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A Proposal for Micromachined Accelerometer, based on a Contactless Suspension with Zero Spring Constant

Kirill V. Poletkin, *Member, IEEE*, Aleksandr I. Chernomorsky, and Christopher Shearwood

Abstract—In this paper, a micromachined accelerometer, based on a contactless suspension with a zero spring constant which is a new challenge that provides possibility to significantly increasing accuracies of the micromachined inertial sensor is proposed. Minimization of the spring constant of the contactless suspension is achieved by combining inductive and electrical contactless suspensions. To study the conditions required to eliminate the spring constant of the suspension and achieve stable levitation of the accelerometer proof mass, a mathematical model of the suspension is developed. It is shown that such a suspension can be developed in principle.

Index Terms—inertial sensor; micro-electro-mechanical system; contactless suspension; levitation.

I. INTRODUCTION

MICROMACHINED contactless suspension technology has attracted a lot of attention from many research groups over the past decade to the design a new generation of micromachined inertial sensors which can be applied to tactical- and inertial-grade applications [1]–[3]. Eliminating the mechanical attachment or contact friction between, in particular, the proof mass of a MEMS accelerometer and its case is the major issue addressed by such technology. This issue leads to on one hand a decreasing value of mechanical-thermal noise and, consequently, increasing the sensitivity of the sensors and their accuracies [4]–[7]. On the other hand an extension of the lifetime and, long-term stability of sensors becomes possible [8], [9].

Moreover, under certain conditions, an electrical spring constant supported by the suspension, in particular, along an input axis of the accelerometer can be minimized or completely eliminated, at the same time keeping the proof mass at an equilibrium point. These possibilities provided by a contactless suspension have not been studied yet. However, it is a new challenge that leads to significantly increasing accuracies of micromachined inertial sensors.

Indeed, let us consider a micromachined accelerometer consisting of a proof mass (PM) that can be suspended by

a mechanical or magneto-electrical contactless suspension. The mass of the PM is denoted by m , the effective spring constant of the suspension is denoted by c and the damping coefficient defined by the friction between the PM and the gas surrounding the PM is μ . Hence, the behavior of the PM can be described by a second - order transfer function, as follows:

$$\frac{y(s)}{a(s)} = \frac{1}{s^2 + \frac{\mu}{m}s + \frac{c}{m}}, \quad (1)$$

where s is the Laplace operator, a is the external acceleration, y is the displacement of the PM. The static sensitivity of the accelerometer is

$$\frac{y}{a} = \frac{m}{c}. \quad (2)$$

As soon as the spring constant trends to zero ($c \rightarrow 0$), the static sensitivity (2) dramatically increases and, in the limiting case becomes infinitely large ($m/c \rightarrow \infty$). Consequently, the accelerometer transfer function (1) can be rewritten as:

$$\frac{y(s)}{a(s)} = \frac{1}{s\left(s + \frac{\mu}{m}\right)}. \quad (3)$$

It is important to note that in Eq. (3) the integration of the external acceleration a is obtained by eliminating the spring constant c [10].

Obviously, due to the infinitely large sensitivity in such an accelerometer performance, it can operate only under closed-loop control when the feedback forces back the PM to the original equilibrium position. It is known that the static loop gain of a closed loop accelerometer can be defined as [11], [12]:

$$K_l = \frac{K_{PO}K_C K_F}{c}, \quad (4)$$

where K_{PO} is the gain of the pick-off circuit, K_C is the controller gain, K_F is the feedback gain. In view of $c \rightarrow 0$, it follows from (4) that $K_l \rightarrow \infty$. As a result, the steady-state errors in such an accelerometer after eliminating the spring constant of the suspension will be significantly smaller in comparison with an accelerometer in which the spring constant of the suspension is preserved. Thus, elimination of the spring constant of the suspension provides on the one hand, a dramatic increase in the static sensitivity of the sensor and, on the other hand, significantly decreasing the steady-state errors of the closed loop sensor. Due to these facts, it is expected that applying a suspension with zero spring constant

Kirill V. Poletkin is with the School of Mechanical & Aerospace Engineering, Nanyang Technological University, Singapore, 639798 Singapore (Phone: (+65) 67904050; e-mail: kpoletkin@ntu.edu.sg).

Aleksandr I. Chernomorsky is with the Department of Automatic Complexes of Orientation and Navigation Systems, Moscow Aviation Institute (State Technical University), Moscow, 125993, GSP-3, Russia. (e-mail: lab305@mai.ru).

Christopher Shearwood is with the School of Mechanical & Aerospace Engineering, Nanyang Technological University, Singapore, 639798 Singapore (Phone: (+65)6790 5524; e-mail: mcshearwood@ntu.edu.sg).

Fig. 1. Electro-mechanical scheme of the micromachined contactless suspension: E_1 , E_2 , E_3 , and E_4 are electrodes.

to a micromachined inertial sensor will significantly increase their accuracies.

In this paper, a micromachined contactless suspension with a zero spring constant is proposed. Minimization of the spring constant of the contactless suspension is achieved by combining inductive and electrical contactless suspensions. To study the conditions required to eliminate the spring constant of the suspension and achieve stable levitation, in particular, of the accelerometer proof mass, a mathematical model of the suspension is developed. It is shown that such a suspension can be developed in principle.

II. KINEMATICS AND OPERATING PRINCIPLE OF THE SUSPENSION

Let us consider the following electro-mechanical, micromachined contactless suspension shown in Fig. 1, which can be used, in particular, to describe an accelerometer. The disc-shaped PM (to be fabricated from a conducting material) is suspended at the equilibrium position characterized by the origin O by means of an inductive contactless suspension. Alternating current i passing through the coil creates in space a variable magnetic flux, which intercepts the surface area of the PM and induces a current, i_2 .

In addition, the system of fixed electrodes E_1 , E_2 , E_3 , and E_4 , as shown in Fig. 1, creates an electrical field around the suspended PM. Electrodes E_3 and E_4 are grounded, while potentials u_1 and u_2 are applied to electrodes E_1 and E_2 respectively. The resultant electrostatic forces F_{e1} and F_{e2} created by the electrical field and acting on the top and bottom of the PM surfaces help provide a minimization of the spring constant of the inductive suspension along the vertical Oy axis. The conditions required for the complete elimination of the spring constant will be considered later. It is assumed that the PM has only a linear displacement along the Oy axis, that is denoted by y from the origin O which is located in the center characterized by the distance h between the top and bottom electrodes.

III. MATHEMATICAL MODEL OF THE SUSPENSION

Lagrange-Maxwell equations are used to compile the mathematical model of the suspension under consideration. At first, let us choose the generalized coordinates of the system.

Fig. 2. Scheme of the electric part of suspension: F_{e1} and F_{e2} are the resultant electrostatic forces.

The suspension can be divided into two parts, namely the electric part, consisting of a system of electrodes E_1 , E_2 , E_3 , E_4 and PM, and the electromagnetic part which is the coil and PM. These parts are assumed to be electrically independent of each other, hence they can be considered separately.

The electrode E_k and the nearest part of the surface of PM to this electrode is considered to be a plane capacitor with capacity C_k , which is dependent on the displacement of PM, where $k = 1..4$ (see, Fig. 2). Assuming that the area of all electrodes are the same and equal to A_e , then from the equation for a plane capacitor, we have

$$C_1 = C_3 = \frac{A}{h-y}; C_2 = C_4 = \frac{A}{h+y}, \quad (5)$$

where $A = \varepsilon_0 \varepsilon A_e$, ε_0 is the relative permittivity, ε is the dielectric constant. According to the design of the electric part of the suspension shown in Fig. 2, the electric circuit can be represented as shown in Fig. 3. The capacitor $C(y)$ shown in Fig. 3 is the sum of capacitors and defined as follow

$$C(y) = C_3 + C_4 = \frac{2Ah}{h^2 - y^2}. \quad (6)$$

The charges e_1 and e_2 , on the first and second electrodes respectively, can be taken as the generalized coordinates of the electric part of the suspension. Note that the current flowing through the capacitor $C(y)$ is equal to $\dot{e}_1 + \dot{e}_2$, therefore the charge on the capacitor $C(y)$ is $e_1 + e_2$ [13].

The PM is suspended at the equilibrium position by means of the electromagnetic part of the suspension, the coil of which is fed by an alternating current $i = ie^{j\omega t}$, where ω is the a

Fig. 3. Electric circuit of the electric part of the suspension: e_1 and e_2 are charges on the electrodes E_1 and E_2 , respectively.

Fig. 4. Electric circuit of the electromagnetic part of the suspension: L_1 and L_2 are self inductances of the coil and PM, respectively.

high frequency, $j = \sqrt{-1}$ is an imaginary unit. Since a mutual inductance denoted by M_{12} between the coil and PM occurs, a current is induced within the PM denoted by i_2 , with the same frequency ω (see, Fig. 4). The currents i and i_2 are taken as generalized coordinates of the electromagnetic part of the suspension.

The linear displacement y of the PM is taken as a generalized coordinate of the mechanical part of the suspension. Hence, the following set of the Lagrange-Maxwell equations, describing the system under consideration, can be written as

$$\begin{cases} -\frac{\partial L}{\partial e_1} + \frac{\partial \Psi}{\partial \dot{e}_1} = u_1; & -\frac{\partial L}{\partial e_2} + \frac{\partial \Psi}{\partial \dot{e}_2} = u_2; \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{i}} \right) + \frac{\partial \Psi}{\partial i} = 0; & \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{i}_2} \right) + \frac{\partial \Psi}{\partial i_2} = 0; \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} + \frac{\partial \Psi}{\partial \dot{y}} = F_y, \end{cases} \quad (7)$$

where $L = T(\dot{y}) - \Pi(y) + W_m(y, i, i_2) - W_e(y, e_1, e_2)$ is the Lagrange function of the system, $T(\dot{y})$ and $\Pi(y)$ are the kinetic and potential energies of the system, respectively, $W_m(y, i, i_2)$ and $W_e(y, e_1, e_2)$ are energies stored in the magnetic and electric fields, respectively, $\Psi(\dot{y}, i, i_2, \dot{e}_1, \dot{e}_2)$ is the dissipation function of the system, $F_y = ma$ is the generalized force acting on the PM.

The kinetic energy of the PM linear movement is

$$T = \frac{1}{2} m \dot{y}^2. \quad (8)$$

The potential energy is

$$\Pi = mgy. \quad (9)$$

Taking into account that the mutual inductance M_{12} is dependent on y , the energy stored in the magnetic field of electromagnetic part of the suspension can be written as

$$W_m = \frac{1}{2} L_1 i^2 + M_{12}(y) i i_2 + \frac{1}{2} L_2 i_2^2, \quad (10)$$

where L_1 and L_2 are the self inductances of the coil and PM, respectively. The energy stored in the electric part of the suspension is

$$\begin{aligned} W_e &= \frac{e_1^2}{2C_1} + \frac{e_2^2}{2C_2} + \frac{(e_1 + e_2)^2}{2C} \\ &= \frac{e_1^2}{2A} (h - y) + \frac{e_2^2}{2A} (h + y) + \frac{(e_1 + e_2)^2}{4Ah} (h^2 - y^2). \end{aligned} \quad (11)$$

Hence, the Lagrange function becomes

$$\begin{aligned} L &= \frac{1}{2} m \dot{y}^2 - mgy + \frac{1}{2} L_1 i^2 + M_{12}(y) i i_2 + \frac{1}{2} L_2 i_2^2 \\ &\quad - \frac{e_1^2}{2A} (h - y) - \frac{e_2^2}{2A} (h + y) - \frac{(e_1 + e_2)^2}{4Ah} (h^2 - y^2). \end{aligned} \quad (12)$$

Neglecting the resistances in the electric part of the suspension, the dissipation function of the system can be written as follow

$$\Psi = \frac{1}{2} R_1 i^2 + \frac{1}{2} R_2 i_2^2 + \frac{1}{2} \mu \dot{y}^2. \quad (13)$$

In practice, μ can be controlled by evacuation of the packaged device.

Substituting (12) and (13) into (7), we have

$$\begin{cases} \frac{h-y}{A} e_1 + \frac{h^2-y^2}{4Ah} (e_1 + e_2) = u_1; \\ \frac{h+y}{A} e_2 + \frac{h^2-y^2}{4Ah} (e_1 + e_2) = u_2; \\ L_1 \frac{di}{dt} + \frac{dM_{12}(y)}{dy} \dot{y} i_2 + M_{12}(y) \frac{di_2}{dt} + R_1 i = 0; \\ L_2 \frac{di_2}{dt} + \frac{dM_{12}(y)}{dy} \dot{y} i + M_{12}(y) \frac{di}{dt} + R_2 i_2 = 0; \\ m \ddot{y} + \mu \dot{y} + mg - \frac{dM_{12}(y)}{dy} i i_2 \\ \quad - \frac{e_1^2}{2A} + \frac{e_2^2}{2A} - \frac{(e_1 + e_2)^2}{2Ah} y = F_y. \end{cases} \quad (14)$$

Equation (14) is the set of nonlinear equations describing the behavior of the PM suspended by the proposed contactless suspension.

IV. CONDITION FOR THE STABLE LEVITATION OF THE PROOF MASS

Let us examine the condition under which the PM occupies an equilibrium position at the point O , located at a height, h , above the coil. In other words, the condition corresponding to the stable levitation of the PM in an alternating magnetic field. In this case, the problem is reduced to defining the static position of the PM that is characterized by the height h at which the electromagnetic forces created by the inductive suspension compensate the gravitational force of the PM mass. Hence, the influence of the PM velocity $\dot{y}$, acceleration $\ddot{y}$ and the generalized force F_y acting on the PM are neglected. The mutual inductance M_{12} is considered to have a function dependence on h . Also, it is assumed that electric part of the suspension is disconnected from the electricity supply. Based on these assumptions, set (14) can be rewritten as follow:

$$\begin{cases} L_1 \frac{di}{dt} + M_{12}(h) \frac{di_2}{dt} + R_1 i = 0; \\ L_2 \frac{di_2}{dt} + M_{12}(h) \frac{di}{dt} + R_2 i_2 = 0; \\ mg - \frac{dM_{12}(h)}{dh} i i_2 = 0. \end{cases} \quad (15)$$

The current i feeding the coil is supplied by a current generator, and consequently, it is assumed to be constant. Using the second equation of set (15), the current i_2 can be expressed in term of i as follow

$$i_2 = -i \frac{\sqrt{\omega^4 L_2^2 + \omega^2 R_2^2}}{\omega^2 L_2^2 + R_2^2} M_{12}(h) e^{j\phi}, \quad (16)$$

where $\phi = \arctan \frac{R_2}{\omega L_2}$. Analysis of Eq. (16) shows that, firstly, the flow direction of i_2 is in the opposite direction to i , secondly, a phase shift ϕ exists between the currents i_2 and i , caused by the electric resistance of the PM conducting material. For normal operation of the inductive suspension, the phase shift ϕ must be minimized, made by the following condition:

$$\omega L_2 \gg R_2. \quad (17)$$

As a rule, fulfilling (17) is provided by adjusting the frequency ω . If condition (17) is held, Eq. (16) can be simplified as

$$i_2 = -i \frac{1}{L_2} M_{12}(h) e^{j\phi}. \quad (18)$$

Substituting (18) into the third equation of set (15), the following equation is obtained:

$$mg + \frac{dM_{12}(h)}{dh} \frac{M_{12}(h)}{L_2} i^2 = 0. \quad (19)$$

Hence, Eq. (19) defines the height h at which the PM occupies the equilibrium position, at the origin O .

On the other hand, at the equilibrium point, the Lagrange-Dirichlet function for this system reaches a maximum:

$$F(h) = \Pi(h) - W_m(h, i) \rightarrow \max. \quad (20)$$

Hence, the second h -derivative of the function $F(h)$ must have a negative sign and accounting for (19), can be written as

$$\frac{d}{dh} \left(mg + \frac{dM_{12}(h)}{dh} \frac{M_{12}(h)}{L_2} i^2 \right) < 0. \quad (21)$$

Differentiating the last equation, Eq. (21) becomes

$$\frac{i^2}{L_2} \left[\frac{d^2 M_{12}(h)}{dh^2} M_{12}(h) + \left(\frac{dM_{12}(h)}{dh} \right)^2 \right] < 0. \quad (22)$$

Since the sign of (22) is dependent only on the expression within the brackets, the final condition for stable levitation of the PM can be written:

$$\frac{d^2 M_{12}(h)}{dh^2} M_{12}(h) + \left(\frac{dM_{12}(h)}{dh} \right)^2 < 0. \quad (23)$$

Let us study the behavior of the PM near to the equilibrium point O . It is assumed that the linear displacement of the PM y is small in comparison with h , hence the following inequality can be written:

$$\frac{y}{h} \ll 1. \quad (24)$$

Because of (24), the function of the mutual inductance $M_{12}(y)$ can be extended by a Taylor series at the point h and, neglecting second-order terms, becomes

$$M_{12}(y) = M_{12}(h) + M_y y, \quad (25)$$

where $M_y = \left. \frac{dM_{12}(y)}{dy} \right|_{y=h}$.

Substituting (25) into the last equation of set (14) and taking into account (18) and (19), the differential equation of the linear displacement of the PM near to the equilibrium point

Fig. 5. Scheme of the electromagnetic part of the suspension: F_m - are the resultant electromagnetic forces created by the inductive suspension.

O (in the absence of the electric part of the suspension) can be written as

$$m\ddot{y} + \mu\dot{y} + \frac{M_y^2}{L_2} i^2 y = F_y. \quad (26)$$

Equation (26) is a linear model of the inductive suspension. The spring constant provided by the inductive suspension is directly proportional to the square M_y and current i feeding the coil and it is inversely proportional to the self-inductance of the PM L_2 .

The self-inductance of the PM is dependent on the induced current circuit within the PM. Due to the high frequency of the supply current i and the disk-shaped of the PM, the current circuit can be considered to be in the form of a closed ring that has a radius r_{mp} , as shown in Fig. 5. The mutual induction of two coaxial rings can be described by the following function [14], [15]:

$$M_{12}(y) = \mu_0 r_c \left[\ln \frac{8r_c}{\sqrt{y^2 + d^2}} - 2 \right], \quad (27)$$

where μ_0 is the permeability, r_c is the radius of the coil, $d = r_c - r_{pm}$. Replacing in (27) y by h and substituting into (23), the condition for the stable levitation of the disk-shaped PM becomes

$$-\frac{\mu_0^2 r_c^2}{(h^2 + d^2)^2} \left[(d^2 - h^2) \left(\ln \frac{8r_c}{\sqrt{h^2 + d^2}} - 2 \right) - h^2 \right] < 0. \quad (28)$$

Once again the sign of the expression is dependent on the terms within the brackets which must always be positive or

$$(d^2 - h^2) \left(\ln \frac{8r_c}{\sqrt{h^2 + d^2}} - 2 \right) - h^2 > 0. \quad (29)$$

Thus, condition (29) is necessarily fulfilled for the stable levitation of the disk-shaped PM in an alternating magnetic field induced by the ring-shaped coil.

It follows from the analysis of (29) that the value of d must not be equal to zero or in other words, the radii of the coil and PM must not be equal. In the framework of the feasible system, it can be shown that the function $\ln \frac{8r_c}{\sqrt{h^2 + d^2}} - 2$ is always positive and much greater than one; hence, Eq. (29)

can be rewritten as

$$\frac{h^2}{d^2} < \frac{\ln \frac{8r_c}{\sqrt{h^2+d^2}} - 2}{\ln \frac{8r_c}{\sqrt{h^2+d^2}} - 1}. \quad (30)$$

Then, Eq. (30) can be reduced to

$$\frac{h^2}{d^2} < 1 \text{ or } h < d. \quad (31)$$

Equation (31) shows that the levitation height h of the PM is limited by d .

In addition, let us define the value of the spring constant of the inductive suspension. For this case, we have

$$M_y = \left. \frac{dM_{12}(y)}{dy} \right|_{y=h} = -\mu_0 r_c \frac{h}{d^2}. \quad (32)$$

The value of the spring constant of the inductive suspension can be defined as

$$c = \left(\mu_0 r_c \frac{h}{d^2} \right)^2 \frac{i^2}{L_2}, \quad (33)$$

where the self-inductance of the disk-shaped PM can be calculated as

$$L_2 = \mu_0 r_{pm} \left[\ln \frac{16r_{pm}}{t_{pm}} - 2 \right], \quad (34)$$

where t_{pm} is the thickness of the PM disk [14].

V. COMPENSATION OF THE SPRING CONSTANT OF THE SUSPENSION

Assuming that the PM is stably levitated (conditions (23) and (31) are held), the behavior of the PM within the electric field created by the system of the electrodes E_1 , E_2 , E_3 , and E_4 can be described by the following set:

$$\begin{cases} \frac{h-y}{A} e_1 + \frac{h^2-y^2}{4Ah} (e_1 + e_2) = u_1; \\ \frac{h+y}{A} e_2 + \frac{h^2-y^2}{4Ah} (e_1 + e_2) = u_2; \\ m\ddot{y} + \mu\dot{y} + \frac{M_y^2}{L_2} i^2 y - \frac{e_1^2}{2A} + \frac{e_2^2}{2A} - \frac{(e_1 + e_2)^2}{2Ah} y = F_y. \end{cases} \quad (35)$$

Using the first and second equations of (35), the charges e_1 and e_2 can be expressed in terms of the potentials u_1 and u_2 as follows

$$e_1 = \frac{A}{4h} \left(\frac{3h-y}{h-y} u_1 - u_2 \right); \quad e_2 = \frac{A}{4h} \left(\frac{3h-y}{h-y} u_2 - u_1 \right). \quad (36)$$

Substituting (36) into the last equation of set (35) and rearranging the equation, the following expression can be written

$$m\ddot{y} + \mu\dot{y} + \frac{M_y^2}{L_2} i^2 y - \frac{A}{4} \left[\frac{u_2^2}{(h+y)^2} - \frac{u_1^2}{(h-y)^2} \right] = F_y. \quad (37)$$

In view of (24) and the fact that the potentials u_1 and u_2 are assumed to be equal to each other, Eq. (37) can be linearized and simplified as follow

$$m\ddot{y} + \mu\dot{y} + \left[\frac{M_y^2}{L_2} i^2 - \frac{Au^2}{h^3} \right] y = F_y. \quad (38)$$

Fig. 6. Dependence of the dimensionless spring constant c_s on the potential u .

Thus, the linear model describing the behavior of the disk-shaped PM suspended by the contactless suspension based on combining inductive and electrical suspensions is obtained. Analysis of model (38) reveals that the spring constant of the suspension is defined by the difference between the two terms, namely, the spring constants of the inductive (the first term within the brackets) and electric (the second term within the brackets) suspensions. Note that the spring constant of the electric suspension has a negative sign and its value is inversely proportional to the cubic of the levitation height. To minimize or completely eliminate the spring constant of the suspension the following condition has to be fulfilled

$$\frac{M_y^2}{L_2} i^2 - \frac{Au^2}{h^3} \simeq 0. \quad (39)$$

From the point of view the stability of the suspension, condition (39) must not be negative.

The developed methodology can be applied to the experimental results of Williams [8]. In this prototype of the inductive suspension, the disk-shaped PM of radius $r_{pm}=250 \mu\text{m}$ and thickness $t_{pm}=10 \mu\text{m}$ was suspended to a height $h=2 \mu\text{m}$. The measured spring constant along the vertical direction was $4 \cdot 10^{-3} \text{ N}\times\text{m}^{-1}$, with coil current i of 0.35 A. Note that the same value of the spring constant of the inductive suspension calculated by (33), under the same parameters of the PM, the levitation height and current is provided by a coil of radius $r_c=265 \mu\text{m}$.

It is assumed that this inductive suspension is provided by the system of electrodes depicted in Fig. 1. For further analysis, a dimensionless spring constant is introduced:

$$c_s = \frac{c_m - c_e}{c_m}, \quad (40)$$

where $c_m = \frac{M_y^2}{L_2} i^2$ and $c_e = \frac{Au^2}{h^3}$. In the case under consideration, the c_m is assumed to be equal to $4 \cdot 10^{-3} \text{ N}\times\text{m}^{-1}$, and the area of the electrode is to be $9.82 \cdot 10^{-8} \text{ m}^2$ which is calculated by the following equation $A_e = (\pi r_{pm}^2)/2$.

Let us plot the dependence of the dimensionless spring constant c_s against the potential u applied to the electrodes E_1 and E_2 as shown in Fig. 6. Figure 6 shows that when the potential u equals $u_0 = 0.1960 \text{ V}$, the spring constant of the suspension is reduced to zero. The suspension is stable when

Fig. 7. Dependence of the static sensitivity of the suspension on the potential u .

$u < u_0$, and unstable when $u > u_0$. It is important to note that the value of the applied potential, u , is a tenth of the voltage to eliminate the spring constant of the suspension.

The inverse value of the dimensionless spring constant $1/c_s$ characterizes the static sensitivity to the measuring acceleration, a . Its dependence on the potential u is shown in Fig. 7. An increase in sensitivity by one order of magnitude occurs after a decrease in the spring constant of the suspension by 90 %. At the point $u = u_0$, the static sensitivity becomes infinitely large due to the complete elimination of the spring constant.

VI. CONCLUSIONS

In this paper, a micromachined accelerometer, based on a contactless suspension with a zero spring constant is proposed. This leads to, on one hand, a dramatic increase to the static sensitivity of the inertial sensor and, on the other hand, a significant reduction of the steady-state error of the sensor in closed loop operation. Minimization of the spring constant of the proposed contactless suspension is achieved by combining inductive and electrical contactless suspensions.

The mathematical model is developed to study conditions associated with the spring constants elimination as well as the levitational stability. Analysis of the model allows us to define the conditions for stable levitation of an inductive, contactless suspension in general. In particular case, the condition for the stable levitation of the disk-shaped PM in an alternating magnetic field from a ring-shaped coil is obtained. This condition predicts that for stable levitation of the disk-shaped PM, the radii of the coil and the PM must not be equal to each other and that the height h of levitation of the PM is limited by the value of the difference between the radii of the coil and the PM.

Based on the data of the experimental study of the inductive contactless suspension prototype developed by Shearwood's group, the performance of the proposed contactless suspension is illustrated theoretically. It is shown that the required applied potential for the minimization of the spring constant is a tenth of the voltage. For this particular case, the complete elimination of the spring constant occurs when the applied potential is equal to 196 mV.

REFERENCES

- [1] N. Yazdi, F. Ayazi, and K. Najafi, "Micromachined inertial sensors," in *proc. of the IEEE*, vol. 86, no. 8, 1998, pp. 1640–1659.
- [2] Editors, "Advances in navigation sensors and integration technology," RTO/NATO, Tech. Rep., February 2004.
- [3] K. Liu, W. Zhang, W. Chen, K. Li, F. Dai, F. Cui, X. Wu, G. Ma, and Q. Xiao, "The development of micro-gyroscope technology," *J. Micromech. Microeng.*, vol. 19, p. 113001, 2009.
- [4] T. Gabrielson, "Mechanical-thermal noise in micromachined acoustic and vibration sensors," *IEEE Trans. Electron. Devices*, vol. 40, no. 5, pp. 903–909, 1993.
- [5] R. Toda, N. Takeda, T. Murakoshi, S. Nakamura, and M. Esashi, "Electrostatically levitated spherical 3-axis accelerometer," in *Micro Electro Mechanical Systems, 2002. The Fifteenth IEEE International Conference on*. IEEE, 2002, pp. 710–713.
- [6] R. Houlihan and M. Kraft, "Modelling of an accelerometer based on a levitated proof mass," *J. Micromech. Microeng.*, vol. 12, p. 495, 2002.
- [7] K. V. Poletkin, A. I. Chernomorsky, and C. Shearwood, "A proposal for micromachined dynamically tuned gyroscope, based on contactless suspension," *IEEE Sensors J.*, vol. PP, no. 99, pp. 1–1, Dec. 2011.
- [8] C. Williams, C. Shearwood, and P. Mellor, "Modeling and testing of a frictionless levitated micromotor," *Sensor. Actuat. A-Phys.*, vol. 61, pp. 469–473, 1997.
- [9] F. Han, L. Wang, Q. Wu, and Y. Liu, "Performance of an active electric bearing for rotary micromotors," *J. Micromech. Microeng.*, vol. 21, p. 085027, 2011.
- [10] J. Van de Vegte and J. Vegte, *Feedback control systems*. Prentice-Hall, 1994.
- [11] V. Mel'nikov, *Electromechanical transducers based on quartz glass*. M.: Mashinostroenie, 1984 (in Russian).
- [12] M. Kraft, C. Lewis, and T. Hesketh, "Closed-loop silicon accelerometers," in *Circuits, Devices and Systems, IEE Proc.*, vol. 145, no. 5. IET, 1998, pp. 325–331.
- [13] Y. G. Martynenko, *Analytical dynamics of electromechanical systems*, I. V. Novogilova, Ed. Moscow Power Engineering Institute, 1984 (in Russian).
- [14] F. Grover, *Inductance calculations: working formulas and tables*. Dover Pubns, 2004.
- [15] C. Paul, *Inductance: loop and partial*. Wiley-IEEE Press, 2009.

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Kirill V. Poletkin received the diploma (Hons.) of electromechanical engineer majoring in Aviation devices and measuring computing complexes in 2001 from Nizhny Novgorod State Technical University and the Ph.D. degree from Moscow Aviation Institute (State University of Aerospace Technologies), Russia in 2007.

Currently, he is a Research Fellow in the School of Mechanical & Aerospace Engineering, Nanyang Technological University, Singapore. He was previously with Giesecke & Devrient GmbH (G&D), JSC Temp-Avia, Russian Federal Nuclear Center (VNIITF). His research interests are in inertial navigation sensors, dynamically turned gyroscopes, micro electro mechanical accelerometers and gyroscopes.

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Alexsandr I. Chernomorsky received the Diploma of Engineer and Ph.D. degree from Moscow Aviation Institute in 1964 and 1968 respectively.

Currently, he is full professor and Deputy Director of Department Automatic complexes of orientation and navigation systems in Moscow Aviation Institute (State University of Aerospace Technologies). Also he is member of international academy Navigation and Motion Control. His active research interests include gravity inertial devices and orientation and navigation systems of movable objects. He is the author of over 200 scientist works and 60 patents.

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Christopher Shearwood received a first class honours degree in Physics and a PhD in Solid State Physics from the Leeds University in 1984 and 1988, respectively.

In 1995, he was appointed Senior Fabricator in the MEMS Unit of the Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield. In 1998, he then moved to Singapore to work in the Micro- Machines Laboratory, Nanyang Technological University. His research interests include MEMS and magnetic thin film technology. He

is presently an Associate Professor at NTU, Singapore.