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A Proposal for Micromachined Dynamically Tuned Gyroscope, Based on Contactless Suspension

Kirill V. Poletkin, *Member, IEEE*, Aleksandr I. Chernomorsky, and Christopher Shearwood

Abstract—In this paper, the operating principle of a micromachined, dynamically tuned gyroscope, based on a contactless suspension is discussed and its mathematical model is derived. Dynamical analysis based on this mathematical model is conducted. The analysis shows that such a gyroscope can be created in principal and provides a value for the gyroscope gain to measuring angular rate which is several orders of magnitude greater in comparison with existed prototypes of the micromachined gyroscope based on a contactless suspension.

Index Terms—gyroscope; micro-electro-mechanical system; rotor tuning condition; inertial sensors.

I. INTRODUCTION

MICROMACHINED GYROSCOPES (MMG) have already attracted a great deal of attention giving rise to a new generation of inertial sensors for measuring angular rate with significant small-size, lower-weight, lower - power consumption, lower-cost, higher reliability, and compatibility with hostile environments. Advantages of MMG allow them to be deployed in a wide range of applications such as ride stabilization and rollover detection in automotive area; consumer electronic application, such as video-camera stabilization, virtual reality, and inertial mice for computer; robotics applications; children's toys; a wide range of military applications, and etc. [1], [2]. In fact, the achieved performance level of MMG is rate-grade as a result of developments over the last decade [3]. However, the rapidly increasing list of MMG applications requires tactical- and inertial-grade performance levels which represent a formidable challenge.

Almost all MMG can be described as a mechanical oscillator in which a silicon Proof Mass (PM) is vibrated sinusoidally in plane to form the Coriolis force acting on the PM which is caused by the rotation of that plate at some angular rate. In this case, the MMG consists of the PM that transforms the measuring angular rate to its linear or angular displacement and a system of Spring Elements (SE) which provides the established degrees-of-freedom to the PM.

It is significant that there is a mechanical link between the PM and the case of the gyroscope by means of the SE system which provides a source of error caused by changes in

both the mechanical properties and geometrical parameters of the SE itself, as well as the influence of the gyroscope case during the gyroscope operational process. The results of both theoretical and experimental investigations, in particular, those presented in [4]–[9] show that the error sources associated with the mechanical link is one of the major error sources that limits accuracy, and as a consequence, prevent applications of MMG in areas that required tactical- or inertial- performance.

The levitation of the PM by means of a Contactless Suspension (CS) is an effective way to eliminate the above mentioned imperfection, namely the mechanical link between the PM and the case of the gyroscope and, as a result, reduce the influence that these error sources produce on the output of the MMG. Although a number of unique solutions have been suggested to provide a CS, most methods used in the MMG are based on either electromagnetic induction [10] or electrostatic suspension [11]. As well as allowing passive suspension, these schemes also provide a mechanism for the rotation of the PM to high angular velocities, often only limited by viscous drag. For example, it has been reported that for a disk-shape rotor, of diameter 0.5 mm, the maximum possible speed of rotation achievable in air is 100,000 RPM [12]. Note, that in vacuum, the maximum possible speed of rotor rotation can be significantly greater than in air due to the reduction of viscous drag.

The first prototype of a micromachined Inductive CS (ICS) was described in [10] with some experimental results and a suggested application as a MMG. Additional experimental results of the MMG based on the same ICS concept, in which the rotation of disk-shaped rotor with speed of ≈ 1000 RMP in air was provided, were presented in [13]. A modified version of the ICS was proposed in [14] based on a rotation of the disk-shaped rotor with a speed of ≈ 3000 RPM in vacuum.

The application of Electrostatic CS (ECS) in MMG has been studied by Tokimec and Tohoku universities since 1993. They developed different prototypes of MMG based on ECS, whose rotors were disc-shaped and ring-shaped and which detected 3-axis linear acceleration and 2-axis angular rate. In the prototype, a rotation speed of 10,000 RMP was achieved for a 5 mm diameter silicon, ring-shaped rotor [15].

However, in prototypes of MMG based on CS, the speed of rotation of the gyroscope rotor is far from the maximum theoretical value. As a result, the highest accuracy of angular rate measurement cannot be achieved.

Since the mechanism that provides the CS also makes possible rotation of MMG rotor, an alternative strategy for improving the accuracy of such gyroscopes is the development of a dynamically tuned gyroscope, in which the angular

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Fig. 1. Kinematics of micromachined DTG: XYZ is the fixed coordinate frame relative to the gyroscope case; $x_r y_r z_r$ is the rotating coordinate frame related to the rotating electromagnetic field; Ω is the steady angular rate of rotation of the electromagnetic field and rotors.

stiffness supported by a CS is used. It is possible, given the experimentally achieved speed of rotor rotation in currently developed MMG based on CS, and under meeting the rotor tuning condition, that an increased gyroscope gain would lead to several orders of magnitude improvement in the measured angular rate.

In this paper, the operating principle of a micromachined, Dynamically Tuned Gyroscope (DTG) based on a CS is considered and its mathematical model developed. Based on this model, the dynamical analysis of a micromachined DTG is conducted. This analysis allows us to evaluate the anticipated gyroscope gain of such a micromachined DTG to the measurement of angular rate. In particular, a comparative analysis between the gyroscope gain of the micromachined DTG and the developed prototype of a MMG based on ICS [13] is conducted. It shows that the micromachined DTG gain, evaluated at a rotor rotation speed of 1000 RMP, is five orders of magnitude greater than gyroscope gain of a MMG based on ICS alone.

II. KINEMATICS AND OPERATING PRINCIPLE

The kinematics of a micromachined DTG are illustrated in Fig. 1. The micromachined DTG consists of two rotors, namely an inner and outer one, that are linked together by a pair of torsional springs. The inner rotor is located within an electromagnetic field and suspended there contactlessly at its equilibrium position. Both rotors are considered as rigid bodies in this study. The position of the center of mass of the inner rotor is characterized by the origin O which is also the origin of the coordinate frame (CF) XYZ , fixed to the gyroscope case. Rotation of the rotor system, with respect to the gyroscope case, is provided by the rotating electromagnetic field with a steady angular rate Ω along the Z axis. The Z axis is set as the gyroscope rotational axis. Let us define the rotating CF $x_r y_r z_r$ which is rotated together with the rotating electromagnetic field and the origin of which coincides with the origin O . The Z and z_r axes are coincident,

Fig. 2. Coordinate frames $x_1 y_1 z_1$ (denoted by green color) and xyz (denoted by blue color) are fixed to the inner and outer rotors, respectively.

geometrically. In this study, a steady state is assumed, therefore the speed of the rotating electromagnetic field and rotors are the same and equal to Ω along the Z axis under the absence of the measuring angular rate (the unperturbed state). In the unperturbed state, the x_r axis of the rotating CF coincides with the axis of torsions as shown in Fig. 1. It is assumed that the static and dynamic unbalances of the rotors and their linear displacements are negligible and the centers of the rotors inertia coincides with the origin O .

Let us assign a CF $x_1 y_1 z_1$ to the inner and a CF xyz to the outer rotors so that the axes of these CF coincide with the principal axes of inertia of the rotors, respectively, and the x_1 and x axes are directed along the axis of the torsional springs, illustrated in Fig. 2. The position of the CF $x_1 y_1 z_1$ with respect to the rotating CF $x_r y_r z_r$ is defined by the angles q_1 and q_2 , and the position of the CF xyz by the angles q_1 and q_3 , respectively.

Note that the angular coordinate q_2 defines the angular displacement of the inner rotor relative to the x_1 axis and the angle between the surfaces $x_1 z_r$, and $x_1 z_1$ or $x_1 y_r$, and $x_1 y_1$. While the angular coordinate q_3 defines the angular displacement of the outer rotor relative to the x axis and the angle between surfaces $x_1 z_r$, and xz or $x_1 y_r$, and xy . It is seen that angular coordinates q_2 and q_3 are independent of each other, since the surfaces $x_1 z_r$, and $x_1 y_r$ do not take part in the angular displacement of the inner as well as the outer rotors.

The rotation of the gyroscope case with the measuring angular rate, which is defined by vector $\bar{\omega}$ lying on the XY surface of the micromachined DTG and having projections ω_X and ω_Y along axes X and Y , respectively induces motion in the outer rotor relative to the output axes y_r and x (characterized by angular coordinates q_1 and q_3 , respectively) which characterizes the value of the measuring input angular rate $\bar{\omega}$.

To define the position of the outer rotor with respect to the fixed CF XYZ , let us introduce the angles α and β which

Fig. 3. Determination of the position of the outer rotor relative to the fixed CF: $X'Y'Z'$ is the ancillary CF, rotating relative to the X axis.

characterize the position of the xy surface with respect to the CF XYZ as shown in Fig. 3(a). The angle α defines the angular displacement of the outer rotor relative to the X axis. The angle β defines the angular displacement of the outer rotor relative to the Y' axis which lies on the YZ surface.

Since it is assumed that angular displacements q_1 , q_3 , α and β are small, so the dedicated triangles in Fig. 3(b) can be considered as plane triangles, and the angles between the sides of the triangles q_1 , q_3 and α , β are 90° ; hence the relationship between angles q_1 , q_3 , α and β can be written as:

$$\left. \begin{aligned} \alpha &= q_3 \cos \Omega t - q_1 \sin \Omega t; \\ \beta &= q_3 \sin \Omega t + q_1 \cos \Omega t. \end{aligned} \right\} \quad (1)$$

Thus, the angular displacements of the outer rotor with respect to the rotating CF (characterized by q_1 , and q_3) as well as the fixed CF (characterized by α and β) are the measure of the input rotation of a gyroscope case or the input rates which can be finally transformed into the electrical output signal by mean of capacitive angular sensors.

III. MATHEMATICAL MODEL

To develop a mathematical model of the micromachined DTG the second kind Lagrange's equations are used. In the framework of the problem, the micromachined DTG has four-degrees-of-freedom: the angle of rotors spin Ωt and angles of rotations of rotors q_1 , q_2 and q_3 relative to the rotating CF. The angle Ωt is a cyclical angle; with a constant value of Ω , it allows us to construct the following three equations for the description of the micromachined DTG motion, taking q_1 , q_2 , and q_3 as the generalized coordinates. Hence, we can write:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = -\frac{\partial \Pi}{\partial q_i} - \frac{\partial \Phi}{\partial \dot{q}_i}, \quad (i = 1, 2, 3), \quad (2)$$

where T and Π are the kinetic and potential energies of the system; Φ is the dissipation function; q_i , $\dot{q}_i$ are the generalized coordinates and velocities, respectively.

The kinetic energy of the system under consideration is the sum of the kinetic energies of the inner and outer rotors and can be written as:

$$T = \frac{1}{2} (J_{x1}\omega_{x1}^2 + J_{y1}\omega_{y1}^2 + J_{z1}\omega_{z1}^2) + \frac{1}{2} (J_x\omega_x^2 + J_y\omega_y^2 + J_z\omega_z^2), \quad (3)$$

where J_{x1} , J_{y1} , J_{z1} and ω_{x1} , ω_{y1} , ω_{z1} are the three central principal moments of inertia about the x_1 , y_1 , z_1 axes and projections of angular rate of the inner rotor on the same axes, respectively; J_x , J_y , J_z and ω_x , ω_y , ω_z are the three central principal moments of inertia about the x , y , z axes and projections of angular rate of the outer rotor on the same axes, respectively.

According to the kinematics of the micromachined DTG, for projections of angular rates of rotors on axes of CF $x_r y_r z_r$, $x_1 y_1 z_1$ and xyz , the following equations are true:

$$\left. \begin{aligned} \omega_{xr} &= \omega_X \cos \Omega t + \omega_Y \sin \Omega t; \\ \omega_{yr} &= -\omega_X \sin \Omega t + \omega_Y \cos \Omega t; \\ \omega_{zr} &= \Omega; \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \omega_{x1} &= \omega_{xr} \cos q_1 - \omega_{zr} \sin q_1 + \dot{q}_2; \\ \omega_{y1} &= (\omega_{yr} + \dot{q}_1) \cos q_2 \\ &\quad + (\omega_{xr} \sin q_1 + \omega_{zr} \cos q_1) \sin q_2; \\ \omega_{z1} &= -(\omega_{yr} + \dot{q}_1) \sin q_2 \\ &\quad + (\omega_{xr} \sin q_1 + \omega_{zr} \cos q_1) \cos q_2; \end{aligned} \right\} \quad (5)$$

$$\left. \begin{aligned} \omega_x &= \omega_{xr} \cos q_1 - \omega_{zr} \sin q_1 + \dot{q}_3; \\ \omega_y &= (\omega_{yr} + \dot{q}_1) \cos q_3 \\ &\quad + (\omega_{xr} \sin q_1 + \omega_{zr} \cos q_1) \sin q_3; \\ \omega_z &= -(\omega_{yr} + \dot{q}_1) \sin q_3 \\ &\quad + (\omega_{xr} \sin q_1 + \omega_{zr} \cos q_1) \cos q_3. \end{aligned} \right\} \quad (6)$$

The CS provides the angular displacement of the inner rotor with the finite angular stiffness which is the same for all equatorial axes of the rotor; the value of this stiffness is

denoted by c_s . Hence, the potential energy of system can be written as follow:

$$\Pi = \frac{1}{2}c_s q_1^2 + \frac{1}{2}c_s q_2^2 + \frac{1}{2}c_t (q_3 - q_2)^2, \quad (7)$$

where c_t is the stiffness of the torsional springs.

The dissipation function is described as:

$$\Phi = \frac{1}{2}k_s \dot{q}_1^2 + \frac{1}{2}k_s \dot{q}_2^2 + \frac{1}{2}k_t (\dot{q}_3 - \dot{q}_2)^2, \quad (8)$$

where k_s and k_t are the damping coefficients of the CS and the torsional springs, respectively. To write the linear mathematical model of the micromachined DTG, let us substitute (3), (7) and (8), accounting for the kinematical relationships (4), (5) and (6) into Eq. (2). Since the angles q_1 , q_2 and q_3 are small, second order terms can be neglected and the mathematical model describing the motion relative to the rotating CF can be written as:

$$\left. \begin{aligned} & (J_{y1} + J_y) \ddot{q}_1 + k_s \dot{q}_1 \\ & + \left(c_s + \left[(J_{z1} + J_z) - (J_{x1} + J_x) \right] \Omega^2 \right) q_1 \\ & - (J_{z1} - J_{y1} - J_{x1}) \Omega \dot{q}_2 \\ & - (J_z - J_y - J_x) \Omega \dot{q}_3 = m_{q1}; \\ & (J_{z1} - J_{y1} - J_{x1}) \Omega \dot{q}_1 + J_{x1} \ddot{q}_2 + (k_s + k_t) \dot{q}_2 \\ & + \left(c_s + [J_{z1} - J_{y1}] \Omega^2 \right) q_2 \\ & - k_t \dot{q}_3 - c_t q_3 = m_{q2}; \\ & (J_z - J_y - J_x) \Omega \dot{q}_1 - k_t \dot{q}_2 - c_t q_2 \\ & + J_x \ddot{q}_3 + k_t \dot{q}_3 \\ & + \left(c_t + [J_z - J_y] \Omega^2 \right) q_3 = m_{q3}, \end{aligned} \right\} \quad (9)$$

where

$$\begin{aligned} m_{q1} &= \left[(J_{z1} + J_z) + (J_{y1} + J_y) - (J_{x1} + J_x) \right] \\ &\quad \times \Omega (\omega_X \cos \Omega t + \omega_Y \sin \Omega t); \\ m_{q2} &= \left[J_{z1} + J_{x1} - J_{y1} \right] \\ &\quad \times \Omega (\omega_X \sin \Omega t - \omega_Y \cos \Omega t); \\ m_{q3} &= \left[J_z + J_x - J_y \right] \Omega (\omega_X \sin \Omega t - \omega_Y \cos \Omega t). \end{aligned} \quad (10)$$

Note that if the angular displacements of the inner rotor q_1 and q_2 , relative to the y_r and x_1 axes, respectively, are absent ($q_1 = q_2 = 0$, $\dot{q}_1 = \dot{q}_2 = 0$), the last equation of set (9) describes the dynamics of a rotor vibratory gyroscope exactly [16]–[20]. On the other hand, if only the angular displacement of the inner rotor q_2 relative to the x_1 axis is absent ($q_2 = 0$, $\dot{q}_2 = 0$), the first and third equations of set (9) are the same as a classic model of a dynamically tuned gyroscope with gimbal [18], [21], [22].

IV. MODIFICATION OF THE MODEL

From a practical viewpoint, let us analyze the motion of the micromachined DTG for the case which follow from the analysis of the experimental study of the MMG prototypes based on CS [13], [15], [23]. For this case the following assumptions can be made:

$$J_x = J_y < J_z; \quad J_{x1} = J_{y1} < J_{z1}. \quad (11)$$

Also, the measuring angular rate vector $\bar{\omega}$ is steady and its projections are $\omega_X = \text{const}_1$; $\omega_Y = \text{const}_2$.

For further analysis, model (9) can be rewritten as follow:

$$\left. \begin{aligned} & J \ddot{q}_1 + k_s \dot{q}_1 + C_{d1} q_1 + h_1 \dot{q}_2 + h \dot{q}_3 = m_{q1}; \\ & -h_1 \dot{q}_1 + J_{x1} \ddot{q}_2 + 2(k_s + k_t) \dot{q}_2 + C_{d2} q_2 \\ & \quad - k_t \dot{q}_3 - c_t q_3 = m_{q2}; \\ & -h \dot{q}_1 - k_t \dot{q}_2 - c_t q_2 \\ & \quad + J_x \ddot{q}_3 + k_t \dot{q}_3 + C_{d3} q_3 = m_{q3}, \end{aligned} \right\} \quad (12)$$

where

$$\begin{aligned} m_{q1} &= (J_{z1} + J_z) \Omega (\omega_X \cos \Omega t + \omega_Y \sin \Omega t); \\ m_{q2} &= J_{z1} \Omega (\omega_X \sin \Omega t - \omega_Y \cos \Omega t); \\ m_{q3} &= J_z \Omega (\omega_X \sin \Omega t - \omega_Y \cos \Omega t), \end{aligned} \quad (13)$$

and $J = J_{y1} + J_y$; $h_1 = (2J_{y1} - J_{z1}) \Omega$; $h = (2J_y - J_z) \Omega$;
 $C_{d1} = c_s + \left[(J_{z1} + J_z) - (J_{x1} + J_x) \right] \Omega^2$;
 $C_{d2} = c_s + [J_{z1} - J_{y1}] \Omega^2$; $C_{d3} = c_t + [J_z - J_y] \Omega^2$.

It is assumed that the value of angular stiffness of the CS is much greater than value of the dynamic stiffness and the following inequalities become true:

$$\left. \begin{aligned} & c_s \gg \left[(J_{z1} + J_z) - (J_{x1} + J_x) \right] \Omega^2; \\ & c_s \gg c_t. \end{aligned} \right\} \quad (14)$$

If conditions (14) are satisfied, the CS is best described as "hard". Due to (14) and the fact, that the value of the moment of inertia of the inner rotor is less than the value of one of the outer rotor, the influence of the motion of the inner rotors relative to the x_1 axis on the motion of the outer rotor is considered negligible. Hence, the mathematical model (12) can be simplified as follow:

$$\left. \begin{aligned} & J \ddot{q}_1 + k_s \dot{q}_1 + C_{d1} q_1 + h \dot{q}_3 = m_{q1}; \\ & -h \dot{q}_1 + J_x \ddot{q}_3 + k_t \dot{q}_3 + C_{d3} q_3 = m_{q3}. \end{aligned} \right\} \quad (15)$$

Note that the model described by set (15) is similar to the model of the rotor vibratory gyroscope considered in [24].

New variables are introduced:

$$U = q_1 \sqrt{J}; \quad V = q_3 \sqrt{J_x}. \quad (16)$$

Using the variables defined in Eq. (16), set (15) can be rewritten as:

$$\left. \begin{aligned} & \ddot{U} + \mu_1 \dot{U} + m_1 U + n \dot{V} = \frac{1}{\sqrt{J}} m_{q1}; \\ & -n \dot{U} + \ddot{V} + \mu_3 \dot{V} + m_3 V = \frac{1}{\sqrt{J_x}} m_{q3}, \end{aligned} \right\} \quad (17)$$

where $\mu_1 = k_s/J$; $\mu_3 = k_t/J_x$; $m_1 = C_{d1}/J$;
 $m_3 = C_{d3}/J_x$; $n = h/\sqrt{J_x J}$. Taking Laplace transformations of (17), under zero initial conditions, set (17) can be written in a complex notation as follow:

$$\left. \begin{aligned} & (s^2 + \mu_1 s + m_1) U(s) + n s V(s) = \frac{1}{\sqrt{J}} m_{q1}(s); \\ & -n s U(s) + (s^2 + \mu_3 s + m_3) V(s) = \frac{1}{\sqrt{J_x}} m_{q3}(s), \end{aligned} \right\} \quad (18)$$

where s is the Laplace operator. The solution of set (18) is:

$$\begin{aligned} U(s) &= \frac{(s^2 + \mu_3 s + m_3) \frac{1}{\sqrt{J}} m_{q1}(s) - n s \frac{1}{\sqrt{J_x}} m_{q3}(s)}{(s^2 + \mu_1 s + m_1)(s^2 + \mu_3 s + m_3) + n^2 s^2}; \\ V(s) &= \frac{(s^2 + \mu_1 s + m_1) \frac{1}{\sqrt{J_x}} m_{q3}(s) + n s \frac{1}{\sqrt{J}} m_{q1}(s)}{(s^2 + \mu_1 s + m_1)(s^2 + \mu_3 s + m_3) + n^2 s^2}. \end{aligned} \quad (19)$$

Thus, Eq. (19) describes the motion of the micromachined DTG relative to the rotating CF in the "hard" CS case.

V. ANALYSIS OF THE MODEL

Let us study the behavior of the mathematical model of the micromachined DTG (19) under the presence of the measuring angular rate lying on the measuring surface of the gyroscope to define the micromachined DTG gain for the "hard" CS case, relative to both the rotating, as well as the fixed CF.

Equations (19) can be rewritten as:

$$\left. \begin{aligned} q_1(s) &= W_1(s) \frac{1}{J} m_{q1}(s) - W_2(s) \frac{1}{\sqrt{J_x J}} m_{q3}(s); \\ q_3(s) &= W_2(s) \frac{1}{\sqrt{J_x J}} m_{q1}(s) + W_3(s) \frac{1}{J_x} m_{q3}(s), \end{aligned} \right\} \quad (20)$$

where

$$\begin{aligned} W_1(s) &= \frac{(s^2 + \mu_3 s + m_3)}{(s^2 + \mu_1 s + m_1)(s^2 + \mu_3 s + m_3) + n^2 s^2}; \\ W_2(s) &= \frac{n s}{(s^2 + \mu_1 s + m_1)(s^2 + \mu_3 s + m_3) + n^2 s^2}; \\ W_3(s) &= \frac{(s^2 + \mu_1 s + m_1)}{(s^2 + \mu_1 s + m_1)(s^2 + \mu_3 s + m_3) + n^2 s^2}. \end{aligned} \quad (21)$$

It is considered that the measuring angular rate creates moments $m_{q1}(s)$, and $m_{q3}(s)$ acting on the inner and outer rotors with a frequency that is equal to the spin speed of the rotor Ω . Hence, let us substitute $j\Omega$ for s into (21), to study the magnitude and phase of these transfer functions. We have:

$$\begin{aligned} |W_1(j\Omega)| &= \frac{\sqrt{(-\Omega^2 + m_3)^2 + \mu_3^2 \Omega^2}}{\sqrt{[\Omega^4 - (m_1 + m_3 + n^2 + \mu_1 \mu_3) \Omega^2 + m_1 m_3]^2 + [(\mu_1 + \mu_3) \Omega^3 + (m_3 \mu_1 + m_1 \mu_3) \Omega]^2}}; \\ \arg(W_1(j\Omega)) &= \arctan\left(\frac{\mu_3 \Omega}{-\Omega^2 + m_3}\right) - \arctan\left(\frac{(\mu_1 + \mu_3) \Omega^3 + (m_3 \mu_1 + m_1 \mu_3) \Omega}{\Omega^4 - (m_1 + m_3 + n^2 + \mu_1 \mu_3) \Omega^2 + m_1 m_3}\right); \end{aligned} \quad (22)$$

$$\begin{aligned} |W_2(j\Omega)| &= \frac{n \Omega}{\sqrt{[\Omega^4 - (m_1 + m_3 + n^2 + \mu_1 \mu_3) \Omega^2 + m_1 m_3]^2 + [(\mu_1 + \mu_3) \Omega^3 + (m_3 \mu_1 + m_1 \mu_3) \Omega]^2}}; \\ \arg(W_2(j\Omega)) &= \frac{\pi}{2} - \arctan\left(\frac{(\mu_1 + \mu_3) \Omega^3 + (m_3 \mu_1 + m_1 \mu_3) \Omega}{\Omega^4 - (m_1 + m_3 + n^2 + \mu_1 \mu_3) \Omega^2 + m_1 m_3}\right); \end{aligned} \quad (23)$$

$$\begin{aligned} |W_3(j\Omega)| &= \frac{\sqrt{(-\Omega^2 + m_1)^2 + \mu_1^2 \Omega^2}}{\sqrt{[\Omega^4 - (m_1 + m_3 + n^2 + \mu_1 \mu_3) \Omega^2 + m_1 m_3]^2 + [(\mu_1 + \mu_3) \Omega^3 + (m_3 \mu_1 + m_1 \mu_3) \Omega]^2}}; \\ \arg(W_3(j\Omega)) &= \arctan\left(\frac{\mu_1 \Omega}{-\Omega^2 + m_1}\right) - \arctan\left(\frac{(\mu_1 + \mu_3) \Omega^3 + (m_3 \mu_1 + m_1 \mu_3) \Omega}{\Omega^4 - (m_1 + m_3 + n^2 + \mu_1 \mu_3) \Omega^2 + m_1 m_3}\right). \end{aligned} \quad (24)$$

Applying $\omega_Y = 0$ and taking into account (22), (23), and (24), equation (20) for the steady motion of the outer rotor of the micromachined DTG relative to the rotating $(x_r y_r z_r)$ CF can be rewritten as:

$$\begin{aligned} q_1(t) &= |W_1(j\Omega)| \frac{J_z + J_{z1}}{J} \Omega \omega_X \cos(\Omega t + \arg(W_1(j\Omega))) \\ &\quad - |W_2(j\Omega)| \frac{J_z}{\sqrt{J J_x}} \Omega \omega_X \sin(\Omega t + \arg(W_2(j\Omega))); \\ q_3(t) &= |W_3(j\Omega)| \frac{J_z}{J_x} \Omega \omega_X \sin(\Omega t + \arg(W_3(j\Omega))) \\ &\quad + |W_2(j\Omega)| \frac{J_z + J_{z1}}{\sqrt{J J_x}} \Omega \omega_X \cos(\Omega t + \arg(W_2(j\Omega))); \end{aligned} \quad (25)$$

Equations (25) describe the motion of the untuned gyroscope. To define the rotor tuning condition, let us assume that $\mu_1 = 0$, $\mu_3 = 0$ and, setting the denominator of transfer functions to zero, we have:

$$\Omega^4 - (m_1 + m_3 + n^2) \Omega^2 + m_1 m_3 = 0. \quad (26)$$

The solution of (26) are the two natural frequencies:

$$\Omega_1 = \sqrt{\frac{m_1 + m_3 + n^2 + \sqrt{(m_1 + m_3 + n^2)^2 - 4m_1 m_3}}{2}}, \quad (27)$$

and

$$\Omega_2 = \sqrt{\frac{m_1 + m_3 + n^2 - \sqrt{(m_1 + m_3 + n^2)^2 - 4m_1 m_3}}{2}}. \quad (28)$$

Furthermore, let us define the parameters m_1 , m_3 and n^2 as follow. The parameter m_1 can be written as:

$$m_1 = c_s / J + \kappa_g \Omega^2, \quad (29)$$

where $\kappa_g = (J_z + J_{z1} - J_x - J_{x1}) / (J_x - J_{x1})$ is the gyroscope constructive parameter. Parameters m_3 and n^2 can be expressed as:

$$\begin{aligned} m_3 &= c_t / J_x + \kappa \Omega^2; \\ n^2 &= J_x (1 - \kappa)^2 \Omega^2 / J, \end{aligned} \quad (30)$$

where $\kappa = (J_z - J_x) / J_x$ is the outer rotor constructive parameter. Note, that both κ_g and κ are approximately equal to one. For example, the aluminium disk-shaped rotor whose diameter 500 μm and thickness 10 μm has a value of κ equal to 0.9998. According to (14) and the previous discussion, the following inequality between the parameters of the gyroscope can be written:

$$m_1 \gg m_3 \gg n^2. \quad (31)$$

In equations (27) and (28), Ω_1 is determined by the parameters of the CS and Ω_2 by the mechanical parameters of the torsional springs, respectively. Hence, to tune the micromachined DTG, the rotors spin speed Ω should be equal

to the second natural frequency. The rotor tuning condition becomes:

$$\Omega - \Omega_2 = 0. \quad (32)$$

The speed of rotor spin which is satisfied by (32) is denoted by $\tilde{\Omega}$. Accounting to (31), condition (32) can be written as:

$$\tilde{\Omega} \approx \sqrt{m_3}, \quad (33)$$

or, using equation (30)

$$\tilde{\Omega} \approx \sqrt{\frac{c_t}{J_x(1-\kappa)}}. \quad (34)$$

The last equation is the classic condition for rotor tuning of a rotor vibratory gyroscope.

If the condition (32) is held, then the magnitude and phase of the transfer functions (22), (23) and (24) become:

$$|W_1(j\tilde{\Omega})| \approx \frac{1}{m_1}; \arg(W_1(j\tilde{\Omega})) \approx 0; \quad (35)$$

$$|W_2(j\tilde{\Omega})| \approx \frac{n}{\mu_3 m_1}; \arg(W_2(j\tilde{\Omega})) \approx 0; \quad (36)$$

$$|W_3(j\tilde{\Omega})| \approx \frac{1}{\mu_3 \tilde{\Omega}}; \arg(W_3(j\tilde{\Omega})) \approx \arctan\left(\frac{\mu_1 \tilde{\Omega}}{m_1}\right) - \frac{\pi}{2}. \quad (37)$$

It is assumed that the values of the damping coefficients $\frac{\mu_1}{\pi}$ and μ_3 are small. Hence, we can use $\arg(W_3(j\tilde{\Omega})) = -\frac{\pi}{2}$. Substituting (35), (36), and (37) into (25), we have:

$$\begin{aligned} q_1(t) &= \frac{J_z + J_{z1}}{m_1 J} \tilde{\Omega} \omega_X \cos(\Omega t) \\ &\quad - \frac{n}{\mu_3 m_1} \frac{J_z}{\sqrt{J J_x}} \tilde{\Omega} \omega_X \sin(\Omega t); \\ q_3(t) &= -\frac{J_z J_x}{\mu_3 m_1} \omega_X \cos(\Omega t) \\ &\quad + \frac{n}{\mu_3 m_1} \frac{J_z + J_{z1}}{\sqrt{J J_x}} \tilde{\Omega} \omega_X \cos(\Omega t); \end{aligned} \quad (38)$$

Equations (38) describe the steady motion of the outer rotor of the tuned micromachined DTG relative to the rotating CF. Note that in the second equation of set (38) the value of the second term is much less than the first one due to $m_1 \gg n\tilde{\Omega}$ (see (29), (30) and (14)); hence it can be neglected.

Taking into account (1) the steady motion of the outer rotor of the tuned micromachined DTG relative to the fixed (XY) CF can be written as follow:

$$\begin{aligned} \alpha(t) &= -K_g \omega_X - K_g \omega_X \cos(2\tilde{\Omega}t) - K_{cc2} \omega_X \\ &\quad - K_{cc1} \omega_X \sin(2\tilde{\Omega}t) + K_{cc2} \omega_X \cos(2\tilde{\Omega}t); \\ \beta(t) &= -K_g \omega_X \sin(2\tilde{\Omega}t) + K_{cc1} \omega_X \\ &\quad + K_{cc1} \omega_X \cos(2\tilde{\Omega}t) - K_{cc2} \omega_X \sin(2\tilde{\Omega}t); \end{aligned} \quad (39)$$

where $K_g = \frac{J_z}{2\mu_3 J_x}$ is the tuned micromachined DTG gyroscope gain for the "hard" CS case, $K_{cc1} = \frac{J_z + J_{z1}}{2m_1 J} \tilde{\Omega}$ and $K_{cc2} = \frac{n}{\mu_3 m_1} \frac{J_z + J_{z1}}{\sqrt{J J_x}} \tilde{\Omega}$ are the cross-coupling coefficients.

VI. COMPARATIVE ANALYSIS BETWEEN GYROSCOPE GAIN OF THE TUNED MICROMACHINED DTG AND THE DEVELOPED PROTOTYPE OF MMG BASED ON CONTACTLESS SUSPENSION

Setting the generalized coordinate q_3 and principal moments of inertia of the outer rotor (J_z , J_x) to zero, the mathematical model of a MMG based on CS arises from (12) as follow:

$$\left. \begin{aligned} J_{x1} \ddot{q}_1 + k_s \dot{q}_1 + C_{d1} q_1 + h_1 \dot{q}_2 &= m_{q1}; \\ -h_1 \dot{q}_1 + J_{x1} \ddot{q}_2 + k_s \dot{q}_2 + C_{d2} q_2 &= m_{q2}. \end{aligned} \right\} \quad (40)$$

Let us rewrite (40) in complex notation:

$$\left. \begin{aligned} (J_{x1} s^2 + k_s s + C_{d1}) q_1(s) + h_1 s q_2(s) &= m_{q1}(s); \\ -h_1 s q_1(s) + (J_{x1} s^2 + k_s s + C_{d1}) q_2(s) &= m_{q2}(s). \end{aligned} \right\} \quad (41)$$

The solution of set (41) is as follows:

$$\begin{aligned} q_1(s) &= W_1(s) m_{q1}(s) - W_2(s) m_{q2}(s); \\ q_2(s) &= W_1(s) m_{q2}(s) + W_2(s) m_{q1}(s), \end{aligned} \quad (42)$$

where

$$\begin{aligned} W_1(s) &= \frac{J_{x1} s^2 + k_s s + C_{d1}}{(J_{x1} s^2 + k_s s + C_{d1})^2 + h_1^2 s^2}; \\ W_2(s) &= \frac{h_1 s}{(J_{x1} s^2 + k_s s + C_{d1})^2 + h_1^2 s^2}. \end{aligned} \quad (43)$$

Setting in (43) $s = j\Omega$, let us write the magnitude and phase of transfer functions $W_1(s)$, $W_2(s)$. Taking into account the definition of parameters C_{d2} and h_1 , we have:

$$\begin{aligned} |W_1(j\Omega)| &= \frac{\sqrt{(-(2J_{x1} - J_{z1})\Omega^2 + c_s)^2 + k_s^2 \Omega^2}}{\sqrt{(-(2J_{x1} - J_{z1})\Omega^2 c_s + c_s^2 - k_s^2 \Omega^2)^2 + 4k_s^2 \Omega^2 (-(2J_{x1} - J_{z1})\Omega^2 + c_s)^2}}; \\ \arg(W_1(j\Omega)) &= \arctan\left(\frac{k_s \Omega}{-(2J_{x1} - J_{z1})\Omega^2 + c_s}\right) \\ &\quad - \arctan\left(\frac{2k_s \Omega (-(2J_{x1} - J_{z1})\Omega^2 + c_s)}{-2(2J_{x1} - J_{z1})\Omega^2 c_s + c_s^2 - k_s^2 \Omega^2}\right); \end{aligned} \quad (44)$$

$$\begin{aligned} |W_2(j\Omega)| &= \frac{(2J_{x1} - J_{z1})\Omega^2}{\sqrt{(-(2J_{x1} - J_{z1})\Omega^2 c_s + c_s^2 - k_s^2 \Omega^2)^2 + 4k_s^2 \Omega^2 (-(2J_{x1} - J_{z1})\Omega^2 + c_s)^2}}; \\ \arg(W_2(j\Omega)) &= \frac{\pi}{2} \\ &\quad - \arctan\left(\frac{2k_s \Omega (-(2J_{x1} - J_{z1})\Omega^2 + c_s)}{-2(2J_{x1} - J_{z1})\Omega^2 c_s + c_s^2 - k_s^2 \Omega^2}\right). \end{aligned} \quad (45)$$

Assuming $k_s = 0$, setting the denominator of transfer functions (44) and (45) to zero, we have:

$$-2(2J_{x1} - J_{z1})\Omega^2 c_s + c_s^2 = 0. \quad (46)$$

The solution of (46) is the natural frequency of a MMG based on CS:

$$\tilde{\Omega} = \sqrt{\frac{c_s}{2(2J_{x1} - J_{z1})}}. \quad (47)$$

Two operating modes can be considered, namely the first mode, when the rotor spin speed is much less than the natural frequency $\tilde{\Omega}$ of the MMG and the second mode, when the

rotor spin speed is equal to the natural frequency $\tilde{\Omega}$. However, the second mode is not used in the developed prototypes of MMG and is therefore not considered here.

For the first mode ($\Omega \ll \tilde{\Omega}$), we have:

$$|W_1(j\Omega)| \approx \frac{1}{c_s}; \arg(W_1(j\Omega)) \approx 0; \quad (48)$$

$$|W_2(j\tilde{\Omega})| \approx \frac{(2J_{x1} - J_{z1})\Omega^2}{c_s^2} = \frac{J_{x1}(1 - \kappa_1)\Omega^2}{c_s^2}; \quad (49)$$

$$\arg(W_2(j\tilde{\Omega})) \approx \frac{\pi}{2},$$

where $\kappa_1 = (J_{z1} - J_{x1})/J_{x1}$ is the constructive parameter of MMG. Hence,

$$|W_1(j\Omega)| \gg |W_2(j\tilde{\Omega})|, \quad (50)$$

and the contribution of term $W_2(s)$ in Eq. (42) are negligible.

Then, for $\omega_y = 0$, the equations of angular displacements of a gyroscope rotor relative to the rotating CF are

$$q_1(t) = \frac{J_z \Omega}{c_s} \omega_X \cos(\Omega t); \quad (51)$$

$$q_2(t) = \frac{J_z \Omega}{c_s} \omega_X \sin(\Omega t);$$

Substituting (51) into (1), let us write the equations of angular displacements of a gyroscope rotor relative to the fixed (XYZ) CF as follows:

$$\alpha = 0; \beta = \frac{J_z \Omega}{c_s} \omega_X. \quad (52)$$

Following from (52), the gyroscope gain can be defined by:

$$K_g = \frac{J_z \Omega}{c_s} \quad (53)$$

To compare the gyroscope gain to the measuring angular rate of micromachined DTG and MMG based on CS, a dimensionless coefficient Ψ is introduced, as the ratio of a gyroscope gain of the micromachined DTG to the MMG based on CS.

In particular, the MMG based on an inductive CS is considered. Results of the experimental study are presented in [13]. In this prototype, the aluminum rotor of diameter 500 μm and thickness 10 μm is rotated with a spin speed of 105 s^{-1} . The ICS provides the angular displacement of the rotor with the finite angular stiffness which is equal to $c_s = 8.0 \times 10^{-11}$ N m rad^{-1} . Values of principal moments of inertia of the rotor are: $J_{z1} = 1.78 \times 10^{-16}$ kg m^2 and $J_{x1} = 8.90 \times 10^{-17}$ kg m^2 .

It is assumed that the micromachined DTG is fabricated as in work [13]. To do so, let us link, in particular, the outer silicon rotor whose outer and inner diameters are 900 μm and 600 μm , respectively and thickness is 10 μm to this inner aluminum rotor via silicon torsional springs whose value of stiffness satisfies a rotor tuning condition for spin speed 105 s^{-1} . Principal moments of inertia of the outer rotor are: $J_z = 1.50 \times 10^{-15}$ kg m^2 and $J_x = 7.50 \times 10^{-16}$ kg m^2 .

For these parameters of the micromachined DTG, the inequality (14) is held. Therefore, we have the case of the "hard"

CS. Taking into account (39), (53) and the notations adopted in (17); the coefficient Ψ can be written as:

$$\Psi = \frac{J_z c_s}{2J_{z1} k_t \tilde{\Omega}}. \quad (54)$$

As can be seen from (54), the dimensionless coefficient Ψ is dependent on the values of the principal moments of rotors. The larger the principal moment of rotor, the greater the gyroscope gain. Hence, this variable can be excluded from consideration and Eq. (54) becomes:

$$\Psi = \frac{c_s}{2k_t \tilde{\Omega}}. \quad (55)$$

The damping coefficient of the torsional springs k_t is expressed in term of the quality factor, and then Eq. (55) can be rewritten as:

$$\Psi = \frac{c_s Q}{2J_x \tilde{\Omega}^2}, \quad (56)$$

where Q is the quality factor of the torsional springs - outer rotor oscillating system. The value of the quality factor in micromechanical silicon torsional springs is about $Q = 0.77$ to 1.37×10^5 [25]. Hence,

$$\Psi = \frac{8.0 \times 10^{-11} \times 0.77 \times 10^5}{2 \times 7.50 \times 10^{-16} \times 105^2} = 3.70 \times 10^5 \gg 1. \quad (57)$$

Thus, the micromachined DTG increases the gyroscope gain to the measuring angular rate in comparison with prototypes of the MMG based on CS, in particular for the ICS by five orders of magnitude. Due to the mechanical decoupling between inner and outer rotors of the micromachined DTG that occurs in the tuned gyroscope, the sensitivity of the linear angular displacement of the gyroscope outer rotor to measuring angular rate is significantly increased. This fact leads to an improvement the signal-to-noise ratio of capacitive angular sensors signal processing. As a result, the accuracy of these type of gyroscopes is expected to be higher.

VII. CONCLUSION

In this paper, the mathematical model of a micromachined, dynamically tuned gyroscope, based on a contactless suspension, is developed as an alternative strategy for improving the accuracy of micromachined gyroscope. The dynamical analysis of it for the case in which the CS provides "hard" electrical springs is conducted. The analysis shows that such a gyroscope can measure the rotation of a gyroscope case in principle and with gyroscope gain which is much larger than one of the existing prototypes of MMG based on CS. This fact leads to an improvement in signal-to-noise ratio of the pick-off sensors signal processing. Hence, the expected accuracy of such type of gyroscopes is expected to be higher.

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