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k-Connectivity in Shadowing Wireless Multi-Hop Networks

"Lili Zhang and "Boon-Hee Soong

*^aSchool of Electrical & Electronic Engineering
Nanyang Technological University
50 Nanyang Drive, Singapore 637 553
Email: {llzhang; EBHSOONG}@ntu.edu.sg*

Abstract

In this paper, we study the network performance of a wireless multi-hop network in terms of the successful link probability and k -connectivity probability by taking into account shadowing. We provide the closed-form analytic results of the successful link probability, and henceforth k -connectivity, in the context of shadowing multi-hop networks. Given a certain number of nodes spatially distributed according to a Poisson point process, we examine the critical density to keep the probability that a node is k -connected close to 1. The impacts of the shadowing on the k -connectivity are illustrated and discussed. The results can be practically used for the design of a wireless multi-hop network.

I. INTRODUCTION

Considerable studies have discussed the issues such as the multi-path routing and capacity in wireless multi-hop ad hoc and sensor networks, among which connectivity [1] [2] [3] is a fundamental property and design metric. In some applications, we have to ensure a certain k -connectivity for the purpose of route diversity in multi-hop networks to enhance the reliability of data communication as well as satisfy the QoS requirements. Hence, it is practical for a network designer to investigate connectivity to provide insight to the design of wireless multi-hop networks. To date, connectivity problem has received quite a lot of attentions in the context of ad hoc and sensor networks. However, most existing work relies on a simplistic channel propagation model with the assumption that two nodes are connected if and only if their distance is less than a deterministic transmission radius. Unfortunately, wireless channel is suffering from the severe propagation impairments such as path loss, multi-path fading and shadowing [6]. As a result, the communication range may not be deterministic. Actually, it is more reasonable to study the k -connectivity performance with respect to the impact of channel impairment characteristics.

Previous work [4] analyzes the connectivity of multi-hop radio networks in a log-normal shadowing environment and gives the tight lower bound for the minimum nodes density that is necessary to obtain an almost surely connected network. [5] derives the outage probability by taking into account the shadowing and Rayleigh fading. [6] studies the impact of interferences on the connectivity of a large-scale ad-hoc network, using percolation theory and shows that there is a critical value of interference coefficient above which the network is made of disconnected clusters of nodes. [7] investigates the isolation probability and thereby the coverage and connectivity probability by considering the shadowing and fading and further exploits the chan-

nel randomness by means of diversity. However, all these existing work only consider 1-connectivity or isolation probability, and none of them investigate k -connectivity ($k > 1$) property. More recently, [8] derives the minimal transmission range that could create an almost surely k -connected network for a given density. [9] studies the asymptotic critical transmission radius for k -connectivity and asymptotic critical neighbor number for k -connectivity in a wireless ad hoc network. However, they consider only a simplistic channel model and ignore the severe channel impairment characteristics. It is not complete without capturing the channel characteristics that has the direct bearing on the link connectivity.

In order to account for a realistic channel model, we, hence, develop a generic mathematical model, by capturing the channel impairment characteristics, and present an analytical procedure for the computation of the k -connectivity in an ad hoc network in the presence of log-normal shadowing. We argue that the shadowing increases the successful link probability and thereby k -connectivity. Given that nodes are distributed according to a Poisson point process, we compute the probability that there is at least k neighbors for a node, and thereby the total probability of k -connectivity for the entire network. This yields sight into how one designer should set the density to ensure the k -connectivity. Alternatively, when the density is certain, it reveals what is the probability to achieve k -connectivity.

The remainder of the paper is organized as follows. Section II describes the system model in our study. Section III analyzes the successful link probability and thereby k -connectivity probability in the corresponding radio propagation model in the presence of the log-normal shadowing. Section IV does the conclusion.

II. SYSTEM MODEL

Let us consider a wireless multi-hop network where nodes are randomly distributed in the plane according to a homogeneous two-dimensional Poisson process with average density λ . Then the probability $P(N, k)$ of finding k nodes within the region with radius R is

$$P(N, k) = \frac{e^{-\lambda\pi R^2} (\lambda\pi R^2)^k}{k!}, k \geq 0,$$

where $N = \lambda\pi R^2$. As nodes are uniformly distributed over the region, we have the probability density function (pdf) of the distance r between a chosen transmitter-receiver pair follows

$$f_R(r) = \frac{2r}{R^2}, 0 \leq r \leq R$$

For the sake of simplicity, we assume that all nodes are with the same transmission power.

Wireless transmissions are severely impaired by the multi-path propagation effect. When a receiver receives multiple attenuated and time-delayed versions of a transmitted signal, with the additional corruption by noise and interference, the transmitted signal might be enhanced, thereby translating into the increasing SINR, or weakened, thereby translating into the decreasing SINR. This is called multi-path fading and can be further divided into large-scale fading and

small-scale fading. The shadowing phenomena considers the case that the received signal strength may be different due to the different propagation conditions in their surroundings even though the distance between two transmitter-receiver pairs is the same. Hence, it is referred to as large-scale fading. We, here, consider the shadowing in the channel model, to analyze k -connectivity.

III. K-CONNECTIVITY ANALYSIS

We first apply a result on the property of geometric random graphs [10], i.e., in a random distributed geometric graph of n nodes, if the corresponding links are added in turn to the empty graph and n is large enough, then the probability that the resulting graph becomes k -connected almost approaches 1 at the moment that it achieves a minimum degree of k . Let d_{min} denote the minimal number of neighbors that a node can have a connection, i.e., a minimum degree per node, then one has

$$\begin{aligned} \mathcal{P}(\text{A homogenous ad hoc network is } k\text{-connected}) \\ \approx \mathcal{P}(d_{min} \geq k) \end{aligned}$$

with k being a predefined value of connectivity. Hence, we seek to investigate the probability $\mathcal{P}(d_{min} \geq k)$ referring to it.

Let p_s denote the successful link probability, then we can express $\mathcal{P}(d_{min} \geq k)$ as a double sum over j and m , i.e.,

$$\mathcal{P}(d_{min} \geq k) = \sum_{m=k}^{\infty} \sum_{j=k}^m \lim_{R \rightarrow \infty} \mathcal{P}(N, i) C_i^j (p_s)^j (1-p_s)^{i-j} \quad (1)$$

Hence, next, we derive p_s in the presence of various channel conditions.

A. The Simplistic Channel Model

Let P_t and P_r denote respectively the transmission power and the receiving signal strength. Then, we have

$$P_r = \iota(d)P_t,$$

where $\iota(d)$ is the path loss with d being the distance between the transmitter and receiver. Two nodes can communicate directly with each other only when

$$P_r \geq \mu_t,$$

where μ_t is the receiving threshold value for successful packet reception. In the simplistic path loss model, we have $\iota(d) = \frac{K}{d^\alpha}$, where K is a constant determined by antenna height and gain, and α is the path-loss exponent. The path loss exponent α depends on the environment and can

vary between 2 in free space and 6 in heavily built urban areas. Hence, given R_0 as the transmission range in the absence of shadowing, we have $R_0 = (\frac{KP_t}{\mu_t})^{1/\alpha}$.

In the simplistic channel model, two nodes can communicate only when they are within the transmission range of each other. Thus, the probability that a randomly chosen node has at least k neighbors is

$$\sum_{i=k}^{\infty} \frac{e^{-\lambda\pi R_0^2} (\lambda\pi R_0^2)^i}{i!}$$

B. The Channel Model in the Presence of Shadowing

In this subsection, we use a more realistic channel model, which considers path loss with large-scale shadowing. The log-normal shadowing process has been widely adopted to model the shadow fade attenuations. Then, with the assumption that the shadow fade attenuations between any two chosen transmitter-receiver pairs follow the *independent and identical distributed* (i.i.d.) log-normal shadowing process, we have

$$P_r = \iota(d)P_t 10^{z/10},$$

where $10^{z/10}$ is the log-normal distributed shadow fade attenuation. The pdf of the log-normal shadowing variable z is

$$f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right) \quad (2)$$

with σ being the log-normal spread, i.e., the standard deviation of the Gaussian distribution that describes the shadowing phenomenon.

We, henceforth, have the probability that there is a connection between two nodes as

$$\begin{aligned} P_s &= \mathcal{P}(R \leq R_0 \exp(\beta z / \alpha)) \\ &= \int_{-\infty}^{\infty} f_Z(z) dz \int_0^{\min\{R, R_0 \exp(\beta z / \alpha)\}} f_R(r) dr \\ &= \frac{R_0^2 \exp(\frac{2\beta^2 \sigma^2}{\alpha^2})}{R^2} \end{aligned} \quad (3)$$

according to [12], where $\beta = \ln 10 / 10$ and R is a very large value.

Substituting Eq. 3 into Eq. 1, we have $P(d_{min} \geq k)$ in the presence of log-normal shadowing as

$$\begin{aligned}
& \mathcal{P}(d_{min} \geq k) \\
&= \sum_{m=k}^{\infty} \sum_{j=k}^m \lim_{R \rightarrow \infty} \mathcal{P}(N, i) C_i^j \left(\frac{R_0^2 \exp(\frac{2\beta^2 \sigma^2}{R^2})}{R^2} \right)^j \\
& \quad \left(1 - \frac{R_0^2 \exp(\frac{2\beta^2 \sigma^2}{R^2})}{R^2} \right)^{i-j}
\end{aligned} \tag{4}$$

Fig. 1 shows $P(d_{min} \geq 3)$ versus the node density for various shadowing spread when $\alpha = 3.5$, $K = 10$ and $R_0 = 100$. We observe that $P(d_{min} \geq 3)$ increases steeply toward 1 as the average density increases. For instance, average node density $\lambda = 10^{-4}$ creates a connected network with the $P(d_{min} \geq 3) = 0.80$, when $\sigma = 6$. If we increase λ to $2 * 10^{-4}$, then we obtain $P(d_{min} \geq 3) = 0.99$. This is consistent with the conclusion drawn in [8]. Meanwhile, we note that with the shadowing factor σ increasing, there is a bigger 3-connectivity probability for the same density and other system parameters. This is due to that there is higher probability of being able to communicate with nodes at farther distances as shadowing becomes serious. The increases in the average node degree directly translate to the increases in the connectivity. We, hence, may draw the conclusion that the presence of log-normal shadowing increases the probability of k -connectivity, as have been argued in previous results [7] on coverage and isolation probability. We plot 3-connectivity probability for the varying values of $\sigma = 1, 3, 6, 8, 10$. The simplistic channel model actually corresponds to the log-normal shadowing with a marginal value of σ approaching 0.

Fig. 2 shows $P(d_{min} \geq 3)$ versus shadowing spread for various path loss exponent when $\lambda = 10^{-4}$, $K = 10$ and $R_0 = 100$. With the shadowing increasing, there is a bigger $P(d_{min} \geq 3)$ and under the same shadowing, $P(d_{min} \geq 3)$ decreases as path loss factor α increases. We observe that for a big value of σ , 3-connectivity almost equals to 1, which indicates that the path loss α can not have much impact for the 3-connectivity for the big shadowing. As the connectivity coincides with the average node degree, we have the conclusion that the path loss plays a big role in k -connectivity properties only for a range of medium values of σ , rather than for a big or small σ . This is also consistent with the conclusion in [7].

IV. CONCLUSION

In this paper, we study the network performance in terms of the successful link probability and k -connectivity probability by taking into account shadowing in wireless multi-hop networks. We show that k -connectivity probability increases as the log-normal spread σ increases. Given a certain number of nodes spatially distributed according to a Poisson process, we examine the critical density to keep the k -connectivity of the network close to 1. These results can be practically used by a network designer to estimate the critical density to ensure a certain network connectivity.

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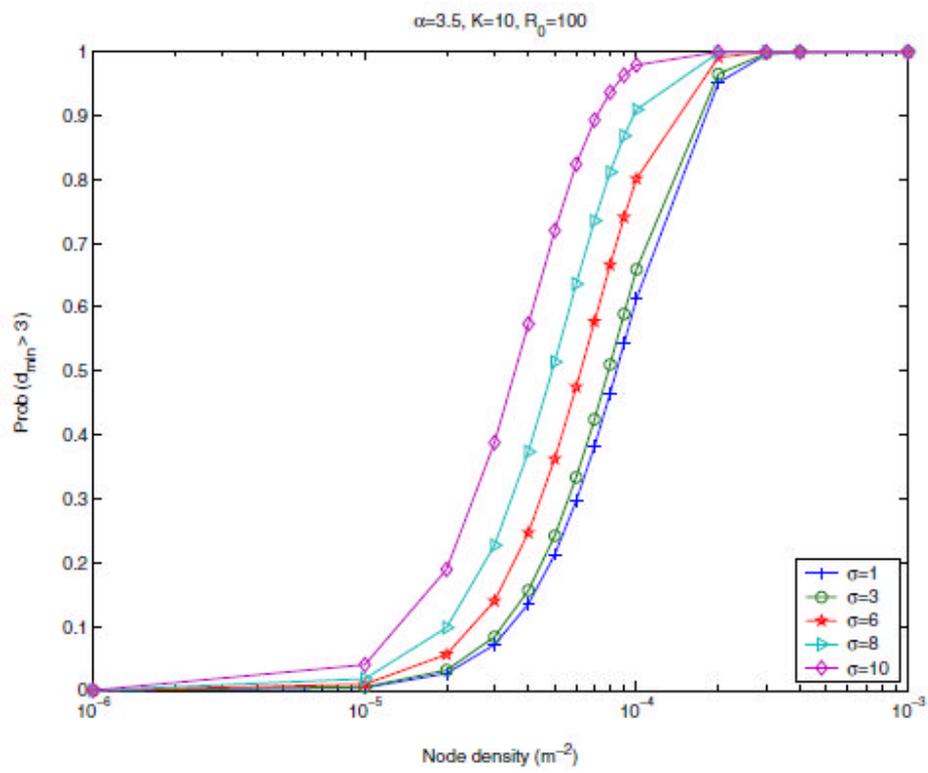


Figure 1

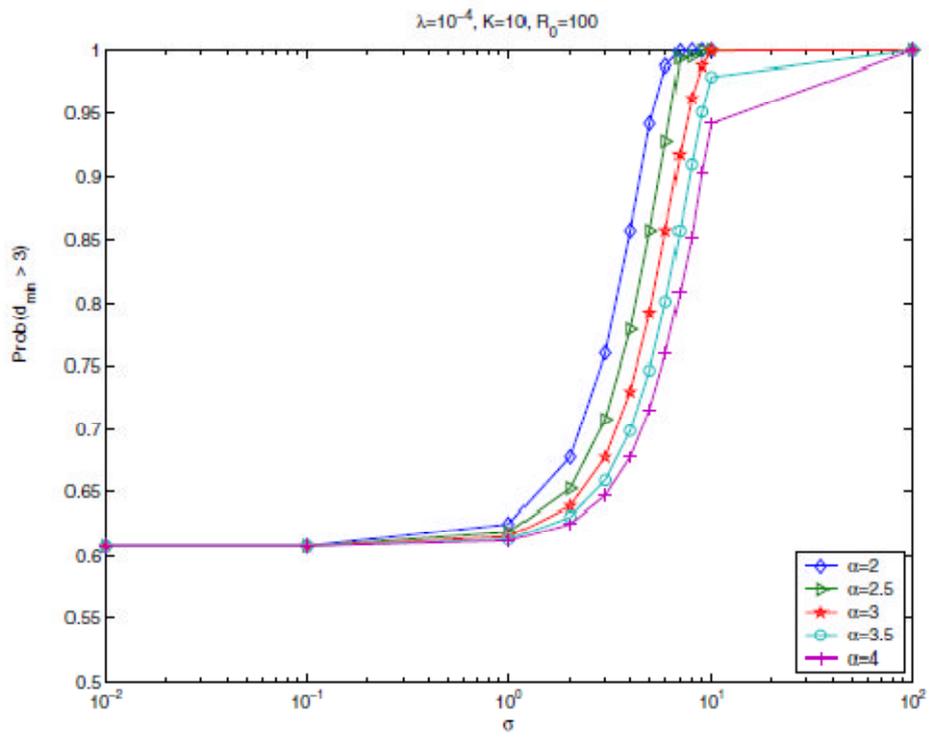


Figure 2