Rainfall and sampling uncertainties: A rain gauge perspective

Gabriele Villarini, Pradeep V. Mandapaka, Witold F. Krajewski, and Robert J. Moore

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[1] Rain gauge networks provide rainfall measurements with a high degree of accuracy at specific locations but, in most cases, the instruments are too sparsely distributed to accurately capture the high spatial and temporal variability of precipitation systems. Radar and satellite remote sensing of rainfall has become a viable approach to address this problem effectively. However, among other sources of uncertainties, the remote-sensing based rainfall products are unavoidably affected by sampling errors that need to be evaluated and characterized. Using a large data set (more than six years) of rainfall measurements from a dense network of 50 rain gauges deployed over an area of about 135 km$^2$ in the Brue catchment (south-western England), this study sheds some light on the temporal and spatial sampling uncertainties: the former are defined as the errors resulting from temporal gaps in rainfall observations, while the latter as the uncertainties due to the approximation of an areal estimate using point measurements. It is shown that the temporal sampling uncertainties increase with the sampling interval according to a scaling law and decrease with increasing averaging area with no strong dependence on local orography. On the other hand, the spatial sampling uncertainties tend to decrease for increasing accumulation time, with no strong dependence on location of the gauge within the pixel or on the gauge elevation. For the evaluation of high resolution satellite rainfall products, a simple rule is proposed for the number of rain gauges required to estimate areal rainfall with a prescribed accuracy. Additionally, a description is given of the characteristics of the rainfall process in the area in terms of spatial correlation.


1. Introduction

[2] Rainfall is a phenomenon characterized by high variability both in space and time [e.g., National Research Council, 1998; Krajewski et al., 2003], which makes its measurement difficult. Even though rain gauges provide accurate rainfall measurements, these are only representative for a limited spatial extent. Over the vast majority of the globe, rain gauge networks are too sparse (or completely missing) to capture the variability of the precipitation systems in space and time. An alternative approach is offered by remote sensing of rainfall. For instance, radar/rainfall estimation provides rainfall values over large areas every five to ten minutes with a spatial resolution as small as 1 km$^2$, while better global coverage but coarser spatial and/or temporal resolutions are achieved by means of satellite-based rainfall estimation. However, there are several sources of uncertainties associated with remote sensing of rainfall due to the combined effects of not fully understood physical processes, parameter estimation, and the measuring devices themselves. In addition to these errors, there are both temporal and spatial sampling uncertainties, the characterization of which is the main focus of this study.

[3] The temporal sampling error (TSE) is defined as the error resulting from repeated temporal gaps in the rainfall observations typical of satellite or radar coverage. Several authors have investigated this issue, mostly using radar data. For a tropical region and for a 2.5° × 2.5° pixel, Laughlin [1981] found that sampling every six hours would result in estimating the mean monthly rainfall with a sampling error (defined as the standard deviation in percentage of the mean) of 5%. Using a space-time model able to reproduce the statistical properties of the GATE rainfall, Bell et al. [1990] found that TSE could be approximated by a normal distribution. For 1-km resolution radar data, Fabry et al. [1994] found that the mean absolute error in hourly rainfall with a sampling interval of five minutes was about 20% and that TSE decreases with increasing pixel size. Using rain gauge measurements, Steiner [1996] investigated the temporal sampling uncertainties in monthly areal mean rainfall, providing a relation to estimate TSE for a given rain amount, domain size, and sampling frequency. For monthly accumulations, Nystuen [1998] found that a 1-min sample every 10 min resulted in a standard deviation in the rainfall estimate equal to 10% of the average accumulation. For 1-km pixels and accumulation times of 5 and 10 min, Jordan et al. [2000] found that the temporal sampling error is between 30% and 60% of...
the mean areal rainfall for a sampling interval of 5 min, depending on the rainfall event. Further, the impact of spatial averaging reduces these errors more significantly in the case of scattered rather than widespread rainfall. Bell and Kundu [2000] showed that the temporal sampling error is approximately proportional to the square root of the monthly averaged rainfall rate. Steiner et al. [2003] found that the rainfall sampling uncertainties scale inversely proportional to rainfall, spatial domain size, time integration, and directly proportional to sampling interval. The validity and applicability of such a relation was investigated by Gebremichael and Krajewski [2004]. Gebremichael and Krajewski [2005] models the distribution of the temporal sampling errors for several sampling intervals, rainfall accumulations and pixel sizes. For more references about TSE, the reader is pointed to Astin [1997] and Gebremichael and Krajewski [2004].

[4] The spatial sampling error (SSE) is defined as the error resulting from approximating an areal estimate using point measurements. In most studies concerning the evaluation of radar-rainfall estimates, rain gauge measurements have been used as approximations of the true rainfall. However, rain gauges introduce a sampling mismatch as large as nine orders of magnitude when the catch area is compared with the size of a pixel of a typical radar-rainfall map [e.g., Ciach and Krajewski, 1999]. Austin [1987] noted that, even in the case of perfect rain gauge and radar measurements, there would still be discrepancies due to sampling. In the literature there is a multitude of studies investigating this issue and its effects on radar and rain gauge comparisons. For the case of uniformly distributed rain gauges, Zawadzki [1973] derived an analytical description of the errors and the fluctuations of areal rainfall. To quantify the uncertainties associated with the approximation of the true areal rainfall with the average of point measurements, Rodrı́guez-Iturbe and Mejı́a [1974] introduced the variance reduction factor (VRF). For Florida in summer, Seed and Austin [1990] found that one gauge per 200 km² would be necessary to estimate mean daily rainfall with an approximate error of 20%. At the hourly scale in England, Kitchen and Blackall [1992] showed that the root mean square error due to the spatial and temporal differences is as large as 150% of the mean rainfall. Introducing the error variance separation method, Ciach and Krajewski [1999] estimated that at the hourly scale the variance of the uncertainties associated with the approximation of the true areal rainfall using rain gauge measurements is approximately 50% of the overall discrepancies between radar and rain gauges. According to Habib and Krajewski [2002] for Florida rainfall, the spatial sampling error accounts for approximately 40% to 80% of the overall disagreement between radar and rain gauge. For a 2-km pixel with eight rain gauges, Wood et al. [2000] found that the standard error associated with measurements by a single device increases with increasing areal rainfall values. Using nine rain gauges in a 500 × 500 m² pixel, Jensen and Pedersen [2005] computed that there can be up to 90% variation among rainfall values recorded at neighboring stations.

[5] From the above brief review of the literature, it is clear that both the spatial and temporal sampling uncertainties have been studied for a long time owing to their relevance for remote sensing of rainfall. However, some aspects of the sampling errors remain poorly explored. Most of the limitations of the previous studies can be ascribed to two main aspects: the reference data (including the length of the data set) and the spatiotemporal scales considered.

[6] The vast majority of studies concerning the temporal sampling error in satellite rainfall products use radar data to obtain the true or reference rainfall values. However, radar data should be regarded only as an approximation, as they are affected by several sources of uncertainties of their own, and typically not accounted for. Also, several of these studies infer the characteristics of TSE based on small data sets. Similarly, even though many works have investigated the spatial sampling error in terms of the possible effects on the comparisons between radar and rain gauges, in most of the cases these results are undermined either by a small sample size or by an inaccurate areal estimate. Moreover, most of the studies focused on TSE at large spatial and temporal scales (monthly and 2.5° × 2.5°). The impact of this error for accumulation times of hydrologic interest (on the order of one to three hours) still needs further investigation.

[7] This paper aims at addressing the aforementioned issues and limitations by employing a large data set of rain gauge measurements with high temporal resolution (one minute) from a dense network (50 rain gauges in an approximately 135 km² basin). The advantage of using this type of information to study TSE and SSE is significant. Even though we will be making the assumption that 1-minute rain gauge measurements approximate remote sensing based rain-rate maps, we will be able to accurately estimate the true rainfall value as a continuous accumulation. For the SSE, we will take advantage of the density of the network to compute an areal estimate which can be considered as a very good approximation of the true areal value, against which we compare the measurements from single gauges. Additionally, the spatial and temporal scales we address are much finer than those in several other studies, ranging from hourly to three-hourly accumulations over 2- to 6-km pixels.

[8] Another element often overlooked is the statistical characterization of the rainfall process in the study area. The benefits of such investigation are twofold: (1) it allows a better understanding of the sampling errors in relation to the rainfall processes in the area, together with some indications about the transferability of the results; and (2) it provides more insight into the rainfall process itself.

[9] The paper is structured as follows. Following this Introduction, section 2 describes the data and basic characteristics of rainfall in the area. Section 3 presents the results concerning the spatial and temporal sampling errors, while section 4 summarizes and discusses the important points made in this article.

2. Data

[10] This study is based on more than six years of rain gauge data (from September 1993 to April 2000) from a dense network of 50 gauges deployed during the HYdrological Radar EXperiment (HYREX [Moore et al., 2000]) within the 135 km² Brue basin in south-west England (Figure 1, upper panel). The data set collected in the basin is unprecedented and rather little explored. It can support a wide range of studies from development and evaluation of
distributed hydrologic models to specialized rainfall studies such as this one.

The basic data are available as number of tips in 10-second intervals and the size of each individual tip is 0.2 mm. These characteristics fulfill the recommendations in terms of sampling interval and bucket size made by Habib et al. [2001b] for the reduction of sampling errors in tipping-bucket rain gauge measurements. We have aggregated this information to obtain 1-minute accumulations (using a tip counting approach), which will be the smallest temporal integration time investigated in this study. These data have been quality controlled [Wood et al., 2000] and we have considered in the analysis only the periods that have passed the quality control check.

[12] We highlight the following features from the 2 × 2 km² regular grid shown in Figure 1 (upper panel): there are 20 pixels with one gauge, seven pixels with two rain gauges, and two superdense pixels with eight gauges each. The rain gauges in the two superdense pixels were deployed according to a “diamond-within-a-square” configuration, chosen because of its optimal properties in terms of the estimation of mean rainfall over the 2-km pixels [Moore et al., 2000]. The pixels with two rain gauges are aligned according to the direction of the prevailing storms and orthogonal to the orography of the area. For a more detailed description of this network, see Moore et al. [2000].

[13] To evaluate the dependence of the sampling errors on areal size, we have considered three pixel sizes (Figure 1, upper panel): (1) 2-km, represented by the two super-dense pixels (8 gauges); (2) two 4-km pixels with 13 and 12 gauges in each of them; and (3) two 6-km pixels with 17 and 18 gauges. Throughout this study, we estimated areal averages only if there were at least 6, 10, and 13 rain gauges working at the same time for the 2-km, 4-km, and 6-km pixels respectively. The catchment can be divided into two regions, based on its topography: a western part, characterized by a flat terrain, and an eastern region with more relief (Figure 1). Henceforth, we will refer to the former as Area 1 (west of 366-km Easting), and to the latter as Area 2 (east of 366-km Easting). Even though there is less than 300-m difference in elevation between the two regions, we can evaluate the impact of the small scale topography on the sampling uncertainties. The rain gauge network presents unique features that make it particularly valuable. Not only are the data available for a long time period at a very fine temporal integration scale, but their configuration allows the characterization of the rainfall process at small and medium spatial scales: as shown in Figure 1 (middle panel), the intergauge distances range from approximately 500 m to 15 km. In general, the precipitation in the Brue catchment lacks a strong seasonal cycle. As shown in Figure 1 (bottom panel), the monthly averages tend to be around 80 mm, with slightly smaller (larger) values during the summer (winter).

2.1. Rainfall Spatial Correlation

[14] To describe spatial dependencies of the rainfall process, investigation of its spatial correlation is a widely used approach. For instance, this measure of spatial dependence is relevant to additive error variance characterization for spatial processes. The estimation of the spatial correlation has been performed using Pearson’s product-moment correlation coefficient $r$:

$$r = \frac{E[XY] - E[X]E[Y]}{\sqrt{(E[X^2] - E[X]^2) \cdot (E[Y^2] - E[Y]^2)}}$$

where $E[\cdot]$ is expectation operator, and $(X, Y)$ are a bivariate random sample.

[15] Habib et al. [2001a] reported that for nonnormal data, the Pearson’s estimates result in biased estimates. They proposed a methodology to compute unbiased estimates of
the correlation coefficient when rainfall is distributed according to a mixed lognormal distribution. However, we have looked at the distribution of the rainfall process in this area and found that its tail is much lighter than the corresponding mixed lognormal one. For this reason, we have not applied this procedure in this study.

[16] In Figure 2 we have plotted the results for the spatial correlation for several accumulation times. The results have been parameterized using a three-parameter exponential function for the spatial correlation at separation distance $h$:

$$ r(h) = c_1 \exp \left( -\frac{h}{c_2} \right)^{c_3} \quad c_1 \in [-1, 1], \; c_2 > 0, \; c_3 \in [0, 2] $$

(2)

where $c_1$ represents the nugget, $c_2$ is the correlation distance and $c_3$ is the shape factor.

[17] As expected, for all cases, larger accumulation times yield correlations with longer spatial dependence, in agreement with other studies [e.g., Krajewski et al., 2003; Ciach and Krajewski, 2006].

[18] Figure 3 summarizes the time dependence of the three coefficients in (2). First, focusing on the nugget, it has a value of about 0.5 at the one-minute scale and it tends to rapidly increase up to 0.96 for the five-minute scale and then asymptotically increases for larger integration times. The nugget $c_1$ gives us information about the small scale variability of the process as well as the measurement errors [e.g., Journel and Huijbregts, 1978]. The closer $c_1$ is to 1.0, the smaller the variability and the measurement errors. Compared to Ciach and Krajewski [2006], the values of $c_1$ are smaller, probably due to the high quality of the Piconet data obtained by using dual rain gauges at each location. The correlation distance $c_2$ gives us information about the distance at which the process decorrelates. As the integration time increases, the correlation distance increases from approximately 5 km to 150 km. The rate of increase is smaller for the timescales up to about 30 min and then it tends to increase faster. Finally, for the shape factor $c_3$, which controls the behavior of the correlation function at the small scale (near zero distance), one can notice an S-shape pattern in its increase from 0.4 to roughly 1 (corresponding to a pure exponential function). In general, the coefficients of the spatial correlation function estimated in this study are different from those obtained by Ciach and Krajewski [2006]. These differences could be due to several reasons: quality of the rain gauge data (double versus single rain gauge), range of intergauge distances (maximum range close to 4 km versus a largest distance of about 15 km), sample size (14 months versus over six years), analyzed period (mostly spring and summer versus the entire data set), and location (Oklahoma versus England).
From the results in Figure 3 it is clear how the nugget and the shape factor are more sensitive to short accumulation times. On the other hand, for larger timescales, the correlation distance seems to be the most influenced parameter. Given the regularity in the dependence of these three parameters with timescale, we have fitted them in parametric form:

\[
\begin{align*}
c_1(\tau) &= 0.51 + 0.47 \cdot \left\{ 1 - \exp \left[ - \left( \frac{X}{0.32} \right)^{1.49} \right] \right\} \\
c_2(\tau) &= \exp(1.84 + X) \\
c_3(\tau) &= 0.92 - \frac{0.52}{1 + \exp \left( \frac{X}{0.24} - 3.83 \right)}
\end{align*}
\]  

(3)

where \(\tau\) is the timescale in minutes, and \(X\) is equal to \(\log(\tau)\). As shown in Figure 3, the functions in (3) fit the data very well.

2.2. Accuracy of the Areal Estimates

To quantify the accuracy of the pixel-averages, we have used the variance reduction factor [e.g., Rodriguez-Iturbe and Mejía, 1974; Morrissey et al., 1995; Krajewski et al., 2000]. This factor quantitatively represents the uncertainties associated with the approximation of the true areal value with the average of point measurements, and it depends on the spatial correlation of the sampled process, the network density and configuration. Let \(R_T\) be the true areal rainfall and \(R_{G,i}\) the rainfall measured by the gauge \(i\) in the sampling domain. If there are \(n\) gauges within the pixel, the areal average can be estimated as

\[
\bar{R}_T = \frac{1}{n} \sum_{i=1}^{n} R_{G,i}
\]  

(4)

For \(E[R_T] = E[\bar{R}_T]\), the error variance for the above estimate can be expressed as

\[
Var[R_T - \bar{R}_T] = E[(R_T - \bar{R}_T)^2] = Var[R_{G,i}] \cdot VRF
\]  

(5)

where \(Var[\cdot]\) denotes the variance operator, \(Var[R_{G,i}]\) is the point variance and \(VRF\) is the variance reduction factor defined as:

\[
VRF = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \rho(d_{ij}) \delta(i) \delta(j)
\]

\[
- \frac{2}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \rho(d_{ij}) \delta(i) + \frac{1}{m} + \frac{2}{mn} \sum_{i=1}^{m} \sum_{j=i+1}^{n} \rho(d_{ij})
\]

(6)

The domain on which the areal value is estimated is divided into \(m\) boxes and the Kronecker function \(\delta(\cdot)\) indicates whether the box \(i\) contains a gauge. The term \(\rho(d_{ij})\) represents the spatial correlation of the rainfall for a separation distance \(d_{ij}\) between the boxes \(i\) and \(j\).

[22] To summarize, equation (5) gives the mean squared error of the areal estimate (4) in terms of the variance reduction factor. For more details on the VRF, the reader is pointed to Morrissey et al. [1995] and Krajewski et al. [2000].

[24] In Figure 4 is plotted VRF as a function of the accumulation time, for the three pixels of interest and the whole basin: for all cases the variance reduction factor is much smaller than 5%, which means that the mean squared error is smaller than 5% of the point variance. In general, as VRF tends to zero, the variance of the differences between true areal and areal-averaged rainfall tends to zero as well, and therefore \(\bar{R}_T\) is a very good approximation of \(R_T\).

Intuitively, one would expect that VRF decreases monotonically with increasing accumulation time. However, for all the pixel sizes, VRF tends to increase at very short accumulation scales, and then to steadily decrease. A possible reason for the peak is the presence of a nugget significantly different from 1 at small temporal resolutions. To support this conclusion, we have estimated the VRF setting \(c_1\) equal to 0.99 independent of the accumulation time: the variance reduction factor is a monotonically decreasing function of the timescale, as shown in Figure 4. For pixels larger than 4 km\(^2\), the two superdense pixels tend to appear as clusters, with a larger impact on the 4-km pixels than on the others. The presence
of these clusters, together with the nugget effect could explain the results in Figure 4.

Equation (6) involves the spatial correlation of the true rainfall. However, at short timescales, we have a nugget significantly different from 1 which could be due to measurement errors and the small scale variability of the rainfall process. Since it is not possible to separate their relative contribution to \( c_1 \), at short timescales it is likely that the VRF may suffer from these uncertainties.

2.3. Small Scale Intermittency of Rainfall

Another rainfall characteristic that we have investigated is the small scale intermittency. Since nonrainy areas smaller than the pixel size can be embedded in precipitation systems at larger scales, it is possible that a rain gauge does not detect rainfall, even though radar or satellite report a rainfall value larger than zero, given the much larger sampling volume. Therefore, as in Habib and Krajewski [2002], we are interested in the variability of rainfall within a pixel. Given \( t \) rain gauges, we define the conditional zero-rainfall probability as the probability that one rain gauge does not detect rainfall, given that a positive amount of rainfall has been detected by any other of the \( t \) gauges. Since we are interested in zero point rainfall probability conditioned on nonzero areal rainfall, the requirements in terms of minimum number of gauges working at the same time are still valid (if this requirement is satisfied, any nonworking gauges are flagged as missing values and not considered). In Figure 5 we have plotted the results of our analysis, focusing on the 2-km superdense pixel in Area 1 (similar results were obtained for Area 2) and the entire basin. For the 2-km pixel can be seen a decrease in this probability from about 0.75 at the 1-minute scale, to about 0.35 at the three-hourly scale. Therefore for a 4-km² pixel at the hourly scale, there is about 40% probability that one rain gauge does not detect rainfall, given that some rainfall has fallen in any other part of the pixels. As expected, the values of the conditional zero-rainfall probability tend to be larger for increasing pixel size: for a 192-km² pixel, it ranges from about 0.92 at the 1-min scale to about 0.65 at the 3-hourly accumulation. Compared to Habib and Krajewski [2002], we observed a higher conditional zero-rainfall probability. However, a direct comparison may be misleading given the

Figure 4. Variance reduction factor (VRF) as a function of the accumulation time. The results for the 2-km, 4-km, and 6-km pixel are similar for both of the areas. The solid line represents the VRF computed from the estimated spatial correlation, while the dotted line was obtained by setting \( c_1 \) equal to 0.99.
much higher rain gauge density in this study, different pixel sizes, and significantly different rainfall regime.

3. Sampling Uncertainties

[28] After characterizing the rainfall spatial dependence as function of temporal accumulation scale for the Brue catchment, the next step is to begin investigating the sampling uncertainties. First, we need to define the temporal and spatial sampling uncertainties in mathematical terms. The temporal sampling error represents the uncertainties resulting from temporal gaps in the rainfall observations. To define it, first consider the temporal sampling error for hourly accumulations. Let $H$ represent the reference rainfall value, computed as the sum of sixty 1-min areal-averaged accumulations or “scans”. On the other hand, let $S$ represent the sampled rainfall, whose values are estimated from a noncontinuously sampled data set. For instance, for a sampling interval of 10 min we would have 6 “scans” and $S$ is obtained by assuming that in each 10-min period the rainfall is uniform. The impact of different sampling intervals is investigated here for three accumulation times (1 h, 2 h, and 3 h) and for three different pixel sizes (2 km, 4 km, and 6 km).

[29] The spatial sampling error is the error caused by approximating an areal estimate with the average of point measurements. We define $H$ as the value obtained by averaging the rain gauge measurements within a pixel, and $S$ as the measurement from a single device. This source of uncertainties is investigated here for increasing accumulation times (from 1-min to daily) and pixel sizes (2 km, 4 km, and 6 km). Additionally, the impact of the rain gauge elevation and distance from the center of pixels is explored.

[30] Therefore we will refer to $H$ as the “true” or reference value and to $S$ as the estimate affected by either temporal or spatial sampling errors. To quantitatively evaluate these sampling errors, we will use the normalized root mean squared error (NRMSE) and normalized mean absolute error (NMAE) defined as follows:

$$\text{NRMSE} = \sqrt{\frac{\sum_{i=1}^{N} (H_i - S_i)^2}{N}}$$  \tag{7}

$$\text{NMAE} = \frac{\sum_{i=1}^{N} |H_i - S_i|}{N}$$  \tag{8}

where $N$ is the sample size.

[31] In Figure 6 we have plotted the temporal sampling uncertainties as a function of sampling interval for 1-h (left panels), 2-h (central panels), and 3-h (right panels) accumulation times. These results refer to Area 1, but similar
results and conclusions can be extended to Area 2. NRMSE
and NMAE tend to increase for increasing sampling interval. This result is easily justifiable since, for increasing
sampling interval, a smaller number of “scans” are avail-
able, and therefore less information is used toward the
computation of the accumulated value. One very interesting
feature from these plots is the presence of scaling behavior
(log-log domain). In Table 1 we have summarized the
estimated coefficients of the regression lines. Finally, the
impact of the sampling interval tends to decrease with
accumulation time: for instance, for a 2-km pixel, sampling
every four minutes at the hourly scales would be compara-
tive to sampling every 10 min at the 3-hourly scale in terms
of NRMSE. Comparing the behavior for different pixel
sizes, we observe that larger pixel sizes tend to have smaller
uncertainties.

Besides looking at the variation of the spatial sam-
ppling error with accumulation time, we also investigated its
dependence on other factors. In Figure 8 the behavior of this
error is plotted as a function of the distance from the center
of the pixel (left panels) and elevation (right panels) for the
2-km pixels, for different accumulation times: no strong
dependence on either of them is evident. These results refer
to the two superdense 2-km pixels but similar conclusions
can be drawn for the other spatial resolutions as well.

Given the spatial and temporal scales investigated
so far, the results discussed until now are mostly related to
radar-rainfall estimation. Nevertheless, rain gauge mea-
surements are also widely used for the evaluation and
validation of satellite-based rainfall products [e.g., Villarini
and Krajewski, 2007]. In this case, the resolutions are
much coarser compared to the radar products and these
results are not directly applicable. Therefore to have a
better understanding of the requirements in terms of
number of rain gauges for accurate areal estimation, we
have considered the whole basin as a 12 km by 16 km
pixel, and time accumulations from 15-min to daily. To the
best of our knowledge, in the literature there are only few
studies providing some simple rules on this issue, but all
of them for much coarser scales (monthly and 2.5° ×
2.5°). According to Xie and Arkin [1995], five rain gauges

Figure 6. Dependence of the temporal sampling error on the sampling interval for Area 1, for different
accumulation times and pixel sizes. Similar results were obtained for Area 2.
are enough to estimate areal rainfall within 10% of the areal value. For the same spatiotemporal resolution, Krajewski et al. [2000] recommended a minimum of 25 gauges operating for the whole period in order to minimize systematic and random uncertainties. At smaller spatiotemporal scales (e.g., three-hourly and $0.25^\circ \times 0.25^\circ$ satellite product), Villarini and Krajewski [2007] used 23 rain gauges to evaluate a pixel of the TRMM Multisatellite Precipitation Analysis (TMPA) three-hourly $0.25^\circ \times 0.25^\circ$ satellite product [Huffman et al., 2007] in Oklahoma and their VRF was within 3%. Nevertheless, a “rule of thumb” in terms of number of rain gauges for these finer-scale satellite products is still missing.

[35] To fill this gap, we have considered the true areal value $H$ as obtained by averaging the measurements from the 50 gauges within the catchment. Given $k$ gauges, $S$ is computed as the average of the corresponding $k$ measurements. For $k \in [2, 48]$, there are at least 1225 possible combinations (for $k = 1$ and $k = 49$, there are only 50 different values, while for $k = 50$ only one). Figure 9 plots the behavior of NRMSE as a function of number of gauges for four accumulation times (15-min, hourly, 3-hourly, and daily). Each boxplot is based on 1000 possible combina-

<table>
<thead>
<tr>
<th>Table 1. Summary of the Intercept $a$ (Values on the Left) and of the Slope $b$ (Values on the Right) of the Fitted Lines in Figure 6 for Area 1 (Upper Table) and Area 2 (Lower Table)</th>
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<tbody>
<tr>
<td>Hourly Pixel Size</td>
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<tr>
<td>NRMSE</td>
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<td>NMAE</td>
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<td>2-Hourly Pixel Size</td>
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<td>NRMSE</td>
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<td>3-Hourly Pixel Size</td>
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<td>NRMSE</td>
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<td>NMAE</td>
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**Figure 7.** Dependence of the spatial sampling error on the accumulation for Area 1, for three pixel sizes. Similar results were obtained for Area 2.
tions (with the exception of the first one, based on 50 possible values). The limits of the whiskers represent the 5th and 95th percentile, the lower and upper margins of the boxes the 25th and 75th percentiles, while the lines and the squares within them are the median and the mean respectively. There is a tendency for the error to decrease for increasing number of gauges. Moreover, the width of the 90% confidence interval decreases as $k$ increases. Finally, for larger accumulation times, the number of rain gauges required to estimate the areal rainfall with a certain degree of accuracy tend to decrease. For instance, at the 15-min and hourly scale we would need over 25 gauges to estimate the true areal rainfall with 20% accuracy. On the other hand, at the three hourly and daily scales, this condition is less restrictive: for the same accuracy level, 15 and 4 gauges are necessary, respectively.

The application of these results to satellite products providing rainfall accumulations over an area is clear. However, their applicability to products providing instantaneous measurements from a single satellite overpass is not straightforward and depends on the temporal scale that best approximates the “instantaneous” estimate (see Villarini and Krajewski [2007] for a discussion of this issue).

4. Conclusions and Discussion

Rainfall is characterized by high spatiotemporal variability. Rain gauges have the capability of measuring it accurately but only in a specific location. Because of the sparseness of the rain gauges, or even the absence of any ground station, in the vast majority of the globe, remote-sensing of rainfall has become a viable tool to capture the variability of the precipitation systems. However, these estimates are affected by several sources of uncertainties, among which spatial and temporal sampling errors play a significant role. For this reason, taking advantage of a large data set available for a very dense rain gauge network, we investigated this source of uncertainty from a rain gauge perspective. We presented several results related to both of these errors, as well as to the rainfall process itself. A summary of findings are set down below.

(1) The spatial correlation of the rainfall process increases for increasing accumulation time. With respect to the nugget effect, the measurement errors and the small scale variability of rainfall substantially decrease for accumulation times larger than five minutes.

(2) The small scale intermittency of rainfall could affect the comparison between rain gauge measurements and remote-sensing based rainfall estimates.

(3) The temporal sampling error (TSE) increases with increasing sampling interval following a scaling behavior. It also decreases with increasing pixel size and accumulation time.

(4) The spatial sampling error (SSE) decreases with increasing accumulation time. Increasing pixel size tends to
increase the magnitude of this error. Additionally, SSE does not seem to strongly depend on the elevation or on the distance of the rain gauges from the pixel center.

(5) For evaluation of satellite products, to estimate areal rainfall (pixel of about 200 km²) within 20% of its true value, respectively over 25, around 25, 15, and 4 gauges are necessary at the 15-min, hourly, 3-hourly, and daily scale.

One issue that we have to discuss is the impact of rain gauge measurement errors, and in particular of using a 0.2 mm tipping bucket, in the measurement of low rainfall values, which are frequent in this region. As shown in the literature [e.g., Habib et al., 2001b; Ciach, 2003] these errors tend to decrease for increasing rainfall rate and accumulation time. Since this region is characterized by frequent low rainfall rates, it is likely that the results for the rainfall spatial correlation (and consequently for the VRF) at short timescale (up to 15 min) may be corrupted by measurement uncertainties, as also suggested by the behavior of the nugget. Quantifying their impact is not an easy task, since stratification of the data into weak and strong rainfall is highly sensitive to the selected threshold [Ciach and Krajewski, 2006]. In the same way, it is likely that our results on the small scale variability of rainfall are affected by these instrumental errors.

Moreover, the procedure used to convert tips into rainfall could also have introduced additional uncertainties. By using a tip interpolation scheme rather than a tip counting approach (as in this study), Ciach [2003] showed a significant reduction of the instrumental errors, especially at small timescales. To apply this method, we would need the time of each tip but in this case, we only have the number of tips in a 10-second interval. Therefore here we have assumed that the tips were evenly distributed within each 10-second period and applied the tip interpolation procedure in order to assess the difference. However, we observed no significant differences in the results with respect to the tip counting scheme.

Following earlier studies [e.g., Steiner, 1996; Steiner et al., 2003; Gebremichael and Krajewski, 2004], we found scaling relations between temporal sampling errors and sampling frequency. Compared to other studies that employed radar data at a much coarser spatiotemporal scale [e.g., Steiner et al., 2003; Gebremichael and Krajewski, 2004], the magnitude of the slopes of the regression lines (Table 1) for TSE are smaller, ranging between 0.7 and 0.8. Possible explanations could be found in the much finer spatial and temporal scales, the different climatic regions, and the different type of sensors (rain gauge versus radar).

This work showed that the spatial sampling error tends to be large, especially at the small temporal integration scales. Therefore the approximation of the true rainfall

Figure 9. Dependence of the accuracy of basin-averaged rainfall estimates on the number of gauges for four accumulation times.
with rain gauge measurements is reasonable only for dense networks and/or large temporal integration scales. If short timescales are of interest and one gauge is used to approximate the areal estimates, it would be necessary to account for the spatial sampling uncertainties describing their full statistical distribution.

[47] This study also provides some indications in terms of the number of rain gauges necessary to estimate areal rainfall with a certain degree of accuracy. This information is valuable in studies concerning the evaluation and validation of satellite products at fine spatiotemporal scales. Nevertheless, since the estimates from many of these products are on a larger scale (about 625-km² pixels), these indications represent an optimistic lower bound. Additionally, this rule-of-thumb is directly applicable only to satellite products representing temporal accumulations over a certain area, while its application to “instantaneous” ones is not straightforward.

[48] One of the elements that may require further discussion is the transferability of these results to other regions. On the basis of the results concerning the characteristics of the rainfall process, these results could be transferred to other midlatitude parts of the world, but not to tropical regions, characterized by a much stronger convective activity and much higher variability of rainfall. However, the verification of this statement depends on the availability of comparable (both in terms of quality and temporal duration) data sets.

[49] Finally, we have been able to perform this study thanks to the vision of the people behind the HYREX project. It is our firm opinion that rain gauge networks of such density and quality are still lacking and need to be developed for improving our knowledge of the rainfall process and its measurement. Clearly, limited accuracy of rainfall estimation methods constrains development and evaluation of rainfall forecasting methods.

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W. F. Krajewski, P. V. Mandapaka, and G. Villarini, IIHR-Hydrosience and Engineering, The University of Iowa, C. Maxwell Stanley Hydraulics Laboratory Iowa City, IA 52242–1585, USA. (gabrielle-villarini@uiowa.edu)

R. J. Moore, Centre for Ecology and Hydrology, Wallingford, Oxfordshire, OX108BB, UK.