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Numerical study of the formation process of ferrofluid droplets

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This paper numerically investigates the influence of a uniform magnetic field on the droplet formation process at a microfluidic flow focusing configuration. The mathematical model was formulated by considering the balance of forces such as interfacial tension, magnetic force, and viscous stress across the liquid/liquid interface. A linearly magnetizable fluid was assumed. The magnetic force acts as a body force where the magnetic permeability jumps across the interface. The governing equations were solved with finite volume method on a Cartesian fixed staggered grid. The evolution of the interface was captured by the particle level set method. The code was validated with the equilibrium steady state of a ferrofluid droplet exposed to a uniform magnetic field. The evolution of the droplet formation in a flow focusing configuration was discussed. The paper mainly analyzes the effects of magnetic Bond number and the susceptibility on the velocity field and the droplet size. The droplet size increased with increasing magnetic strength and susceptibility. © 2011 American Institute of Physics. [doi:10.1063/1.3614569]

I. INTRODUCTION

The emerging field of droplet-based microfluidics has numerous applications in life sciences and analytical chemistry. Droplet-based labs on a chip have been used for drug delivery,¹ biological analysis,² and chemical analysis.³ Manipulation tasks such as mixing, sorting, and transportation can be performed on droplet-based platforms.^{4,5} Monodispersed droplets are generated in microfluidic devices by injecting immiscible fluids into microchannel configurations such as T-junction,^{6,7} flow focusing,⁸ or co-flowing.⁹ The droplet size and the formation frequency can be controlled by adjusting the flow rates, the flow rate ratio, the viscosities, and the interfacial tension of the two phases. Droplet formation processes in the different microfluidic configurations has been numerically studied in recent years.^{10–15}

Manipulation of microdroplets in a microfluidic network can be achieved passively by arranging the microchannel geometry. For example, a loop was used to realize coding/decoding of a droplet train.¹⁶ With a similar concept, droplet traffic at a T-junction was regulated using a bypass.¹⁷ Through appropriate microfluidic network designs, logical operations of AND, OR, and NOT can also be performed on a droplet train.¹⁸ Droplet velocities can be regulated with microfluidic ratchets.¹⁹

Active concepts for droplet manipulation rely on an external physical field. For instance, a temperature field induces an interfacial tension gradient on a droplet. Since the Marangoni force causes the droplet to move toward a place with lower surface energy, active control can be achieved by using a nonuniform temperature field. The temperature field is generated either by resistive heaters⁴ or by focused laser beam.^{20–22} Heaters were used to transport liquid plugs along a capillary.²³ Baroud *et al.* controlled processes such as droplet formation, sorting, and fusion using a single focused laser spot.^{21,22} Laser patterns can also be used to guide the droplet

by the orientation of the patterns.²⁴ The above thermocapillary phenomenon can be solved numerically by coupling the energy equation with two-phase flow modeling using the finite volume method.²⁵ Manipulation with thermocapillarity is relatively slow. In contrast, electrowetting allows manipulating droplets with a fast response and low energy consumption. Electrowetting is based on the change of the contact angle under an applied electric field.²⁶ Active manipulation tasks such as droplet splitting, merging, transporting, and formation were realized with electrowetting.²⁷

A magnetic field can also be used for manipulating droplets. In this case, a droplet with magnetic properties such as a ferrofluid droplet is needed. Ferrofluids consist of ferromagnetic nanoparticles such as magnetite (Fe_3O_4) coated by a layer of surfactants and suspended in a carrier liquid. A magnetic field can deform a ferrofluid drop.²⁸ Ferrofluid droplet can be moved by employing planar coils.²⁹ Recently, our group reported the effect of magnetowetting on a sessile ferrofluid droplet.³⁰ A permanent magnet was used to control the formation process of ferrofluid droplets at a microfluidic T-junction.³¹ All these experimental observations require numerical models to further explain the interaction between ferrohydrodynamics and capillarity.

The mathematical formulation for hydrodynamics of ferrofluid (ferrohydrodynamics) was discussed by Rosensweig.²⁸ The problem of deformation of a ferrofluid droplet in a uniform magnetic field was numerically studied by coupling the magnetic field, the free surface, and the fluid flow in two-dimensional (2D) (Ref. 32) and three-dimensional (3D) (Ref. 33) models. The magnetic force can be defined as a body force acting on the interface. A ferrofluid droplet falling down a non-magnetic fluid was modeled by Korlie *et al.*³⁴ In all these studies, linearly magnetizable fluids were considered. The shape of a droplet of a non-linear magnetic fluid was investigated by Afkhami *et al.*³⁵

In contrast to the previous numerical works on ferrofluid droplets, this paper reports the development of a numerical model to study the formation process of ferrofluid droplets in a flow-focusing configuration under the influence of a magnetic field. The magnetization vector of the magnetic fluid is assumed to be parallel to the magnetic field. We first discuss the governing equations of the magnetic field and the fluid flow. Just as interfacial tension, the magnetic force induced by the jump of permeability was coupled in the Navier-Stokes (N-S) equation. The particle level set method was used to track the moving interface between two immiscible phases. Next, the numerical method used in this paper is introduced. The magnetic potential of a permeable sphere was solved numerically to validate the code. The equilibrium shape of a ferrofluid drop was obtained by using the particle level set method under the consideration of both the magnetic field and the flow field. The droplet formation in the flow focusing configuration was tested and compared with the volume of fluid (VOF) method to further validate our numerical scheme. The effects of magnetic field strength represented by the magnetic bond number and the susceptibility of the ferrofluid was investigated and discussed.

II. MATHEMATICAL MODELS

A. Physical field

The Maxwell equations for nonconducting fluids are

$$\vec{\nabla} \times \vec{H} = 0 \quad (1)$$

and

$$\vec{\nabla} \cdot \vec{B} = 0, \quad (2)$$

where $\vec{H}$ is the magnetic field strength and $\vec{B}$ is the magnetic flux density. We consider a ferrofluid domain Ω_f surrounded by a nonmagnetizable medium Ω/Ω_f . The fields $\vec{B}$ and $\vec{H}$ satisfy

$$\vec{B} = \begin{cases} \mu_0(\vec{H} + \vec{M}), & \text{if } \Omega_f \\ \mu_0(\vec{H}), & \text{if } \Omega/\Omega_f \end{cases}. \quad (3)$$

μ_0 is the permeability constant of the free space and its value is $4\pi \times 10^{-7}$ N/A². $\vec{M}$ denotes the magnetization of the ferrofluid. The magnetization of the ferrofluid is assumed to be a linear function of the magnetic field strength

$$\vec{M} = \chi_m \vec{H}, \quad (4)$$

where χ_m is the ferrofluid susceptibility. The magnetic permeability of the ferrofluid is defined as $\mu_1 = \mu_0(1 + \chi_m)$. Thus, the flux density inside the ferrofluid is $\vec{B} = \mu_1 \vec{H}$. Introducing the magnetic scalar potential ψ in the form of $\vec{H} = -\nabla\psi$ yields

$$\nabla \cdot [(1 + \chi_m)\nabla\psi] = 0. \quad (5)$$

The permeability jumps across the interface between two immiscible phases. Thus, the scalar potential changes as the interface is moving. The permeability can be solved within

the whole computational domain based on the harmonic mean

$$\frac{1}{1 + \chi_m} = \frac{1 - H}{1 + \chi_{m-}} + \frac{H}{1 + \chi_{m+}}. \quad (6)$$

The magnetic force is calculated by

$$\vec{F}_m = -\frac{1}{2}\mu_0|\vec{H}|^2\nabla\chi_m, \quad (7)$$

where $\nabla\chi_m = \frac{d\chi_m}{d\phi}\nabla\phi$ and

$$\frac{d\chi_m}{d\phi} = \frac{(1 + \chi_{m+})(1 + \chi_{m-})(\chi_{m+} - \chi_{m-})D(\phi)}{[(1 + \chi_{m+}) + H(\chi_{m-} - \chi_{m+})]^2}. \quad (8)$$

The magnetic force only acts on the interface where the discontinuity in magnetic permeability takes place and vanishes when the permeability is constant. The magnetic field boundary condition satisfies

$$\vec{H} \cdot \hat{n} = -\frac{\partial\psi}{\partial n}, \quad \forall \vec{x} \in \partial\Omega. \quad (9)$$

Therefore, the two-phase microfluidic system can be described by a single modified N-S equation coupled with interfacial force and magnetic force

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla \cdot (\rho\vec{u}\vec{u}) = -\nabla p + \nabla \cdot [\mu(\nabla\vec{u} + \nabla\vec{u}^T)] - \sigma\kappa\hat{n}_f D(\phi) - \frac{1}{2}\mu_0|\vec{H}|^2\nabla\chi_m, \quad (10)$$

where the third term on the right side of the equation is the interfacial force F_σ , with σ the interface tension, κ the curvature, $\hat{n}_f$ the normal vector to the interface, and $D(\phi)$ the delta function that is zero everywhere except near the interface (Ref. 19). The gravity is neglected in this model. An incompressible and unsteady media is assumed

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\vec{u}) = 0. \quad (11)$$

The jumps of the properties ρ and μ near the interface were smoothed with Heaviside function in the whole computational domain.³⁶

B. Particle level set method

A level set function $\phi(\vec{x}, t)$ was introduced over the whole computational domain to describe the shortest normal distance from the interface. The interface was represented by $\phi(\vec{x}, t) = 0$. The different signs “+” and “−” are used to distinguish the two liquid phases. The evolution of the interface between two immiscible phases was governed by the level set method³⁷ on a fixed rectangular grid with a grid size of Δx .

$$\phi_t + \vec{u} \cdot \nabla\phi = 0. \quad (12)$$

A reinitialization procedure is employed to maintain the level set function to be a signed distance function to the interface with some degree of accuracy

$$\frac{\partial \phi^*}{\partial t^*} = \text{sign}(\phi_0)(1 - |\nabla \phi^*|) \quad (13)$$

and

$$\phi_0(\vec{x}) = \phi(\vec{x}, 0). \quad (14)$$

For numerical purposes, it is useful to smooth the signed function as

$$\text{sign}(\phi_0) = \phi / \sqrt{\phi^2 + |\nabla \phi|^2 \Delta x}. \quad (15)$$

The steady-state solution of the equation will converge to the actual distance and satisfy $|\nabla \phi| = 1$.

The particle level set method (PLS) presented by Enright *et al.*³⁸ and Tran and Udaykumar³⁹ is employed to alleviate the mass loss problem in the under-resolved region. Spherical particles are employed to track the characteristic information and to reconstruct the interface properly in a 3D space. The radii of the particle are within a range of $r_{\min} < r_p < r_{\max}$ ($r_{\min} = 0.1\Delta x$ and $r_{\max} = 0.5\Delta x$). The center locations are $\vec{x}_p$. The particles are randomly seeded into the cells within a band of $b_{\max}$ from the interface ($b_{\max} = 2\Delta x$ in 3D space, and $b_{\max} = 3\Delta x$ in 2D space). Fig. 1(a) shows the result of a random distribution of the particles in the 2D case. These particles are subsequently attracted to its corresponding region. The negative particles will be attracted to the Ω_c ($\phi \leq 0$) region; the positive particles are attracted to the Ω_d ($\phi > 0$) region, Fig. 1(b). The boundary separating the two signed particles is the interface. In order to move the particle in the normal direction to the interface, ϕ_g is introduced

$$\phi_g = r_{\min} + \text{ran}(\text{iseed}) \bullet (b_{\max} - b_{\min}), \quad (16)$$

where $\text{ran}(\text{iseed})$ is a random value between 0 and 1. Thus, $\phi_g \in (\pm b_{\min}, \pm b_{\max})$. The particles are attracted to the iso-contour ϕ_g by³⁸

$$\vec{x}_{\text{new}} = \vec{x}_p + \lambda (\phi_g - \phi(\vec{x}_p)) \vec{N}(\vec{x}_p). \quad (17)$$

The level set value $\phi(\vec{x}_p)$ and the unit normal $\vec{N}(\vec{x}_p)$ are obtained by the trilinear interpolation method from the values of $\phi(\vec{x}, t)$ and $\hat{n}_f$ at the eight neighboring corner nodes of the underlying grid. Initially with $\lambda = 1$, the new particle has to satisfy two conditions: (a) $\vec{x}_{\text{new}}$ does not escape the calcu-

lation domain and (b) $\phi(\vec{x}_{\text{new}}) \in (\pm b_{\min}, \pm b_{\max})$. Otherwise, λ is to be reduced by half and Eq. (17) is repeated. If the run fails to meet both conditions after a maximum iteration number of 20, the particle is deleted. The next particle is randomly seeded until the particle number is 64 for the 3D case and 16 for the 2D case. The particle radius is defined as

$$r_p = \begin{cases} r_{\max}, & \text{if } S_{\text{new}} > r_{\max} \\ S_{\text{new}}, & \text{if } r_{\min} \leq S_{\text{new}} \leq r_{\max} \\ r_{\min}, & \text{if } S_{\text{new}} < r_{\min} \end{cases}, \quad (18)$$

where $S_{\text{new}} = \text{sign}(\phi(\vec{x}_{\text{new}}))\phi(\vec{x}_{\text{new}})$. Particle overlapping is permitted. The mean distance between particles is generally less than Δx . These particles offer sub-cell resolution, i.e., they are more accurate in representing the interface than the level-set function. Therefore, these particles can be used to correct the level-set function. A successful particle attachment is shown in Fig. 1(b).

The particles are advected by the velocity field via³⁸

$$\frac{d\vec{x}_p}{dt} = \vec{u}(\vec{x}_p), \quad (19)$$

where $\vec{u}(\vec{x}_p)$ is the particle velocity. The velocities are obtained by trilinear interpolation method from the fluid velocities. In each time iteration step, $\phi(\vec{x}, t)$ is calculated from Eqs. (12) and (13) first. Subsequently, the positions of particles are advected to their new spatial locations by applying the second-order TVD Runge-Kutta scheme to Eq. (19). In the well-resolved regions, the particles will follow the motion of the interface. However, if the level-set solution is not adequately accurate, the particles may escape across the interface to the opposite side during the advection process. It is therefore necessary to perform particle correction strategies to rebuild the positive region and the negative region. One correction technique was presented by Enright *et al.*³⁸ The weak points of this method were pointed out by Tran and Udaykumar³⁹ and Wang *et al.*,⁴⁰ who proposed improved methods. The negative and positive particles that happen to be in the opposite side of the interface by a distance more than its radius are identified as E^- and E^+ , respectively. The level-set functions corresponding to the signed particles, ϕ^- and ϕ^+ , are updated as

$$\phi^+ = \begin{cases} \max_{\forall p \in E^+} (|\phi^+|, |\phi_p|), & \text{if } \phi^+ > 0 \\ -\min_{\forall p \in E^+} (|\phi^+|, |\phi_p|), & \text{if } \phi^+ < 0 \end{cases} \quad (20)$$

and

FIG. 1. 2D particle seeding (a) initial random seeding and (b) after attachment procedure (black indicates positive particles and gray indicates negative particles).

FIG. 2. Particle reseeding procedure (a) before reseeding, (b) after reseeding.

$$\phi^- = \begin{cases} \min_{\forall p \in E^-} (|\phi^-|, |\phi_p|), & \text{if } \phi^- > 0 \\ -\max_{\forall p \in E^-} (|\phi^-|, |\phi_p|), & \text{if } \phi^- < 0 \end{cases} \quad (21)$$

where $\phi_p = \text{sign}[\phi(\vec{x}_p)](r_p - |\vec{x} - \vec{x}_p|)$. $\phi(\vec{x}_p)$ is the trilinear interpolation result from the values underlying the grid nodes. Finally, particle correction on the level-set function ϕ is

$$\phi = \begin{cases} \phi^-, & \text{if } |\phi^+| > |\phi^-| \\ \phi^+, & \text{if } |\phi^+| \leq |\phi^-| \end{cases} \quad (22)$$

After a few time iteration steps, because of the interface movement and particle correction, the particles are either removed or added into the cell, as shown in Fig. 2(a). Reseeding algorithm is necessary to control the particle distribution at fixed time iteration steps. The particles should be deleted if they are beyond the computational domain or located in the cell without a corner in a narrow bond b_{max} . If the number of particle is less than 64 (in the 3D case), additional particles are seeded into the cell randomly. If more than 64, sort the values of $\text{sign}(\phi(\vec{x}_{new}))\phi(\vec{x}_{new}) - r_p$ in ascending order and delete the larger ones by heap sorting. Fig. 2(b) shows the completed reseeding results from the poor particle distribution depicted in Fig. 2(a). The reseeding algorithm is not a perfect method to smooth the interface. It is necessary to randomly seed and attract the particles repeatedly when the reseeding algorithm failed to maintain the smoothness of the interface.

C. Numerical method

The N-S Eq. (10), continuum Eq. (11), and magnetic potential Eq. (5) are solved on the Cartesian staggered grid by a finite volume method. The velocity-pressure coupling utilizes the SIMPLER algorithm.⁴¹ The combined convection-diffusion effect is predicted by the Power Law. The time integration used a fully implicit scheme. The level set Eq. (11) and the reinitialization Eq. (12) are solved by constructing high-order weighted essentially non-oscillatory (WENO) schemes with local Lax-Friedrichs flux⁴² and Godunov flux,⁴³ respectively, and advected by total variation diminishing (TVD) Runge-Kutta methods.⁴⁴ To save the computational time, a narrow band procedure is implemented near the interface.⁴⁵

According to the given initial conditions $\vec{u}^n$, ϕ^n , and ψ^n , coupling of the physical fields and the interface evolution is carried out based on the concept depicted in Fig. 3.

FIG. 3. Coupled interface, magnetic field, and flow field interactions.

FIG. 4. The magnetic potential for a permeable sphere placed in a uniform magnetic field.

III. RESULTS AND DISCUSSION

A. Magnetic field validation

The magnetic potential was obtained by numerically solving Eq. (5). We consider a permeable sphere exposed to a uniform magnetic field of strength H_0 . The permeability ratio of two media is assumed as $\mu_0/\mu_1 = 1/6$. The contour of magnetic potential in and around a permeable sphere was calculated and compared with the exact solution⁴⁶ in Fig. 4. Our numerical results agree well with the exact solution. The magnetic potential varies across the interface where the permeability jumps in value and is identical where the permeability is constant.

To validate the coupling of the magnetic field, the velocity field, and the evolution of the interface, the steady-state shape of the ferrofluid drop was analyzed.⁴⁷ We consider a ferrofluid drop suspended in a miscible non-magnetic fluid. Initially, the drop has a spherical shape due to the minimized interfacial tension. An external uniform magnetic field was then supplied. The drop is stretched in the direction of the uniform magnetic field until it reaches the stable equilibrium shape. The velocities of the computational domain relax to zero in the steady state. Therefore, the drop deformation depends on the balance of the magnetic force and the interfacial force. Their ratio is described by the dimensionless magnetic Bond number $Bm = \mu_0 V^{1/3} \chi_m H_0^2 / (2\sigma)$, where V is the volume of the droplet and H_0 is magnetic field strength. The equilibrium shapes are compared with results reported by Lavrova,⁴⁷ as shown in Fig. 5. The ratios between

FIG. 5. The equilibrium shape of a ferrofluid drop in a magnetic field compared with the simulation results by Lavrova.

semimajor and semiminor are 2.08, 2.30, 2.42, and 2.54 corresponding to 20, 30, 40, and 60 cells, respectively. The numerical result approximates to the Lavrova result of 2.77 and the theoretical result of 2.45 (Ref. 47) with grid refinement. Therefore, the grid independence study is of vital importance in the future calculation.

1. Droplet formation

Fig. 6 shows the geometry of a flow-focusing configuration. The configuration consists of two side inlets and a single main inlet and an outlet. The channel width as well as height L is taken as the characteristic length. In the device used in the later experiment, this length is $100\ \mu\text{m}$. Two immiscible liquids introduced into the inlets act as the continuous phase and the dispersed phase. The droplets of dispersed phase 1 were formed because of the squeezing of continuous phase 2. Inlet velocities were assumed as fully developed with u_1 and v_2 . To save computational time, only one quarter of the domain was calculated due to the symmetry of the channel geometry and the flow field. The symmetric boundary conditions are applied to the symmetry surfaces. No-slip conditions are imposed on all the solid walls. The outlet uses outflow boundary condition.

In this case study, The mean velocities of the main channel and the lateral channel are $u_1 = 0.0025$ and $v_2 = 0.01$. Both fluids have same densities and viscosities, and they are $\rho_1 = \rho_2 = 1000\ \text{kg/m}^3$ and $\mu_1 = \mu_2 = 1\ \text{mPa}\cdot\text{s}$. The interfacial tension is $\sigma = 1\ \text{mN/m}$. The present results are compared and agree well with results from VOF method. The normalized droplet volumes $V^* = V/L^3$ are shown in Table I. Our numerical method uses a staggered grid to approach the flow focusing channel, while the VOF method can define the real channel structure shown in Fig. 6. The difference can be reduced with the grid refinement. The number of control volumes $184 \times 62 \times 26$ will be used in the following simulation.

2. Magnetic field influence on the droplet formation process

The formation of ferrofluid droplets in a flow focusing configuration was first investigated without the magnetic force. The density and viscosity of ferrofluid are $\rho_1 = 1100\ \text{kg/m}^3$ and $\mu_1 = 2 \times 10^{-3}\ \text{mPa}\cdot\text{s}$. The properties of the oil are $\rho_2 = 960\ \text{kg/m}^3$ and $\mu_2 = 96 \times 10^{-3}\ \text{mPa}\cdot\text{s}$. The interfacial tension is $\sigma = 12\ \text{mN/m}$. The flow rate of each oil inlet is $Q_2 = 10\ \mu\text{l/h}$. The water flow rate is $Q_1 = 5\ \mu\text{l/h}$. Thus, the mean velocities are $u_1 = 2.78 \times 10^{-4}\ \text{m/s}$ and $v_2 = u_1/2$. The droplet formation process in the flow

FIG. 6. Schematic of the microfluidic flow-focusing configuration.

FIG. 7. Sequence of droplet formation for (a) experimental results and (b) numerical results on x-y plane ($Ca = 2.22 \times 10^{-3}$, $Re = 2.78 \times 10^{-4}$, $Q_2 = 10\ \mu\text{l/h}$, $Q_1 = 5\ \mu\text{l/h}$).

TABLE I. Drop volume comparison between VOF results and LSM results.

VOF			LSM			
Cell number	Δt	V^*	Cell number	Δt	V^*	Difference (%)
14 280	2E-5	0.3127	93 $\times$ 32 $\times$ 14	2E-5	0.28038	10.3
68 663	1E-5	0.29986	116 $\times$ 40 $\times$ 17	2E-5	0.28656	4.4
117 560	5E-6	0.29633	184 $\times$ 62 $\times$ 26	1E-5	0.28828	2.7

FIG. 8. (Color online) The behavior of ferrofluid flow without the magnetic field with section for the plane $y = 2.075 L$ ($Ca = 2.22 \times 10^{-3}$, $Re = 2.78 \times 10^{-4}$, $Q_2 = 10 \mu\text{l/h}$, $Q_1 = 5 \mu\text{l/h}$).

FIG. 9. (Color online) Velocity field changes in the droplet forming process without magnetic field effect for the symmetry x-y plane ($Ca = 2.22 \times 10^{-3}$, $Re = 2.78 \times 10^{-4}$, $Q_2 = 10 \mu\text{l/h}$, $Q_1 = 5 \mu\text{l/h}$).

FIG. 10. Velocity field in the magnetic field in the droplet forming process for symmetry x-y plane ($B_m = 0.1$, $\chi_{m+} = 8$).

focusing configuration was calculated with the following dimensionless numbers:

$$Ca = \frac{\mu_2 u_2}{\sigma}, \quad Re = \frac{\rho_2 u_2 L}{\mu_2}, \quad \mu^* = \frac{\mu_1}{\mu_2}, \quad \rho^* = \frac{\rho_1}{\rho_2},$$

$$u^* = \frac{u_1}{u_2}, \quad t^* = \frac{tu_2}{L}. \quad (23)$$

The values of the capillary number and the Reynolds number are $Ca = 2.22 \times 10^{-3}$ and $Re = 2.78 \times 10^{-4}$.

The simulated evolution of the formed droplet is shown in Fig. 7. The numerical results are compared with the experimental results. The microfluidic device used in the experiments was fabricated with the same method presented by our group.³¹ Details on the experiments will be reported elsewhere. The discrepancy occurs after the droplet breakup at $t = 187$ ms, marked with a circle (Fig. 7(a), $t = 187$ ms, 207 ms); the tip of the ferrofluid is sharper in the experiment. Due to swelling of the device material (PDMS, polydimethylsiloxane) when exposed to silicone oil,^{48,49} the rough

channel wall may result in stronger friction and affect the shape of the tip. The discrepancy can also possibly cause from the deformation of the microchannel.⁵⁰ The swelling effect was not considered in our numerical model. The 3D numerical results with one slice plane located at the narrowest throat of the junction are depicted in Fig. 8. The droplet

FIG. 11. The various forces the ferrofluid tip suffered in the droplet forming process in the flow focusing channel. The magnetic field is enforced.

FIG. 12. (Color online) The behavior of ferrofluid flow in the magnetic field with section for the plane $y = 2.075 L$ ($B_m = 0.1$, $\chi_{m+} = 8$).

neck blocks the throat at $t = 47$ ms and then decreases gradually.

The forces acting on the ferrofluid tip are the viscous force F_μ , the pressure difference F_p , and the interfacial tension force F_σ . F_μ is caused by the viscous stress acting on the interface and is proportional to the tip area and the velocity gradient. Together with F_p , the viscous force F_μ provides a squeezing action on the tip to move it downstream. On the contrary, the interfacial tension force F_σ keeps the ferrofluid

tip from moving forward. As shown in Fig. 8, at the beginning of the formation process ($t = 0, 47$ ms), the curvature of the tip is smaller and results in a larger capillary force. The capillary force is proportional to the curvature and the interfacial tension. A higher pressure is needed to move the tip through the throat of the junction. As a result, the throat is blocked as the dispersed phase flows through. The velocities inside the throat orientate upstream instead of downstream (see Fig. 9 $t = 47$ ms). Because the ferrofluid blocks almost

FIG. 13. (Color online) Non-dimensional volume of droplet against susceptibility χ_{m+} on the condition of $B_m = 0.1$.

FIG. 14. The magnetic effect on the droplet size.

the entire throat, F_p increases dramatically at the region outside ferrofluid thread. The tip is pushed to move further forward. The curvature of the tip becomes smaller as the tip grows and weakens the capillary force. Thus, the direction of the fluid velocity located inside the rectangular window at $t = 116$ ms reverses to the opposite direction (Fig. 9) resulting in competition between F_p , F_μ , and F_σ inside the ferrofluid thread. This process happens instantaneous, accompanying a couple of opposite velocities increases dramatically from $t = 166$ ms to $t = 173$ ms marked with the rectangular window in Fig. 9. The thinner thread can weaken the interfacial tension effect and increase its corresponding curvature. At this moment, the pressure difference is very high inside the thread. Under the stretching action, the ferrofluid inside the thread is pushed to separate in both opposite directions until the weaker F_σ is no longer strong enough to hold the tip. The thread becomes thinner and breaks up to form a droplet at $t = 187$ ms.

The magnetic force has a significant effect on the droplet formation process. When the microfluidic device is exposed to a uniform magnetic field and the direction is parallel with the ferrofluid flow direction, the ferrofluid flow is exposed to a coupled effect of interfacial tension, pressure difference, viscous force, and magnetic stretching force. Magnetic bond number B_m and the susceptibility χ_{m+} are used to represent the effect of magnetic field on droplet formation process. The susceptibility χ_{m+} represents the response of the ferrofluid to an applied magnetic field. The different χ_{m+} values denote the different ferrofluid types. The magnetic bond number $B_m (= \mu_0 LH^2/\sigma)$ describes the ratio between the magnetic force and the interfacial tension force. The magnetic bond number increases by increasing field strength H .

Fig. 10 shows the simulated velocity field during the droplet formation process in a uniform magnetic field with $B_m = 0.1$ and $\chi_{m+} = 8$. It takes much longer for the ferrofluid tip to complete the formation process as compared to the process depicted in Fig. 8. The formation process now includes the additional magnetic force F_m , besides F_σ , F_μ , and F_p . The magnetic force F_m acts as a drag force on the tip. This force is acting on the entire interface where the susceptibility jumps and is zero inside each fluid. This force is related to the magnetic field strength and the magnetic susceptibility, Eq. (7). The direction changes of flow velocity inside the throat happen between $t = 43$ ms and $t = 108$ ms. As the channel throat is blocked, as shown in Fig. 10 ($t = 108$ ms), the pressure drop F_p increases near the region upstream of the throat. However, because of the forward drag force F_m acting on the tip downstream, the whole tip is elongated. The elongation is more serious at increasing χ_{m+} (see Fig. 13). As a result, the tip does not exhibit the spherical shape as in the case without the magnetic field shown in Fig. 8. The thread is relatively longer too (Fig. 10, $t = 223$ ms). Thus, from $t = 108$ ms to 288 ms in Fig. 10, the velocity direction inside the thread keeps orientating downstream. The magnitude of the velocities increases gradually until the breakup moment. The pressure outside the thread increases slowly. The curvature of the thread is small and changes slowly because of the high pressure inside the thread. These

behaviors increase the forming time and enlarge the ferrofluid tip. Finally, a bigger droplet is formed when the interfacial tension force cannot withhold the ferrofluid. The formed droplet has a shape of an ellipsoid due to the stretching effect of the magnetic force ($t = 295$ ms to 309.4 ms in Fig. 12).

The droplet size depends on both the magnetic field strength and the magnetic susceptibility. Fig. 13 shows the relationship between the volume V^* of the formed droplet and the susceptibility. The nonlinear relationship shows a higher sensitivity at the higher susceptibility. The visible change in droplet size with the increase of magnetic field strength is shown in Fig. 14.

IV. CONCLUSIONS

This paper numerically describes the formation process of a ferrofluid droplet in a flow-focusing configuration with and without the magnetic effect. In the absence of the magnetic field, a couple of opposite flow velocities appear in the flow focusing channel as a result of interaction between the pressure drop, viscous drag force, and interfacial tension. The pressure drop and the viscous drag force push the ferrofluid forward, while the interfacial tension keeps the tip from moving forward. In the presence of a magnetic field, the ferrofluid tip was pulled forward by the additional magnetic force. The thread and the tip become longer resulting in a longer formation time. The flow velocity inside the thread also increases. In the presence of a magnetic field, the forming droplet behaves in a different way as compared to the case without the magnetic field. Further simulations were carried out to investigate the influence of the magnetic Bond number and the susceptibility on the droplet size. The higher the magnetic Bond number and susceptibility are, the larger is the volume of the formed droplet. The present numerical method can be employed to study the droplet formation in other channel geometries.

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