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An Analysis of k -Connectivity in Shadowing and Nakagami Fading Wireless Multi-Hop Networks

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Abstract

In this paper, we develop k -connectivity in wireless multi-hop networks in the presence of both the log-normal shadowing and Nakagami- m fading. The formula are generally applicable in a variety of realistic physical situations. Based on the results, we are able to find the critical node density to satisfy node k -connectivity almost with probability 1. In addition, k -connectivity shows significant difference with distinct channel conditions, which necessitates a generic result. The result is useful for the design of a wireless multi-hop network in practice.

I. Introduction

Connectivity is a fundamental property as well as a performance metric in wireless multi-hop ad hoc and sensor networks [1] [2] [3]. In a number of scenarios, we must satisfy k —connectivity ($k \sim 1$) to enhance the communication reliability as well as to guarantee the QoS requirements. Here, k —connectivity refers to the property that a random node has at least k neighbors. Since wireless channel is suffering from various severe propagation impairments, k —connectivity performance is normally evaluated under different channel conditions.

In the literature, several studies have investigated 1- connectivity issue in multi-hop networks. The study in [4] analyzes the connectivity in a log-normal shadowing environment and presents the tight lower bound of the minimum nodes density to obtain an almost surely connected network. [5] reports the impact of interferences on the connectivity and shows that there is a critical value of interference coefficient above which the network is made of disconnected clusters of nodes. [6] develops the connectivity by considering shadowing and fading simultaneously. Recently, some works studies the characteristics of k -connectivity with simplistic channel model in case k is greater than 1. The study in [7] derives the minimal transmission range that can ascertain an almost surely k —connected network for a given density. [8] reports the asymptotic critical transmission radius for k —connectivity and asymptotic critical neighbor number for k —connectivity. However, it is well-known that wireless transmissions are severely impaired by path-loss, multi-path fading and also large-scale shadowing. Consequently,

the existed results with a simplistic channel model assumption is inappropriate and the developed results may not applicable in complicated signal fluctuation situations.

In this paper, we will develop an analytical model and formulate k -connectivity with a generic channel model by taking into account both the large-scale log-normal shadowing and small-scale Nakagami- m fading. The result indicates that the shadowing effect increases the k —connectivity. The k —connectivity increases with larger m in the Nakagami- m fading. Illustrative examples are presented to demonstrate the interaction between the node density and the probability that there is at least k neighbors for a node. This relationship is useful for network designer to set the critical node density to guarantee the k —connectivity in practice.

II. System Model

Let us consider a wireless multi-hop network where nodes are randomly distributed in the plane according to a homogeneous two-dimensional Poisson process with average density λ . Let N denote the circular region with radius R . Then, $N = \lambda\pi R^2$. Let $P(N, k)$ denote the probability of finding k nodes within the circular region N .

$$\mathcal{P}(N, k) = \frac{e^{-\lambda\pi R^2} (\lambda\pi R^2)^k}{k!}, k \geq 0$$

As nodes are uniformly distributed over the region, we have the probability density function (pdf) of the distance r between a transmitter-receiver pair.

$$f_r(r) = \frac{2r}{R^2}, 0 \leq r \leq R$$

We assume that all nodes have the same transmission power.

III. k -Connectivity Analysis

For a geometric random graph, we refer to the following theorem in [9]. In a random distributed geometric graph of n nodes, if the corresponding links are added in turn to the empty graph and n is large enough, then the probability that the resulting graph becomes k —connected almost approaches 1 at the moment that it achieves a minimum degree of k . Let d_{min} denote the minimal number of neighbors that a node has a connection, i.e., a minimum degree per node. Then, we have

$$\begin{aligned} &\mathcal{P}(\text{A homogenous ad hoc network is } k\text{—connected}) \\ &\approx \mathcal{P}(d_{min} \geq k) \end{aligned} \tag{1}$$

where k is the predefined value of connectivity. Hence, we need to develop the result for the probability $P(d_{min} \geq k)$ above. Let p_s denote the successful link probability. Accounting the wireless channel effect, we express $P(d_{min} \geq k)$ as,

$$\mathcal{P}(d_{min} \geq k) = \sum_{i=k}^{\infty} \sum_{j=k}^i \lim_{R \rightarrow \infty} \mathcal{P}(N, i) C_i^j p_s^j (1 - p_s)^{i-j} \quad (2)$$

In the following, we will derive p_s in the presence of various channel conditions.

A. The Simplistic Channel Model

Let d represent the distance between the transmitter and receiver. In the simplistic path loss model, we have $\xi(d) = \frac{K}{d^\alpha}$, where K is a constant determined by antenna height and gain. α denotes the path-loss exponent, which depends on the environment and can vary between 2 in free space and 6 in urban areas. Let P_t and P_r denote the transmission power and the received signal power, respectively. Then, we have

$$P_r = \xi(d) P_t \quad (3)$$

Two nodes can communicate directly with each other only when

$$P_r \geq \mu_t \quad (4)$$

where μ_t denotes the power threshold for successful packet reception. Hence, given R_0 as the transmission range in the absence of fading and shadowing, we have $R_0 = (\frac{K P_t}{\mu_t})^{1/\alpha}$. In the simplistic channel model, two nodes can communicate only when they are within the transmission range of each other. Thus, the probability that a randomly chosen node has at least k neighbors is given by

$$\sum_{i=k}^{\infty} \frac{e^{-\lambda \pi R_0^2} (\lambda \pi R_0^2)^i}{i!}$$

B. Path-Loss and Log-normal Shadowing

In this subsection, we develop the formula for k -connectivity in the presence of path-loss and also large-scale log-normal shadowing. The log-normal shadowing process has been widely used to model the shadow fading attenuations. We assume that the shadow fading between any

transmitter-receiver pair follows the independent and identical distributed (i.i.d.) log-normal process, we have

$$P_r = \xi(d)P_t 10^{z/10},$$

where $10^{z/10}$ refers to the log-normal distributed shadowing. The pdf of the log-normal random variable z is expressed as

$$f_z(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right) \quad (5)$$

where σ denotes the log-normal spread, i.e., the standard deviation of the Gaussian distribution that describes the shadowing phenomenon. Thereafter, the probability that there is a connection between two nodes is given by [10]

$$\begin{aligned} p_s &= \mathcal{P}(R \leq R_0 \exp(\beta z / \alpha)) \\ &= \int_{-\infty}^{\infty} f_z(z) dz \int_0^{\min\{R, R_0 \exp(\beta z / \alpha)\}} f_r(r) dr \\ &= \frac{R_0^2 \exp(\frac{2\beta^2 \sigma^2}{\alpha^2})}{R^2} \end{aligned} \quad (6)$$

where $\beta = \ln 10 / 10$ and R is a very large value.

Substituting (6) into (2), we obtain the probability of k -connectivity $P(d_{min} \geq k)$ in the presence of both path-loss and log-normal shadowing

$$\mathcal{P}(d_{min} \geq k) = \sum_{i=k}^{\infty} \sum_{j=k}^i \lim_{R \rightarrow \infty} \mathcal{P}(N, i) C_i^j \left(\frac{R_0^2 \exp(\frac{2\beta^2 \sigma^2}{\alpha^2})}{R^2} \right)^j \times \left(1 - \frac{R_0^2 \exp(\frac{2\beta^2 \sigma^2}{\alpha^2})}{R^2} \right)^{i-j} \quad (7)$$

$$= \sum_{i=k}^{\infty} \sum_{j=k}^i \sum_{l=1}^{i-j} \lim_{R \rightarrow \infty} \frac{e^{-\lambda \pi R^2} (\lambda \pi)^i R^{2l} (R_0^2 \exp(\frac{2\beta^2 \sigma^2}{\alpha^2}))^{i-l}}{j! l! (i-j-l)!} \quad (8)$$

Fig. 1 shows $P(d_{min} \geq 3)$ in terms of the node density with different shadowing spread. It is observed that $P(d_{min} \geq 3)$ increases sharply toward 1 as the average density increases. For instance, when $\sigma = 6$, if average node density λ increases from 10^{-4} to $2 * 10^{-4}$, we obtain $P(d_{min} \geq 3)$ from 0.80 to 0.99. This is consistent with the conclusion in [7]. Meanwhile, the greater shadowing factor σ leads to higher 3-connectivity probability. This is because there is higher

probability of the ability to communicate with nodes at farther distances as shadowing becomes serious. The higher node degree directly results in the increases in the connectivity. Hence, the presence of log-normal shadowing increases the probability of k -connectivity.

Fig. 2 shows $P(d_{min} \geq 3)$ in terms of shadowing spread with different path loss exponent. With the increasing shadowing spread, there is a higher $P(d_{min} \geq 3)$. Moreover, greater path loss exponent leads to lower $P(d_{min} \geq 3)$. For a sufficient big σ (e.g. $\sigma > 10$ in this example), 3-connectivity almost equals to 1, indicating that the path loss α can not have much impact for the 3-connectivity in such case. Alternatively, it is summarized that the path loss effect on k -connectivity is substantial for a medium value σ , however insignificant for too big or small σ .

C. Log-normal Shadowing and Nakagami-m Fading

In this section, we will develop the formula for k -connectivity in taking into account both the large-scale shadowing and small-scale fading. For the fading, we employ a generic Nakagami- m fading distribution to derive a generally applicable results. Nakagami- m is a distribution function with two-parameter m and $\bar{\gamma}$ and can provide the best fit to the statistics of signals transmitted over multi-path channels in a land-mobile and indoor-mobile environments. The instantaneous received carrier to noise ratio γ on a Nakagami- m fading channel is a continuous stochastic variable with gamma distribution. The pdf of γ is given by

$$p_{\gamma}(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp(-m\frac{\gamma}{\bar{\gamma}})$$

where $\bar{\gamma}$ is the average received power, m is the Nakagami fading parameter ($m \geq 1/2$) and $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$, ($z \geq 0$). As m increases, the fading becomes less severe. As a special case of $m = 1$, the Nakagami- m distribution represents the Rayleigh fading. The case $m = 1/2$ shows the worst one-side Gaussian fading. The probability that there is a connection between two nodes is derived as

$$\begin{aligned} & \mathcal{P}(\gamma \geq \mu_t) \\ &= \int_{\mu_t}^{\infty} p_{\gamma}(\gamma) d\gamma \\ &= 1 + \left(\frac{\bar{\gamma}}{\mu_t}\right)^m \left(\frac{\Gamma(m, \frac{m\mu_t}{\bar{\gamma}})}{\Gamma(m)} - 1\right) \\ &= \begin{cases} \exp(-\mu_t/\bar{\gamma}), & m = 1 \\ \frac{1}{\bar{\gamma}} \exp(-2\mu_t/\bar{\gamma})(2\mu_t + \bar{\gamma}), & m = 2 \\ \frac{1}{3\bar{\gamma}^3} \exp(-4\mu_t/\bar{\gamma}) \\ \quad (32\mu_t^3 + 24\mu_t^2\bar{\gamma} + 12\mu_t\bar{\gamma}^2 + 3\bar{\gamma}^3), & m = 4 \end{cases} \end{aligned} \tag{9}$$

where $\bar{\gamma} = \frac{K P_t 10^{z/10}}{d^\alpha} = \frac{\mu_t R_0^\alpha 10^{z/10}}{d^\alpha}$. Define $\psi = \frac{d^\alpha}{R_0^\alpha 10^{z/10}}$.

The equation above can be simplified to

$$\mathcal{P}(\gamma \geq \mu_t) = \begin{cases} \exp(-\psi), & m = 1 \\ \exp(-2\psi)(2\psi + 1), & m = 2 \\ \frac{1}{3}\exp(-4\psi) \\ (32\psi^3 + 24\psi^2 + 12\psi + 3), & m = 4 \end{cases} \quad (10)$$

Therefore, for sufficiently large R, we have

$$\begin{aligned} p_s &= \int_{-\infty}^{\infty} \mathcal{P}(\gamma \geq \mu_t) f_z(z) dz \int_0^{\min\{R, R_0 \exp(\beta z/\alpha)\}} f_r(r) dr \\ &= \int_{-\infty}^{\infty} \mathcal{P}(\gamma \geq \mu_t) f_z(z) \frac{R_0^2 \exp(\frac{\beta^2 z^2}{\alpha^2})}{R^2} dz \end{aligned} \quad (11)$$

Substituting (11) into (2), we can calculate the k -connectivity in the presence of both log-normal shadowing and Nakagami- m fading.

Fig. 3, Fig. 4 and Fig. 5 show $P(d_{\min} \geq 3)$ versus the node density when $\alpha = 3.5$, $K = 10$ and $R_0 = 100$ with shadowing channel condition and shadowing plus various fading channel conditions, for $\sigma = 1$, $\sigma = 3$ and $\sigma = 6$ respectively. We observe that the probability that the minimal degree is bigger than 3 decreases with the additional fading, compared to with only shadowing, for various values of σ . For instance, in Fig. 5, when $\lambda = 10^{-4}$, $P(d_{\min} \geq 3)$ is 0.80, 0.60, 0.68 and 0.72 for the shadowing, shadowing plus Rayleigh fading and Nakagami- m fading with $m = 2$ and $m = 4$, respectively. Correspondingly, the minimal average node density $1.75 * 10^{-4}$, $2.08 * 10^{-4}$, $1.95 * 10^{-4}$ and $1.87 * 10^{-4}$ are respectively required to ensure that the probability of the minimal degree bigger than 3 exceeds 0.95. This suggests that Nakagami- m fading reduces the connectivity properties of the network, which holds for any σ . However, with m increases, $P(d_{\min} \geq 3)$ also increases, which implies that the fading does not change much the connectivity properties. We also note that as σ increases, $P(d_{\min} \geq 3)$ monotonously increases for shadowing plus a certain fading channel conditions. In order to achieve that a node has at least 3 neighbors with probability 0.95, the average density $2.91 * 10^{-4}$, $2.76 * 10^{-4}$ and $2.08 * 10^{-4}$ are required for the case of shadowing plus Rayleigh fading in Fig. 3, Fig. 4 and Fig. 5 respectively. This further shows that shadowing increases connectivity properties, in the presence of a certain fading.

Fig. 6 shows that for the increasing range of mean degree k , the probability $P(d_{\min} \geq k)$ decreases, for various kinds of shadowing plus fading phenomena under a certain density with

the system parameters $\sigma = 3$, $\alpha = 3.5$, $K = 10$ and $R_0 = 100$. For instance, $P(d_{min} \geq k)$ is almost 1 for $k = 1$ when $\lambda = 2 * 10^{-4}$ with only shadowing, while decreases to 0.96, 0.81 and 0.51 as k increases to 3, 5 and 7 respectively. This coincides with the conclusion in [7] that considers neither shadowing nor fading. We observe that for a certain value of k , increasing node density causes a bigger value of $P(d_{min} \geq k)$, e.g., with shadowing plus Nakagami-m fading ($m=4$), $\lambda = 10^{-4}$ for $k = 3$ can achieve that the probability of the minimal degree bigger than 3 is about 0.57, whereas $\lambda = 2 * 10^{-4}$ can achieve that at about 0.95. This yields the same conclusion as subsection B.

IV. Conclusion

In this paper, we study the network performance in terms of the successful link probability and k -connectivity probability by taking into account both the shadowing and fading in wireless multi-hop networks. The result indicates that k -connectivity probability increases with higher log-normal spread σ . We investigate the impacts of the fading characteristics and system parameters on the k -connectivity property. Given a number of nodes spatially distributed in an area, the critical density can be obtained to satisfy the k -connectivity almostly approaching to 1. The methodology and the results are useful for multi-hop network practical design in the presence of path-loss, large-scale shadowing and also small-scale fading.

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Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6