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2011

Poletkin, K. V., Chernomorsky, A. I., & Shearwood, C. (2011). Selection of Series Compensation Parameters for Closed Loop, Rotor Vibratory Gyroscope. IEEE Sensors Journal, 12 (5), 1384-1392.

<https://hdl.handle.net/10356/94355>

<https://doi.org/10.1109/JSEN.2011.2169784>

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Selection of Series Compensation Parameters for Closed Loop Rotor Vibratory Gyroscope

Kirill V. Poletkin, *Member, IEEE*, Aleksandr I. Chernomorsky, and Christopher Shearwood

Abstract—This article presents an approach to selection of Series Compensation (SC) parameters for a Rotor Vibratory Gyroscope (RVG) in closed loop operation (RVG) to meet necessary quality of dynamical and static characteristics of such gyroscope output. Proposed approach is based on developed mathematical model of a RVG as two axis control system with modulation. The analysis of this model allows us to reveal some peculiarities of one, for instance, effect of “limited quality factor”. Influence of these observed peculiarities on dynamical and static characteristics of a RVG are studied and reflected in proposed approach. Finally, the selection of SC parameters by means of proposed approach is shown by example for construction scheme of RVG, in particularly, designed by the Arzamas Research & Production Joint Stock Company “Temp-Avia”.

Index Terms—Inertial Sensor, Gyroscope, Angular rate, Two axis control system, Modulation.

I. INTRODUCTION

FOR autonomous navigation and orientation of many modern, highly dynamic movable objects Strapdown Inertial System (SIS), directly mounted onto the object, are used. Typically, the sensory element of the SIS that provides measurements of angular rate is a gyroscope [1], [2]. This gyroscope has to meet a wide range of metrological, operational, technical as well as economical requirements which are often difficult to balance. One of the advanced gyroscopes, applied to the above mentioned scenarios is the Rotor Vibratory Gyroscope in closed loop operation [3]–[8], and [9] which has the capability of fully satisfying the technical and economical requirements for SIS of such movable objects. In present a RVG performance provides tactical grade under the following service condition, in particular, which are the temperature range from minus 60 to plus 50 degrees, the linear broadband random vibration with amplitude 6.5g and frequency range from 5 Hz to 2000 Hz. At the same time the cost of sensor unit based on such gyroscope is less than 1500\$.

A schematic of a RVG is illustrated in Fig. 1. It consists of a Motor Supply (MS), Signal Processing System (SPS), and a gyroscope. The gyroscope includes a Driving Motor (DM), a dual axis Angular displacement Sensor (AS), a dual channel electromagnetic Torque Sensor (TS), and a rotor. The rotor

Fig. 1. Schematic of a rotor vibratory gyroscope: $O\xi\eta\zeta$ - coordinate frame fixed to the casing of gyroscope; $Oxyz$ - coordinate frame fixed to the rotor of gyroscope; Ω - angular rate of the motor shaft; α - the angle deflection of rotor along axis Ox ; ω_ξ and ω_η - the projections of measured angular rate on respective measuring axis $O\xi$ and $O\eta$.

is mounted on the shaft of the DM by means of a Spring Element (SE) which includes a ring-type magnetic core and two magnets of the same geometry.

The operating principle of a RVG is described as follows: an electromotive force within each coil of the AS is induced by the vibratory motion of the rotor magnets, the deflection of this vibratory motion in one turn is proportional to the projections of the measured angular rate along axes $O\xi$ and $O\eta$ that are fixed to the gyroscope casing. This induced voltage is then fed into the SPS. Each TS coil is connected to a load resistor, R_L , and the potential drop, U_{out} , across R_L is proportional to the projections of the measured angular rates ω_ξ and ω_η on the respective measuring axis. To simplify schematic of a RVG on the Fig. 1 is shown operation of one channel measuring of angular rate projection on the axis $O\eta$ since other channel has the same operating principle and functional blocks.

Presently, the RVG that, in particular, is designed by Arzamas Research & Production JSC “Temp-Avia”, provides two axis measuring angular rate with accuracy less than 10 deg/h and maximum angular rate value up to 600 deg/s. Hence, the dynamic range of the RVG achieves value of up to 10^6 made possible only by closed loop operation of the RVG. In general, the influence of the closed loop operation on the output signal behavior of inertial sensors, in particular angular rate sensor, is well documented in the literature [10], [11]. Namely, the

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Fig. 2. Block Diagrams of RVG in closed loop operation: $K_0 = J_x(1 + \chi)\Omega$; $W_g(s)$ - transfer function of the gyroscope; $\Delta_\Sigma = \Delta_1 \sin(\Omega t) + \Delta_2 \cos(\Omega t)$.

closed loop operation decreases statical and dynamical output signal errors and increases the dynamical range of an inertial sensor. The similar effect is present in the RVG. Moreover, as shown in work [12], by redistribution of direct measurement series coefficients of RVG in closed loop operation provides a reduction of the gyroscope thermal bias drift.

At the same time, the necessary quality of dynamical and statical characteristics of the RVG output are provided by series compensation which is part of the SPS. It is obvious that selection of SC parameters is important component to support output quality of a RVG on the whole. However, a RVG in closed loop operation can be represented as a two axis control system with modulation which in general is described by the transfer function with the complex coefficients. Directly the using of known techniques of linear control theory where the coefficients of the transfer function are real numbers to design series compensation of a RVG is not possible. Therefore, the parameters of SC are defined experimentally by designers. However, experimental approach to selection of SC parameters is suitable to well known and already developed RVG and is not acceptable to new RVG construction scheme, in particular, suggested by designers of “Temp-Avia” company [13].

This article presents a low cost approach to the selection of SC parameters to meet the requirement quality of dynamical and statical characteristics of a RVG. To propose mentioned above approach the mathematical model of RVG as a two axis control system is developed. Analysis of its peculiarities allows us adapt known techniques of linear control theory to study the dynamical and statical characteristic of a RVG as a two axis system with modulation. In particular, the effect of “limited quality factor” is considered and influence of it on the dynamical and statical characteristics of a RVG is taken into account by proposed approach. Finally, application of approach to the selection of SC parameters is shown by examples for construction scheme of the RVG which are designed by “Temp-Avia” company.

II. BLOCK DIAGRAM AND MATHEMATICAL MODEL OF RVG IN CLOSED LOOP OPERATION

To obtain mathematical model of the RVG, let's assume that the angular deflections of the rotor are Hookean, the rate of the shaft rotation Ω is constant and that the static and

dynamical unbalances of the rotor are negligible. The angle of deflection of the rotor along the Ox axis is given by α . Hence, a simplified model of the angular displacement of the rotor with respect to the Ox axis of the spring element is as follows:

$$\begin{aligned} \delta_x \left(J_x \ddot{\alpha} + \Omega^2 \alpha (J_z - J_y) + \mu \dot{\alpha} \right) + \alpha &= \\ &= \delta_x \left(\Delta_\xi \cos(\Omega t) + \Delta_\eta \sin(\Omega t) \right), \end{aligned} \quad (1)$$

where δ_x is the flexibility of the SE along Ox axis; J_x , J_y , and J_z denote the three central principal moment of the rotor relative to $Oxyz$; μ is the damping coefficient of the SE and

$$\begin{aligned} \Delta_\xi &= -K_0 \omega_\eta - J_x \dot{\omega}_\xi - M_{TS\xi}; \\ \Delta_\eta &= K_0 \omega_\xi - J_x \dot{\omega}_\eta - M_{TS\eta}, \end{aligned} \quad (2)$$

where $K_0 = J_x(1 + \chi)\Omega$ and $\chi = (J_z - J_y)/J_x$ is the gyroscope constructive parameter. Equation (2) represents the sum of the moments from measuring angular rate and acceleration, and the torques $M_{TS\xi}$, $M_{TS\eta}$ that are produced by the TS along the fixed axes $O\xi$ and $O\eta$.

The projections β_ξ and β_η of small angular deflection α of the rotor on the fixed axes $O\xi$ and $O\eta$, respectively,

$$\begin{aligned} \beta_\xi &= \alpha \cos(\Omega t), \\ \beta_\eta &= \alpha \sin(\Omega t) \end{aligned} \quad (3)$$

are received by the AS. The input of SPS is fed by the output of the AS which is proportional to the angles β_ξ and β_η . The SPS then produces a feedback signal for the TS coils.

The dynamical characteristic of the SPS, AS, and TS can be described by transfer functions $W_{SPS}(s)$, $W_{TS}(s)$, and $W_{AS}(s)$, respectively, where s is a Laplace operator.

An examination of equations (1) and (2) reveals us that projection ω_ξ of measuring angular rate creates gyroscopic moment relative to axis $O\eta$ that is defined as measurement axis of first channel. Let's denote the indexes of the projections of moment, angular deflection of rotor on this axis and output potential drop across load resistor (after signal processing of respective angular deflection) by the number one. Analogously, the projection ω_η of measuring angular rate creates gyroscopic moment relative to axis $O\xi$ that is defined as measurement axis of second channel and denote a index by number two for

Fig. 3. Block Diagrams of RVG in closed loop operation in complex coordinates.

projections of moment, angular deflection of rotor on this axis and also output potential drop.

According to these new notations and equations (1), (2), and (3), the block diagram of the RVG in closed loop operation can now be presented as a two axis control system with modulation as shown on Fig. 2.

To study this presented system, let's define complex vectors for the coordinates of the two channels section of block diagram (see Fig. 2) [14]

$$\begin{aligned}\bar{\omega} &= \omega_\xi + j\omega_\eta, & \bar{M}_{TS} &= M_{TS1} + jM_{TS2}, \\ \bar{\Delta} &= \Delta_1 + j\Delta_2, & \bar{\beta} &= \beta_1 + j\beta_2, \\ \bar{U}_{out} &= U_{out1} + jU_{out2},\end{aligned}$$

where $j = \sqrt{-1}$. A relationship exists between these complex vectors that can be described by the following system of equations:

$$\left. \begin{aligned}\Delta_\Sigma(s) &= j\frac{1}{2}[\bar{\Delta}^*(s+j\Omega) - \bar{\Delta}(s-j\Omega)], \\ \alpha(s) &= W_g(s)\Delta_\Sigma(s), \\ \bar{\beta}(s) &= j\alpha(s+j\Omega), \\ \bar{U}_{out}(s) &= W_{SPS}(s)\bar{\beta}(s), \\ \bar{\Delta}(s) &= (K_0 - jsJ_x)\bar{\omega}^*(s) - \bar{M}_{TS}(s), \\ \bar{M}_{TS}(s) &= W_{TS}(s)\bar{U}_{out}(s),\end{aligned}\right\} \quad (4)$$

where “*” denotes complex conjugation. An equivalent block diagram which corresponds to the system of equations given in (4) is shown in Fig.3.

A closed loop equation for the error vector $\bar{\Delta}(s)$ can be obtained by using the same system of equations (4). We have

$$\begin{aligned}[1 + D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) W_g(s + j\Omega)] \bar{\Delta}(s) \\ - D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) W_g(s + j\Omega) \bar{\Delta}^*(s + j2\Omega) \\ = (K_0 - jsJ_x) \bar{\omega}^*(s).\end{aligned} \quad (5)$$

Here D_0 denotes the gain of the opened loop system and is given by:

$$D_0 = \frac{1}{2} k_g k_{TS} k_{AS} k_{SPS}, \quad (6)$$

where k_g , k_{TS} , k_{AS} , and k_{SPS} are the gain of gyroscope, TS, AS, and SPS respectively. Analysis of (5) shows that the relationship between the error vector $\bar{\Delta}(s)$ and measuring angular rate vector $\bar{\omega}(s)$ is described by linear equation with periodic coefficient $e^{-j2\Omega}$. To further analysis, Eq. (5) can be transformed to stationary form in which the periodic coefficient is reduced to the input of the system. Finally we will obtain an equivalent linear equation for the error vector with reduced right part that includes harmonic component $e^{-j2\Omega}$.

To transform this equation to a stationary form, the first, let's obtain an equation that is the complex conjugate of equation (5) by replacing complex coordinates and transfer

functions of (5) by the complex conjugate of them. Then substituting there $s + j2\Omega$ for s and multiplying both parts by $D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) W_g(s + j\Omega)$, we have

$$\begin{aligned}[1 + D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) W_g(s + j\Omega) + D_0 W_{TS}(s + j2\Omega) \\ \times W_{SPS}(s + j2\Omega) W_{AS}(s + j2\Omega) W_g(s + j\Omega)] \bar{\Delta}(s) \\ = [1 + D_0 W_{TS}(s + j2\Omega) W_{SPS}(s + j2\Omega) W_{AS}(s + j2\Omega) \times \\ \times W_g(s + j\Omega)] (K_0 - jsJ_x) \bar{\omega}^*(s) + D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) \\ \times W_g(s + j\Omega) (K_0 + j(s + j2\Omega) J_x) \bar{\omega}(s + j2\Omega).\end{aligned} \quad (7)$$

Thus, obtained Eq. (7) is stationary form of (5) and can be considered as a linear equation with reduced right part which includes harmonic component $e^{-j2\Omega}$. To simplify Eq.(7) let's define

$$\begin{aligned}W_l(s) &= D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) W_g(s + j\Omega) \\ &+ D_0 W_{TS}(s + j2\Omega) W_{SPS}(s + j2\Omega) \\ &\times W_{AS}(s + j2\Omega) W_g(s + j\Omega);\end{aligned} \quad (8)$$

$$\begin{aligned}R(s) &= 1 + D_0 W_{TS}(s + j2\Omega) W_{SPS}(s + j2\Omega) \\ &\times W_{AS}(s + j2\Omega) W_g(s + j\Omega);\end{aligned} \quad (9)$$

$$\begin{aligned}S(s) &= D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) \\ &\times W_g(s + j\Omega) (K_0 + j(s + j2\Omega) J_x);\end{aligned} \quad (10)$$

$$\begin{aligned}A(s) &= D_0 W_{TS}(s) W_{SPS}(s) W_{AS}(s) \\ &\times W_g(s + j\Omega).\end{aligned} \quad (11)$$

Substituting (8), (9), and (10) into (7), the equation of stationary form under closed loop can be abbreviated as:

$$\begin{aligned}[1 + W_l(s)] \bar{\Delta}(s) &= R(s) \bar{\omega}^*(s) \\ &+ S(s) \bar{\omega}(s + j2\Omega).\end{aligned} \quad (12)$$

Note that $A(s)$ is the transfer function of a opened loop system without non-stationary part, in other words without terms with harmonical coefficients with double frequency of rotation. $W_l(s)$ is transfer function of reduced part of system. The right part of equation (12) consists of a stationary term with measuring angular rate $\bar{\omega}^*$ and non-stationary term $\bar{\omega}(s + j2\Omega)$. From (12) can easily be obtained, the equation of closed loop for output signal $\bar{U}_{out}$

$$\begin{aligned}[1 + W_l(s)] W_{TS}(s) \bar{U}_{out}(s) \\ = A(s) (K_0 + j(s + j2\Omega) J_x) \bar{\omega}^*(s) - S(s) \bar{\omega}(s + j2\Omega),\end{aligned} \quad (13)$$

which corresponds to mathematical model of the RVG in closed loop operation.

III. PECULIARITIES OF MATH MODEL OF RVG IN CLOSED LOOP OPERATION AS TWO AXIS CONTROL SYSTEM WITH MODULATION

The mathematical model given in equation (13) allows by using the known techniques of linear control theory to study stability and evaluate dynamical characteristic of a closed loop system in response to a unit step input. However, at the same time, account has to be taken of the peculiarities of a two axis control system with modulation.

As a rule, for tracing chart of response to a unit step input or step response (SR) in one axis linear control system is used real part of response frequency of closed loop $P(\omega)$ [14]–[17], where ω - frequency of exiting input signal. Note that $P(\omega)$ is symmetrical relatively vertical axis passing through point $\omega = 0$. It means fulfilment of condition

$$P(\omega) = P(-\omega) \quad (14)$$

and equation of SR in time domain can be obtained in the form of

$$h(t) = \frac{2}{\pi} \int_0^{+\infty} \frac{P(\omega)}{\omega} \sin(\omega t) d\omega. \quad (15)$$

However, in the case of a two axis control system, the coefficients of transfer functions are complex numbers, and not real. Therefore, the condition described by (14) does not hold, that is

$$P(\omega) \neq P(-\omega). \quad (16)$$

Therefore, for the tracing of SR in a two axis control system with modulation, account has to be taken of the real part of frequency response tracing in both the positive $P(\omega)$, as well as the negative $P(-\omega)$ frequency domains. Hence the equation of SR for a two axis system is as follows [14]:

$$h(t) = \frac{1}{\pi} \int_0^{+\infty} \frac{P(\omega)}{\omega} \sin(\omega t) d\omega + \frac{1}{\pi} \int_0^{+\infty} \frac{P(-\omega)}{\omega} \sin(\omega t) d\omega. \quad (17)$$

For studying stability it is useful to use the reduced transfer function of the linear part of the system $W_l(p)$. The Nyquist stability criterion applying to $W_l(p)$ can be easy obtained and becomes

$$\begin{cases} \delta \arg [1 + W_l(s)] = 2k\pi, \\ s = j\omega (-\infty < \omega < +\infty), \end{cases} \quad (18)$$

where k - number of zeros of polynomial $P_l(s)$, that is the denominator of the transfer function $W_l(s)$, in the right half plane. This criteria is convenient especially since $W_l(s)$ is symmetrical relative to real axis cutting at the point $\omega = -\Omega$ or mathematically it can be represented as follows

$$W_l(j\omega) = W_l(j(-2\Omega - \omega))^*. \quad (19)$$

Hence, the frequency response of $W_l(s)$ can be plotted only in frequency range $-\infty < \omega < -\Omega$ (or $-\Omega > \omega < +\infty$) instead of the whole frequency range $-\infty < \omega < +\infty$ which is necessary for plotting frequency response of system that is not symmetric.

One of parameters which characterizes quality of a closed loop system is the quality factor Q_s that is equal to ratio of

Fig. 4. Reduced block diagram of RVG in closed loop operation.

Fig. 5. Rearranged block diagram of RVG in closed loop operation.

steady-state value of output quantity to steady-state value of its error. In particular, for the considering system, it can be written as:

$$Q_s = \frac{\bar{U}_{out}(s)}{\bar{\Delta}(s)}. \quad (20)$$

In examining a two axis system with modulation takes place so called effect of limitation of value of quality factor Q_s . This effect consists in that value of quality factor Q_s is not equal to gain of opened loop system and it is limited even through gain D_0 of the opened loop system approach to infinity.

To examine this effect, let obtain equation of closed loop which shows dependence of the RVG output voltage $\bar{U}_{out}(s)$ on the error $\bar{\Delta}(s)$. Based on (12) and (13), this equation can be written as follows

$$R(s)W_{TS}(s)\bar{U}_{out}(s) = A(s)\bar{\Delta}(s) - S(s)\bar{\omega}(s + j2\Omega). \quad (21)$$

Fig. 4 shows the reduced block diagram of RVG in closed loop operation which satisfies equation (21). This block diagram can be rearranged as shown in Fig. 5 to join the stationary term of measuring angular rate $\bar{\omega}^*(s)$ and non-stationary term $\bar{\omega}(s + j2\Omega)$ in one input. For block diagram shown on Fig. 5 there are equations below

$$\frac{\bar{U}_{out}(s)}{\bar{\Delta}_1(s)} = \frac{A(s)}{R(s)W_{TS}(s)}, \quad (22)$$

$$\bar{\Delta}_1(s) = \bar{M}_{\Sigma g} - W_{TS}(s)\bar{U}_{out}(s), \quad (23)$$

$$\begin{aligned} \bar{M}_{\Sigma g}(s) = & \left(K_0 + j(s + j2\Omega)J_x \right) \bar{\omega}^*(s) \\ & - S(s)\bar{\omega}(s + j2\Omega), \end{aligned} \quad (24)$$

where $\bar{M}_{\Sigma g}$ - equivalent input gyroscopic torque, $\bar{\Delta}_1(s)$ - reduced error;

$$\frac{\bar{U}_{out}(s)}{\bar{M}_{\Sigma g}(s)} = \frac{W_r(s)}{1 + W_r(s)} \frac{1}{W_{TS}(s)}, \quad (25)$$

where

$$W_r(s) = \frac{A(s)}{R(s)} \quad (26)$$

Fig. 6. Bode plots for $W_l(s)$ without series compensator.

is transfer function of opened loop reduced system. Taking into account (26), equation (22) can be rewritten as

$$\frac{\bar{U}_{out}(s)}{\Delta_1(s)} = \frac{W_r(s)}{W_{TS}(s)}. \quad (27)$$

Hence, the equation for quality factor of system can be written as:

$$Q_s = \frac{W_r(s)}{W_{TS}(s)} \Big|_{s=0} = \frac{W_{TS}(s)W_{SPS}(s)W_{AS}(s)W_g(s+j\Omega)}{\frac{1}{D_0} + W_{TS}(s+j2\Omega)W_{SPS}(s+j2\Omega)W_{AS}(s+j2\Omega)W_g(s+j2\Omega)} \Big|_{s=0}. \quad (28)$$

As the gain of the opened loop system D_0 approaches infinity,

$$\lim_{D_0 \rightarrow \infty} Q_s = \frac{W_{TS}(s)W_{SPS}(s)W_{AS}(s)}{W_{TS}(s+j2\Omega)W_{SPS}(s+j2\Omega)} \times \frac{1}{W_{AS}(s+j2\Omega)} \Big|_{s=0}. \quad (29)$$

The calculated limit from equation (29) is the maximum value of the quality factor $Q_{s \max}$ in closed loop operation of RVG. Analysis of equation (29) shows that value of the quality factor Q_s is limited even though gain D_0 tends to infinity. At the same time maximum value $Q_{s \max}$ is not dependent on D_0 but it is a function of parameters of SPS and, particularly, series compensation.

Equation (29) can be simplified by assuming that AS and TS are not inertial blocks. This means that $W_{AS}(s) = k_{AS}$ and $W_{TS} = k_{TS}$. In this case, $Q_{s \max}$ becomes

$$Q_{s \max} = \frac{W_{SPS}(s)}{k_{TS}W_{SPS}(s+j2\Omega)} \Big|_{s=0}. \quad (30)$$

The effect of the limited value of quality factor Q_s of RVG in closed loop operation at physical point of view can be explained by passing signal on double frequency of rotation through system. As a result in closed loop of RVG appears error component that is independent of gain D_0 .

IV. APPROACH TO SELECTION OF SERIES COMPENSATION PARAMETERS

An examination of the stability of a RVG in closed loop operation without SC by using the transfer function of reduced part system $W_l(s)$ reveals that the system is situated on the edge of stability (Fig. 6).

The estimation also indicates that by applying a SC using phase lag

$$W_c(s) = \frac{T_{c1}s + 1}{T_{c2}s + 1}, \quad (31)$$

where T_{c1} and T_{c2} are time constants, the required quality of control of the RVG in closed loop operation, under defined value of time constants of the SC will be met.

Therefore in development process of approach parameters of the SC (31) are selected to meet the required quality statical and dynamical characteristic of RVG.

The step response of RVG as behavior output voltage under input action of step function $\mathbf{1}(t)$ - measuring angular rate can be defined. Neglect inertance of the TS ($W_{TS}(s) = k_{TS}$) and additive element jsJ_x of coefficient (24) of measuring angular rate because its influence on dynamical and statical characteristic of RVG is insignificantly. The mentioned above assumptions allow us to obtain step response of closed loop system in dimensionless value without taking into account coefficient K_0/k_{TS} . Hence, we can represent results in more comfortable visual form and easier compare with each other. To obtain step response in absolute value let step response function $h(t)$ multiply by scale coefficient $K_{sc} = \frac{K_0\omega_a}{k_{TS}}$, where ω_a - magnitude value of step input.

An analysis of the required specifications of a rotor vibratory gyroscope for highly dynamic movable objects indicate that the required values of the indexes of dynamical quality and statical accuracy are a maximum overshoot σ of up to 40%; a settling time t_s no more than 100 ms; a bandwidth f_b more than 50 Hz and a statistical error Ξ of no more than 0.7%.

The input data used for the selection parameters of the SC are the central principal moment of the rotor J_x , J_y , and J_z ; the quality factor of the SE Q_{SE} ; c - constant of the SE (value of c is selected by the rotor tuning condition); and gains of the TS and AS. Depending on the design of the SPS, the transfer function $W_{SPS}(s)$ is chosen accordingly.

The SC (31) is part of the SPS and is included in transfer function $W_{SPS}(s)$. An approach for the selection of the parameters of the SC T_{c1} and T_{c2} is as follows.

The **first step**: the parameters of the open loop transfer function of reduced system $W_r(s)$ describing by (26) are defined. For this reason, the natural frequency of the rotor oscillation (without rotation of the DM) and internal friction coefficient μ (1) are calculated by following equation :

$$f = \sqrt{\frac{c}{J_x}}, \quad \mu = \frac{c}{fQ_{SE}}. \quad (32)$$

Next, the rotor tuning frequency Ω is calculated [18]:

$$\Omega^2 = \frac{1}{J_x(1 - \chi)\delta_x}, \quad (33)$$

where $\chi = (J_z - J_y)/J_x$ gyroscope constructive parameter. Parameters of $W_r(s)$ is dependent also on T_{c1} and T_{c2} .

The **second step**: an estimate is made of T_{c1} and T_{c2} that provides an overshoot σ^* of no more then 40%, to a first approximation. For this reason we evaluate equation

$$\sigma^* = \frac{P_{\max} - P(0)}{P(0)}, \quad (34)$$

where

$P(0) = \Re \left[\frac{W_r(0)}{1+W_r(0)} \right]$, $P_{\max} = \max \left(\Re \left[\frac{W_r(\omega)}{1+W_r(\omega)} \right] \right)$. At the same time, according to [14] σ^* is close overshoot σ . Then we plot chart $\sigma^* = \sigma^*(T_{c1}, T_{c2})$. Based on this dependence, a domain of rational values of T_{c1} and T_{c2} is chosen.

The **third step**, more accurate rational values of T_{c1} and T_{c2} are defined. Using equation (30) the dependence of maximum quality factor $Q_{s \max}$ on T_{c1} and T_{c2} that are chosen from domain of rational values obtained previously is developed.

Based on (25) and (27) the statical error of the RVG in closed loop operation is as follows

$$\Xi = \frac{\bar{\Delta}_1(s)}{\bar{M}_{\Sigma g(s)}} \Big|_{s=0} = \frac{1}{1 + W_r(s)} \Big|_{s=0}. \quad (35)$$

Taking into account (28) the relationship between statical error Ξ and the quality factor Q_s can be rewritten as

$$\Xi = \frac{1}{1 + Q_s W_{TS}(s)} \Big|_{s=0}. \quad (36)$$

Analysis of (36) indicates that the larger the quality factor Q_s , the smaller statical error Ξ . In particular, to provide a statical error Ξ of less than 0.7%, the maximum value of the quality factor Q_s should be

$$Q_{s \max} \geq \frac{1}{0.007 k_{TS}}. \quad (37)$$

And so in this step, a subregion of rational values of T_{c1} and T_{c2} providing to a first approximation both the requirement value of overshoot and the requirement value of statical error are found.

Finally, in the **fourth step**, the recommended values of T_{c1} and T_{c2} are selected by plotting the step responses based on (17). Analysis of these results provides a statical error Ξ of no more than 0.7%, overshoot σ no more than 40%, settling time t_s no more than 100 ms and bandwidth f_b more than 50 Hz. The relationship between bandwidth and settling time are estimated by equations [14], [17] to be:

$$f_b \leq \frac{4\pi}{t_s} s^{-1}, \text{ or } f_b \leq \frac{2}{t_s} \text{ Hz}. \quad (38)$$

A plot of the frequency and phase response of the dimensionless transfer function $\frac{W_r(s)}{1+W_r(s)}$ is made so that the index of oscillation can be evaluated to allow a more accurate defining bandwidth of RVG under closed loop operation.

V. RESULTS OF SERIES COMPENSATION PARAMETERS SELECTION FOR ROTOR VIBRATORY GYROSCOPE

Selection of the compensators series (31) parameters of RVG in closed loop operation with SPS based on low pass filter can be made using this approach. The values of central

principal moments are $J_x = 0.824 \cdot 10^{-7} \text{ kg}\cdot\text{m}^2$, $J_y = 0.316 \cdot 10^{-7} \text{ kg}\cdot\text{m}^2$, $J_z = 1.117 \cdot 10^{-7} \text{ kg}\cdot\text{m}^2$. The stiffness and quality factor of SE are $c = 0.0063 \text{ N}\cdot\text{m}$ and $Q = 1000$, respectively. The gain of the AS is $k_{AS} = 10 \text{ V/rad}$. The gain of TS is $k_{TS} = 10^{-4} \text{ N}\cdot\text{m/V}$ for measuring angular rate range up to 180 grad/s. These rate sensor properties correspond to a RVG developed by the Arzamas Research & Production Joint Stock Company "Temp-Avia" shown on Fig. 7 with service electronic and sensor unit for autonomous movable object in which such RVG is used.

Fig. 7. The Rotor Vibratory Gyroscope - 1 with the Service Electronic - 2 and the Sensor Unit - 3.

Using Eq. (32), the self-frequency rotor f and the damping coefficient μ are calculated to be 43 Hz and $2.42 \cdot 10^{-8} \text{ N}\cdot\text{m}\cdot\text{s}$, respectively. The rotor tuning frequency is calculated by using Eq. (33) to be equal to $\Omega = 260.6 \text{ Hz}$.

The Low Pass Filter (LPF) fits with a second order system, described by the transfer function:

$$W_f(s) = \frac{1}{T_f^2 s^2 + 2T_f \xi s + 1}, \quad (39)$$

where $T_f = 0.0026 \text{ s}$, $\xi = 0.16$.

Based on Eq. (1) transfer function of the gyroscope is as follow:

$$W_g(s) = \frac{1}{J_x s^2 + \mu s + [(J_z - J_y)\Omega^2 + c]}, \quad (40)$$

Eq.(40) can be rewritten as:

$$W_g(s) = \frac{1}{c_r (T^2 s^2 + 2\zeta T s + 1)}, \quad (41)$$

where $c_r = [(J_z - J_y)\Omega^2 + c]$ is the reduced stiffness; $T = \sqrt{J_x/c_r}$ is the time constant of the gyroscope; $\zeta = \mu/(2\sqrt{J_x c_r})$ is the damping coefficient. By substituting $s + j\Omega$ for s into (41) the transfer function of the gyroscope can

Fig. 8. Dependence overshoot $\sigma^* = \sigma^*(T_{c1}, T_{c2})$ on variation of parameters compensators series.

Fig. 9. Behavior of $Q_{s \max}(T_{c1}, T_{c2})$ in preliminary chosen domains of rational values of parameters compensators series.

be rewritten as:

$$W_g(s + j\Omega) = \frac{1}{c_r \left(T^2(s + j\Omega)^2 + 2\zeta T(s + j\Omega) + 1 \right)}. \quad (42)$$

Notice that the gyroscope is tuned when $\Omega = 1/T$. The transfer function of gyroscope (42) as multiplication of type block of two axis control system with complex coefficients [14]. The roots of the denominator in Eq. (42) are

$$\begin{aligned} s_1 &= -\zeta \frac{1}{T} - j \left(\Omega - \frac{1}{T} \sqrt{1 - \zeta^2} \right), \\ s_2 &= -\zeta \frac{1}{T} - j \left(\Omega + \frac{1}{T} \sqrt{1 - \zeta^2} \right), \end{aligned}$$

hence:

$$\begin{aligned} &c_r \left(T^2(s + j\Omega)^2 + 2\zeta T(s + j\Omega) + 1 \right) = \\ &= \frac{T^2 c_r}{T_1 T_2} (T_1 s + j + \alpha_1) (T_2 s + j + \alpha_2), \end{aligned} \quad (43)$$

where

$$\begin{aligned} T_1 &= 1 / \left(\Omega - \frac{1}{T} \sqrt{1 - \zeta^2} \right), T_2 = 1 / \left(\Omega + \frac{1}{T} \sqrt{1 - \zeta^2} \right), \\ \alpha_1 &= \zeta \frac{1}{T} / \left(\Omega - \frac{1}{T} \sqrt{1 - \zeta^2} \right), \alpha_2 = \zeta \frac{1}{T} / \left(\Omega + \frac{1}{T} \sqrt{1 - \zeta^2} \right). \end{aligned} \quad (44)$$

Thus,

$$\begin{aligned} W_g(s + j\Omega) &= \\ &= \frac{k_g}{(T_1 s + j + \alpha_1) (T_2 s + j + \alpha_2)}, \end{aligned} \quad (45)$$

where $k_g = T_1 T_2 / (T^2 c_r)$. According to (45) transfer function of gyroscope can be represented as series of two aperiodic blocks with complex constants.

The transfer function of SPS can be written as multiplication of SC and LPS transfer functions in the following way:

$$W_{SPS}(s) = W_f(s)W_c(s). \quad (46)$$

By taking into account Eq. (45), Eq. (46) and earlier introduced notations, the transfer functions $A(s)$ and $R(s)$ can be rewritten as:

$$A(s) = D_0 \frac{T_{c1}s + 1}{(T_1s + j + \alpha_1)(T_2s + j + \alpha_2)} \times \frac{1}{(T_f^2s^2 + 2T_f\xi s + 1)(T_{c2}s + 1)}, \quad (47)$$

$$R(s) = 1 + D_0 \frac{T_{c1}(s + j2\Omega) + 1}{(T_{c2}(s + j2\Omega) + 1)} \times \frac{1}{(T_f^2(s + j2\Omega)^2 + 2T_f\xi(s + j2\Omega) + 1)} \times \frac{1}{(T_1s + j + \alpha_1)(T_2s + j + \alpha_2)}. \quad (48)$$

By substituting (47) and (48) into (26), finally, the transfer function of the reduced system $W_r(s)$ is obtained.

Following the second step used select the compensators series parameters, a plot of the functional dependence $\sigma^* = \sigma^*(T_{c1}, T_{c2})$ is given in Fig. 8(a). On Fig. 8(b), the thick lines shows the edges of T_{c1} and T_{c2} values rational domains that are denoted by Roman numerals I and II where overshoot σ^* to a first approximation is no more then 40%.

According to third step of this approach, the quality factor of system Q_{smax} is defined to provide the statical error of RVG in closed loop operation of less then 0.7%. By taking into account the value of k_{TS} mentioned above and by substituting it into Eq. (37), the following is obtained:

$$Q_{smax} \geq 1.4 \times 10^6. \quad (49)$$

Fig. 10. Defined more exactly rational values of T_{c1} and T_{c2} .

Fig. 11. Subregion (grey area) of rational values of T_{c1} and T_{c2} providing to a first approximation both the requirement value of overshoot and the requirement value of statical error.

By using Eq. (30) and Eq. (46) the dependence of maximum value of quality factor Q_{smax} of closed loop system on parameters T_{c1} and T_{c2} can be written as:

$$Q_{smax}(T_{c1}, T_{c2}) = \frac{(T_{c2}j2\Omega + 1)}{k_{TS}(T_{c1}j2\Omega + 1)} \times (-T_f^2 4\Omega^2 + 2T_f\xi j2\Omega + 1), \quad (50)$$

An examination of the behavior of $Q_{smax}(T_{c1}, T_{c2})$ with domains I ($0.001 \text{ s} < T_{c1} < 0.07 \text{ s}$ and $0.002 \text{ s} < T_{c2} < 0.02 \text{ s}$) and II ($0.03 \text{ s} < T_{c1} < 0.09 \text{ s}$ and $T_{c2} < 0.002 \text{ s}$) that have been defined in second step can be made (see Fig. 8(b)). The results are shown in Fig. 9.

Analysis of the results (see Fig. 9) shows that the quality factor Q_{smax} can be achieved in domain I (see Fig. 9(a)), but in domain II the value of Q_{smax} is less than 1.4×10^6 by an order of magnitude. Therefore, domain II can be excluded from further consideration. The domain of T_{c1} and T_{c2} which satisfies condition (49) is shown in Fig. 10. The edge of this domain is denoted by the thick lines.

By using the results of the dependencies $\sigma^* = \sigma^*(T_{c1}, T_{c2})$ (see Fig. 8) and $Q_{smax} = Q_{smax}(T_{c1}, T_{c2})$ (see Fig. 9) study, a plot of the subregion of rational values of T_{c1} and T_{c2} provides, to a first approximation, both the requirement values of overshoot and statical error. The obtained subregion of rational values of T_{c1} and T_{c2} is shown in Fig. 11, where this subregion is denoted by grey color.

Values of $T_{c1} = 0.02 \text{ s}$ and $T_{c2} = 0.004 \text{ s}$ are chosen from subregion and substituting into $W_r(s)$. The step response can then be plotted using Eq. (17). The step response of RVG in closed loop for selected parameters of SC T_{c1} and T_{c2} is shown in Fig. 12 (dotted line).

Fig. 12 shows us that step response satisfies requirement excluding overshoot σ value of one is more than 40%. Let correct obtained result in order to decrease value of overshoot

Fig. 12. The step response of RVG in closed loop for $T_{c1} = 0.02$ s and $T_{c2} = 0.004$ s (dotted line); its characteristics: $\sigma_1 = 56\%$, $t_{s1} = 55$ ms, $\Xi_1 = 0.65\%$; for $T_{c1} = 0.01$ s and $T_{c2} = 0.003$ (firm line); its characteristics: $\sigma_2 = 33\%$, $t_{s2} = 35$ ms, $\Xi_2 = 0.67\%$.

σ . According to Fig. 8(b), to decrease overshoot a move into “darker” region has to be made.

In this case let define parameters of SC as $T_{c1} = 0.01$ s, $T_{c2} = 0.003$ s. The step response of RVG in closed loop which satisfies such parameters is shown in Fig. 12 by solid line.

Thus, the selected parameters of SC $T_{c1} = 0.01$ s, $T_{c2} = 0.003$ s provide required quality of step response of RVG in closed loop, that is overshoot $\sigma = 33\%$, settling time $t_s = 35$ ms, statical error $\Xi = 0.67\%$ (see Fig. 12). Evaluation of bandwidth which can be calculated by Eq. (38) gives $f_b = 60$ Hz.

Finally, a plot of frequency response of RVG in closed loop, using transfer function $\frac{W_r(s)}{1+W_r(s)}$ to estimate amplitude of structural resonance and define more exactly bandwidth of gyroscope. Since the bandwidth is less than 2Ω , the influence of non-stationary term $R(s)$ on behavior transfer function of opened loop reduced system $W_r(s)$ is neglected. In this case $W_r(s) \simeq A(s)$.

The frequency response of RVG in closed loop is represented in Fig. 13. For this gyroscope with an amplitude of structural resonance of 1.38, the bandwidth is 65 Hz.

As a result, following suggested approach to selection of parameters of series compensation using phase lag for rotor vibratory gyroscope in closed loop operation, the characteristics of rate sensor are as following: parameters of SC $T_{c1} = 0.01$ s, $T_{c2} = 0.003$ s; overshoot $\sigma = 33\%$; settling time $t_s = 35$ ms; statical error $\Xi = 0.67\%$; the bandwidth is 65 Hz and the amplitude of structural resonance is 1.38 are obtained.

Additionally, the obtained result of selection of SC parameters was evaluated by experimentally. The experiment was conducted with the four gyroscopes designed by JSC “Temp-Avia” with mechanical characteristics mentioned in Sec. V. Each of them was taken separately and then connected to the same SPS based on the low pass filter. Then parameters of SC using phase lag are adjusted to meet requirements of specification have mentioned in Sec. IV. Experimental results show

Fig. 13. The frequency response of RVG in closed loop: amplitude of structural resonance is 1.38 and bandwidth is 65 Hz.

that parameters of SC using phase lag lie in following ranges: T_{c1} is between 0.013 and 0.024 s and T_{c2} is between 0.0029 and 0.0043 s. The dispersion in the experimental evaluation of parameters of SC is due to inaccuracy in performance of mechanical part of RVG. Analysis of obtained experimental data shows quite enough correlation results of approach with experiments.

VI. CONCLUSION

Mathematical model of RVG as a two axis control system with modulation is developed which then is reduced to stationary form and allows to study dynamical characteristics of RVG based on theory one axis control systems.

Peculiarities of mathematical model of RVG in closed loop is considered. In particular, for plotting step response of RVG in closed loop we need to take into account behavior of real part of frequency response of transfer function that describes dynamic characteristic of RVG in closed loop as positive as well as negative frequency range. As shown $W_l(s)$ transfer function of reduced part of system is symmetrical that allow to simplify study of stability by plotting bode diagram of transfer function $W_l(j\omega)$ only in frequency range $-\infty < \omega < -\Omega$ or $-\Omega < \omega < \infty$.

The effect of limitation of value of system quality factor Q_s is examined. This effect consists in that value of quality factor Q_s is not equal to gain of opened loop system and it is limited even through gain D_0 of the opened loop system approach to infinity.

Finally, the approach to selection of parameters of series compensation using phase lag for rotor vibratory gyroscope in closed loop operation based on results of study of its mathematical model have obtained above is developed which provides requirement values of overshoot, settling time, statical error, and the bandwidth. In particular, suggested approach allows to select parameters of compensators series such as phase lag for RVG designed by JSC “Temp-Avia”.

ACKNOWLEDGMENT

The authors acknowledge with thanks the support of the Arzamas Research & Production Joint Stock Company "Temp-Avia".

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