferometry for vibration analysis of small objects has been presented. The effects on reconstructed amplitude and phase information of the vibrating objects were studied for different recording distances when the object wave was illuminated with a diverging beam. There appear to be two different phenomena that seem to control the amplitude and phase parts. Following convention, an increase in the recording distance increases the sampling of the interference pattern between the reference and object

wave, resulting in increased sensitivity to the amplitude-modulated time-averaged fringes. On the other hand, double-exposure fringes are controlled by the speckle size, which increases with the recording distance and thus reduces sensitivity to mean deformation fringes. This study has implications for the use of lensless in-line digital holography for the vibration analysis of small objects such as microelectromechanical systems diaphragms.

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