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Extending the Concept of IIR Filtering to Array Processing Using Approximate Spatial IIR Structure

Fuxi Wen, Boon Poh Ng and Vinod Veera Reddy

Abstract—Various array processing techniques applied to uniform linear arrays are involuntarily realized using structures that are analogous to finite impulse response (FIR) filters. This observation leads to the following question: “can we extend infinite impulse response (IIR) filtering to array processing?”.

In this paper, we introduce the concept of IIR array in spatial domain. Note that IIR array here does not mean time-domain IIR filtering for array beamforming which is commonly understood. This paper is dedicated to the study of an alternate approach for array signal processing which defines IIR structure in spatial domain.

To illustrate the applicability of the concept of IIR array, we propose a new direction-of-arrival (DOA) estimation technique as well as a beamformer with the spatial domain IIR array implementation. The performance of the proposed methods are comparable to the existing techniques. These illustrations are intended to introduce a new approach which potentially can offer more degrees of freedom to control the performance of the array and reduce the complexity of the system for a desired performance.

Index Terms—Array signal processing, infinite impulse response (IIR) array, direction-of-arrival (DOA) estimation.

I. INTRODUCTION

ARRAY processing has been widely used in applications such as sonar, radar and wireless communications. The developments in research over several decades are consolidated in Van Veen and Buckley (1988), Trees (2002), Krim and Viberg (1996) and Godara (1997). Two predominant topics in this community are direction-of-arrival (DOA) estimation and beamforming. While the former estimates the directions of unknown sources from the recorded sensor snapshots, a beamformer employs these estimates to form a beam along the desired source direction and simultaneously suppresses the interferences. This is indeed a spatio-temporal filtering which filters signals corresponding to a specific direction.

Techniques for DOA estimation have evolved from conventional methods such as Capon (Capon 1969) and maximum entropy (ME) (Therrien 1992) to subspace-based methods such as multiple signal classification (MUSIC) (Schmidt 1986) and estimation of signal parameter via a rotational invariant technique (ESPRIT) (Roy and Kailath 1989). The most primitive form of beamforming is the delay-and-sum beamformer

which enhances the narrowband signal from a specific direction. Capon beamformer or minimum variance distortionless response beamformer (MVDR) has been a more versatile beamformer with improved performance and provision for multiple constraints.

Array beamforming can be interpreted as a spatial filter which allows signals from specific directions to pass through it while suppressing the interference signals. This is analogous to a time-domain digital filter which allows signals within its passband. Therefore, the response of the array beamformer (beampattern) is equivalent to the frequency response of the time-domain digital filter as will be shown in Section II. This equivalence arises because a digital filter samples the signal in time-domain whereas an array samples the signal in spatial domain.

With such close relation between array snapshots and discrete-time signals, it is interesting to explore the application of concepts and algorithms developed in different streams of research for array processing. It is well acclaimed that digital filters with desired frequency response can be implemented with either finite impulse response (FIR) structure or infinite impulse response (IIR) structure. In array processing, a one-to-one correspondence of these filter structures with array structures has not been exhaustively studied. While an FIR filter structure processes uniformly sampled discrete-time signal, a uniform linear array (ULA) structure samples the signal at uniform spatial displacement and the corresponding weights process similar to an FIR filter (Van Veen and Buckley 1988). Therefore, the FIR filter response is analogous to the array beampattern. The feasibility of applying FIR filter design techniques in array processing has been explored recently by considering special types of FIR filter design techniques, such as frequency-response masking (FRM) (Liu and Lin 2004, 2006) and interpolated FIR (IFIR) (Vaidyanathan and Weng 2010). In Xiao et al. (2004) a wideband DOA technique is proposed based on the correlation between the sensor measurements of a ULA and a sine or cosine waveform with known frequency.

TABLE I
ANALOGY BETWEEN FILTERING AND BEAMFORMING.

Digital Filter (Time Domain)	Array Beamformer (Spatial Domain)
FIR	Uniform Linear Array
IIR	? ?

Following the analogy between FIR filter structure and ULA, the question in hand is “what is the array structure in spatial domain that is analogous to IIR filter structure?”.

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Table I shows the correspondence between the structures available in temporal and spatial processing. Completion of this table with an equivalent array structure may enable us to extend several filter designs for the design of beamformer. This can potentially extend the advantages of IIR structure to array processing in the following ways: (1) IIR processing provides increased degrees of freedom over conventional array processing to control the array performance. Moreover, the well-established IIR filter design theory can be used to design an IIR array. (2) In comparison with FIR filters, the number of IIR filter taps required for a desired performance is much smaller. Likewise, for a desired array response, we hope that an array designed with IIR structure requires a smaller array aperture than a conventional array. This is reflected in a reduction in cost and complexity of the entire system.

In this paper, we present the possibility of replacing FIR structure with approximate IIR structure for DOA estimation and beamforming applications. We propose a new DOA estimation technique which incorporates a virtual feedback resulting in an IIR structure. While the time delay between the taps of the discrete-time IIR spatial structure in the feedback and feedforward networks is a constant proportional to sampling interval, the analogous delay in spatial domain corresponds to the relative propagation delay across the sensor array which is proportional to the source direction. In the IIR array structure, array snapshots can be directly used for the moving average part of the filter. In order to develop the feedback section of the IIR structure, we propose a virtual structure with an approximate propagation delay (which is dependent on the source direction as discussed). The performance of the proposed technique is observed to have improved performance over the Capon method. This work may provide a platform for a possible subspace based DOA estimation for IIR array structure.

Furthermore, we propose a primitive beamformer with an IIR array implementation. Although the configuration presented in this work is not optimal, the scope of this contribution is to prove the concept of IIR structured beamformer. In order to implement a genuine IIR array, it is necessary to have spatial feedback which can be realized using the concept of subarrays. The output of previous subarray can be used for spatial feedback. The stability of the IIR structure is ensured by using a notch filter structure (Nehorai 1985; Pei and Tseng 1994; Shynk 1986; Stoica and Nehorai 1988; Tseng and Pei 2001; Xiao et al. 2001, 2005; Zhou and Li 2004) which constrains the zeros and poles to lie within the unit circle. Although the beamformer capability is achieved, the subarray feedback requires an increased array size.

The paper is organized as follows. In section II, we review the equivalence between an FIR filter and a uniform linear array. A new DOA estimation technique using spatial IIR notch filter structure is presented in Section III. Section IV discusses the possibility and limitations of IIR structure in spatial domain. Furthermore, we present an implementation of IIR array processing for beamforming application. Conclusions are drawn in Section V.

Notations: To distinguish the input and output signals of the filters in time domain and snapshots in spatial domain,

different time indexes are used. Time index n is used for samples of temporal filters and t is used to denote the time index of snapshots. For example, $x(n)$ denotes the input data sample of a filter at time index n , $x_k(t)$ denotes the snapshot of sensor k at time index t . Superscript $*$ represents complex conjugate, superscript H denotes Hermitian transpose and superscript T denotes transpose, respectively. $E[\cdot]$ is the expectation operator and $\arg(\cdot)$ is an operator to return the phase of a complex number.

II. EQUIVALENCE BETWEEN AN FIR FILTER AND A ULA

In this section, we review the equivalence observed between a temporal FIR filter structure and a ULA in Van Veen and Buckley (1988).

Fig. 1. Spatial sampling of the incoming wavefront by a linear array.

Fig. 1 depicts a ULA of m sensors used for processing narrowband signals. The propagating wavefields in space are sampled by the sensors and are linearly combined to give the array output,

$$y(t) = \sum_{k=0}^{m-1} w_k^* x_k(t) = \mathbf{w}^H \mathbf{x}(t), \quad (1)$$

where $\mathbf{w} = [w_0, w_1, \dots, w_{m-1}]^T$ (for the case of adaptive array, $\mathbf{w}$ is data dependent and is updated as a function of time) and $\mathbf{x}(t) = [x_0(t), x_1(t), \dots, x_{m-1}(t)]^T$ are the weight and data vectors, respectively. It is conventional to multiply the data by conjugates of the weights to simplify notation.

Let θ denote the direction measured w.r.t. the array axis, $\tau_k(\theta)$ is the time delay on sensor k w.r.t. the reference and ω_0 is temporal frequency. The beamformer response is given by

$$H(\theta, \omega_0) = \sum_{k=0}^{m-1} w_k^* e^{j\omega_0 \tau_k(\theta)} = \mathbf{w}^H \mathbf{d}(\theta, \omega_0), \quad (2)$$

where $\mathbf{d}(\theta, \omega_0)$ is known as the steering vector. The correspondence between FIR filtering and beamforming is closest when operated at a single temporal frequency ω_0 and the array geometry is linear and equi-spaced. Now

$$\tau_k(\theta) = \frac{d_k \cos(\theta)}{c} = k \frac{d \cos(\theta)}{c}, \quad (3)$$

where $\{d_k, k = 0, 1, \dots, m-1\}$ is the distance between the k^{th} element and the endfire array element chosen as the reference, d is the inter-element spacing, c is the propagation speed.

Fig. 2. Tapped-delay line implementation of an FIR filter.

For comparison, we now study a temporal $(m-1)^{th}$ order FIR filter shown in Fig. 2. The sampled input signal is delayed by a factor $z^{-1} = e^{-j\omega}$ where ω is the normalized frequency and the delayed samples are linearly combined to get the filter output,

$$y(n) = \sum_{k=0}^{m-1} w_k^* x(n-k). \quad (4)$$

The FIR filter response is given by

$$H(\omega) = \sum_{k=0}^{m-1} w_k^* e^{-j\omega k} = \mathbf{w}^H \mathbf{d}(\omega). \quad (5)$$

Here, the vector $\mathbf{d}(\omega)$ describes the phase of the complex sinusoid at each tap in the FIR filter.

From (2) and (5), the similarity in the structure of a ULA and an FIR filter is apparent. Thus, temporal frequency in an FIR filter corresponds to the cosine of direction in a narrowband, linear, equi-spaced array.

III. DOA ESTIMATION USING SPATIAL IIR NOTCH STRUCTURE

This section provides a study of adaptive IIR notch filter followed by a new DOA estimation technique incorporating adaptive IIR notch filter structure. Comparisons with the Capon and ME methods are shown along with the performance analysis.

A. Adaptive IIR Notch Filtering

A complex m^{th} order IIR notch filter is characterized by the following transfer function (Ko and Li 1994),

$$H(z) = \frac{1 + a_1^* z^{-1} + a_2^* z^{-2} + \dots + a_m^* z^{-m}}{1 + r a_1^* z^{-1} + r^2 a_2^* z^{-2} + \dots + r^m a_m^* z^{-m}}, \quad (6)$$

where $0 < r < 1$, is a constant close to 1, which controls the notch bandwidth and $\{a_1, \dots, a_m\}$ are the weights of the filter. The stability of the IIR notch filter is ensured if all the poles are present within the unit circle.

The complex adaptive IIR notch filtering algorithm (Pei and Tseng 1994) can be used to estimate the weights of the filter. For a complex input signal $x(n)$, the output of the filter is given by

$$y(n) = \frac{A(z^{-1})}{A(rz^{-1})} x(n), \quad (7)$$

where z^{-1} is the unit delay operator, i.e., $z^{-1}x(n) = x(n-1)$. From (7), it is possible to write the following recursive difference equation:

$$y(n) = x(n) + \sum_{k=1}^m a_k^* x(n-k) - \sum_{k=1}^m r^k a_k^* y(n-k). \quad (8)$$

With

$$\boldsymbol{\varphi}(n) = \begin{bmatrix} x(n-1) - r y(n-1) \\ \vdots \\ x(n-m) - r^m y(n-m) \end{bmatrix},$$

the output can be rewritten as

$$y(n) = x(n) + \boldsymbol{\psi}^H(n) \boldsymbol{\varphi}(n), \quad (9)$$

where $\boldsymbol{\psi}(n) = [a_1(n), \dots, a_m(n)]^T$ are the adaptive weights of the IIR notch filter when it is implemented adaptively.

The purpose of using a notch filter is to block the signal present in a particular band. This can be achieved by minimizing the output power of the filter

$$F = E[|y(n)|^2]. \quad (10)$$

The adaptation of filter weights using the gradient descent algorithm is given by

$$\boldsymbol{\psi}(n+1) = \boldsymbol{\psi}(n) - \mu \boldsymbol{\phi}(n) y^*(n), \quad (11)$$

where $\boldsymbol{\phi}(n) = [\frac{\partial y(n)}{\partial a_1^*(n)}, \dots, \frac{\partial y(n)}{\partial a_m^*(n)}]^T$ and $0 < \mu \leq 1$ is the stepsize. Since this algorithm has a very slow convergence to the optimal solution, the inverse of the data correlation matrix $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^H\}$, can be incorporated to accelerate the convergence as follows:

$$\boldsymbol{\psi}(n+1) = \boldsymbol{\psi}(n) - \mu \mathbf{R}^{-1}(n) \boldsymbol{\phi}(n) y^*(n). \quad (12)$$

In order to reduce the computation complexity in evaluating $\mathbf{R}^{-1}$, $\mathbf{P}(n) = \mathbf{R}^{-1}(n)$ is directly updated using matrix inversion lemma:

$$\mathbf{P}(n) = \frac{1}{\lambda} \left[\mathbf{P}(n-1) - \frac{\mathbf{P}(n-1) \boldsymbol{\phi}(n) \boldsymbol{\phi}^H(n) \mathbf{P}(n-1)}{\lambda + \boldsymbol{\phi}^H(n) \mathbf{P}(n-1) \boldsymbol{\phi}(n)} \right]. \quad (13)$$

where λ is the forgetting factor, $0 < \lambda < 1$.

B. The Proposed DOA Estimation Technique

With the equivalence between a discrete-time FIR filter and a ULA studied in Section II, we now introduce an array whose output is obtained from an IIR structure as shown in Fig 3. In order to ensure stability, the above discussed adaptive IIR notch filters is used. The sensor outputs are weighted with the coefficients of the numerator of $H(j\omega)$ in (14). The desired time delay for the auto regressor is obtained approximately which will be discussed later.

To explain this new structure, we consider a ULA with 3 elements which can identify a maximum of 2 directions. We can write an equivalent transfer function for this structure as

$$H(j\omega) = \frac{1 + a_1^* e^{-j\omega\tau(\theta)} + a_2^* e^{-j2\omega\tau(\theta)}}{1 + r a_1^* e^{-j\omega\tau(\theta)} + r^2 a_2^* e^{-j2\omega\tau(\theta)}}, \quad (14)$$

where $\tau(\theta)$ is the inter-element time delay for the source from direction θ .

Fig. 3. The proposed structure for DOA estimation.

Considering the first sensor to be the reference, the snapshots at time index t can be written as

$$x_k(t) = x(t - k\tau(\theta)), k = 0, 1, 2. \quad (15)$$

In order to obtain the feedback component, the array output

$$y_k(t) = y(t - k\tau(\theta)), k = 0, 1, 2, \quad (16)$$

is passed through the virtual delay elements shown in Fig 3. The output signal at time t , based on (8), is given by

$$y(t) = x(t) + \sum_{k=1}^2 \left(a_k^* x(t - k\tau(\theta)) - r^k a_k^* y(t - k\tau(\theta)) \right). \quad (17)$$

Similar to (9), we can write the above equation as

$$y(t) = x(t) + \psi^H(t) \varphi(t). \quad (18)$$

For fast convergence, we use the adaptive algorithm shown in (12) as

$$\psi(t+1) = \psi(t) - \mu \phi(t) \mathbf{P}(t) y^*(t), \quad (19)$$

where $t+1$ denotes one sample delay, $\mathbf{P}(t)$ is the inverse correlation matrix which can be updated using (13) with $\mu = 1$.

For the IIR notch structure used, the coefficients in the numerator and denominator have to be constrained such that the structure is stable. This requires the zeros of the filter structure to be situated on the unit circle and the poles to be placed inside the unit circle (Nehorai 1985). While the DOA of the sources can be estimated by plotting the spatial spectrum, a non-search procedure is to estimate source DOAs from the phase of complex roots of (14), i.e., by setting,

$$1 + a_1^* \exp(-j\omega\tau(\theta)) + a_2^* \exp(-j2\omega\tau(\theta)) = 0. \quad (20)$$

The relationship between the phase of the complex roots α and DOAs θ are $\alpha = 2\pi d \cos(\theta)$, where d is the inter-element spacing of the ULA with regard to the signal wavelength. The complex adaptive notch filter algorithm explained above is summarized in Table II.

TABLE II
COMPLEX ADAPTIVE IIR NOTCH ALGORITHM

Notations:

$$[x_0(t), \dots, x_k(t)] = [x(t), \dots, x(t - k\tau(\theta))]$$

$$[y_0(t), \dots, y_k(t)] = [y(t), \dots, y(t - k\tau(\theta))]$$

$$[\phi_0(t), \dots, \phi_k(t)] = [\phi(t), \dots, \phi(t - k\tau(\theta))]$$

$$[\mathbf{P}_0(t), \mathbf{P}_1(t)] = [\mathbf{P}(t), \mathbf{P}(t - \tau(\theta))]$$

$$\psi(t) = [a_1(t), a_2(t), \dots, a_k(t)]^T$$

Initialization:

For $t = 1, \dots, k$

$$\lambda(t) = 0.1, \lambda_1 = 0.55, \lambda_\infty = 0.95$$

$$r(t) = 0.1, r_1 = 0.55, r_\infty = 0.95$$

$$y(t), \phi(t), \psi(t) = 0, \mathbf{P}(t) = \mathbf{I}$$

Main iteration loop:

For $t = k+1, \dots, N$, where N is the number of snapshots.

$$\varphi(t) = [\varphi_1(t), \dots, \varphi_k(t)]^T$$

$$\varphi_i(t) = x_i(t) - r^i y_i(t), i = 1, 2, \dots, k$$

$$y(t) = x(t) + \psi^H(t) \varphi(t)$$

$$\phi(t) = \varphi(t) + r a_1^* \phi_1(t) + \dots + r^k a_k^* \phi_k(t)$$

$$\mathbf{P}(t) = \frac{1}{\lambda} [\mathbf{P}_1(t) - \frac{\mathbf{P}_1(t) \phi(t) \phi^H(t) \mathbf{P}_1(t)}{\lambda + \phi^H(t) \mathbf{P}_1(t) \phi(t)}]$$

$$\psi(t+1) = \psi(t) - \mathbf{P}(t) \phi(t) y^*(t)$$

$$\lambda(t+1) = \lambda_1 \lambda(t) + (1 - \lambda_1) \lambda_\infty$$

$$r(t+1) = r_1 r(t) + (1 - r_1) r_\infty$$

C. Simulation Results

In this section, we evaluate the proposed method with simulated and experimental data. A ULA formed by three omnidirectional sensors with half-wavelength inter-element spacing is used to estimate two source directions. Additive white Gaussian noise (AWGN) is added to the simulated sensor outputs and the signal-to-noise ratio (SNR) is 30 dB.

In the first simulation, the array acquires data from two closely-spaced sources situated at 50° and 60° direction with respect to the array axis. The proposed algorithm operates on 1000 array snapshots to obtain DOA estimates. Fig. 4 shows the spatial spectrum of the proposed technique averaged over 500 trials in comparison with the Capon and ME methods. Since the concept of subspace is not yet defined for IIR structure, we limit our comparison to conventional methods such as the Capon and ME methods. In Fig. 4, it is interesting to note that the resolution of the proposed method is better than the Capon method which fails to resolve the two sources and is comparable with that of ME method. We notice that the spectrum of the proposed technique in source-free directions is better than the other two techniques. This phenomenon can be attributed to the increase in degrees of freedom obtained from the IIR structure over the FIR structure adopted by

the other techniques. The degrees of freedom available for Capon and ME methods are utilized to give distinct peaks along the source directions. However, besides providing the distinct source direction peaks in the spatial spectrum, the proposed technique can suppress the spectrum floor along all other directions due to the increased degrees of freedom.

In order to study this side-lobe suppression performance, we simulate another example where the sources are located at 60° and 90° direction with respect to the array axis. Since the source are farther apart, the Capon method and the ME method are expected to have better response along the source-free directions. Fig. 5 asserts this understanding, where the spatial spectrum of Capon method and the ME method have improved spatial spectrum floor. However, we notice that the proposed technique has a better flat response along the source-free directions than the other techniques.

We next subject experimentally acquired data from 3 sensors to the proposed method, the Capon method and the MUSIC algorithm. The sources are situated in the far-field region at angles 44° and 90° from the array axis. Since the angular separation between the sources is large, Fig. 6 shows that all the techniques succeed in estimating accurate directions with as low as 50 snapshots. We observe that all the techniques have similar performance. In order to study the effect of increase in the number of snapshots, we plot the spatial spectrum for the same scenario for 100 snapshots in Fig. 7. The comparison of the spatial spectra in Fig. 6 and Fig. 7 unveils that the performance of all the techniques improve with the number of snapshots. It is interesting to know that the floor of spatial spectrum for the proposed technique is better than the other two techniques which can be credited to the increase in the degrees of freedom obtained by the IIR structure.

Fig. 4. DOA estimation using different methods with sources from 50° and 60° .

D. Performance Analysis

The performance of the proposed DOA estimator can be analyzed on the same lines as that provided in Xiao et al. (2001) and Xiao et al. (2005). We here obtain a closed form solutions of bias and MSE for the proposed estimator. For

Fig. 5. DOA estimation using different methods with sources from 60° and 90° .

Fig. 6. DOA estimation using real data, 50 snapshots are used.

Fig. 7. DOA estimation using real data, 100 snapshots are used.

convenience, a first order IIR notch filter is considered. Its transfer function is given by

$$H_N(z) = \frac{1 + a^* z^{-1}}{1 + r a^* z^{-1}}, \quad (21)$$

where $r \in (0, 1)$ is a pole contraction factor which controls the notch bandwidth of the filter and a is the filter coefficient. Here, this first order transfer function corresponds to a 2 sensor array. For the nulling purpose, it is necessary that $a = -\exp(-j\pi \cos(\theta)) = -\exp(-j\omega)$, with θ being the source direction. The output of the sensors (a snapshot) is given by

$$\mathbf{x}(t) = A(\theta)s(t) + v(t), \quad (22)$$

where θ is the direction of the signal, $A(\theta) = [1 \ a]^T$ is the steering vector of the array corresponds to a source at direction θ , $v(t)$ is an additive white Gaussian noise with variance δ_v^2 .

Using the steepest descent algorithm explained in Section III(B)(11), the coefficient is updated adaptively as follows:

$$\hat{a}(t+1) = \hat{a}(t) - \mu\phi(t)y^*(t), \quad (23)$$

where $\hat{a}(t)$ is the filter (DOA estimator) coefficient estimated at time instant t , the gradient signal $\phi(t)$ is the output of the gradient filter (Pei and Tseng 1994)

$$H_S(z) \simeq z^{-1} - rz^{-1}H(z) = \frac{(1-r)z^{-1}}{1+raz^{-1}}. \quad (24)$$

The filter coefficient $\hat{a}(t)$ will converge to its ideal value a_0 when

$$f(\hat{a}) = E[\phi(t)y(t)] = 0. \quad (25)$$

Using the Taylor series expansion, $f(\hat{a})$ can be approximated in the vicinity of a_0 as follows:

$$\begin{aligned} f(\hat{a}) &\simeq f(a_0) + (\hat{a} - a_0)f'(a)|_{a=a_0} + (\hat{a} - a_0)^2 f''(a)|_{a=a_0} \\ &= f(a_0) + \delta_a f'(a)|_{a=a_0} + \delta_a^2 f''(a)|_{a=a_0}. \end{aligned} \quad (26)$$

Here, $\delta_a = \hat{a} - a_0$, is the estimation error of the filter coefficient. The expansion terms with $\{\delta_a^k | k \geq 3\}$ are neglected.

The bias of the DOA estimator is given by

$$E[\delta_a(\infty)] = \frac{\eta_{12}E[\delta_a^2(\infty)]}{\eta_{11}} - \frac{R_{12}}{\eta_{11}}, \quad (27)$$

where $E[\delta_a^2(\infty)]$ is given in (28), R_{12} is given in (66), η_{11} and η_{12} are given in (68) and (69), respectively. The detailed derivation is given in the Appendix. The first term of (27) is associated with the estimated mean square error (MSE) which converges to a small value by choosing a small and proper step size. However the second term is a constant indicating that the DOA estimate obtained by the proposed method is biased.

Fig. 8 shows the plot of bias as a function of the number of snapshots for two SNR values of 20dB and 30dB. We observe that the bias estimated over 500 trials for the proposed method follows closely with the theoretical values. The bias of ME estimator is observed to be better than the bias of DOA estimates obtained by the proposed technique.

The MSE of the algorithm is given by:

$$E[\delta_a^2(\infty)] = \frac{\frac{R_{12}^2}{\eta_{11}} - \mu(\frac{\eta_{23}}{\eta_{11}}R_{12} - \eta_{24})}{\eta_{21} + \frac{\eta_{12}R_{12}}{\eta_{11}} - \mu(\eta_{22} + \frac{\eta_{12}\eta_{23}}{\eta_{11}})}, \quad (28)$$

where R_{12} is given in (66), η_{11} and η_{12} are given in (68) and (69), respectively. η_{21} , η_{22} , η_{23} and η_{24} are given in (75), (77),

Fig. 8. Bias of the IIR notch estimator.

Fig. 9. Standard deviation of the IIR notch estimator.

(78) and (79), respectively. The detailed derivation is given in the Appendix.

Fig. 9 shows the standard deviation for these two methods along with the Capon algorithm. It can be observed that the variance of the proposed method is better than the other two methods.

In summary, the proposed method has better noise suppression than the other conventional methods such as Capon and ME. Resolution of MUSIC is observed to be better than the proposed method. The reason being that the proposed method does not use subspace-based mechanism. The order selection problem observed in other DOA estimation techniques is also observed in this method. The order mismatch issue which can arise with less number of sources present in the scenario, can be solved by considering more snapshots.

Though the performance of the IIR notch method is limited, the purpose here is to introduce the IIR structure (in spatial domain) for array processing which has not been used in the literature. It is now appropriate to question whether the structure in Fig 3 is truly an IIR array. The next section explains the approximation made for this structure.

IV. IMPLEMENTATION OF IIR ARRAY PROCESSING

With the array assumed to be situated in the farfield of directional sources, the planar wavefront is spatially sampled by the sensors. The sensor output captures the effect of signal propagation as composite time delay $\tau(\theta)$ obtained due to multiple sources in the scenario. For the case where a ULA with 3 sensors acquires signal from a single source, the time delays with respect to the reference sensor are $\tau(\theta)$ and $2\tau(\theta)$.

The discrete-time IIR filter is characterized with poles and zeros. The positioning of zeros, or equivalently the nulls along the signal direction blocks the signal from that direction resulting in minimum array output power. The poles of IIR filter transform to spatial feedback for spatial domain IIR structure. In order to provide feedback, the array output has to be spatially delayed by a factor of $\tau(\theta)$. However, the output $y(t)$ is obtained by compensating for all the spatial delays and there is no means to preserve this information. Therefore, in Section III, we have provided an approximate temporal delay instead of a delay corresponding to the source direction to obtain $y_1(t)$ and $y_2(t)$.

In order to implement an IIR array, spatial feedback has to be provided. This can be realized using the concept of subarrays. The output of preceding subarray can be interpreted as a spatially delayed output of the subarray of interest and can be used for the IIR structure. The following details the implementation of this structure.

In Section II, the equivalence between the output of a ULA and an FIR filter output is discussed. Likewise, we here present the design of an IIR structure using ULA for beamforming application. This provides a prototype of the spatial IIR array.

Fig. 10. The structure of the IIR array.

Consider a ULA with sufficiently large number of identical sensors. We divide this array into overlapping subarrays of 3 sensors each, such that the sensors $\{0, 1, 2\}$ form the first sub array, sensors $\{1, 2, 3\}$ form the second subarray and so on as shown in Fig.10. For an array of n elements, $n - 2$ subarrays of 3-element ULA can be formed. For each subarray, a second order constrained IIR notch array processing structure is implemented. The output of the $(n - 2)^{th}$ subarray (the subarray with the snapshot $[x_{n-2}(t) \ x_{n-1}(t) \ x_n(t)]^T$ being processed) at time t is given by

$$y_n(t) = x_n(t) + a_1^* x_{n-1}(t) + a_2^* x_{n-2}(t) - r a_1^* y_{n-1}(t) - r^2 a_2^* y_{n-2}(t). \quad (29)$$

For the first subarray, the feedback components $y_1(t)$ and $y_2(t)$ are zeros reducing $y_3(t)$ to conventional FIR structured beamformer outputs. For the second subarray, the feedback component $y_2(t)$ is zero. For any $n > 4$, the output of the $(n - 2)^{th}$ subarray is obtained using spatial feedback from the previous subarray unlike the structure adopted for DOA estimation where approximate temporal delay was used instead.

It is important to note that $y_n(t)$ is obtained using $y_{n-1}(t)$ and $y_{n-2}(t)$, likewise, $y_{n-1}(t)$ from $y_{n-2}(t)$ and $y_{n-3}(t)$, and so on. This effectively makes the impulse response of the last subarray to depend on all the subarray outputs and hence establishes the IIR structure. The output of the last subarray is thus the output of the entire IIR structured array. It is apparent from (29) that, a second order IIR array can be implemented with a minimum of 5 sensors. From other experiments, it was noticed that an array with larger aperture is necessary for the algorithm to converge.

In order to develop an IIR beamformer using spatial feedback, consider the response of one of the 3-element subarrays given by

$$H(f, \theta) = \frac{1 + a_1^* e^{-j2\pi f \tau(\theta)} + a_2^* e^{-j2\pi f 2\tau(\theta)}}{1 + r a_1^* e^{-j2\pi f \tau(\theta)} + r^2 a_2^* e^{-j2\pi f 2\tau(\theta)}}. \quad (30)$$

The position of the zeros in the transfer function is critical in either suppressing the signal or filtering it. If the zero corresponding to a source direction is positioned within the unit circle, the array will introduce a deep null along the direction, thus blocking the signal. On the contrary, if the zero corresponding to a source is located outside the unit circle, the signal passes through the filter without any distortion. Therefore, a beamformer can be designed by positioning the zeros of the IIR filter such that only the desired source is beamformed while suppressing all the interferences. With $z = e^{j2\pi f \tau(\theta)}$, the zeros for the notch filter can be evaluated with

$$1 + a_1^* z^{-1} + a_2^* z^{-2} = (1 - c_1 z^{-1})(1 - c_2 z^{-1}) = 0. \quad (31)$$

Since the zero in the desired look direction θ_0 should be placed outside the unit circle,

$$|1 - c_1 z^{-1}| > 0 \Rightarrow |c_1| > 1. \quad (32)$$

For a half-wavelength spacing ULA, we have $z = e^{j\pi \cos(\theta)}$ and hence $c_1 = r_c e^{j\pi \cos(\theta)}$, where $r_c > 1$ is a constant which ensures that the zero is situated outside the unit circle. From (31) and (32), it can be shown that

$$a_2^* = -c_1 a_1^* - c_1^2. \quad (33)$$

The weight a_1 can be adaptively estimated by following the algorithm in Table II. This results in suppressing the interference and bypassing the desired signal.

We validate this structure for beamforming with an simulation where the narrowband desired signal impinges the array from the broadside of the 32-element ULA in the presence of an interference from 60° . The desired signal is a complex amplitude modulated (AM) signal, while the interference is a random signal with Gaussian distribution. Sensor noise is simulated with AWGN at an SNR of 30dB. The array is

divided into overlapping subarrays with each subarray forming a ULA of 3 sensors. The real part of the desired signal and one of the sensor outputs are shown in Fig. 11(a) and Fig. 11(b), respectively. The IIR structured beamformer output is shown in Fig. 11(c). It can be observed that the desired signal is reconstructed at the array output with significant interference suppression.

The array directional response in Fig. 12(a) shows that the IIR structure introduces a deep null along the interference direction and simultaneously maintains a flat response along the look direction. The poles and zeros of the IIR notch beamformer are shown in Fig. 12(b). We note that one of the zeros of the system transfer function is outside the unit circle while the other zero is situated on the unit circle. While the position of the zero corresponding to the source direction is constrained, the other zero as well as the positions of poles are adaptively regulated by the remaining parameters. The phase of the zeros and poles implicitly relate to the signal direction which requires further investigation.

We observe that the constrained notch beamformer does not have any signal cancellation problem. Initial tests reveal that the method is more robust when compared to conventional arrays such as Capon beamformer. As is evident, one of the limitations of this structure is the necessity of recursive spatial components, thus requiring a large aperture. However, the main aim of this work has been to identify the possibility of extending IIR filtering to IIR array beamforming. With afore explained structure, we infer that IIR arrays are indeed realizable for practical applications. This provides the scope for further research towards the development of efficient IIR array structure for array processing which can improve the performance and cost of the system.

Fig. 11. The (a) desire signal, (b) input signal on sensor 1 and (c) output signal of the IIR array.

V. CONCLUSION

This work studies the possibility of using IIR structure for array processing. A new IIR structure-based DOA estimation technique has been proposed. The performance of the proposed method is better than the conventional DOA estimation methods, such as Capon and ME. Furthermore, an IIR array

Fig. 12. The spatial response and the z-plane of the IIR array.

beamformer is implemented in the spatial domain using sub-arrays. The results presented in this paper demonstrate the possibility of extending IIR filtering to IIR array processing, but with the following challenges:

- (i) Choice of optimal IIR structure for array processing.
- (ii) Extension of subspace techniques to IIR structure to enhance the performance of the IIR structure based DOA estimation techniques.
- (iii) Algorithm development for IIR beamformer synthesis.

APPENDIX PROOF FOR THE BIAS AND MSE OF THE PROPOSED METHOD

Using the Taylor series expansion on the IIR notch filter $H_N(z)$ and the gradient filter $H_S(z)$ in the vicinity of a_0 , $H_N(z)$ and $H_S(z)$ defined in (21) and (24) respectively, can now be expressed as

$$H_N(z) \approx \left. \frac{\partial H_N(z)}{\partial a} \right|_{a=a_0} (\hat{a} - a_0) + \left. \frac{\partial^2 H_N(z)}{\partial a^2} \right|_{a=a_0} (\hat{a} - a_0)^2, \quad (34)$$

$$H_S(z) \approx \left. \frac{\partial H_S(z)}{\partial a} \right|_{a=a_0} (\hat{a} - a_0) + \left. \frac{\partial^2 H_S(z)}{\partial a^2} \right|_{a=a_0} (\hat{a} - a_0)^2, \quad (35)$$

where the first order and second order gradient of $H_N(z)$ and $H_S(z)$ are given by

$$\frac{\partial H_N(z)}{\partial a} = \frac{(1-r)z^{-1}}{(1+raz^{-1})^2}, \quad (36)$$

$$\frac{\partial^2 H_N(z)}{\partial a^2} = \frac{r(1-r)z^{-2}}{(1+raz^{-1})^3}, \quad (37)$$

$$\frac{\partial H_S(z)}{\partial a} = z^{-1} - \frac{r(1-r)z^{-2}}{(1+raz^{-1})^2}, \quad (38)$$

$$\frac{\partial^2 H_S(z)}{\partial a^2} = \frac{r^2(1-r)z^{-3}}{(1+raz^{-1})^3}. \quad (39)$$

If the expansion terms higher than the second order of $H_N(z)$ are neglected, we have

$$H_N(z) \approx \frac{(1-r)z^{-1}}{(1+raz^{-1})^2}(\hat{a} - a_0) + \frac{r(1-r)z^{-2}}{(1+raz^{-1})^3}(\hat{a} - a_0)^2, \quad (40)$$

$$H_N(e^{j\omega_0}) \approx \frac{(1-r)e^{-j\omega_0}}{(1+rae^{-j\omega_0})^2}(\hat{a} - a_0) + \frac{r(1-r)e^{-j2\omega_0}}{(1+rae^{-j\omega_0})^3}(\hat{a} - a_0)^2 = H_{11}e^{j\omega_{11}}\delta_a + H_{12}e^{j\omega_{12}}\delta_a^2, \quad (41)$$

where H_{11} , ω_{11} , H_{12} and ω_{12} are defined as

$$H_{11} = \left| \frac{(1-r)e^{-j\omega_0}}{(1+ra_0e^{-j\omega_0})^2} \right|, \quad (42)$$

$$\omega_{11} = \arg \left(\frac{(1-r)e^{-j\omega_0}}{(1+ra_0e^{-j\omega_0})^2} \right), \quad (43)$$

$$H_{12} = \left| \frac{r(1-r)e^{-j2\omega_0}}{(1+ra_0e^{-j\omega_0})^3} \right|, \quad (44)$$

$$\omega_{12} = \arg \left(\frac{r(1-r)\exp^{-j2\omega_0}}{(1+ra_0e^{-j\omega_0})^3} \right). \quad (45)$$

If the expansion terms higher than second order of $H_S(z)$ are neglected, then

$$H_S(z) \approx z^{-1} - \frac{r(1-r)z^{-2}}{(1+raz^{-1})^2}(\hat{a} - a_0) + \frac{r^2(1-r)z^{-3}}{(1+raz^{-1})^3}(\hat{a} - a_0)^2, \quad (46)$$

$$H_S(e^{j\omega_0}) \approx e^{-j\omega_0} - \frac{r(1-r)e^{-j2\omega_0}}{(1+rae^{-j\omega_0})^2}(\hat{a} - a_0) + \frac{r^2(1-r)e^{-j3\omega_0}}{(1+rae^{-j\omega_0})^3}(\hat{a} - a_0)^2 = e^{-j\omega_0} - H_{21}e^{j\omega_{21}}\delta_a + H_{22}e^{j\omega_{22}}\delta_a^2, \quad (47)$$

where H_{21} , ω_{21} , H_{22} and ω_{22} are defined as

$$H_{21} = \left| \frac{r(1-r)e^{-j2\omega_0}}{(1+ra_0e^{-j2\omega_0})^2} \right|, \quad (48)$$

$$\omega_{21} = \arg \left(\frac{r(1-r)e^{-j2\omega_0}}{(1+ra_0e^{-j2\omega_0})^2} \right), \quad (49)$$

$$H_{22} = \left| \frac{r^2(1-r)e^{-j3\omega_0}}{(1+ra_0e^{-j\omega_0})^3} \right|, \quad (50)$$

$$\omega_{22} = \arg \left(\frac{r^2(1-r)e^{-j3\omega_0}}{(1+ra_0e^{-j\omega_0})^3} \right). \quad (51)$$

As a result, the output signal $y(t)$ of the notch filter $H_N(z)$ with input signal $x(t)$ defined in (22) is given by

$$y(t) = H_{11}\delta_a(t)A(\omega_0 - \omega_{11})s(t) + H_{12}\delta_a^2(t)A(\omega_0 - \omega_{12})s(t) + v_1(t), \quad (52)$$

where $v_1(t)$ is the output of $v(t)$ defined in (22) through the notch filter $H_N(z)$. The variance of noise $v_1(t)$ of the notch filter is calculated by

$$E[v_1^2(t)] = \frac{\delta_v^2}{2\pi j} \oint_C H_N(z)H_N(z^{-1})z^{-1}dz = \frac{\delta_v^2}{2\pi j} \oint_C \frac{z+a_0}{z+ra_0} \frac{1+a_0z}{1+ra_0z} \frac{1}{z} dz \quad (53)$$

where δ_v^2 is the variance of noise $v(t)$ which is defined in (22). From (53) we can see that the three poles of $E[v_1^2(t)]$ are $z_0 = 0$, $z_1 = -ra_0$ and $z_2 = -\frac{1}{ra_0}$.

Applying the residue theorem to the above equation, one has

$$Res(z_0) = \frac{1}{r}, \quad (54)$$

$$Res(z_1) = \frac{ra_0^2 - 1}{ra_0(1 - r^2a_0^2)}, \quad (55)$$

$$Res(z_2) = \frac{ra_0^2 - 1}{ra_0(1 + r^2a_0^2)}, \quad (56)$$

where $Res(z_0)$, $Res(z_1)$ and $Res(z_2)$ are the residues of the three poles. We then have

$$\delta_{v1}^2 = \delta_v^2 \{ Res(0) + Res(z_1) + Res(z_2) \} = \frac{\delta_v^2}{r} + \frac{2(ra_0^2 - 1)}{ra_0(1 - r^2a_0^2)(1 + r^2a_0^2)} \delta_v^2. \quad (57)$$

The output signal of $x(t)$ through the gradient filter $H_S(z)$ at steady state is given by

$$\varphi(t) = s(t) - H_{21}\delta_a(t)A(\omega_0 - \omega_{21})s(t) + H_{22}\delta_a^2(t)A(\omega_0 - \omega_{22})s(t) + v_2(t), \quad (58)$$

where $v_2(t)$ is the output of $v(t)$ defined in (22) through the gradient filter $H_S(z)$. The variance of $v_2(t)$ can be calculated by the method used in (53)

$$E[v_2^2(t)] = \frac{\delta_v^2}{2\pi j} \oint_C H_S(z)H_S(z^{-1})z^{-1}dz = \frac{\delta_v^2}{2\pi j} \oint_C \frac{(1-r)^2}{(z+ra_0)(1+ra_0z)} dz, \quad (59)$$

where $z_1 = -ra_0$, $z_2 = -\frac{1}{ra_0}$, again applying the theory of residues to the above equation, one has

$$Res(z_1) = \frac{1-r^2}{1-r^2a_0^2}, \quad (60)$$

$$Res(z_2) = \frac{-ra_0(1-r^2)}{1-r^2a_0^2}, \quad (61)$$

then

$$\delta_{v2}^2 = \delta_v^2 \{ Res(z_1) + Res(z_2) \} = \frac{(1-r^2)}{1+ra_0}. \quad (62)$$

It is seen that the variance δ_{v2}^2 will be very small as long as the poles of the notch filter are varying close to the unit circle (r is close to unit), implying that the additive noise at the filter input passes the notch filter with little change in energy.

The correlation between $v_1(t)$ and $v_2(t)$ can be expressed by

$$\begin{aligned} E[v_1(t)v_2(t)] &= \frac{\delta_v^2}{2\pi j} \oint_C H_N(z)H_S(z^{-1})z^{-1}dz \\ &= \frac{\delta_v^2}{2\pi j} \oint_C \frac{(z+a_0)}{(z+ra_0)} \frac{(1-r)}{(1+ra_0z)} dz. \end{aligned} \quad (63)$$

Similarly, using the theory of residues, the new residues are

$$Res(z_1) = \frac{a_0(1-r)^2}{1-r^2a_0^2}, \quad (64)$$

$$Res(z_2) = \frac{(1-r)(1-ra_0^2)}{1-r^2a_0^2}, \quad (65)$$

which leads to

$$\begin{aligned} R_{12} &= E[v_1(t)v_2(t)] \\ &= \delta_v^2 \{Res(z_1) + Res(z_2)\} \\ &= \frac{[1+a_0(1-r)-ra_0^2](1-r)}{1-r^2a_0^2} \delta_v^2. \end{aligned} \quad (66)$$

A. Bias Analysis of the Proposed DOA Estimator

We first define $\delta_a = \hat{a} - a_0$ as the estimation error of the filter coefficient. The instant value can be estimated by (23). Taking expectation on both side of (23), we get

$$\begin{aligned} E[\delta_a(t+1)] &= E[\delta_a(t)] - \mu E[y(t)\varphi(t)] \\ &\simeq \left\{1 - \mu H_{11}A(\omega_0 - \omega_{11})s^2(t)\right\} E[\delta_a(t)] \\ &\quad + \mu \left\{[H_{11}H_{21}A(2\omega_0 - \omega_{11} - \omega_{21}) \right. \\ &\quad \left. - H_{12}A(\omega_0 - \omega_{12})]s^2(t)\right\} E[\delta_a^2(t)] - \mu R_{12} \\ &= (1 - \mu\eta_{11})E[\delta_a(t)] + \mu\eta_{12}E[\delta_a^2(t)] \\ &\quad - \mu R_{12}, \end{aligned} \quad (67)$$

where $y(t)$ and $\varphi(t)$ are given in (52) and (58) and

$$\eta_{11} = H_{11}A(\omega_0 - \omega_{11})s^2(t), \quad (68)$$

$$\begin{aligned} \eta_{12} &= [H_{11}H_{21}A(2\omega_0 - \omega_{11} - \omega_{21}) \\ &\quad - H_{12}A(\omega_0 - \omega_{12})]s^2(t), \end{aligned} \quad (69)$$

where H_{11} is given in (42), H_{12} and H_{21} are given in (44) and (48), respectively. Terms of $\{\delta_a^k(t) | k \geq 3\}$ are ignored for analytical simplicity as they are very small in comparison with $E[\delta_a(t)]$ and $E[\delta_a^2(t)]$. At the steady state, using

$$E[\delta_a(t+1)]|_{t \rightarrow \infty} = E[\delta_a(t)]|_{t \rightarrow \infty} = E[\delta_a(\infty)], \quad (70)$$

and

$$E[\delta_a^2(t)]|_{t \rightarrow \infty} = E[\delta_a^2(\infty)], \quad (71)$$

we have

$$E[\delta_a(\infty)] = \frac{\eta_{12}E[\delta_a^2(\infty)]}{\eta_{11}} - \frac{R_{12}}{\eta_{11}}, \quad (72)$$

where R_{12} is given in (66), η_{11} and η_{12} are given in (68) and (69), respectively. The expression for $E[\delta_a^2(\infty)]$ is derived in (82) in the following section.

B. MSE Analysis of the Proposed DOA Estimator

The MSE is defined as

$$\begin{aligned} E[\delta_a^2(t+1)] &= E[(\delta_a(t) - \mu y(t)s(t))^2] \\ &= E[\delta_a^2(t)] - 2\mu E[\delta_a(t)y(t)s(t)] \\ &\quad + \mu^2 E[y^2(t)s^2(t)] \\ &= E[\delta_a^2(t)] - 2\mu I_1 + \mu^2 I_2, \end{aligned} \quad (73)$$

μ is the step size, I_1 and I_2 are derived as follows:

$$\begin{aligned} I_1 &\simeq H_{11}s^2(t)A(\omega_0 - \omega_{11})E[\delta_a^2(t)] + R_{12}E[\delta_a(t)] \\ &= \eta_{21}E[\delta_a^2(t)] + R_{12}E[\delta_a(t)], \end{aligned} \quad (74)$$

where

$$\eta_{21} = H_{11}s^2(t)A(\omega_0 - \omega_{11}). \quad (75)$$

$$\begin{aligned} I_2 &\simeq s^2(t)\delta_{v_1}^2 + \delta_{v_1}^2\delta_{v_2}^2 + 2R_{12}^2 \\ &\quad + 4s^2(t)R_{12}H_{11}A(\omega_0 - \omega_{11})E[\delta_a(t)] \\ &\quad - 2s^2(t)\delta_{v_1}^2H_{21}A(\omega_0 - \omega_{21})E[\delta_a(t)] \\ &\quad + s^2(t)\delta_{v_1}^2H_{21}^2A(2\omega_0 - 2\omega_{21})E[\delta_a^2(t)] \\ &\quad + 2s^2(t)\delta_{v_1}^2H_{22}A(\omega_0 - \omega_{22})E[\delta_a^2(t)] \\ &\quad - 4s^2(t)R_{12}H_{11}H_{21}A(2\omega_0 - \omega_{11} - \omega_{21})E[\delta_a^2(t)] \\ &\quad + 4s^2(t)R_{12}H_{12}A(\omega_0 - \omega_{12})E[\delta_a^2(t)] \\ &\quad + s^4(t)H_{11}^2A(2\omega_0 - 2\omega_{11})E[\delta_a^2(t)] \\ &\quad + s^2(t)\delta_{v_2}^2H_{11}^2A(2\omega_0 - 2\omega_{11})E[\delta_a^2(t)] \\ &= \eta_{22}E[\delta_a^2(t)] + \eta_{23}E[\delta_a(t)] + \eta_{24}, \end{aligned} \quad (76)$$

where

$$\begin{aligned} \eta_{22} &= s^2(t)\delta_{v_1}^2H_{21}^2A(2\omega_0 - 2\omega_{21}) + 2s^2(t)\delta_{v_1}^2H_{22}A(\omega_0 - \omega_{22}) \\ &\quad - 4s^2(t)R_{12}H_{11}H_{21}A(2\omega_0 - \omega_{11} - \omega_{21}) \\ &\quad + 4s^2(t)R_{12}H_{12}A(\omega_0 - \omega_{12}) + s^4(t)H_{11}^2A(2\omega_0 - 2\omega_{11}) \\ &\quad + s^2(t)\delta_{v_2}^2H_{11}^2A(2\omega_0 - 2\omega_{11}), \end{aligned} \quad (77)$$

$$\begin{aligned} \eta_{23} &= 4s^2(t)R_{12}H_{11}A(\omega_0 - \omega_{11}) \\ &\quad - 2s^2(t)\delta_{v_1}^2H_{21}A(\omega_0 - \omega_{21}), \end{aligned} \quad (78)$$

$$\eta_{24} = s^2(t)\delta_{v_1}^2 + \delta_{v_1}^2\delta_{v_2}^2 + 2R_{12}^2. \quad (79)$$

The terms of $\{\delta_a^k(t) | k \geq 3\}$ are all ignored for analytical simplicity. The mean square error is given by

$$\begin{aligned} E[\delta_a^2(t+1)] &= (1 - \mu\eta_{21} + \mu^2\eta_{22})E[\delta_a^2(t)] \\ &\quad - (\mu R_{12} - \mu^2\eta_{23})E[\delta_a(t)] + \mu^2\eta_{24}, \end{aligned} \quad (80)$$

where η_{21} is given in (75), R_{12} is given in (66), η_{22} , η_{23} and η_{24} are given in (77), (78) and (79), respectively.

At steady state,

$$\begin{aligned} (\eta_{21} - \eta_{22})E[\delta_a^2(\infty)] &= -(R_{12} - \mu\eta_{23})E[\delta_a(\infty)] \\ &\quad + \mu\eta_{24}. \end{aligned} \quad (81)$$

Substitute $E[\delta_a(\infty)]$ into the above equation, we get the estimation MSE of the algorithm as follows:

$$E[\delta_a^2(\infty)] = \frac{\frac{R_{12}^2}{\eta_{11}} - \mu(\frac{\eta_{23}}{\eta_{11}}R_{12} - \eta_{24})}{\eta_{21} + \frac{\eta_{12}R_{12}}{\eta_{11}} - \mu(\eta_{22} + \frac{\eta_{12}\eta_{23}}{\eta_{11}})}, \quad (82)$$

where R_{12} is given in (66), η_{11} and η_{12} are given in (68) and (69), respectively. η_{21} , η_{22} , η_{23} and η_{24} are given in (75), (77), (78) and (79), respectively. Substitute $E[\delta_a^2(\infty)]$ back to (72), the bias can be finally evaluated.

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