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Generation of shock-free pressure waves in shaped resonators by boundary driving

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Investigation of high amplitude pressure oscillations generated by boundary driving in shaped resonators has been carried out both theoretically and experimentally. In the theoretical modeling, the acoustic resonance in an axisymmetric resonator is studied by the Galerkin method. The resonator is exponentially expanded and the boundary driving is provided by a piston at one end. The pressure wave forms, amplitudes, resonance frequencies, and ratio of pressures at the two ends of the resonator are calculated for various expansion flare constants and driving strengths. These results are partially compared with those generated by shaking the resonator. They are also verified in the experiment, in which an exponentially expanded resonator is connected to a speaker box functioning as the piston. The experiment is further extended to a horn-shaped resonator with a rectangular cross section. The boundary driving in this case is generated by a circular piezoelectric disk, which forms one sidewall of the resonator cavity. The characteristics of axisymmetric resonators, such as the resonance frequency and amplitude ratio of pressures at the two ends, are observed in this low aspect ratio rectangular resonator with the sidewall driving. © 2007 Acoustical Society of America. [DOI: 10.1121/1.2713716]

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I. INTRODUCTION

High amplitude pressures can be generated by shaking shaped resonators whose cross-section areas are expanded along the axial dimensions. This has been demonstrated by both experiments¹ and theoretical modeling.^{2–6} In the experiments conducted by Lawrenson *et al.*,¹ standing wave overpressures in excess of 340% ambient pressure were recorded by shaking shaped resonators. Ilinskii *et al.*² developed a one-dimensional model to analyze the nonlinear standing waves in shaped resonators. The large amplitude pressures, wave form distortion, and resonance frequency shift were calculated and compared with the experimental results. This one-dimensional model was later improved by including the shear viscosity term in the momentum equation.^{3,4} Ilinskii *et al.*³ studied the energy losses and the quality factors of shaped resonators. Luo *et al.*⁴ investigated the effect of resonator dimensions. An analytical study was conducted by Hamilton *et al.*⁵ to predict and explain the dependence of the nonlinear frequency response on resonator shapes. Erickson and Zinn⁶ proposed a procedure to solve the one-dimensional wave equation for exponentially expanded resonators by the Galerkin method. The resonance characteristics are discussed in terms of expansion flare constant. Other than shaking the shaped resonators, generation of high amplitude pressure waves in a tube with an array of Helmholtz resonators are also reported.^{7,8}

The driving method to generate high amplitude pressures in shaped resonators has been so far focused on shaking the entire resonators, probably because the entire shaking is more efficient in power delivery¹ compared to other meth-

ods, such as piston driving. In order to broaden the potential of this technology in other engineering applications, such as miniature or microfluidic devices,^{9,10} it is necessary to explore other driving mechanisms for various resonator configurations. The boundary driving in these applications may be a better way than shaking to generate useful dynamic pressures, which motivated the present study. In this study, generation of high amplitude pressure in shaped resonators by boundary driving was investigated both theoretically and experimentally. The theoretical modeling using Galerkin's method was performed on an axisymmetric resonator with one end connected to a piston. Calculated results, including the resonance frequencies, pressure amplitudes at two ends of the resonator, and their ratios were compared with measured data from the experiment, which was conducted on an axial symmetric resonator. The experimental study was further extended to a horn-shaped resonator with low aspect ratio rectangular cross sections. The boundary driving in this case was from a piezoelectric disk which formed the sidewall of the resonator cavity. The resonance frequency and pressures were measured.

II. MODELING

A. Basic equations and boundary conditions

We consider an axisymmetric resonator shown in Fig. 1. The resonator has length l and radius $r(x)$ which is a function of x . The cross-sectional area of the resonators is expanded exponentially along its axis. The resonator is sealed at the small end ($x=0$) and connected to a piston at the other end ($x=l$). The configuration of the resonator is very similar to the one previously used by Luo *et al.*,⁴ except that there is no body shaking force in the present case. The

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FIG. 1. The exponentially expanded resonator with a piston driving at one end.

dimensionless nonlinear wave equation previously used by Luo *et al.*⁴ for the resonator under shaking is written here:

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial T^2} - \frac{1}{\Omega^2 \pi^2} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi}{\partial X} - \frac{1}{\Omega^2 \pi^2} \frac{\partial^2 \Phi}{\partial X^2} + \frac{2}{\Omega} \frac{\partial^2 \Phi}{\partial X \partial T} \frac{\partial \Phi}{\partial X} \\ + \frac{(\gamma-1)}{\Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi}{\partial T} \frac{\partial \Phi}{\partial X} + \frac{(\gamma-1)}{\Omega} \frac{\partial \Phi}{\partial T} \frac{\partial^2 \Phi}{\partial X^2} \\ - \frac{G_B}{\pi^3 \Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial^2 \Phi}{\partial T \partial X} - \frac{G_B}{\pi^3 \Omega} \frac{\partial^3 \Phi}{\partial T \partial X^2} + \frac{G_S}{R \Omega^{1/2}} \frac{\partial \Phi}{\partial T} \\ = F_{\text{shaking}}, \end{aligned} \quad (1)$$

where

$$\begin{aligned} F_{\text{shaking}} = -\frac{1}{\Omega^2} \left\{ A \frac{\partial \Phi}{\partial X} + \Omega \frac{\partial A}{\partial T} X + (\gamma-1) \left\{ \frac{1}{R^2} \frac{dR^2}{dX} \right\} \right. \\ \left. \times \frac{\partial \Phi}{\partial X} A X + (\gamma-1) \frac{\partial^2 \Phi}{\partial X^2} A X \right\} \end{aligned} \quad (2)$$

consists of the terms related to the dimensionless shaking acceleration A . In Eqs. (1) and (2),

$$X = \frac{x}{l}, \quad T = \omega t, \quad S = \frac{\pi r^2}{l^2}, \quad R = \frac{r}{l}, \quad (3)$$

$$\omega_0 = \frac{\pi c_0}{l}, \quad A = \frac{a}{l \omega_0^2},$$

$$G_B = \frac{(\zeta + 4\eta/3) \pi \omega_0}{\rho_0 c_0^2}, \quad G_S = \sqrt{\frac{2\eta}{\pi \rho_0 c_0 l}}, \quad (4)$$

$$\Omega = \frac{\omega}{\omega_0}, \quad \Phi = \frac{\varphi}{l^2 \omega_0},$$

where c_0 is the speed of sound, ζ and η are coefficients of viscosity, $a = a_0 \sin(\omega t)$ is acceleration associated with the shaking, and φ is velocity potential. By setting $A=0$, F_{shaking} will be zero in Eq. (1), and the dimensionless wave equation for the present case is obtained as

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial T^2} - \frac{1}{\Omega^2 \pi^2} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi}{\partial X} - \frac{1}{\Omega^2 \pi^2} \frac{\partial^2 \Phi}{\partial X^2} + \frac{2}{\Omega} \frac{\partial^2 \Phi}{\partial X \partial T} \frac{\partial \Phi}{\partial X} \\ + \frac{(\gamma-1)}{\Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi}{\partial T} \frac{\partial \Phi}{\partial X} + \frac{(\gamma-1)}{\Omega} \frac{\partial \Phi}{\partial T} \frac{\partial^2 \Phi}{\partial X^2} \\ - \frac{G_B}{\pi^3 \Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial^2 \Phi}{\partial T \partial X} - \frac{G_B}{\pi^3 \Omega} \frac{\partial^3 \Phi}{\partial T \partial X^2} \\ + \frac{G_S}{R \Omega^{1/2}} \frac{\partial \Phi}{\partial T} = 0. \end{aligned} \quad (5)$$

The piston velocity at $x=l$ in Fig. 1 is $u(t) = u_0 \sin(\omega t)$, and its dimensionless form is

$$U(T) = U^* \sin(T), \quad (6)$$

where $U^* = u_0/l\omega_0$ is the velocity amplitude of the piston, indicating the driving strength. The boundary conditions in dimensionless forms are therefore

$$\left. \frac{\partial \Phi}{\partial X} \right|_{X=0} = 0, \quad \left. \frac{\partial \Phi}{\partial X} \right|_{X=1} = U^* \sin(T). \quad (7)$$

Since the piston is oscillating at the right end, the resonator length l in the present case is measured from $x=0$ to the average position of piston. The nonlinear wave equation (5), together with boundary conditions (7) may be solved by assuming that Φ consists of two parts, i.e.,

$$\Phi = \Phi_A + \Phi_B, \quad (8)$$

where Φ_A is a new unknown function to be solved, which satisfies homogeneous boundary conditions

$$\left. \frac{\partial \Phi_A}{\partial X} \right|_{X=0} = 0, \quad \left. \frac{\partial \Phi_A}{\partial X} \right|_{X=1} = 0, \quad (9)$$

and Φ_B can be chosen to satisfy the nonhomogeneous boundary conditions (7). By substituting Eq. (8) into Eq. (5), one obtains

$$\begin{aligned} \frac{\partial^2 \Phi_A}{\partial T^2} - \frac{1}{\Omega^2 \pi^2} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi_A}{\partial X} - \frac{1}{\Omega^2 \pi^2} \frac{\partial^2 \Phi_A}{\partial X^2} + \frac{2}{\Omega} \frac{\partial^2 \Phi_A}{\partial X \partial T} \frac{\partial \Phi_A}{\partial X} \\ + \frac{(\gamma-1)}{\Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi_A}{\partial T} \frac{\partial \Phi_A}{\partial X} + \frac{(\gamma-1)}{\Omega} \frac{\partial \Phi_A}{\partial T} \frac{\partial^2 \Phi_A}{\partial X^2} \\ - \frac{G_B}{\pi^3 \Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial^2 \Phi_A}{\partial T \partial X} - \frac{G_B}{\pi^3 \Omega} \frac{\partial^3 \Phi_A}{\partial T \partial X^2} \\ + \frac{G_S}{R \Omega^{1/2}} \frac{\partial \Phi_A}{\partial T} = F_{\text{piston}}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} F_{\text{piston}} = -\frac{\partial^2 \Phi_B}{\partial T^2} + \frac{1}{\Omega^2 \pi^2} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi_B}{\partial X} + \frac{1}{\Omega^2 \pi^2} \frac{\partial^2 \Phi_B}{\partial X^2} \\ - \frac{2}{\Omega} \frac{\partial^2 \Phi_A}{\partial X \partial T} \frac{\partial \Phi_B}{\partial X} - \frac{2}{\Omega} \frac{\partial^2 \Phi_B}{\partial X \partial T} \frac{\partial \Phi_A}{\partial X} - \frac{2}{\Omega} \frac{\partial^2 \Phi_B}{\partial X \partial T} \frac{\partial \Phi_B}{\partial X} \\ - \frac{(\gamma-1)}{\Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi_A}{\partial T} \frac{\partial \Phi_B}{\partial X} - \frac{(\gamma-1)}{\Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \\ \times \frac{\partial \Phi_B}{\partial T} \frac{\partial \Phi_A}{\partial X} - \frac{(\gamma-1)}{\Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial \Phi_B}{\partial T} \frac{\partial \Phi_B}{\partial X} \end{aligned}$$

$$\begin{aligned}
& -\frac{(\gamma-1)}{\Omega} \frac{\partial \Phi_A}{\partial T} \frac{\partial^2 \Phi_B}{\partial X^2} - \frac{(\gamma-1)}{\Omega} \frac{\partial \Phi_B}{\partial T} \frac{\partial^2 \Phi_A}{\partial X^2} \\
& -\frac{(\gamma-1)}{\Omega} \frac{\partial \Phi_B}{\partial T} \frac{\partial^2 \Phi_B}{\partial X^2} - \frac{G_B}{\pi^3 \Omega} \left\{ \frac{1}{S} \frac{dS}{dX} \right\} \frac{\partial^2 \Phi_B}{\partial T \partial X} \\
& -\frac{G_B}{\pi^3 \Omega} \frac{\partial^3 \Phi_B}{\partial T \partial X^2} + \frac{G_S}{R\Omega^{1/2}} \frac{\partial \Phi_B}{\partial T}. \quad (11)
\end{aligned}$$

F_{piston} consists all terms associated with Φ_B relating to the piston motion. Many possible functions can be chosen for Φ_B to satisfy boundary conditions (7). A simple one is taken in the present study,

$$\Phi_B = U^* \frac{X^2}{2} \sin(T). \quad (12)$$

The nonlinear equation (10) for Φ_A , together with the homogeneous boundary conditions (9), are solved by the Galerkin method^{4,6} in this study. Once $\Phi = \Phi_A + \Phi_B$ is obtained, the pressure in the resonator can be evaluated by

$$\begin{aligned}
\frac{P}{P_0} = & \left[1 - (\gamma-1)\pi^2 \left(\Omega \frac{\partial \Phi_A}{\partial T} + \Omega \frac{\partial \Phi_B}{\partial T} + \frac{1}{2} \left(\frac{\partial \Phi_A}{\partial X} + \frac{\partial \Phi_B}{\partial X} \right)^2 \right. \right. \\
& - \frac{G_B}{\pi^3 S} \frac{\partial}{\partial X} \left(S \frac{\partial \Phi_A}{\partial X} + S \frac{\partial \Phi_B}{\partial X} \right) \\
& \left. \left. + \frac{G_S \Omega^{1/2}}{D} (\Phi_A + \Phi_B) \right) \right]^{\gamma(\gamma-1)}. \quad (13)
\end{aligned}$$

The resonance frequency of the resonator can be found in seeking the maximum amplitude of pressure. It can also be estimated from the following formula⁶ for an exponentially expanded resonator:

$$k_n^2 = \left(\frac{\omega_n}{c_0} \right)^2 = \left(\frac{n\pi}{l} \right)^2 + \frac{1}{4} \left(\frac{\alpha}{l} \right)^2 \quad \text{for } n = 1, 2, 3, \dots, \quad (14)$$

where k_n are the nature acoustic wave numbers of different resonance modes and α is the flare constant. For the fundamental mode ($n=1$), the dimensionless resonance frequency is

$$\Omega = \frac{\omega_1}{\omega_0} = \sqrt{1 + \frac{1}{4} \left(\frac{\alpha}{\pi} \right)^2}. \quad (15)$$

B. Calculated results for resonators with piston driving

The cross-sectional area of the resonator is given by

$$S = S_0 e^{\alpha X}, \quad (16)$$

where α is the flare constant indicating the rate of expansion. $\alpha=0$ represents a straight resonator without expansion. The resonators in the following calculation are of the same geometry of $l=0.2$ m, $r_0/l=0.02$, and the gas inside the resonators is air. Figure 2 illustrates the resonance pressure wave forms at the small end ($X=0$) of exponentially expanded resonators with various flare constants. The piston driving strength in this case is $U^*=5 \times 10^{-4}$. It shows sawtooth wave forms with low pressure amplitudes for the

FIG. 2. The wave forms in exponentially expanded resonators with different flare constants. The driving force is $U^*=5 \times 10^{-4}$.

straight ($\alpha=0$) and slightly expanded ($\alpha=2$) resonators, and shock-free wave forms with high pressure amplitudes for highly expanded ($\alpha=4$) resonator. These are typical characteristics in shaped resonators under shaking.

The resonance frequencies of the resonators with different expansion rates α are identified based on the driving frequencies that produce maximum pressure amplitudes. The results are shown in Fig. 3, together with the frequencies calculated by Eq. (15). It is seen that the resonance frequency basically follows Eq. (15) to increase with the expansion rate. A strong driving ($U^*=5 \times 10^{-4}$) increases the resonance frequencies compared to the weak driving ($U^*=5 \times 10^{-6}$), which shows the hardening behavior. The calculated results for $\alpha=5.6$, $\Omega \approx 1.35$ are depicted in Fig. 3 for the purpose of comparison with experimental data.

It is known that high amplitude pressures will be obtained at the small end in a shaped resonator. The ratios of

FIG. 3. The dimensionless resonance frequency of exponentially expanded resonators with different flare constants. The open squares and triangles are the results obtained by solving nonlinear wave equations (10) and (13) at strong and weak driving strengths, respectively. The solid line shows the predicted results by using Eq. (15).

FIG. 4. The ratio of pressure amplitudes at the small end to the big end in exponentially expanded resonators with different flare constants at strong and weak driving forces.

the pressure amplitude at the small end to the pressure amplitude at the big end are calculated for resonators under different expansion rates and driving strengths. The results, plotted in Fig. 4, show that the pressure amplitude ratio increases slowly with the expansion rate in the lower range where the flare constant $\alpha < 5$, and it goes up much faster after $\alpha > 5$, especially under higher strength driving. The pressure amplitude ratio will be used as one of the resonance characteristics for shaped resonators to evaluate the experimental results.

C. Comparison of piston driving with shaking

The piston driving in the present study is compared with the entire shaking on the same resonator shown in Fig. 1. In order to provide a base for the comparison, the strengths in two driving methods must be related. It is assumed that the piston is of the same acceleration as that produced by the shaking on the resonator, i.e., $A^* = U^* \Omega$ (where $A^* = a_0 / l \omega_0^2$).

FIG. 5. The small end pressure wave forms in two exponentially expanded resonators ($\alpha=4$ and 5.75), obtained by the piston driving and shaking, respectively.

TABLE I. Dimensionless acceleration A for shaking and velocity U^* for piston driving in the force set used in Fig. 6.

Force set	1	2	3	4	5
A^*	1×10^{-4}	2×10^{-4}	3×10^{-4}	4×10^{-4}	5×10^{-4}
Ω	1.357	1.361	1.363	1.365	1.367
$U^* = A/\Omega$	7.37×10^{-5}	1.47×10^{-4}	2.20×10^{-4}	2.93×10^{-4}	3.66×10^{-4}

In the case of shaking, the pressures are obtained by solving Eq. (1) under homogeneous boundary conditions at $X=0$ and $X=1$. Figure 5 illustrates the pressure wave forms at the small end of resonators with the flare constant $\alpha=4$ and 5.75 , respectively. The dashed lines are the pressures generated by shaking and the solid lines are produced by piston driving. The driving strengths are $A^* = 1 \times 10^{-4}$ in shaking for both resonators. The corresponding piston velocities are $U^* = 8.42 \times 10^{-5}$ and $U^* = 7.37 \times 10^{-5}$ for $\alpha=4$ and 5.75 resonators, respectively. The piston driving strengths are different for two resonators because of the difference in their resonance frequencies. The results in Fig. 5 show that the piston driving generates similar pressures to that generated by the shaking, in terms of the wave forms and amplitudes. In another comparison, the pressure amplitudes at two ends of a resonator are calculated. The resonator is driven by shaking and piston, respectively, with different strengths. A set of shaking acceleration A^* and the corresponding U^* is denoted as a force set, and five force sets are listed in Table I. The results presented in Fig. 6 show that the piston driving has the same effect as the shaking in generating high amplitude pressures. It should be pointed out that the comparisons conducted here are still qualitative, although the results produced by the two driving methods agree well. This is because the shaking will apply an acceleration uniformly on the gas inside the resonator, while the piston driving only gives the same acceleration to the gas immediately contacting to the piston at $X=1$.

FIG. 6. Comparison of pressures at the small end ($X=0$) and large end ($X=1$) produced by shaking and piston driving. The driving strengths associated with the force sets are listed in Table I.

FIG. 7. (Color online) Schematic illustration of the experimental setup for the axisymmetric resonator.

III. EXPERIMENT

The experimental study consisted of two parts: (1) An axisymmetric resonator driven by a speaker at one end and (2) a low aspect ratio rectangular resonator driven by a piezoelectric disk at one side of the cavity.

A. Experiment setup for axisymmetric resonator

Figure 7 is a schematic illustration of the experiment setup, including a resonator, a speaker box, two pressure sensors, and other supporting equipment. The resonator was made of aluminum alloy and was exponentially expanded with the flare constant $\alpha=5.6$. The resonator had a small end of diameter 8 mm and length 200 mm. The small end of the resonator was sealed and the large end was connected to the speaker box through a 110-mm-diam-hole. A 5 in. 40 W loudspeaker was mounted behind the hole to function as the piston. There was a 1/4 in. microphone behind the speaker to measure the sound pressure levels inside the back space of the box. Since the back space of the box was isolated from the resonator by the speaker diaphragm, the sound pressure level measured there was not directly related to the resonance pressures inside the resonator but served as an indication of the piston driving strength. The signals from a function generator (HP 33120A) were amplified by a power amplifier (CROWN D-75), and then were input to the speaker. Two pressure transducers (PCB 103B12) were fixed at the two ends of the resonator to measure dynamic pressures inside the resonator. The pressure transducer at the small end was denoted as A and the pressures measured from it were denoted as P_A accordingly. Similarly, the pressures measured at the big end were denoted as P_B . The signals from the transducers were received by a signal conditioner (PCB 483A) and analyzed by a signal analyzer (ONO SOKKI CF-5220Z).

B. Experimental results of axisymmetric resonator

The resonance frequency of the resonator was identified by measuring the frequency responses of pressure at the

FIG. 8. The frequency responses measured by pressure P_A at different driving strengths, indicated by the microphone outputs. The resonance frequency is found to be 1100 Hz.

small end under various driving strengths. The results are shown in Fig. 8, in which the sound pressure levels (80, 100, and 120 dB) are measured by the microphone to indicate the driving strength. The resonance frequency was found to be 1100 Hz for all three driving strengths, indicating that the driving in the present study was not strong enough to have significant effect on the resonance frequency. The measured resonance frequency $f=1100$ Hz can be converted to the dimensionless resonance frequency to give $\Omega=1.29$, which is about 4% less than the calculated value shown in Fig. 3. Considering the speaker used in the experiment is not the flat piston assumed in the modeling, the difference of 4% is quite reasonable. The pressures P_A and P_B obtained in the experiment are used to find the ratio of pressure amplitudes between two ends of the resonator, and the results are presented in Fig. 9. It is found from the experimental data that the pressure ratio between two ends of the resonator is about 17.6, which agrees well with the modeling results 17–19, as

FIG. 9. The ratio of pressure amplitudes at the small end to the big end of the resonator measured from the experiment, indicated by open triangles. The dashed line is the mean value ($P_A/P_B=17.6$).

FIG. 10. The pressures at the small end ($X=0$) and the big end ($X=1$) measured in the experiment compared to the calculated values.

indicated in Fig. 4. The results plotted in Fig. 9 show that the pressure amplitude ratios under stronger driving are slightly higher than that under weak driving, which has also been observed in the modeling results shown in Fig. 4.

In order to compare the pressures measured at two ends of the resonator with the calculated values from the modeling, the piston velocity U^* corresponding to the speaker power in the experiment needs to be obtained. This is done based on the fact that the box behind the speaker is sealed, so that the equivalent piston velocity U^* can be assumed to be proportional to the pressure fluctuations behind the speaker, i.e., $U^* = k10^{SPL/20}$, where SPL is the sound pressure level measured by the microphone behind the speaker, k is a constant. In the experiment, ten pressure data from two ends of the resonator were recorded under five different driving SPL values. One datum was used to calibrate for k , so that the piston velocities corresponding to five SPL values were obtained. The calculated pressures under these five piston driving strengths (U^*) were therefore calculated and compared with the experimental data. The results are shown in Fig. 10, in which the predicted pressures at the small end ($X=0$) are compared to the measured values P_A in the experiment, and the predicted pressures at the big end ($X=1$) are compared to P_B measured in the experiment. A good agreement is seen in this comparison.

C. Low aspect ratio rectangular resonator

The boundary driving was further tested by conducting the experiment on a low aspect ratio rectangular resonator. This is to explore the possibility of generating high amplitude dynamic pressures with various driving and resonator configurations, so as to extend their applications, such as in miniature/microfluidic devices. The low aspect ratio configuration is a common feature in miniature fluidic devices fabricated by surface machine technologies. Figure 11 is an assembly drawing of the resonator. The resonator was formed between a cavity plate and a cover plate. The cavity was machined on a brass plate and the depth of the cavity was 3 mm. The profile of the cavity consists of a 3-mm-diam

FIG. 11. Schematic illustration of the low aspect ratio rectangular resonator used in the experiment.

semicircle at the small end, a 50-mm-diam semicircle at the big end, and a section linking two semicircles. The profile of the link section is

$$y = \begin{cases} 1.5e^{0.0583x}, & 0 < x \leq 42.1 \\ \sqrt{25^2 - (60 - x)^2}, & 42.1 < x \leq 60. \end{cases} \quad (17)$$

A piezoelectric disk was mounted, as part of the surface, on the cover plate. The diameter of the disk was 50 mm and the thickness was 0.5 mm. The cover plate was fixed onto the cavity plate, and the resonator was formed with the piezoelectric disk being one sidewall of the resonator cavity. The resonator was under the boundary driving when driving voltages were applied on the piezoelectric disk. The pressure transducers were the same as that used for the axisymmetric resonator: One was located below the small end to give pressure P_A and another was located below the center of the semicircle at the big end to give P_B . The resonance pressures P_A and P_B under different driving voltages, from 2 to 23 V, are plotted out in Fig. 12. It shows that the pressures at both ends are linearly increased with the driving voltages applied on the boundary disk, and the pressures at the small end are much higher than that at the big end, which is a typical characteristic observed in other shaped resonators. The ratio of P_A to P_B is calculated based on the data shown in Fig. 12. The average value for the ratio in this case is found to be 20.2, which is comparable to 17.6 obtained in the exponentially expanded resonator with the flare constant $\alpha=5.6$. The resonance frequency for the low aspect ratio rectangular resonator was measured in the experiment to be 4020 Hz. By taking the distance from the small end to the center of the semicircle at the big end as the characteristic length, which is 60 mm shown in Fig. 11, the dimensionless resonance frequency can be calculated to be $\Omega=1.41$. The measured pres-

FIG. 12. The pressures obtained from the transducers A and B in the low aspect ratio rectangular resonator at different driving voltages applied to the piezoelectric disk. The resonance frequency is 4020 Hz and the average pressure ratio is $P_A/P_B=20.2$.

sure ratio and dimensionless frequency demonstrate that the low aspect ratio rectangular resonator driven by the sidewall has resonance characteristics similar to axisymmetric resonators.

IV. CONCLUSIONS

The generation of high amplitude pressures in shaped resonators by boundary driving has been studied. The configurations of resonators in the study include axisymmetric resonators driven by pistons and a low aspect ratio rectangular resonator driven by a sidewall. In the theoretical modeling, the nonhomogeneous boundary condition associated with the piston driving has been removed by a known function, so that the Galerkin method can be applied. The calculated results, including the resonance frequencies, pressure

wave forms, pressures at the two ends of the resonators and their ratios, are compared with the results generated by shaking the resonators and measured from the experiments. The experimental results on the low aspect ratio rectangular resonator with the sidewall driving show the same resonance characteristics as that observed in axisymmetric resonators. The present study demonstrates that the boundary driving, incorporated with properly designed resonators, could be a promising method to provide useful dynamic pressures for various applications, especially in small scale devices.

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